Finding a Good Price in Opaque Over-the-Counter Markets

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This article offers a dynamic model of opaque over-the-counter markets. A seller searches for an attractive price by visiting multiple buyers, one at a time. The buyers do not observe contacts, quotes, or trades elsewhere in the market. A repeat contact with a buyer reveals the seller's reduced outside options and worsens the price offered by the revisited buyer. When the asset value is uncertain and common to all buyers, a visit by the seller suggests that other buyers could have quoted unattractive prices and thus worsens the visited buyer's inference regarding the asset value. (*JEL* G14, C78, D82, D83)

Trading in many segments of financial markets occurs over-the-counter (OTC). As opposed to centralized exchanges and auctions, opaque OTC markets rely on sequential search and bilateral negotiations. For example, in markets for corporate bonds, municipal bonds, mortgage-backed securities (MBS), asset-backed securities (ABS), and exotic derivatives, firm (executable) prices are usually not publicly quoted. Traders often search for attractive prices by sequentially contacting multiple counterparties. Once a quote is provided, the opportunity to accept quickly lapses. For example, in corporate bond markets, "Telephone quotations indicate a firm price but are only good 'as long as the breath is warm,' which limits one's ability to obtain multiple quotations before committing to trade" (Bessembinder and Maxwell 2008). Even when quotes are displayed on electronic systems, they are often merely indicative and can differ from actual transaction prices.[1] Electronic trading, which makes it easier

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[1] For example, Froot (2008) finds large and persistent disparities between the quoted prices on Thomson Reuters and actual transaction prices. For TRACE-ineligible securities, which include the majority of MBS and ABS, the average transaction-quote disparity is 200 basis points for the bottom third of trades under the quotes and 100 basis points for the top third of trades over the quotes. Ten days after a trade, these disparities only shrink by about half on average. For TRACE-eligible securities, the corresponding transaction-quote disparities are lower, at about 100 and 50 basis points, respectively.

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to obtain multiple quotes quickly, is also limited in the markets for many fixed-income securities and derivatives.[2] Beyond financial securities, markets for bank loans, labor, and real estates are also OTC.

In this article, I develop a model of opaque OTC markets. A seller, say an investor in need of liquidity, wishes to sell an indivisible asset to one of $N > 1$ buyers, say quote-providing dealers. There is no pretrade transparency. The seller must visit the buyers one at a time. When visited, a buyer makes a quote for the asset. The seller may sell the asset to the current potential buyer or may turn down the offer and contact another buyer. Because a buyer does not observe negotiations elsewhere in the market, he faces *contact-order uncertainty*—uncertainty regarding the order in which the competing buyers are visited by the seller. The seller may also make a repeat contact with a previously rejected buyer, such as when a new buyer's quote is sufficiently unattractive.

I show that the potential for a repeat contact creates strategic pricing behavior by quote providers. If the seller and buyers have independent private values for owning the asset,[3] a returning seller faces no adverse price movement caused by fundamental news but invites adverse inference about the price quotes available elsewhere in the market. For example, a seller may initially refuse an unattractive quote from one buyer, only to learn that other buyers' quotes are even worse. In this case, the seller takes into account the likely inference of the original buyer if she contacts him for a second time. Upon a second contact by the seller, the original buyer infers that the seller's outside options are sufficiently unattractive to warrant the repeat contact, despite the adverse inference. In accordance, the buyer revises his offer downward. The natural intuition that a repeat contact signals reduced outside options—and hence results in a lower offer—is confirmed as the first main result of this article.

As the second main result of this article, I show that when buyers have a common valuation of the asset, search induces an additional source of adverse selection. In the model, the seller observes the fundamental value v of the asset, but buyers observe only noisy signals of v. The seller is assumed to randomly choose the order of contacts with the buyers. I also assume that buyers have higher private values for owning the asset than does the seller, so the potential gain from trade is positive.

I show that a buyer's expected asset value conditional on his own signal *and on being visited*, $\mathbb{E}(v \mid \text{signal,visit})$, is strictly lower than the expected asset

[2] For example, SIFMA (2009) finds that electronic trading accounts for less than 20% of European sell-side trading volume for credit and sovereigns. For interest-rate swaps, credit default swaps, and asset-backed securities, the fractions are lower than 10%. Barclay, Hendershott, and Kotz (2006) find that the market share of electronic intermediation falls from 81% to 12% when U.S. Treasury securities go off the run.

[3] We can interpret the private values as "private components" of valuations, relative to a commonly known fundamental value. For example, a buyer of real estate often has an idiosyncratic preference beyond the resale value of the real estate. Hedging demands in financial markets are also likely to be private.

value conditional only on his own signal, $\mathbb{E}(v \mid \text{signal})$, provided that $N \geq 2$. Intuitively, the fact that the asset is currently offered for sale means that nobody has yet bought it, which, in turn, suggests that other buyers may have received pessimistic signals about its fundamental value. Anticipating this "ringing-phone curse,"[4] a buyer may quote a low price for the asset, even if his own signal indicates that the asset value is high.

Perhaps surprisingly, the ringing-phone curse in OTC markets is discovered to be less severe than the winner's curse in first-price auctions, in the sense that a trade is more likely to occur in the OTC market than in the first-price auction in expectation. Intuitively, when a buyer is visited by the seller in the OTC market, he infers that only *already visited* buyers have received pessimistic signals. However, when a buyer wins a first-price auction, he infers that the signals of *all other* buyers are more pessimistic than his. Therefore, a trade is less likely to take place in an auction than in an OTC market. Given the associated gains from trade, an OTC market may be superior to an auction market from a welfare viewpoint, at least within the confines of this model setting.

Moreover, buyers' inferences regarding the asset value are less sensitive to their signals in an OTC market than in a first-price auction. In a first-price auction, a higher signal of a buyer translates into a higher bid and thus a higher probability of winning. In an OTC market, by contrast, due to the lack of simultaneous contacts, a higher signal of a particular buyer does not change the search path of the seller nor the buyers' inference of it.

To the best of my knowledge, this article offers the first model that captures the joint implications of uncertain contact order, bargaining power, adverse selection, and market opacity. The results of this article generate a number of empirical implications. First, a repeat contact in an OTC market tends to worsen price quotes.[5] Second, interaction with quote seekers gives a quote provider valuable information regarding the prices available from competitors, so we expect dealers with larger market shares of trading volume to quote prices that are closer to quotes available elsewhere in the market. Third, in an OTC market a buyer with the highest value among all buyers may not be visited at all and thus may not purchase the asset, so we expect to see more inter-dealer trading when customers cannot simultaneously contact multiple dealers. This suggests that the new Dodd-Frank requirement—to expose standard OTC derivatives in "swap execution facilities" (SEFs) to multiple counterparties—could reduce the market shares of trading volumes captured by intermediaries in affected derivatives. Fourth, for assets with high degrees of information asymmetry, trading relationships improve quoted prices from

[4] I thank Kerry Back for suggesting this intuitive name.

[5] When quote providers cannot observe the trading direction of the quote seeker, a worse price is reflected in a wider bid-ask spread. If the marketwide prices have moved between the original contact and the repeat contact, a worse price applies after adjusting for this marketwide price movement.

frequently visited counterparties only at the cost of worsening price quotes from rarely visited counterparties. Fifth, search-induced adverse selection in OTC markets dampens the sensitivity of quoted prices to payoff-relevant information, compared with centralized auctions. The model thus predicts that allowing simultaneous contacts to multiple counterparties increases the cross-sectional dispersion of quotes, increases price volatility, and speeds information aggregation. These testable implications are particularly relevant for the design and reform of OTC derivative markets as new regulations in the United States and Europe move more of OTC derivatives trading onto electronic platforms.

1. Dynamic Search with Repeat Contacts

There is one quote seeker, say an investor, and $N \geq 2$ ex ante identical quote providers, say dealer banks. Everyone is risk neutral. Without loss of generality, suppose that the quote seeker is a seller and the quote providers are potential buyers. The seller has one unit of an indivisible asset she wishes to sell. The seller's valuation, v_0, and the buyers' valuations, v_i, $i = 1, 2, \ldots, N$, are jointly independent and privately held information.[6] The seller's value of v_0 is binomially distributed with

$$\mathbb{P}(v_0 = V_H) = p_H, \quad \mathbb{P}(v_0 = V_L) = p_L = 1 - p_H, \tag{1}$$

where $V_H > V_L > 0$ and (p_H, p_L) are commonly known constants. The buyers' values have an identical cumulative distribution function $G : [0, \infty) \to [0, 1]$.

The market is over-the-counter. The seller contacts buyers one by one. Contacts are instantaneous and have no costs for the seller.[7] Upon a contact, the selected buyer makes an offer for the asset. The seller cannot counteroffer, but can accept or reject the quote. The inability of the quote seeker to counteroffer is realistic in functioning OTC markets, in which customers rarely have the market power to make offers to the quote-providing dealers. If the seller accepts the quote, then the transaction occurs at the quoted price and the game ends. If she rejects it, then the buyer's quote immediately lapses. After rejecting a quote, the seller may subsequently contact a new buyer, who is randomly chosen with equal probabilities across the remaining buyers and independently of everything else, or may contact an already visited buyer. Upon the next contact, the same negotiation is repeated, and so on. Any contact between two counterparties is unobservable to anyone else. For simplicity, I refer to the nth buyer visited for the first time by the seller as "the nth buyer."

6 Private valuations can stem from inventory positions, hedging needs, margin requirements, leverage constraints, or benefits of control, all of which are likely to be private. A common-value setting is considered in Section 2.

7 This zero-cost assumption allows me to bypass the Diamond paradox (Diamond 1971) and focus on the sequential nature of search, rather than the pecuniary cost of search.

An equilibrium consists of the buyers' quoting strategies and the seller's acceptance or rejection of quotes, with the property that all players maximize their expected net payoffs. In selecting an equilibrium, I focus on a symmetric, perfectly revealing equilibrium in which buyers use the same quote strategies, and a buyer's first quote perfectly reveals what his quotes would be upon subsequent contacts. I further assume that upon each contact, a buyer quotes a price for the sole purpose of trading on that contact, given the option to trade on any subsequent contact, but not for the purpose of "manipulating" the seller's belief about the buyer's valuation. As we discuss shortly, this assumption is unlikely to change the qualitative nature of the results. Finally, I impose two tie-breaking rules:

1. Whenever the expected payoffs of trading versus not trading are equal, a player strictly prefers trading.

2. Whenever two strategies give the same expected payoff to a buyer or seller, the buyer or seller strictly prefers the strategy with a fewer number of contacts.

At his kth contact with the seller, any buyer i bids $\beta_k(v_i)$, where $\beta_k : [0,\infty) \to \mathbb{R}$ is a quoting strategy common to all buyers and is assumed to be right-continuous with left limits.[8] Without loss of generality, we restrict attention to offers that are accepted with strictly positive probability.

We observe three properties of equilibria. First, no buyer strictly increases his offer upon a repeat contact; otherwise, the earlier, lower offer is rejected with probability 1. Thus, for all v_i and k,

$$\beta_k(v_i) \geq \beta_{k+1}(v_i). \tag{2}$$

Second, because contacts are unobservable, the seller does not return to any rejected buyer unless she has visited all remaining buyers.[9] Once the seller has visited all buyers at least once, perfect revelation implies that there is no uncertainty regarding quotes upon subsequent contacts. Because the seller prefers the shortest path (given the tie-breaking rule), the seller's last visit is to the buyer (or one of the buyers) who would make the highest second quote. Thus, the third property of equilibria is that the seller makes two contacts with the same buyer at most.

