TWO HALF-HOUR LECTURES

OUTLINE:

OBJECTIVES OF CONTROL
- METRICS
- INFORMATION

DOMINANT POLES
- SIGNIFICANCE
- ANALYSIS

ROOT LOCUS TECHNIQUES
- BACKGROUND
- MODIFICATION / SYNTHESIS

REALIZABLE CONTROLLERS
- IMPLICATIONS FOR ESTIMATION & SENSING

NESTED LOOP APPROACH
- "SEPARATION"

TIME DELAY

SUMMARY

REFERENCE: PROF. HOW, "INTRODUCTION TO CONTROL DESIGN TECHNIQUES" LECTURE NOTES, 1998
(AVAILABLE AT: http://www.mit.edu/~16.82)
"CLOSED-LOOP SYSTEM"

DISTURBANCE
UNCERTAINTY

PLANT

ACTUATORS

SENSORS

CONTROL
LAW

- WE ARE GIVEN THE PLANT
- WE DESIGN THE "CONTROL SYSTEM"
  (THE SENSORS, CONTROL LAW AND ACTUATORS)
- OUR GOAL IS TO ACHIEVE REQUIREMENTS FOR THE
  DYNAMICS OF THE CLOSED-LOOP SYSTEM

TYPICAL REQUIREMENTS:

- QUANTITATIVE
  - STEP RESPONSE SETTLING TIME $\leq T_1$
  - OVERSHOOT ON STEP $\leq \%$
  - STEADY STATE ERROR $\leq Y$

- QUALITATIVE
  - USE MINIMUM FUEL
  - STAY AS "CLOSE" AS POSSIBLE TO REFERENCE
EXAMPLE: 1-D POINT MASS

ACTUATOR: FORCE ON THE MASS
SENSOR: POSITION OF THE MASS

(FOR SIMPLICITY, USE MASS = 1)

EQUATION OF MOTION: \( \ddot{x} = F \)

FOR CONTROL ANALYSIS, LAPLACE TRANSFORM IS MORE USEFUL

(BACKGROUND REF: 1-10)

\[
x(s) = \frac{1}{s^2} F(s)
\]

THIS IS OUR PLANT TRANSFER FUNCTION

\[
G_p(s) = \frac{x(s)}{F(s)} = \frac{1}{s^2}
\]

WE CAN NOW MODEL OUR CLOSED LOOP SYSTEM IN LAPLACE FORM:

WHERE \( K(s) \) IS OUR CONTROL LAW IN LAPLACE FORM
SIGNIFICANCE OF POLES

WE HAVE THE LAPLACE FORM OF A SYSTEM:

\[ P(s) = \frac{N(s)}{D(s)} \]

CAN FIND ITS POLES (VALUES OF S GIVING D(s) = 0)
AND ITS ZEREOES (VALUES OF S GIVING N(s) = 0)

EACH POLE DETERMINES A COMPONENT OF THE SYSTEM RESPONSE. THE ZEREOES ARE A FUNCTION OF HOW THEY MIX.
WE COULD USE PARTIAL FRACTIONS AND SEPARATE:

\[ P(s) = \frac{A}{s+a} + \frac{B}{s+b} + \frac{C}{s+c} + \ldots \]

TWO IMPORTANT FORMS:

1) POLE ON REAL AXIS: \( \frac{1}{s+a} \Rightarrow e^{-at} \) EXPONENTIAL DECAY

2) COMPLEX CONJUGATE POLE PAIR

\[ \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \Rightarrow DAMPED SINE\]

\[ e^{-\zeta w_n t} \sin(\omega_n \sqrt{1-\zeta^2} t) \]
Figure 1

\[ \text{SYSTEM } P(s) = \frac{s^2 + 110s + 200}{s^3 + 11s^2 + 110s + 100} \]

**Partial Fraction Expansion**

\[ P(s) = \frac{1}{s+1} + \frac{100}{s^2 + 10s + 100} = P_1(s) + P_2(s) \]

**Now look at step responses (at left & right)**

**Can sum the contributions of the poles.**
FOR CONJUGATE POLE PAIR, THE METRICS (TIME DOMAIN) CAN BE RELATED TO THE POLE POSITION (LAPLACE DOMAIN) BY COMMON FORMULAE (SEE 1-15)

STEP SETTLING TIME = \( \frac{4.6}{3 \cos \left( -\frac{71.5}{1 - \frac{30}{3}}\right) } \)

STEP PEAK OVERSHOOT = \( e \)

DOMINANT POLES

A SYSTEM'S RESPONSE IS DOMINATED BY THE POLE OR POLE PAIR CLOSEST TO THE IMAGINARY AXIS.

