1. (35%) The figure shows a circular pulley wheel with mass moment of inertia $I_c$ about the center of rotation, which is the axel of the wheel. The pulley has two different radii. The smaller one is $R/4$ and the larger is $R$. The end of the spring is attached to a cable, which is wrapped around the larger part of the wheel so that the centerline of the spring is always tangent to the wheel at the point of contact.

A cable supporting a weight is wrapped around the smaller radius as shown. Gravity acts. Let $x(t)$ be measure from the zero spring force position of the system.

a. Write down expressions for the total energy in the system, including kinetic and potential energies. Use the coordinate $x = 0$ as the reference position for the gravitational potential energy.

b. Use the principle of the conservation of total energy to find the undamped equation of motion of the system.

c. Find the static equilibrium deflection in terms of the coordinate $x$.

d. Find an expression for the natural frequency.
2. (35%) A colleague is seeking your advice. He has designed a vibration isolation support for a box containing a recording system and a video camera. The whole container has mass, M. It is to sit on a factory floor, which has substantial vertical vibration at 20 Hz. The design specification from the boss is that the isolation system must reduce the vertical vibration of the camera to no more than $\frac{1}{5}$th of the floor motion.

Your colleague showed his boss the final assembled system. The boss immediately measured the static deflection of the camera system on the vibration isolation mounts. The static deflection was 0.002 m. The boss told your colleague that it would not work and to redesign it. Clueless, your colleague has come to you to find out what is wrong.

a. What is wrong with the first design? Come up with a new design that will meet the specifications. To keep it simple, you may initially assume the damping is small. You are expected to find numerical answers for the natural frequency of the system before and after redesign, the static deflection of the box on the supporting springs before and after the redesign, and the design point for the ratio of the excitation frequency to the natural frequency.

b. If the damping is not negligible, explain in words what you would do to compensate for damping so as to still achieve the desired reduction in response.

3. (30%) An ore railway cart in a mine has velocity of 2.0 m/s when it collides with a barrier, which has a spring constant of $10^4$ N/m. The cart has a mass of 1000 kg. When the car hits the barrier it latches (connects) permanently to the end of the spring. The barrier has a linear damping constant $R = 347$ N/(m/s).

a. Write down an expression for the damped response time history of the cart. Let $t=0$ be at the moment of impact, and use as your coordinate system a translational coordinate which has as its origin the initial point of contact with the bumper as shown in the figure.

b. What is the damping ratio of the system after the cart connects to the bumper.

c. Sketch the time history from part 'a'. Label two peaks separated by two periods of motion, and estimate the ratio of the amplitude of these two peaks from the information you have developed in this problem.