8 A function $F : [a,b] \to \mathbb{R}$ is right-continuous at $x \in (a,b)$ if $\lim_{y\downarrow x} F(y) = F(x)$. The function F has a left limit at $x \in (a,b)$ if $\lim_{y\uparrow x} F(y)$ exists.

9 To see why, suppose otherwise, and a seller visits, say Buyer 1, for a kth time ($k \geq 2$) before the first contact to, say, Buyer 2. If the seller accepts Buyer 1's kth quote in equilibrium, then a strictly better strategy for the seller is to visit Buyer 2 before making the kth contact to Buyer 1 because Buyer 2's first quote might be better. If the seller rejects Buyer 1's kth quote in equilibrium, then the seller is no better off than if he had not made the kth contact. In fact, by the tie-breaking rules it is suboptimal for the seller to make the kth contact to Buyer 1 and then reject his kth quote. Therefore, the seller never revisits a buyer unless she has visited all other buyers.

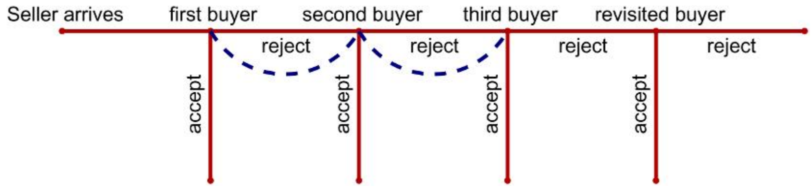

Figure 1
The game tree for $N = 3$
The seller goes from the left to the right and visits buyers one at a time. The seller can accept or reject a quote at any time. The dashed lines link the information sets of three buyers upon the first contact and represent the uncertainty of the buyers regarding the order of contacts. The revisited buyer can be any of the three buyers.

Figure 1 plots the game tree for $N = 3$. The seller contacts the three buyers in sequence and may accept or reject any quote along the path. Upon the first contact, none of the buyers know if they are the first, second, or third buyer to be visited by the seller. If, however, the seller visits a buyer for a second time, then the revisited buyer infers that the other two buyers have quoted sufficiently unattractive prices. Exploiting the seller's reduced outside options, the revisited buyer strictly lowers his quote.

Proposition 1. (Search with repeat contact.) Let $V_0 \geq V_1 \geq \cdots \geq V_N \equiv V_L$ and $R_1 > R_2 > \cdots > R_N \equiv V_L$ be implicitly defined, whenever possible, by

$$R_k = \mathbb{E}\left[\max\left(\beta_1(v_{k+1}), R_{k+1}\right)\right], \quad 1 \leq k \leq N-1, \tag{3}$$

$$(V_k - R_k)\frac{\sum_{j=k}^{N} q_j}{\sum_{j=1}^{N} q_j} = (V_k - R_{k+1})\frac{\sum_{j=k+1}^{N} q_k}{\sum_{j=1}^{N} q_j}, \quad 1 \leq k \leq N-1, \tag{4}$$

$$V_0 - V_H = (V_0 - R_1)\left(1 + \frac{\sum_{j=1}^{N} G(V_0)^{j-1}}{\sum_{j=1}^{N} q_j}\frac{p_H}{p_L}\right)^{-1}, \tag{5}$$

where

$$\beta_1(v_i) = \begin{cases} 0, & \text{if } v_i \in [0, V_L) \\ R_k, & \text{if } v_i \in [V_k, V_{k-1}), 1 \leq k \leq N \\ V_H, & \text{if } v_i \in [V_0, \infty) \end{cases} \tag{6}$$

$$q_k = \prod_{j=1}^{k-1} G(V_j), \quad 1 \leq k \leq N. \tag{7}$$

If a solution $\{V_k\}_{k=0}^{N-1}$ and $\{R_k\}_{k=1}^{N-1}$ to Equations (3)–(5) exists, then the following strategies constitute an equilibrium:

1. The first quote $\beta_1(v_i)$ of buyer i is given by Equation (6).

2. The second quote of buyer i is

$$\beta_2(v_i) = \begin{cases} 0, & \text{if } v_i \in [0, V_L), \\ V_L, & \text{if } v_i \geq V_L. \end{cases} \tag{8}$$

3. A high-value seller accepts a quote of V_H as soon as it is quoted. If all buyers' quotes are lower than V_H, then the high-value seller leaves the market.

4. A low-value seller accepts the first quote of the kth buyer, $1 \leq k \leq N$, if and only if it is no lower than R_k. Otherwise, she rejects the quote and visits a new buyer. If the seller still holds the asset after visiting all buyers, she returns to any buyer whose first quote is no lower than V_L and accepts the revisited buyer's second quote if it is no lower than V_L. If all buyers' first quotes are lower than V_L, then the lower-value seller leaves the market.

In any such equilibrium, $\beta_2(v_i) < \beta_1(v_i)$ as long as $V_L < \beta_1(v_i) < V_H$.

A proof of Proposition 1 is provided in the Appendix. Because we have $2N-1$ equilibrium variables, $\{V_k\}_{k=0}^{N-1}$ and $\{R_k\}_{k=1}^{N-1}$, and $2N-1$ equations, Equations (3)–(5), we expect Equations (3)–(5) to have a unique solution.

I now present an example that illustrates the intuition of Proposition 1. In the general equilibrium characterization, as well as in the following example, the key determinant of a buyer's first quote is whether or not the buyer is willing to "match" a seller's continuation value and prevent the seller from further search.

Example 1. Let $N = 3$, $V_H = 1$, $V_L = 0.4$, and $p_H = p_L = 0.5$. Also let the values of the buyers have the standard exponential cumulative distribution function G. That is, $G(x) = 1 - e^{-x}$. In this equilibrium, $R_1 = 0.76$, $R_2 = 0.63$, $V_0 = 1.21$, $V_1 = 0.87$, and $V_2 = 0.76$, as plotted in Figure 2.

If the value u of a buyer is low, specifically $u \in [0.4, 0.76)$, the buyer is not willing to pay a low-value seller's continuation value. His equilibrium quote of $V_L = 0.4$ is accepted by a low-value seller if and only if the seller has failed to find a good price from the other two buyers—i.e., if the seller has run out of outside options. A buyer with a higher value $u \in [0.76, 0.87)$ is willing to quote a higher price of $R_2 = 0.63$, which is equal to the continuation value of a low-value seller who has one more buyer to visit, i.e., $R_2 = \mathbb{E}[\max(\beta(v_3), V_L)]$. A buyer with value $u \in [0.87, 1.21)$ quotes a price of $R_1 = 0.76$, which is the continuation value of a low-value seller who is

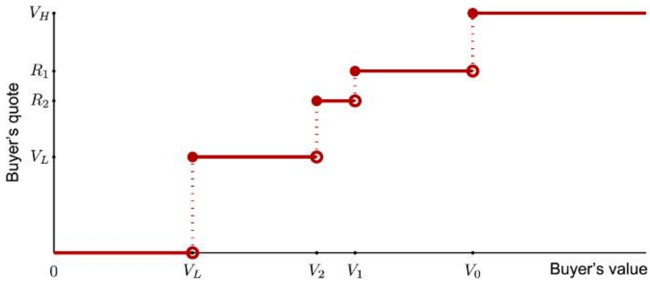

Figure 2
Equilibrium quoting strategy of buyers, for $N=3$
Parameters: $V_H=1$, $V_L=0.4$, and $p_H=p_L=0.5$. The values of the buyers have the cumulative distribution function $G(x)=1-e^{-x}$.

yet to visit either of the other two buyers. That is, $R_1=\mathbb{E}[\max(\beta(v_2), R_2)]$. A quote of R_1 is thus accepted with certainty by a low-value seller. Finally, a buyer with value $u\in[1.21,\infty)$ quotes a price of $V_H=1$ and trades immediately with both types of sellers.

The cutoff values $\{R_k\}$ and $\{V_k\}$ are determined so that a buyer with a value of V_k is indifferent between quoting the higher price of R_k or the lower price of R_{k+1}, as shown in Equation (4). A higher quote is compensated by a higher probability of trade and vice versa.[10]

A key result of Proposition 1 is that a buyer's second quote upon a repeat contact is strictly lower than his first quote. In this example, suppose that the seller has the low value of V_L and that the first quotes of the three buyers are $\beta_1(v_1)=0.63$, $\beta_1(v_2)=0$, and $\beta_1(v_3)=0$, respectively. In equilibrium, these three quotes are all lower than the seller's continuation values at the times of contact and are thus rejected by the seller. After rejecting them, however, the seller learns that it is the first buyer who has the highest value among the three and returns to the first buyer. Upon this repeat contact, the first buyer infers that the seller's value is V_L (as the seller would have otherwise left the market without trading) and that no other buyer has a value above $V_0=1.21$ (as the seller would have otherwise already traded and never returned). Exploiting the seller's reduced bargaining power, this revisited buyer lowers his quote from $R_2=0.63$ to $V_L=0.4$. Without a better outside option, the seller accepts this new, lower quote.

The main intuition of Proposition 1, which leads to a strictly lower quote upon a repeat contact, is likely to be robust to the myopic assumption that buyers do not manipulate the seller's belief. On the one hand, with private values, a buyer has no incentive to manipulate the seller's belief "downward," as such manipulation would make the seller less likely to return. On the other hand, a buyer may manipulate the seller's belief "upward" to encourage the

10 This quoting behavior is analogous to the pricing behavior in the limit-order book model of Rosu (2009), in which a limit order with a better price is more likely to be executed and vice versa.

seller to return. However, such manipulation requires quoting a high price upon the first contact and subsequently *lowering* the quotes upon repeat contacts, which reinforces the effect of Proposition 1.

The ability of quote providers to revise their quotes upon repeat contacts distinguishes the model of this article from existing search models that have perfect recall. In those models, quote providers commit to their original quotes when the quote seeker returns (Quan and Quigley 1991; Biais 1993; de Frutos and Manzano 2002; Yin 2005; Green 2007).[11] In this article, as in functioning OTC markets, a rejected quote immediately lapses. Repeat contacts have zero probability in models that are based on the "random matching" of an infinite number of buyers and sellers, as in Duffie, Gârleanu, and Pedersen (2005, 2007), Vayanos and Wang (2007), and Vayanos and Weill (2008), among others. Infinite-agent models thus miss a key aspect of search markets. In addition, the results here reveal that the sequential nature of search can give rise to strategic pricing behavior that is unfavorable to quote seekers. This perspective complements the traditional focus on a positive pecuniary cost of search (Diamond 1971).

The model of this article also differs from existing bargaining models that have outside options, such as those of Chatterjee and Lee (1998), de Fraja and Muthoo (2000), Gantner (2008), and Fuchs and Skrzypacz (2010), among others. In these models, the contact order is common knowledge and outside options are often exogenous.[12] By contrast, contact-order uncertainty in this article creates information asymmetry regarding the quote seeker's endogenous outside options, which is key to the strategic pricing behavior of quote providers. Moreover, a repeat contact in this article signals a reduced outside option of the quote seeker and worsens the price quotes. This prediction is opposite to those of bargaining models that are based on screening, in which a delay signals a "strong" valuation, and thus a repeat contact (weakly) improves the price offered to the quote seeker (Rubinstein 1982).