* CAUTION: WATCH OUT FOR NEARBY ZEROS, (SEE 2-1)

\[
\frac{1}{(s+1)(s^2 + 15s + 100)} \quad \text{BEHAVES VERY MUCH LIKE} \quad \frac{1}{s+1}
\]

AND

\[
\frac{1}{(s+10)(s^2 + s + 1)} \quad \text{BEHAVES VERY MUCH LIKE} \quad \frac{1}{s^2 + s + 1}
\]
EXAMPLE

\[ P = \frac{100}{(s+1)(s^2 + 10s + 100)} \]

\[ P_1 = \frac{100}{(s^2 + 10s + 100)} \]

\[ P_2 = \frac{1}{(s+1)} \]

\[ P = P_1 \times P_2 \]

STEP RESPONSES OF P AND P2 VERY SIMILAR

⇒ TO ANALYZE P, JUST CONSIDER DOMINANT POLE P2
BACK TO OUR CONTROL PROBLEM

1. A SYSTEM'S RESPONSE IS DOMINATED BY THE POLE(S) CLOSEST TO IMAGINARY AXIS

2. THE POLES OF A SYSTEM CAN BE RELATED TO ITS TIME-DOMAIN RESPONSE (SETTLING TIME ETC.)

SO, WE CAN RELATE OUR CONTROL REQUIREMENTS TO RESTRICTIONS ON POLE LOCATIONS. THE CONTROL PROBLEM IS NOW TO CHOOSE K(s) SUCH THAT THE DOMINANT POLES OF THE CLOSED LOOP SYSTEM ARE IN THE RIGHT REGIONS.

![Diagram of the root locus with peak overshoot and settling time requirements]
WHERE ARE THE CLOSED LOOP POLES?

THE ROOT-LOCUS DIAGRAM TELLS US WHERE
THE CLOSED-LOOP POLES ARE AS A FUNCTION OF
FEEDBACK GAIN

P(s) INCLUDES
K(s) AND G(s)
(CONTROL & PLANT)

(BACKGROUND: SEE 2-5 ONWARDS)
START WITH ROOT LOCUS OF PLANT ALONE, G(s)

EXAMPLE 1  SINGLE INTEGRATOR  G(s) = \frac{1}{s}

APPLY SUFFICIENT GAIN
TO MOVE POLE TO THE LEFT
OF THE REQUIREMENT.

MATLAB
rlocus

MATLAB
ρ = tf(1, [1, 0])

MATLAB
rlocfind(ρ)

[click to 'left of req.]
[displays a gain: use
at least this much]

SIMPLE SCALAR GAIN
DOES THE TRICK:
"PROPORTIONAL FEEDBACK"
That was easy! The location was already on the root locus, not always so...

\[ G(s) = \frac{1}{s^2} \]

**Example 2**  
Double Integrator

**Problem:** No gain value will move poles where we want them.

**Answer:** Add dynamic elements (poles or zeroes) to controller \( K(s) \)

**Typical strategy for adding damping:** PD control

**PD:** Proportional plus derivative

\[ u(t) = K_p y(t) + K_v \frac{dy}{dt} \]

**Laplace**

\[ U(s) = K_p Y(s) + K_v S Y(s) \]

\[ = K_v \left( s + \frac{K_p}{K_v} \right) Y(s) \]

**Adding a zero**

Next lecture: Implications of adding a zero
SUMMARY

OUTLINED A PROCEDURE FOR CONTROLLER SYNTHESIS:

1) MODEL THE PLANT
2) DRAW ITS ROOT LOCUS
3) MARK ON THE REQUIREMENTS FOR DOMINANT POLES
4) CAN A PROPORTIONAL GAIN DO THE JOB?
   - YES ⇒ GREAT, FIND IT & USE IT
   - NO ⇒ NEED TO ADD POLES/ZEROS
     TRY STANDARD FORMS (PD, PI, PID)

REMARKS

- THIS IS A METHOD FOR SYNTHESIS, THERE ARE OTHERS (LOOP-SHAPING WITH BODE PLOTS, STATE SPACE POLE PLACEMENT. SEE REF)
- APPROACH BASED ON ASSUMPTIONS
  A1) NEGLECT ALL BUT DOMINANT POLES
  A2) BELIEVE THE MODEL
  ALWAYS CHECK ASSUMPTIONS: e.g. SIMULATE, TEST