1. (25%) The figure below is the transient decay of a single degree of freedom system given initial conditions.
   a. Use the plot to estimate the natural frequency and the damping ratio of the system. Mark the data points you used on the figure to make these estimates.
   b. Estimate the initial conditions on velocity and displacement. That is estimate $x_0$ and $v_0$. I am not looking for any calculus just for some basic understanding of the response to I.C.'s

![Graph of transient decay](image-url)
2. (25%) Find the equation of motion and the natural frequency of the following system. The mass moment of inertia of a uniform bar about its center of gravity is \( I_{cg} = \frac{mL^2}{12} \), where \( L \) is the total length of the bar. The C.G. is at the center of the bar.

![Diagram of a system with a uniform rigid bar and springs](image)

3. (25%) A device known as a ballistic pendulum is used to estimate the speed of a projectile by measuring the height reached by the pendulum after it is struck by the projectile. Assume the projectile embeds itself in the pendulum and the pendulum is at rest before being hit.

In this case the pendulum is constructed of a concentrated mass \( M = 10.0 \) kg. The length of the pendulum string is \( L = 1.5 \) m. The bullet has mass \( m = 0.05 \) kg.

After being struck by the projectile the pendulum rises to a maximum height of 0.067 m at an angle of 0.3 radians.

a. Find the velocity of the projectile.

b. How would damping influence the accuracy of your estimate. Would your estimate of the velocity tend to be high or low if significant damping were present during the measurement?
4. (25%) A periodic force time history is shown. The first three terms of the Fourier series which represents this force are given by:

\[
F(t) = \frac{8A}{\pi^2} \left( \frac{\sin(\omega_p t)}{1^2} - \frac{\sin(3\omega_p t)}{3^2} + \frac{\sin(5\omega_p t)}{5^2} - \ldots \right)
\]

Where one period of the force, \( \tau = \frac{2\pi}{\omega_p} \).

Let \( A = 10 \) N. This force drives a SDOF oscillator, which has the following properties.

\( M = 2 \) kg, \( K = 12800 \) N/m and the damping ratio is 2% of critical.

The natural frequency of the system is related to the excitation frequency as follows:

\[
\omega_n = 3 \omega_p
\]

a. Find the magnitude and phase of the displacement response of the system to the force term in the Fourier series which drives the system at resonance.

b. What is the magnitude of the force transmitted to the support of this SDOF system as a result of only the frequency component of force found in part (a).
Problem 1 - Worksheet.
Label data points and turn in with the quiz.