In addition to the prediction that quotes worsen with a repeat contact, the results here also suggest that quote providers can learn something about each other's valuation from repeated interactions with quote seekers. Because of this learning, the model predicts that dealer banks that handle larger shares of an OTC market quote prices that are closer to quotes available elsewhere in

[11] Cheng, Lin, and Liu (2008) allow some quote providers to exogenously drop out, but remaining quote providers commit to their original quotes.

[12] For example, Chatterjee and Lee (1998) consider a one-to-one bargaining game and show that when the search cost is sufficiently low, the quote provider may begin by offering a relatively unattractive price, hoping that this price may become acceptable once the outside option of the quote seeker is determined to be worse. Their model, however, assumes that the values of both parties are common knowledge and that the outside offers are exogenously drawn from some distribution function. In this article, both valuations and contact order are uncertain, and outside offers are endogenously determined by quote providers. de Fraja and Muthoo (2000) study a bargaining game between one quote seeker and two quote providers and characterize conditions under which the quote seeker switches between the two quote providers. They nonetheless assume symmetric information regarding the quote seeker's valuation and the order of contact. Another difference is that offers in the model of de Fraja and Muthoo (2000) improve over time, whereas offers in the model of this article deteriorate.

the market, after controlling for other benefits associated with size, such as superior research capabilities, which are not modeled here.

Because repeat contacts reveal valuable information regarding outside options, a financial institution may benefit by keeping a complete history of its interactions with clients. Indeed, many broker-dealers organize their traders by specialization, whereby all transactions of a particular security are handled by one trader. This specialization makes it harder for returning customers to avoid the adverse inference caused by repeated contacts.

The sequential nature of search can cause allocational inefficiency. In the example previously calculated, if $\beta_1(v_1) = R_1 = 0.76$ and $\beta_1(v_2) = \beta_1(v_3) = 1$, then a low-value seller stops searching at the first buyer, even though the other two buyers would quote *higher* prices—an efficiency loss.[13] By contrast, the winner in a centralized auction is the highest bidder, who in general also has the highest value. To the extent that dealers retrade among themselves after dealing with customers, we expect dealers to capture a larger fraction of total trading volume in OTC markets than in auctions. For example, the Commodity Futures Trading Commission (2011) states that "to ensure that multiple participants have the ability to reach multiple counterparties, the Commission proposes to require SEFs [Swap Execution Facilities] to provide that market participants transmit a request for quote to at least five potential counterparties in the trading system or platform." The model of this article suggests that these requirements can increase direct trading among "end-users" and reduce the fraction of trading volume that is conducted through intermediaries.

1.1 Comparative statics

Now we calculate the comparative statics of the equilibrium of Proposition 1, with respect to characteristics of the market and the players. Because Equations (3)–(5) are nonlinear, all equilibria are numerically computed. The parameters are those of Figure 2, unless otherwise specified.

Figure 3 plots the equilibrium quotes and the distribution of quotes as functions of the probability p_H of a high asset value. As p_H increases, it becomes less likely that a quote lower than V_H is ever accepted, so buyers raise their quotes R_1 and R_2, as shown in the left-hand plot. The right-hand plot shows the probability distribution of buyers' quotes. For a randomly selected buyer, the probability that he quotes a price of R_1 is $G(V_0) - G(V_1)$, the probability that he quotes a price of R_2 is $G(V_1) - G(V_2)$, and so on. As p_H increases, a buyer is more likely to quote either a high price of V_H or low prices of R_2 and V_L, but he is less likely to quote an intermediate price of R_1. Intuitively, if a buyer does not have a value that is sufficiently high to result in an immediate trade with a high-value seller, then he is less willing to pay

[13] For example, Ashcraft and Duffie (2007) find that a significant number of loans in the federal funds market are made by lenders who are relatively short of funds themselves.

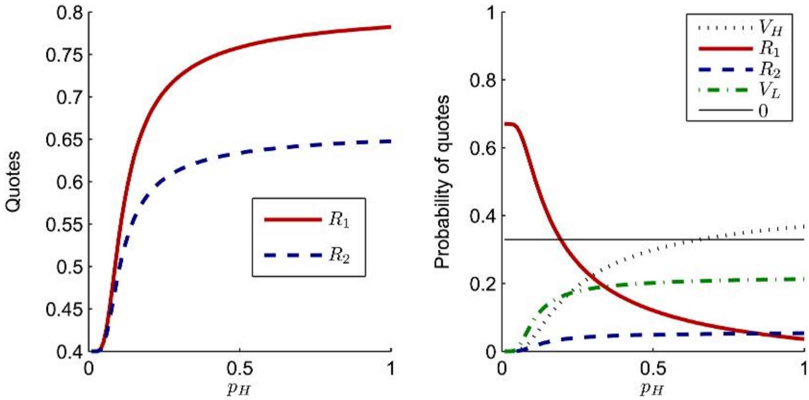

Figure 3

Comparative statics of the equilibrium of Proposition 1, with respect to p_H

Other parameters are those in Figure 2. The left panel plots the equilibrium quotes R_1 and R_2. The right panel plots the equilibrium probability distribution of quotes.

a low-value seller's outside option value of R_1. The buyer would rather trade with a seller whose outside option is reduced or exhausted.

Figure 4 plots the comparative statics, as V_L varies, of the equilibrium of Proposition 1. Clearly, as the low-value seller's value increases, buyers' offers increase, as shown in the left-hand-side plot. An increase in V_L also leads to an increase in the probability that a buyer quotes the highest price, V_H, or the lowest price, 0, as shown in the right-hand-side plot. A buyer's incentive to quote V_H increases because the seller's value increases; the incentive to quote 0 increases because fewer buyers can afford to quote a price of V_L or higher. Quotes of the intermediate prices V_L, R_1, and R_2 decrease in V_L. Intuitively, because a seller's outside option depends on the gap $V_H - V_L$, as V_L converges to V_H a buyer is less able or willing to screen sellers based on their outside options. As V_L converges to V_H, a buyer does not screen at all and quotes V_L or 0 with probability 1.

Figure 5 plots the comparative statics of the equilibrium of Proposition 1, with respect to the distribution of buyers' values. Here, I assume that the buyers' values are exponentially distributed with parameter λ (mean $1/\lambda$), where $\lambda > 0$ is a free parameter. Increasing λ lowers the probability distribution of the buyers in the sense of first-order stochastic dominance. Naturally, the offers R_1 and R_2 increase in buyers' values (decreases in λ), as illustrated in the left-hand-side plot. As λ increases, the gap $R_1 - R_2$ first widens and then shrinks, which suggests that intermediate buyer valuations give buyers the strongest incentive to screen a seller on the basis of her outside options. The right-hand-side plot of Figure 5 reveals that, as the distribution of buyers' values decreases (as λ increases), the probabilities of the three intermediate quotes R_1, R_2, and V_L are all hump shaped. Intuitively, as λ increases from 0, buyers'

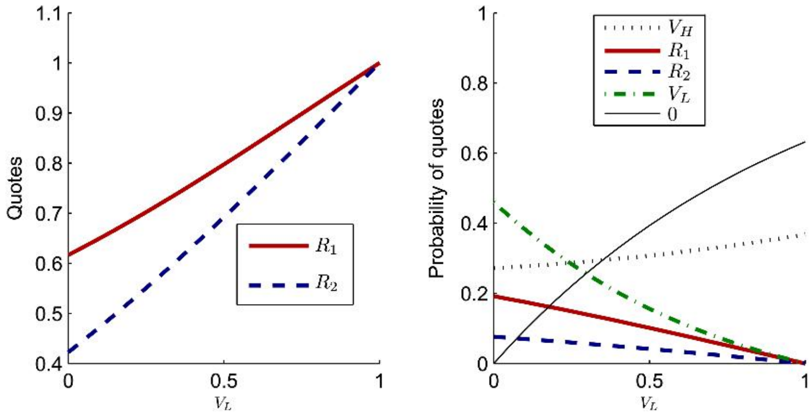

Figure 4
Comparative statics of the equilibrium of Proposition 1, with respect to V_L
Other parameters are those in Figure 2. The left panel plots the equilibrium quotes R_1 and R_2. The right panel plots the equilibrium probability distribution of quotes.

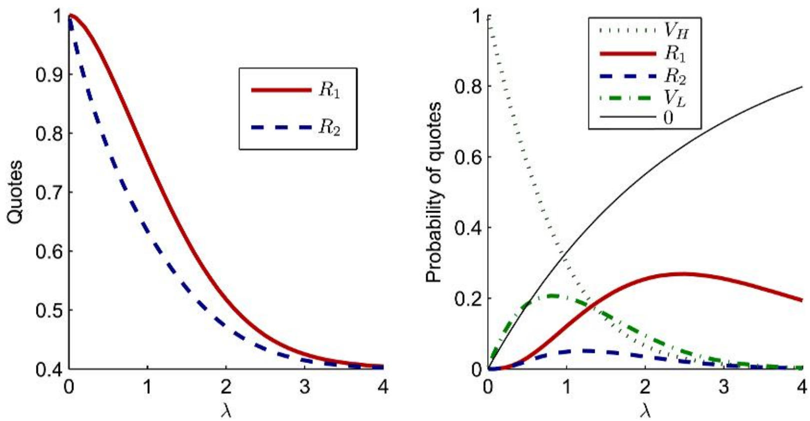

Figure 5
Comparative statics of the equilibrium of Proposition 1, with respect to $\lambda > 0$, where the buyers' values have the cumulative probability distribution function $G(x) = 1 - e^{-\lambda x}$
Other parameters are those in Figure 2. The left panel plots the equilibrium quotes R_1 and R_2. The right panel plots the equilibrium probability distribution of quotes.

values decline, but they are still much higher than V_H on average. Thus, buyers are willing to bid the seller's outside option value. As λ further increases, however, buyers' values further decline and eventually become comparable to the seller's. As a result, buyers become less willing to bid the seller's outside options.

2. Search-induced Adverse Selection

So far, I have analyzed a model of an opaque OTC market in which traders have private values. In this section, I incorporate a common value into the model and examine the interplay between uncertain contact order, market opacity, and adverse selection.

The market structure is that of Section 1. The seller contacts buyers one by one at random and with equal probabilities. Each contact is instantaneous and unobservable to anyone except the two involved counterparties. The seller can sell the asset at any time. I also maintain the two tie-breaking rules; i.e., whenever the expected payoffs are equal, players prefer trading to not trading and prefer fewer contacts.

The fundamental (common) value υ of the asset has a binomial distribution

$$\mathbb{P}(\upsilon = V_H) = p_H, \quad \mathbb{P}(\upsilon = V_L) = p_L = 1 - p_H, \tag{9}$$

for $V_H > V_L \geq 0$. The seller perfectly observes υ, but buyers do not. Instead, conditional on υ, buyers receive i.i.d. signals with a continuously differentiable distribution function $F(\cdot \mid \upsilon) : [0, \bar{s}] \to [0, 1]$, where $0 < \bar{s} < \infty$. For simplicity, I write $F_\theta(s) \equiv F(s \mid \upsilon = V_\theta)$ and $f_\theta(s) \equiv F'_\theta(s)$, for $\theta \in \{H, L\}$. Section 2.2 considers "lumpy" signal distributions, which allow F_θ to have discontinuities. These probability densities satisfy the monotone likelihood ratio property (MLRP)

$$\frac{d}{ds}\left(\frac{f_H(s)}{f_L(s)}\right) > 0, \quad s \in (0, \bar{s}). \tag{10}$$

That is, higher signals are more likely to occur if the asset value is higher. A standard result (see Milgrom 1981) is that the MLRP implies first-order stochastic dominance:

$$F_H(s) < F_L(s), s \in (0, \bar{s}). \tag{11}$$

Furthermore, the seller and the buyers have the same low-outcome valuation V_L for the asset. Conditional on a high value for the asset, the seller values the asset at the fundamental value V_H, while the buyers value the asset at DV_H for some commonly known constant $D > 1$. Thus, $(D-1)V_H$ is the potential gain from trade. The focus of this section is to study the extent to which adverse selection in an OTC market prevents the realization of this gain from trade. I also compare this effect with that of a first-price auction.

Conditional on a signal of s, a regular version of the conditional distribution of υ is uniquely determined by the likelihood ratio

$$\frac{\mathbb{P}(\upsilon = V_H \mid s)}{\mathbb{P}(\upsilon = V_L \mid s)} = \frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)}, \tag{12}$$

with the usual abuse of notation. Here, the first fraction on the right-hand side of Equation (12) is the prior, and the second fraction is the information contained in the signal s. To rule out some trivialities, I further assume that

$$p_H DV_H + p_L V_L < V_H < \frac{p_H f_H(\bar{s}) DV_H + p_L f_L(\bar{s}) V_L}{p_H f_H(\bar{s}) + p_L f_L(\bar{s})}. \tag{13}$$

This condition implies that adverse selection is sufficiently severe that a buyer's ex ante expected valuation for the asset is lower than the high-value seller's value V_H. However, a monopolist buyer who receives the most optimistic signal can nonetheless purchase the asset from a high-value seller at a price of V_H.

Proposition 2. (Search-induced adverse selection.) Under Condition (13), there exists a signal outcome $s^* \in (0, \bar{s})$ that is implicitly defined by

$$J(s^*, N) \equiv \frac{p_H}{p_L} \cdot \frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s^*)^k}{\sum_{k=0}^{N-1} F_L(s^*)^k} = \frac{V_H - V_L}{(D-1)V_H}. \tag{14}$$

If this cutoff signal s^* is unique, there exists an equilibrium in which

1. If a buyer receives a signal of $s \geq s^*$, then he quotes a price of V_H. Otherwise, he quotes a price of V_L.

2. A seller searches through N buyers one by one and accepts the first quote that is at least V_H. If no buyer quotes a price of V_H or higher, then a seller with a high-value asset leaves the market, whereas a seller with a low-value asset accepts a quote of V_L from the last buyer.

In this equilibrium, conditional on being visited by the seller, a buyer with a signal of $s \in [0, \bar{s}]$ assigns the likelihood ratio

$$I_{OTC}(s, N) = \frac{\mathbb{P}(v = V_H \mid s, \text{visit})}{\mathbb{P}(v = V_L \mid s, \text{visit})} = \frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s^*)^k}{\sum_{k=0}^{N-1} F_L(s^*)^k}. \tag{15}$$

Moreover, if $\partial J(s, N)/\partial s > 0$ for all N, then the cutoff signal s^* is unique and strictly increasing in N, and $I_{OTC}(s, N)$ is strictly decreasing in N.

The intuition of the equilibrium of Proposition 2 is simple. The first fraction on the right-hand side of Equation (15) is the prior belief. The second is the information contained in the signal. The third term is the effect of search-induced adverse selection (or the "ringing-phone curse"). Intuitively, because a buyer is more likely to be visited when quotes elsewhere are low, he puts a higher weight on the event $\{v = V_L\}$ than on the event $\{v = V_H\}$, as reflected in the fraction $\sum_{k=0}^{N-1} F_H(s^*)^k / \sum_{k=0}^{N-1} F_L(s^*)^k < 1$. For a buyer, a call is bad news for the value of the asset. Such ringing-phone curse is absent in common-value search models that are based on random matching, in which contacts are exogenous (Duffie, Malamud, and Manso 2010; Chiu and Koeppl 2010).

Clearly, in no equilibrium would a buyer offer a price in the interval (V_L, V_H) because any seller willing to accept such a price must be of low type. When a price of at least V_H occurs in equilibrium, we can show that the equilibrium of Proposition 2 is the unique "cutoff equilibrium." A cutoff equilibrium is represented by a pair (P_H, s^*) with the property that buyers with signals greater than or equal to s^* quote a price of P_H, whereas buyers with signals lower than s^* quote a price of V_L.

Proposition 3. (Equilibrium selection.) Suppose that Condition (13) holds and $N \geq 2$. In any cutoff equilibrium, the high quote P_H is equal to V_H and the cutoff signal s^* is as given by Proposition 2.

We now proceed to analyze the asymptotic behavior of prices as the number N of buyers becomes large. As the market becomes larger, a visiting seller could have contacted more buyers, which, in turn, suggests that more buyers have received low signals. To mitigate this adverse selection, buyers impose an ever higher cutoff signal s^*. In the limit, search-induced adverse selection dominates any informative signal, except for the most optimistic one, $\bar{s}$. Under fairly general conditions, the market breaks down with a strictly positive probability. In expectation, a seller must visit infinitely many buyers before she can find, if at all, a sufficiently good price.

Proposition 4. (Asymptotic behavior of large OTC markets.) Suppose that Condition (13) holds and $\partial J(s, N)/\partial s > 0$ for all N. Then, in the equilibria of Proposition 2,

1. As $N \to \infty$, $s^* \to \bar{s}$.
2. For all $s < \bar{s}$, $\lim_{N\to\infty} \mathbb{E}[v \mid s, \text{visit}] < V_H$.
3. Provided that $f_H(\bar{s})/f_L(\bar{s}) < \infty$, $\lim_{N\to\infty} \mathbb{E}[v \mid \bar{s}, \text{visit}] = V_H$.
4. Suppose that $f_H(\bar{s})/f_L(\bar{s}) < \infty$ and $v = V_H$. Let $x_H \equiv \lim_{N\to\infty} F_H(s^*)^N$ be the probability of a market breakdown in the limit. Then, $x_H \in (0, 1)$ is the smaller root of the equation

$$a x^{f_L(\bar{s})/f_H(\bar{s})} - x = a - 1, \tag{16}$$

where

$$a = \frac{V_H - V_L}{(D-1)V_H} \cdot \frac{p_L}{p_H}.$$

5. Conditional on $v = V_H$, let T_N be the number of buyers being searched in equilibrium in a market with N buyers. Then, $\lim_{N\to\infty} \mathbb{E}(T_N) = \infty$.

Proposition 4 reveals that as long as no signal is infinitely informative, the limiting probability of a market breakdown is strictly positive and depends only on the likelihood ratio $f_H(\bar{s})/f_L(\bar{s})$ on the boundary. This result has a natural counterpart in centralized auctions, as we further discuss in Section 2.1.

Example 2. Let $F_H(s) = s^2$ and $F_L(s) = s$ for $s \in [0, 1]$. Also, let $V_H = 0.6$, $V_L = 0$, $D = 5/3$, and $p_H = p_L = 0.5$. We leave N as a free parameter. It is easy to check that Condition (13) holds. For these parameters, Equation (14) reduces to

$$J(s^*, N) = \frac{2s^*(1 + s^{*N})}{1 + s^*} = \frac{3}{2}.$$

Because $\partial J(s, N)/\partial s > 0$, for each N, a unique cutoff signal s^*, which increases in N, determines the equilibrium strategy of Proposition 2. Figure 6 plots this cutoff signal s^* as a function of N. As N becomes larger, an ever higher cutoff signal is required for a buyer to quote the high price of V_H. With a relatively optimistic signal of $s = 0.9$, a buyer finds it unprofitable to purchase the asset at a price of V_H, as long as $N > 5$. As $N \to \infty$, the market breaks down if all buyers receive signals below s^*, which occurs with the limiting probability

$$\lim_{N\to\infty} F_H(s^*)^N = \lim_{N\to\infty} (s^*)^{2N} = \lim_{N\to\infty} \left(\frac{3 - s^*}{4s^*}\right)^2 = \frac{1}{4},$$

which is the smaller root of Equation (16) (or $1.5\sqrt{x} - x = 0.5$). Whether or not the market breaks down, a high-value seller must search for infinitely many buyers in expectation before she can find a good price.

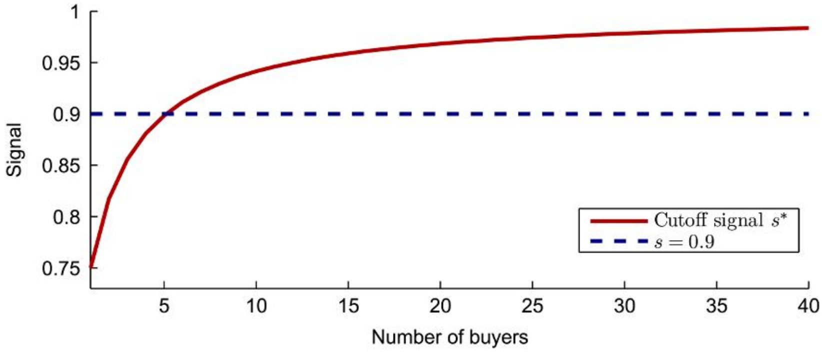

Figure 6
Cutoff signal s^* in the equilibrium of Proposition 2 for $F_H(s) = s^2$ and $F_L(s) = s$, $s \in [0, 1]$
Model parameters: $V_H = 0.6$, $V_L = 0$, $D = 5/3$, and $p_H = p_L = 0.5$.

So far we have considered equilibria in which the seller searches for buyers in a random order. We now consider an equilibrium in which some buyers, such as those with a "relationship" with the seller, take priority over others buyers. For example, the seller can commit to visit a "favored" group of N_1 buyers before visiting the "disfavored" group of the other $N - N_1$ buyers. In each group, the seller assigns a random contact order. The ringing-phone curse becomes less severe in the favored group, each member of which assigns a lower cutoff signal. Buyers in the disfavored group, however, assign a higher cutoff signal because they know that the seller visits the favored group before visiting them. This intuition applies to an arbitrary partition of the buyers, as stated in the following proposition.

Proposition 5. (Concentrating adverse selection by fragmentation.) Suppose that Condition (13) holds and $\partial J(s^*, N)/\partial s^* > 0$ for all N. Suppose that the set $\{1, 2, \ldots, N\}$ of buyers is partitioned into M groups. Denote by $\mathcal{P}_j$ the jth group of buyers, for $j = 1, 2, \ldots, M$. Then, there exists some $\bar{j} \in \{1, 2, \ldots, M\}$ such that the following strategies constitute an equilibrium:

1. For all $j \leq \bar{j}$, let the jth cutoff signal s_j^* be implicitly defined by

$$\frac{p_H}{p_L} \cdot \frac{f_H(s_j^*)}{f_L(s_j^*)} \cdot \frac{\sum_{k=0}^{|\mathcal{P}_j|-1} F_H(s_j^*)^k}{\sum_{k=0}^{|\mathcal{P}_j|-1} F_L(s_j^*)^k} \cdot \prod_{l=1}^{j-1} \frac{F_H(s_l^*)^{|\mathcal{P}_l|}}{F_L(s_l^*)^{|\mathcal{P}_l|}} = \frac{V_H - V_L}{(D-1)V_H}. \quad (17)$$

   A buyer in the jth group quotes a price of V_H if his signal is greater than or equal to s_j^* and quotes a price of V_L if his signal is less than s_j^*. Moreover, the cutoff signals satisfy

$$s_j^* < s_{j+1}^*, \quad 1 \leq j \leq \bar{j} - 1. \quad (18)$$

2. For all $j > \bar{j}$, buyers in the jth group quote a price of V_L.

3. The seller visits the M groups of buyers in the order of $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_M$. Within each group, the seller is assumed to adopt a random search order. The seller sells the asset as soon as she is quoted a price of V_H or higher. If no buyer quotes a price of V_H or higher, then a seller with a high-value asset leaves the market, whereas a seller with a low-value asset accepts a quote of V_L from the last buyer.

The last term on the left-hand side of Equation (17) reflects a buyer's inference that he is only visited because all buyers of the previous $j-1$ groups have received signals below their respective cutoffs. The larger the j, the worse the inference of these buyers regarding the asset value. In short, fragmentation of OTC markets concentrates adverse selection, rather than eliminating it.

Figure 7 plots the equilibrium cutoff signals for $N = 20$ buyers, who are partitioned into a favored group and a disfavored group. Because the favored group is visited first, they assign a strictly lower cutoff signal than that applied without partitioning the buyers. To account for this additional adverse selection, the disfavored group assigns a strictly higher cutoff signal. Both cutoff signals are strictly increasing in the size N_1 of the favored group.

An empirical implication of Proposition 5 is that committing to a favored counterparty improves the prices offered by that counterparty, but worsens the price offered by other counterparties. For example, Bharath, Dahiya, Sauders, and Srinivasan (2011) find that repeated borrowing from the same lender lowers the loan spread offered, but this lending relationship has little benefit when the information of the borrower is relatively symmetric between the borrower and the lender.

Moreover, a market structure in which traders favor certain counterparties over others looks quite similar to that of many OTC-traded assets, such as MBS, ABS, and collateralized debt obligations (CDOs). Both market structures involve creating liquidity by "pooling and tranching" (DeMarzo 2005). In the fragmented market of Proposition 5, the seller "tranches" the pool of counterparties, just as a CDO structure tranches the pool of underlying assets. Liquidity is created in the preferred group of buyers, just like the liquidity created for the senior tranche of a CDO. Adverse selection is, however, transferred and concentrated to other parts of the market. It would be desirable to characterize whether, under general conditions, the seller prefers to favor certain counterparities over others or to treat all counterparties equally, but such a result has not been obtained. Section 2.2 studies an alternative information structure in which, under certain conditions, treating all counterparties equally can be less profitable for the seller than favoring some counterparties.

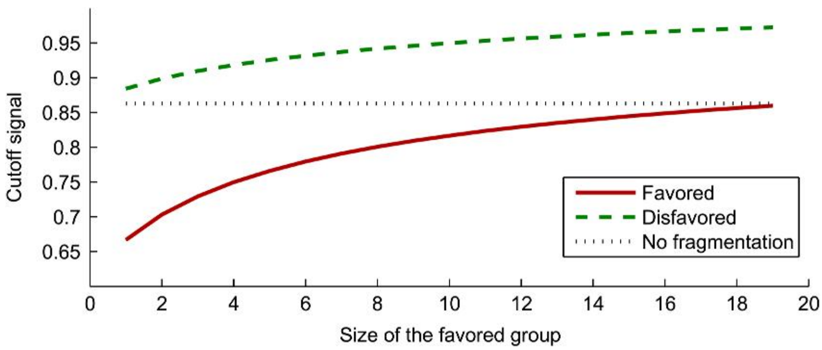

Figure 7
Cutoff signals when $N = 20$ buyers are fragmented into two groups
Distribution functions are $F_H(s) = s$ and $F_L(s) = 1 - (1 - s)^2$ for $s \in [0, 1]$. Other parameters are the same as in Figure 6.

2.1 Ringing-phone curse versus winner's curse

Among centralized trading mechanisms, a first-price auction is a natural counterpart to the OTC market considered in this article. Both forms of market are opaque in the sense that offers are not publicly observed. The key difference is that buyers compete simultaneously in an auction but sequentially in an OTC market.

In a first-price auction, suppose that buyers use a symmetric bidding strategy $\beta_A(s)$ that is strictly increasing in a buyer's signal s. Then, in equilibrium, the winning buyer with a signal of s assigns the likelihood ratio

$$I_A(s,N) = \frac{\mathbb{P}(v = V_H \mid s, \text{win})}{\mathbb{P}(v = V_L \mid s, \text{win})} = \frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)} \cdot \frac{F_H(s)^{N-1}}{F_L(s)^{N-1}}. \tag{19}$$

The likelihood ratio (19) is a natural counterpart to Equation (15). The first two fractions on the right-hand side of Equation (19) represent, as in the OTC market, the buyer's prior belief and the information contained in the signal. The last fraction, $F_H(s)^{N-1}/F_L(s)^{N-1}$, represents the winning bidder's adverse inference that all other $N-1$ buyers' signals are strictly lower than s, the familiar winner's curse.

Proposition 6. (OTC versus first-price auction.) In a first-price auction, there exists a unique cutoff signal $s^A \in (s^*, \bar{s})$ that is implicitly defined by

$$I_A(s^A,N) = \frac{p_H}{p_L} \cdot \frac{f_H(s^A)}{f_L(s^A)} \cdot \frac{F_H(s^A)^{N-1}}{F_L(s^A)^{N-1}} = \frac{V_H - V_L}{(D-1)V_H}. \tag{20}$$

A buyer receiving the signal s^A quotes a price of V_H. This signal s^A is increasing in N. Moreover, for any finite N, the probability $F_H(s^A)^N$ of a market breakdown in a first-price auction is higher than the probability $F_H(s^*)^N$ of a market breakdown in the OTC equilibrium of Proposition 2. As $N \to \infty$,

1. If $f_H(\bar{s})/f_L(\bar{s}) < \infty$, then

$$\lim_{N\to\infty} F_H(s^A)^N = \left(\frac{V_H - V_L}{(D-1)V_H} \cdot \frac{p_L}{p_H} \cdot \frac{f_L(\bar{s})}{f_H(\bar{s})}\right)^{\frac{f_H(\bar{s})}{f_H(\bar{s})-f_L(\bar{s})}} > \lim_{N\to\infty} F_H(s^*)^N. \tag{21}$$

2. If $f_H(\bar{s})/f_L(\bar{s}) = \infty$, then $\lim_{N\to\infty} F_H(s^*)^N = \lim_{N\to\infty} F_H(s^A)^N = 0$.

Perhaps surprisingly, Proposition 6 suggests that the auction market is more likely to break down than is the OTC market. To the extent that a market breakdown prevents gains from trade, the first-price auction is less efficient than the OTC market. Intuitively, because the OTC market does not allow simultaneous contacts, a visited buyer, say Buyer A, infers that only *already visited* buyers received low signals, as reflected in the ratio of weighted sums $\sum_{k=0}^{N-1} F_H(s^*)^k \Big/ \sum_{k=0}^{N-1} F_L(s^*)^k$. By comparison, because bids are simultaneously submitted in an auction, winning an auction unambiguously reveals that the winner's signal is the highest among *all* buyers, as reflected in the ratio $F_H(s)^{N-1}/F_L(s)^{N-1}$. For a buyer receiving a signal of s^*, the likelihood ratio in the auction is smaller than that in the OTC market. Thus, for a buyer to be willing to bid a price of V_H in an auction, it takes a higher cutoff signal, i.e., $s^A > s^*$.

Moreover, because OTC markets and first-price auctions are both pretrade opaque—quotes and bids are not publicly observed—we can interpret these potential market breakdowns as manifestations of the allocational inefficiency of *pretrade market opacity*. Notably, the U.S. Treasury Department introduced a preauction "when-issued" OTC market for treasuries in order to aggregate asymmetrically held information before each treasury auction and thus mitigate the winner's curse effect on auction yields.

The asymptotic inefficiency of the first-price auction relative to the OTC market is closely linked to whether the auction itself aggregates information. A common-value auction aggregates information if the winning bid in the auction converges to the true asset value in probability as N becomes large, as modeled by Wilson (1977), Milgrom (1979), and Kremer (2002), among others. Milgrom (1979) provides a necessary and sufficient condition on the distribution of signals for information aggregation. In the setting of this article, Milgrom's condition is equivalent to $f_H(\bar{s})/f_L(\bar{s}) = \infty$, i.e., signals are *unboundedly informative*. If $f_H(\bar{s})/f_L(\bar{s}) = \infty$, then the auction aggregates information, and both markets are asymptotically efficient. If, however, $f_H(\bar{s})/f_L(\bar{s}) < \infty$, then the auction may not aggregate information, and the auction is less efficient than the OTC market, both for finite N and as N becomes large.

The OTC equilibrium of Proposition 2 differs from that of Lauermann and Wolinsky (2010), who study the interaction between adverse selection and search in a different setting. In their model, not only does the searcher (the better-informed party) make the offers, she also observes the signals of her counterparties. Thus, the equilibrium of Lauermann and Wolinsky (2010) is based on signaling, and the searcher's offers reveal her private information as N becomes large and when some signals are infinitely informative. By contrast, the searcher modeled in this article receives quotes and does not observe the signals of her counterparties. Aside from the benefit of its realism, this specification also allows me to characterize a simple cutoff equilibrium in closed form.

Proposition 6 also implies that a buyer's inference regarding the asset value is more sensitive to his signal in an auction market than in an OTC market. The difference of the logarithms of likelihood ratios $I_A(s, N)$ and $I_{OTC}(s, N)$,

$$\log I_A(s,N) - \log I_{OTC}(s,N) = \log\left(\frac{F_H(s)^{N-1}/F_L(s)^{N-1}}{\sum_{k=0}^{N-1} F_H(s^*)^k / \sum_{k=0}^{N-1} F_L(s^*)^k}\right), \tag{22}$$

is clearly increasing in the signal s. That is, $\log I_A(s,N)$ is a "steeper" function of s than is $\log I_{OTC}(s,N)$, as illustrated in Figure 8.

An empirical implication of Equation (22) is that payoff-relevant information has a smaller impact on quoted prices in an OTC market than in first-price auctions. Allowing simultaneous contacts, as the result suggests, increases the cross-sectional dispersion of quotes. In addition, as information is gradually revealed through time, we also expect simultaneous contacts to increase price volatility and speed price discovery. These implications are relevant in light of recent legislation that moves standard OTC derivative trading into swap execution facilities (SEFs) that allow simultaneous access to multiple counterparties (Commodity Futures Trading Commission 2011; Securities and Exchange Commission 2011).

2.2 Information granularity

So far we have studied an information structure in which signals are infinitely granular, i.e., there is no point mass in the distribution functions of the signals. This short subsection shows that market breakdowns are exacerbated by "lumpy" information.

I start by considering the set that contains cumulative distribution functions that are piecewise continuously differentiable on the support $[0,\bar{s}]$, are weakly increasing, and are right-continuous with left limits. The cumulative conditional distribution functions of signals, $F_\theta : [0,\bar{s}] \to [0,1]$, $\theta \in \{H,L\}$, are drawn from this set. The left limit of $F_\theta(\cdot)$ at s is denoted by $F_\theta(s-) \equiv \lim_{t\uparrow s} F_\theta(t)$. When F_θ is absolutely continuous at s, we keep $f_\theta(s) = F'_\theta(s)$

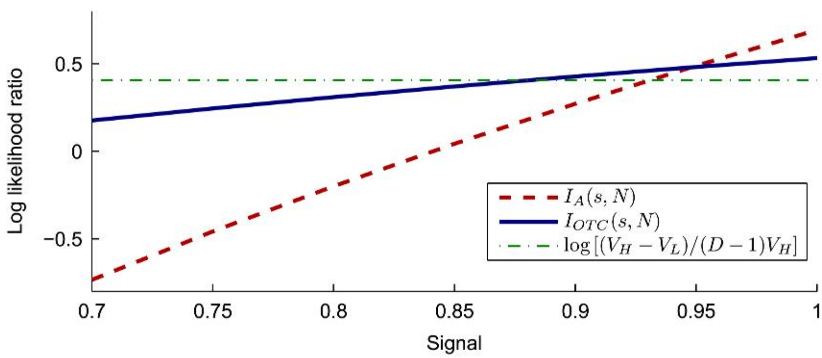

Figure 8
Log likelihood ratio $\log I_A(s,N)$ **and** $\log I_{OTC}(s,N)$ **as a function of** s**, taking** $N=4$
Other parameters and distributions are the same as those of Figure 6.

as usual. When F_θ is discontinuous at s, a pseudo-probability density (mass) function is defined by

$$f_\theta(s) \equiv F_\theta(s) - F_\theta(s-), \quad \text{if } F_\theta(s) > F_\theta(s-),\ s \in (0, \bar{s}]. \tag{23}$$

Finally, I slightly amend the MLRP definition so that it only applies to signals for which the probability distribution (mass) function is positive. That is, for all $s > s'$ such that $f_H(s) > 0$, $f_L(s) > 0$, $f_H(s') > 0$, $f_L(s') > 0$, I assume that

$$\frac{f_H(s)}{f_L(s)} > \frac{f_H(s')}{f_L(s')}. \tag{24}$$

This MLRP definition allows for distribution functions with atoms.[14]

Proposition 7. (Lumpy information in OTC markets.) Suppose that $F_\theta(\bar{s}-) < 1$ for $\theta \in \{H, L\}$ and that Condition (13) holds. In an OTC market, if the seller adopts a random search order, then for sufficiently large N, there exists no cutoff equilibrium in which a buyer quotes a price of at least V_H.

Proposition 7 suggests that, at least in the space of cutoff equilibria, a large OTC market breaks down with a probability approaching 1 as N becomes large. Since there is a point mass at $\bar{s}$, some buyers receive the signal $\bar{s}$ with the limit probability of 1 as $N \to \infty$. Therefore, when a buyer is visited, the fact that no previously visited buyer receives the signal $\bar{s}$ speaks so strongly against the asset quality that the ringing-phone curse dominates any inference from signals, including the signal $\bar{s}$. Instead of having a zero probability of trade, a high-value seller in this case would rather commit to, for example, visiting a particular buyer before others because the probability of selling the asset to that favored buyer is no lower than $f_H(\bar{s}) > 0$, by Condition (13).

A comparison of the asymptotic inefficiencies between the two information structures (those characterized by Proposition 7 and Proposition 4) suggests that information granularity can affect the size of the market for an OTC-traded asset. For example, when the post-trade reporting of transaction prices and trade volumes is less frequent, we expect to find there to be fewer dealers willing to make a market.

Example 3. Consider an information structure with signal outcomes 0 and 1. Let

$$\mathbb{P}(s = 1 \,|\, v = V_H) = \mathbb{P}(s = 0 \,|\, v = V_L) = q, \tag{25}$$

$$\mathbb{P}(s = 0 \,|\, v = V_H) = \mathbb{P}(s = 1 \,|\, v = V_L) = 1 - q, \tag{26}$$

[14] For example, the signal s may be drawn from $\{0, \bar{s}\}$, and the distribution functions satisfy $f_H(\bar{s}) = f_L(0) = q > 1 - q = f_H(0) = f_L(\bar{s})$, for some $q \in (0.5, 1)$.

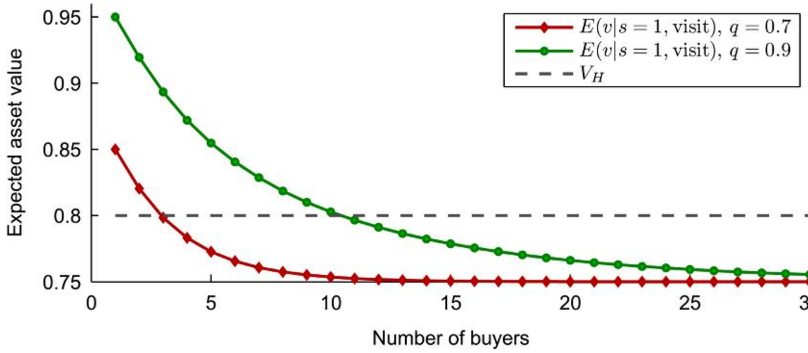

Figure 9
Expected asset value conditional on being visited as a function of N
The seller chooses the order of contacts at random. Parameters: $V_H = 0.8$, $V_L = 0.5$, $D = 1.25$, and $p_H = p_L = 0.5$.

where $q > 0.5$ is the "quality" of the signal. Suppose that there exists a cutoff equilibrium in which the seller searches in a random order and buyers who receive the high signal quote a high price of V_H, while buyers with the low signal quote a low price of V_L. It is easy to show that, upon a contact, a buyer with a high signal forms the inference

$$\frac{\mathbb{P}(v = V_H \mid s = 1, \text{visit})}{\mathbb{P}(v = V_L \mid s = 1, \text{visit})} = \frac{p_H}{p_L} \cdot \frac{1-(1-q)^N}{1-q^N}, \tag{27}$$

which is increasing in q and decreasing in N.

Figure 9 shows how the conditional expected asset value, given the high signal, varies with the number N of buyers for different levels of information quality q. The model parameters are $V_H = 0.8$, $V_L = 0.5$, $D = 1.25$, and $p_H = p_L = 0.5$. As we can see, for a relatively high information quality, $q = 0.7$, a cutoff equilibrium can be supported by two buyers at most. For a very high information quality, $q = 0.9$, a cutoff equilibrium can be supported by ten buyers at most.

Proposition 7 has a natural analogue in first-price auctions, as follows.

Proposition 8. (Lumpy information in first-price auctions.) Suppose that $F_\theta(\bar{s}-) < 1$ for $\theta \in \{H, L\}$ and that Condition (13) holds. In a first-price auction, if the seller chooses the winner from the highest bidders at random and with equal probabilities, then for sufficiently large N, there exists no pure-strategy equilibrium in which a buyer bids a price of at least V_H.

3. Conclusion

This article offers a model of opaque over-the-counter markets. A quote seeker searches for an attractive price by contacting multiple quote providers in sequence, and possibly repeatedly. Under stated conditions, a repeat contact with a counterparty reveals a quote seeker's reduced outside options and worsens the quote from the revisited counterparty.

I also show that the combined effects of market opacity and contact-order uncertainty create a ringing-phone curse that lowers quote providers' inference regarding the value of the asset. Selecting certain counterparties over others improves the prices that are offered by these "favored" counterparties but worsens those by other "disfavored" counterparties. The results further reveal that an OTC market can, in some cases, realize a higher gain from trade than a first-price auction in expectation. Finally, the model predicts that quoted prices are more sensitive to payoff-relevant information in first-price auctions than in OTC markets, suggesting that centralized auctions can provide faster price discovery than OTC markets.

Appendix

Proof of Proposition 1. We prove the proposition by direct verification. First, when the highest quote in the market is V_H, both types of sellers will accept this quote; when all sellers accept a price of V_H, no buyer quotes a price higher than V_H. Second, because a buyer with a value of $u < V_L$ cannot purchase the asset, it is without loss of generality that he quotes zero. Third, if all buyers quote a price of V_L upon the second contact, then a low-value seller will accept, and vice versa. Fourth, given β_1 and β_2, the continuation value (or reserve price) R_k of a low-value seller is given by Equation (3). Moreover, Equation (3) implies that $R_k > R_{k+1}$ for $1 \le k \le N-1$. Fifth, if we take β_1 and β_2 as given (we verify their expressions below), then $\beta_2(v_i) < \beta_1(v_i)$, as long as $\beta_1(v_i) > V_L$.

What remains to be verified is the first quote of a buyer, say Buyer A, with value $u \ge V_L$. Because a buyer does not observe the order of contact, he must infer it. Suppose that buyer A considers quoting a price of, say, $R_N = V_L$. This quote leads to an immediate trade if and only if two conditions hold. First, the seller is of low type conditional on the buyer being visited, i.e., with probability $\mathbb{P}(v_0 = V_L \mid \text{visit})$. Second, conditional on the seller's value being low, Buyer A is the Nth buyer visited, and all previous $(N-1)$ buyers have quoted prices lower than the seller's continuation values, i.e., with probability $q_N = G(V_1)G(V_2)\ldots G(V_{N-1})$. Then, by Bayes' Rule, the probability that Buyer A is the Nth buyer visited, conditional on his being visited and conditional on the seller's value being low, is equal to

$$\mathbb{P}(N\text{-th} \mid \text{visit}, v_0 = V_L) = \frac{q_N \cdot \mathbb{P}(N\text{-th} \mid v_0 = V_L)}{\sum_{j=1}^{N} q_j \cdot \mathbb{P}(j\text{-th} \mid v_0 = V_L)} = \frac{q_N}{\sum_{k=1}^{N} q_j},$$

where the last equality follows from the fact that a seller's search order is random with equal probability $1/N$. The buyer's expected profit of quoting a price of $R_N = V_L$ is thus

$$\Pi(R_N, u) = \mathbb{P}(v_0 = V_L \mid \text{visit})(u - R_N)\frac{q_N}{\sum_{j=1}^{N} q_j}.$$

Similarly, if the buyer quotes a higher price of R_{N-1}, then a low-value seller accepts it if and only if the buyer is either the Nth or the $(N-1)$th buyer visited. The buyer's corresponding profit is

$$\Pi(R_{N-1}, u) = \mathbb{P}(v_0 = V_L \mid \text{visit})(u - R_{N-1})\frac{q_N + q_{N-1}}{\sum_{j=1}^{N} q_j}.$$

Because $\partial\Pi(R_N, u)/\partial u < \partial\Pi(R_{N-1}, u)/\partial u$, the higher the value of the buyer, the more attractive it is to quote the higher price of R_{N-1}, relative to quoting the lower price of R_N. In equilibrium, a buyer with a value of V_{N-1} must be indifferent between the two quotes. That is,

$$(V_{N-1} - R_{N-1})\frac{q_N + q_{N-1}}{\sum_{j=1}^{N} q_j} = (V_{N-1} - R_N)\frac{q_N}{\sum_{j=1}^{N} q_j},$$

which is one equation of Equation (4). We can derive other equations in Equation (4) in a similar manner. Moreover, by Bayes' Rule,

$$\frac{\mathbb{P}(v_0 = V_H \mid \text{visit})}{\mathbb{P}(v_0 = V_L \mid \text{visit})} = \frac{\mathbb{P}(\text{visit} \mid v_0 = V_H) p_H}{\mathbb{P}(\text{visit} \mid v_0 = V_L) p_L} = \frac{\sum_{j=1}^{N} G(V_0)^{j-1}}{\sum_{j=1}^{N} q_j} \frac{p_H}{p_L},$$

and thus

$$\mathbb{P}(v_0 = V_L \mid \text{visit}) = \left(1 + \frac{\sum_{j=1}^{N} G(V_0)^{j-1}}{\sum_{j=1}^{N} q_j} \frac{p_H}{p_L}\right)^{-1}.$$

Then, Equation (5) follows from the fact that a buyer with a value of $u = V_0$ is indifferent between trading with any type of seller at a price of V_H and trading with a low-value seller at a price of R_1. ■

Proof of Proposition 2. To verify the equilibrium of Proposition 2, suppose that players adopt the conjectured strategies and that there is a unique cutoff signal that satisfies Equation (14). Given the seller's acceptance strategy and the random ordering of buyers, a visited buyer assigns a probability of $1/N$ that he is the kth buyer visited, $k = 1, 2, \ldots, N$. That is, the previous $k - 1$ buyers all have received signals below s^*. By Bayes' Rule and the independence of signals, we have

$$\frac{\mathbb{P}(v = V_H \mid s, \text{visit})}{\mathbb{P}(v = V_L \mid s, \text{visit})} = \frac{p_H}{p_L} \cdot \frac{\mathbb{P}(s, \text{visit} \mid v = V_H)}{\mathbb{P}(s, \text{visit} \mid v = V_L)} = \frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)} \cdot \frac{\frac{1}{N}\sum_{k=0}^{N-1} F_H(s^*)^k}{\frac{1}{N}\sum_{k=0}^{N-1} F_L(s^*)^k}.$$

The cutoff signal s^* must imply an expected asset value of V_H or, equivalently, a likelihood ratio of $\frac{V_H - V_L}{(D-1)V_H}$. Thus, s^* must satisfy Equation (14).

It remains to show that such s^* exists. Write the right-hand side of Equation (14) as J^*. From MLRP, for some small $\underline{s} > 0$, $f_H(\underline{s}) \leq f_L(\underline{s})$. By Condition (13), at $s = \underline{s}$,

$$J(\underline{s}, N) < \frac{p_H}{p_L} \cdot \frac{f_H(\underline{s})}{f_L(\underline{s})} \leq \frac{p_H}{p_L} < J^*,$$

and at the upper support $s = \bar{s}$,

$$J(\bar{s}, N) = \frac{p_H}{p_L} \cdot \frac{f_H(\bar{s})}{f_L(\bar{s})} > J^*.$$

So, there exists some $s^* \in (0, \bar{s})$ such that Equation (14) holds. Moreover, if $\partial J(s, N)/\partial s > 0$ for all N, then the cutoff signal s^* is unique.

When s^* is unique, then any buyer who receives a signal below (above) s^* has expected asset value below (above) V_H. A buyer with a signal $s \geq s^*$ has no incentive to quote a price higher than V_H, as V_H is acceptable to any seller. If the buyer deviates and quotes a price that is strictly lower than V_H, then he only buys the asset if the asset is of low value, which implies a nonpositive profit for the buyer. Similarly, a buyer with a signal of $s < s^*$ does not quote V_H, as he otherwise makes a negative expected profit. He has no incentive to deviate to a quote that is strictly lower than V_L, as doing so results in zero profit. Quoting a price higher than V_L but lower than V_H only attracts low-value sellers, for whom a quote of V_L suffices. If all buyers quote a price lower than V_H, i.e., they all quote V_L, then a high-value buyer leaves the market because the asset is worth V_H to her and because already visited buyers will not raise the quotes. (If buyers ever raise their quotes in equilibrium, then earlier, lower quotes become nonserious and are rejected.) This completes the verification of the equilibrium.

We now show that $\partial J(s^*, N)/\partial N < 0$. Observe that for all integers $k > j \geq 0$ and any $s \in (0, \bar{s})$,

$$\frac{F_H(s)^j}{F_L(s)^j} > \frac{F_H(s)^j + F_H(s)^k}{F_L(s)^j + F_L(s)^k} > \frac{F_H(s)^k}{F_L(s)^k}.$$

Iterate it and we get

$$\frac{\sum_{k=0}^{N-1} F_H(s)^k}{\sum_{k=0}^{N-1} F_L(s)^k} > \frac{F_H(s)^N}{F_L(s)^N}.$$

Then,

$$\frac{\sum_{k=0}^{N-1} F_H(s)^k}{\sum_{k=0}^{N-1} F_L(s)^k} > \frac{\sum_{k=0}^{N} F_H(s)^k}{\sum_{k=0}^{N} F_L(s)^k} > \frac{F_H(s)^N}{F_L(s)^N}.$$

If $\partial J(s^*, N)/\partial s > 0$, then by the inverse function theorem,

$$\frac{ds^*}{dN} = -\frac{\partial J(s^*, N)/\partial N}{\partial J(s^*, N)/\partial s^*} > 0.$$

Finally, Equations (14) and (15) imply that for any fixed s,

$$I_{OTC}(s, N) = \frac{f_H(s)/f_L(s)}{f_H(s^*)/f_L(s^*)} \cdot \frac{V_H - V_L}{(D-1)V_H}.$$

By MLRP, $I_{OTC}(s, N)$ is decreasing in s^*, and thus decreasing in N. ■

Proof of Proposition 3. Suppose for contradiction that there is a cutoff equilibrium in which $P_H > V_H$. Consider a buyer, say Buyer A, who receives the cutoff signal s^* and quotes a price of P_H. We first observe that Buyer A must be indifferent between quoting P_H and V_L, since if he were to strictly prefer quoting P_H, a buyer with a signal of $s^* - \epsilon$ for sufficiently small $\epsilon > 0$ would deviate to quote P_H, too. Thus, Buyer A values the asset at P_H.

Consider the seller's response when Buyer A deviates to quote $P_H - \epsilon$ for small $\epsilon > 0$. If the seller rejects this lower quote, her payoff from continued search can increase by ϵ at most. However, with a probability of at least $F_H(s^*)^{N-1}$, the seller cannot find a quote of P_H from the unvisited buyers. In this case, if the seller returns to Buyer A, Buyer A's likelihood ratio of the asset value reduces from

$$\frac{p_H}{p_L} \cdot \frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s^*)^k}{\sum_{k=0}^{N-1} F_L(s^*)^k} \quad \text{to} \quad \frac{p_H}{p_L} \cdot \frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{F_H(s^*)^{N-1}}{F_L(s^*)^{N-1}},$$

where the latter inference reflects the fact that the other $N - 1 \geq 1$ buyers receive signals below s^*. Because Buyer A's original inference gives an expected asset value of P_H, this new inference implies an asset value strictly lower than P_H. Accordingly, the revisited Buyer A reduces his quote by a discrete amount $\Delta(s^*) > 0$. Thus, the seller's cost of rejecting a quote of $P_H - \epsilon$ is at least $F_H(s^*)^{N-1}\Delta(s^*) > 0$. For small enough ϵ, the seller accepts the quote of $P_H - \epsilon$, and the conjectured equilibrium cannot hold. Therefore, the only possible cutoff equilibrium is for the buyers to quote V_H. ■

Proof of Proposition 4. By Proposition 2, $\partial J(s, N)/\partial s > 0$ implies that s^* is strictly increasing in N. Since s^* is bounded above by $\bar{s}$, $\lim_{N\to\infty} s^*$ exists. Suppose for contradiction that the limit is $\bar{s} - \epsilon$ for some $\epsilon > 0$. As $N \to \infty$, $F_H(\bar{s} - \epsilon)^N \to 0$, $F_L(\bar{s} - \epsilon)^N \to 0$, and thus

$$\frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s^*)^k}{\sum_{k=0}^{N-1} F_L(s^*)^k} \to \frac{f_H(\bar{s}-\epsilon)}{f_L(\bar{s}-\epsilon)} \cdot \frac{1 - F_L(\bar{s}-\epsilon)}{1 - F_H(\bar{s}-\epsilon)} < 1,$$

where the last inequality follows from MLRP. Given Condition (13), for some sufficiently large but finite N, Equation (14) cannot hold, so it is a contradiction. Therefore, $s^* \to \bar{s}$ as $N \to \infty$.

For any $s < \bar{s}$, there exists some $\bar{N}$ such that for all $N > \bar{N}$, $s^* > s$. By MLRP and Equations (14) and (15),

$$I_{OTC}(s, N) = \frac{f_H(s)/f_L(s)}{f_H(s^*)/f_L(s^*)} J(s^*, N) < J(s^*, N) = \frac{V_H - V_L}{(D-1)V_H},$$

so $\mathbb{E}(v \mid s, \text{visit}) < V_H$. When $f_H(\bar{s})/f_L(\bar{s}) < \infty$,

$$\lim_{N\to\infty} I_{OTC}(\bar{s}, N) = \lim_{N\to\infty} \frac{f_H(\bar{s})/f_L(\bar{s})}{f_H(s^*)/f_L(s^*)} J(s^*, N) = \frac{V_H - V_L}{(D-1)V_H}.$$

We now calculate $x_H \equiv \lim_{N\to\infty} F_H(s^*)^N$. We let $x_L \equiv \lim_{N\to\infty} F_L(s^*)^N$. By the definition of s^*, we have

$$\frac{V_H - V_L}{(D-1)V_H} = \frac{p_H}{p_L} \cdot \lim_{N\to\infty} \frac{1 - F_H(s^*)^N}{1 - F_L(s^*)^N},$$

where the equality follows from

$$\lim_{N\to\infty} \frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{1 - F_L(s^*)}{1 - F_H(s^*)} = 1$$

by l'Hôpital's Rule. The proof of Proposition 6 shows that $x_H < 1$, so we must have $x_L < 1$ as well. Then, we have

$$1 - x_H = a(1 - x_L).$$

To derive a function of x_H alone, note that

$$\frac{x_H}{x_L} = \lim_{N\to\infty} \frac{F_H(s^*)^N}{F_L(s^*)^N} = \lim_{N\to\infty} e^{N \log \frac{F_H(s^*)}{F_L(s^*)}} = \lim_{N\to\infty} e^{-N\left(1 - \frac{F_H(s^*)}{F_L(s^*)}\right)}.$$

We can further calculate by l'Hôpital's Rule that

$$\lim_{N\to\infty} \frac{1 - \frac{F_H(s^*)}{F_L(s^*)}}{1 - F_H(s^*)} = \frac{f_H(\bar{s}) - f_L(\bar{s})}{f_H(\bar{s})}.$$

Substitute back and we have

$$\frac{x_H}{x_L} = \left(\lim_{N\to\infty} e^{-N(1-F_H(s^*))}\right)^{1 - f_L(\bar{s})/f_H(\bar{s})} = x_H^{1 - f_L(\bar{s})/f_H(\bar{s})}.$$

Thus, $x_L = x_H^{f_L(\bar{s})/f_H(\bar{s})}$. Substituting back into $1 - x_H = a(1 - x_L)$, we see that x_H solves

$$g(x) \equiv a x^{f_L(\bar{s})/f_H(\bar{s})} - x = a - 1.$$

Given $f_L(\bar{s})/f_H(\bar{s}) < 1$, $g(x)$ is strictly concave, and there are two roots of $g(x) = a - 1$ at most. It is easy to show that $g(x)$ achieves its maximum at some x^* that is implicitly defined by

$$\left(x^*\right)^{f_L(\bar{s})/f_H(\bar{s})-1} = \left(a\,\frac{f_L(\bar{s})}{f_H(\bar{s})}\right)^{-1} > 1.$$

So, $x^* < 1$. Since $g(1) = a - 1$, we must have $g(x^*) > a - 1$, and there is a unique root of g between 0 and 1. This root is $x_H < 1$.

Finally, we calculate $\mathbb{E}(T_N)$. Fix an arbitrary N'. Since $s^* \to \bar{s}$, for sufficiently large N the probability $F_H(s^*)^{N'}$ that the first N' buyers receive signals below s^* is at least $1/2$. So, $\lim_{N\to\infty} \mathbb{E}(T_N) > N'/2$ for all N'. That is, $\mathbb{E}(T_N) \to \infty$ as $N \to \infty$. ■

Proof of Proposition 5. It is straightforward that whenever the cutoff signal s_j^* exists, it is given by Equation (17). We now prove Equation (18). Since inferences of the asset value at the cutoff signals are equal, we have

$$\frac{f_H(s_j^*)}{f_L(s_j^*)} \cdot \frac{\sum_{k=0}^{|\mathcal{P}_j|-1} F_H(s_j^*)^k}{\sum_{k=0}^{|\mathcal{P}_j|-1} F_L(s_j^*)^k} \cdot \prod_{l=1}^{j-1} \frac{F_H(s_l^*)^{|\mathcal{P}_l|}}{F_L(s_l^*)^{|\mathcal{P}_l|}}$$
$$= \frac{f_H(s_{j+1}^*)}{f_L(s_{j+1}^*)} \cdot \frac{\sum_{k=0}^{|\mathcal{P}_{j+1}|-1} F_H(s_{j+1}^*)^k}{\sum_{k=0}^{|\mathcal{P}_{j+1}|-1} F_L(s_{j+1}^*)^k} \cdot \prod_{l=1}^{j} \frac{F_H(s_l^*)^{|\mathcal{P}_l|}}{F_L(s_l^*)^{|\mathcal{P}_l|}}. \tag{A1}$$

Suppose by contradiction that $s_{j+1}^* \le s_j^*$. Because $\partial J(s^*, N)/\partial s^* > 0$ for all N, the ratio of the right-hand side of Equation (A1) to the left-hand side is no higher than

$$\frac{\dfrac{\sum_{k=0}^{|\mathcal{P}_{j+1}|-1} F_H(s_j^*)^k}{\sum_{k=0}^{|\mathcal{P}_{j+1}|-1} F_L(s_j^*)^k}}{\dfrac{\sum_{k=0}^{|\mathcal{P}_j|-1} F_H(s_j^*)^k}{\sum_{k=0}^{|\mathcal{P}_j|-1} F_L(s_j^*)^k}} \cdot \frac{F_H(s_j^*)^{|\mathcal{P}_j|}}{F_L(s_j^*)^{|\mathcal{P}_j|}} < 1,$$

which is a contradiction. Thus, $s_{j+1}^* > s_j^*$. ■

Proof of Proposition 6. The implicit definition of s^A simply follows from the fact that a buyer who receives a signal of s^A is just indifferent between bidding V_H and bidding V_L. Because bidding V_L yields an expected profit of 0, bidding V_H also yields an expected profit of 0. The cutoff s^A is unique because $I_A(s, N)$ is monotone in s. Also, since $\partial I_A(s, N)/\partial N < 0$, we have $\partial s^A/\partial N > 0$.

To show $s^A > s^*$, suppose for contradiction that $s^A \le s^*$. Then, we have, for $N \ge 2$,

$$\frac{V_H - V_L}{(D-1)V_H} \le \frac{p_H}{p_L} \cdot \frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{F_H(s^*)^{N-1}}{F_L(s^*)^{N-1}} < \frac{p_H}{p_L} \cdot \frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s^*)^k}{\sum_{k=0}^{N-1} F_L(s^*)^k},$$

contradicting Equation (14). Thus, $s^A > s^*$. The probability of market breakdown then follows accordingly.

When $f_H(\bar{s})/f_L(\bar{s}) < \infty$, by the definition of s^A, we have

$$\lim_{N\to\infty} \frac{F_H(s^A)^N}{F_L(s^A)^N} = \frac{V_H - V_L}{(D-1)V_H} \cdot \frac{p_L}{p_H} \cdot \frac{f_L(\bar{s})}{f_H(\bar{s})} \equiv b. \tag{A2}$$

To calculate the limit of $F_H(s^A)^N$, observe that

$$\lim_{N\to\infty} F_H(s^A)^N = \lim_{N\to\infty} e^{N\log F_H(s^A)} = \lim_{N\to\infty} e^{-N(1-F_H(s^A))},$$

$$b = \lim_{N\to\infty} \frac{F_H(s^A)^N}{F_L(s^A)^N} = \lim_{N\to\infty} e^{N\log\frac{F_H(s^A)}{F_L(s^A)}} = \lim_{N\to\infty} e^{-N\left(1-\frac{F_H(s^A)}{F_L(s^A)}\right)}.$$

Then, the expression of $\lim_{N\to\infty} F_H(s^A)^N$ follows from the fact that

$$\lim_{N\to\infty} \frac{1-F_H(s^A)}{1-\frac{F_H(s^A)}{F_L(s^A)}} = \frac{f_H(\bar{s})}{f_H(\bar{s})-f_L(\bar{s})}.$$

Obviously, $\lim_{N\to\infty} F_H(s^*)^N \le \lim_{N\to\infty} F_H(s^A)^N$ because $F_H(s^*) < F_H(s^A)$ for each N. Moreover, $\lim_{N\to\infty} F_H(s^A)^N < 1$ by Condition (13).

Write $y_H = \lim_{N\to\infty} F_H(s^A)^N$. To show that $y_H > x_H$, it is sufficient to show that $a y_H^{f_L(\bar{s})/f_H(\bar{s})} - y_H > a-1$, where a is given in Proposition 4. Because

$$y_H = \left(a\frac{f_L(\bar{s})}{f_H(\bar{s})}\right)^{\frac{f_H(\bar{s})}{f_H(\bar{s})-f_L(\bar{s})}},$$

we have

$$a y_H^{f_L(\bar{s})/f_H(\bar{s})} - y_H = b^{\frac{f_L(\bar{s})}{f_H(\bar{s})-f_L(\bar{s})}}(a-b) \equiv K(b),$$

where b is given in Equation (A2). Clearly, $K(1) = a-1$. It is easy to show that $K'(z) \le 0$ when $z \ge b$, so $K(b) > K(1) = a-1$, and thus $y_H > x_H$.

Finally, because

$$\frac{p_H}{p_L} \cdot \lim_{N\to\infty} \frac{f_H(s^A)}{f_L(s^A)} \cdot \lim_{N\to\infty} \frac{F_H(s^A)^{N-1}}{F_L(s^A)^{N-1}} = \frac{V_H - V_L}{(D-1)V_H} < \infty,$$

by Equation (19), we must have $\lim_{N\to\infty} F_H(s^A)^N = 0$ when $f_H(\bar{s})/f_L(\bar{s}) = \infty$. In that case, $\lim_{N\to\infty} F_H(s^*)^N = 0$ because $F_H(s^*) < F_H(s^A)$ for each N. ■

Proof of Proposition 7. Suppose for contradiction that there exists an equilibrium in which a buyer with a signal of at least s^* quotes a price of $P_H \ge V_H$. If the seller adopts a random search order, upon a contact by the seller, a buyer with a signal of s^* assigns the likelihood ratio

$$\begin{aligned} I_{OTC}(s^*) &= \frac{p_H}{p_L} \cdot \frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s^*-)^k}{\sum_{k=0}^{N-1} F_L(s^*-)^k} \\ &= \frac{p_H}{p_L} \cdot \frac{1-F_H(s^*-)^N}{1-F_L(s^*-)^N} \cdot \frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{1-F_L(s^*-)}{1-F_H(s^*-)}. \end{aligned}$$

By MLRP, for all $s \ge s^*$, $f_H(s)/f_L(s) \ge f_H(s^*)/f_L(s^*)$, and thus

$$\frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{1-F_L(s^*-)}{1-F_H(s^*-)} \le 1.$$

Then, because $F_\theta(s^*-) \leq F_\theta(\bar{s}-) < 1$,

$$\lim_{N\to\infty} I_{OTC}(s^*) \leq \frac{p_H}{p_L}.$$

That is, $\lim_{N\to\infty} E(v \mid s^*, \text{visit}) < V_H$. Thus, the conjectured equilibrium cannot survive. ■

Proof of Proposition 8. Suppose for contradiction that there exists a pure-strategy equilibrium in the first-price auction. Consider the inference of the winner who receives a signal of $\bar{s}$. Since the seller chooses a winner at random and with equal probabilities if multiple buyers bid the same highest price, the state-θ probability of winning conditional on signal $\bar{s}$ is

$$\mathbb{P}(\text{win} \mid \bar{s}, v = V_\theta) = \sum_{k=1}^{N} \frac{1}{k} \binom{N-1}{k-1} F_\theta(\bar{s}-)^{N-k}(1 - F_\theta(\bar{s}-))^{k-1} = \frac{1 - F_\theta(\bar{s}-)^N}{N f_\theta(\bar{s})},$$

where I have used $f_\theta(\bar{s}) = 1 - F_\theta(\bar{s}-)$. The corresponding likelihood ratio is

$$\frac{\mathbb{P}(v = V_H \mid \bar{s}, \text{win})}{\mathbb{P}(v = V_L \mid \bar{s}, \text{win})} = \frac{p_H}{p_L} \cdot \frac{f_H(\bar{s})}{f_L(\bar{s})} \cdot \frac{\frac{1-F_H(\bar{s}-)^N}{N f_H(\bar{s})}}{\frac{1-F_L(\bar{s}-)^N}{N f_L(\bar{s})}} \to \frac{p_H}{p_L},$$

as $N \to \infty$. By Condition (13), a buyer with the highest signal $\bar{s}$ cannot quote a price of V_H or higher. ■

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