Internet Appendix for “Back-Running: Seeking and Hiding Fundamental Information in Order Flows”

Liyan Yang* Haoxiang Zhu†

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In Yang and Zhu (2017), we have taken the information of the fundamental investor and the back-runner as given. In this internet appendix, we explicitly model information acquisition. Besides showing the robustness of our earlier results, this additional step sheds light on questions such as “Does back-running discourage acquisition of fundamental information?”

Setup of Information Acquisition

We add one period, $t = 0$, before the two-period economy considered in Yang and Zhu (2017). At $t = 0$, the fundamental investor decides the amount of fundamental information she acquires, and the back-runner decides the precision of order flow information he acquires. Specifically, for the fundamental trader, we follow Admati and Pfleiderer (1989), Madrigal (1996), and Bond, Goldstein, and Prescott (2010) and assume that the fundamental investor can pay a cost $C_F(\phi)$ upfront to observe the fundamental value v with probability $\phi \in (0, 1)$. For the back-runner, we follow Verrecchia (1982) and Vives (2008) and assume that the back-runner can pay a cost $C_B\left(\frac{1}{\sigma_\varepsilon^2}\right)$ upfront to observe a signal s of x_1 with precision $\frac{1}{\sigma_\varepsilon^2}$. These information-acquisition decisions are simultaneous. After time 0, the choices of ϕ and σ_ε become public information. In reality, investment in fundamental research, such as hiring analysts, and investment in advanced trading technology, such as high-speed connections to exchanges, are usually observable.

To ensure interior solutions of σ_ε^2 and ϕ, we make the standard technical assumptions: (i) $C_B(\cdot)$ and $C_F(\cdot)$ are increasing and convex; and (ii) $C_B(0) = C_B'(0) = 0, C_B(\infty) = C_B'(\infty) = \infty, C_F(0) = C_F'(0) = 0$, and $C_F(1) = C_F'(1) = \infty$.

*Rotman School of Management, University of Toronto; liyan.yang@rotman.utoronto.ca.
†MIT Sloan School of Management; zhuh@mit.edu.

For simplicity, we assume that at the beginning of period 1, it becomes public knowledge whether the fundamental investor has successfully observed v. It is a standard assumption in Kyle-type models whether such an (fundamentally) informed investor exists. Then, the subsequent game has two possible outcomes:

1. If the fundamental investor observes v, then the economy is the one that we analyzed in Yang and Zhu (2017).
2. If the fundamental investor does not observe v, then as an uninformed investor she will not trade in either period. As a result, the back-runner will not trade in period 2, either, despite receiving the signal of the fundamental investor's (zero) order flow. In this case, only noise traders submit orders, and so the price is $p_1 = p_2 = E(v) = p_0$.

Analysis and Results

Our objective is to find the equilibrium levels of ϕ and σ_ε. These are determined jointly by the period-0 maximization problems of the fundamental investor and the back-runner.

Recall that $\pi_{F,1}$ and $\pi_{F,2}$ denote the realized profits of the fundamental investor in dates 1 and 2, respectively. The fundamental investor's period-0 expected net profit is:

$$\Pi_{F,0} \equiv \phi E\left(\pi_{F,1} + \pi_{F,2}\right) - C_F\left(\phi\right),$$

and her problem is to chose ϕ to maximize $\Pi_{F,0}$, taking her conjectured equilibrium value of σ_ε as given. Because $E\left(\pi_{F,1} + \pi_{F,2}\right)$ does not depend on ϕ, and given the technical assumption on $C_F\left(\phi\right)$, we know that the solution to the fundamental investor's problem is characterized by the first-order condition:

$$E\left(\pi_{F,1} + \pi_{F,2}\right) = C_F'\left(\phi\right).$$

Now we consider the back-runner's information acquisition problem. Recall that

$$\pi_{B,2} \equiv \left(v - p_2\right) d_2$$

is the back-runner's realized period-2 profit. So, his period-0 expected net profit of acquiring order flow information is:

$$\Pi_{B,0} \equiv \phi E\left(\pi_{B,2}\right) - C_B\left(\frac{1}{\sigma_\varepsilon^2}\right).$$

The back-runner takes the equilibrium value ϕ as given and chooses σ_ε to maximize Π_0^B.

The back-runner's choice of σ_ε affects $E(\pi_{B,2})$ through its effect on the equilibrium strategies, $\sigma_z, \beta_{v,1}, \beta_{v,2}, \beta_{x_1}, \beta_{y_1}, \delta_s, \delta_{y_1}, \lambda_1$ and λ_2. Specifically, we can compute

$$E(\pi_{B,2}) = \lambda_2 \left[(\delta_s - \delta_{y_1})^2 \beta_{v,1}^2 \Sigma_0 + (\delta_s - \delta_{y_1})^2 \sigma_z^2 + \delta_s^2 \sigma_\varepsilon^2 + \delta_{y_1}^2 \sigma_u^2 \right],$$

and hence

$$\Pi_{B,0} = \phi \lambda_2 \left[(\delta_s - \delta_{y_1})^2 \beta_{v,1}^2 \Sigma_0 + (\delta_s - \delta_{y_1})^2 \sigma_z^2 + \delta_s^2 \sigma_\varepsilon^2 + \delta_{y_1}^2 \sigma_u^2 \right] - C_B\left(\frac{1}{\sigma_\varepsilon^2}\right).$$

There is an important complication in the solution to the back-runner's information-acquisition problem. Although this problem has an interior solution, as ensured by the cost function $C_B(\cdot)$, the optimal choice of σ_ε cannot in general be guaranteed by setting the first-order derivative to zero. This is because whether the equilibrium has a mixed strategy or a pure strategy (used by the fundamental investor) depends on σ_ε. As σ_ε decreases and drops below the threshold value of $(\sqrt{\sqrt{17-4}/2})\sigma_u$, the equilibrium switches from pure strategy to mixed strategy, giving rise to a kink in $E(\pi_{B,2})$. If the optimal value of σ_ε occurs at the kink, the first-order condition is characterized by two inequalities rather than an equality. (This complication does not apply to the fundamental investor's problem.)

To solve the equilibrium explicitly and numerically, we need explicit functional forms of C_B and C_F. Following Vives (2008), we choose the following parametrization:

$$\begin{aligned} C_B\left(\frac{1}{\sigma_\varepsilon^2}\right) &= k_B\left(\frac{1}{\sigma_\varepsilon^2}\right)^{h_B} = k_B \sigma_\varepsilon^{-2h_B}, \\ C_F(\phi) &= k_F\left(\frac{\phi}{1-\phi}\right)^{h_F}, \end{aligned}$$

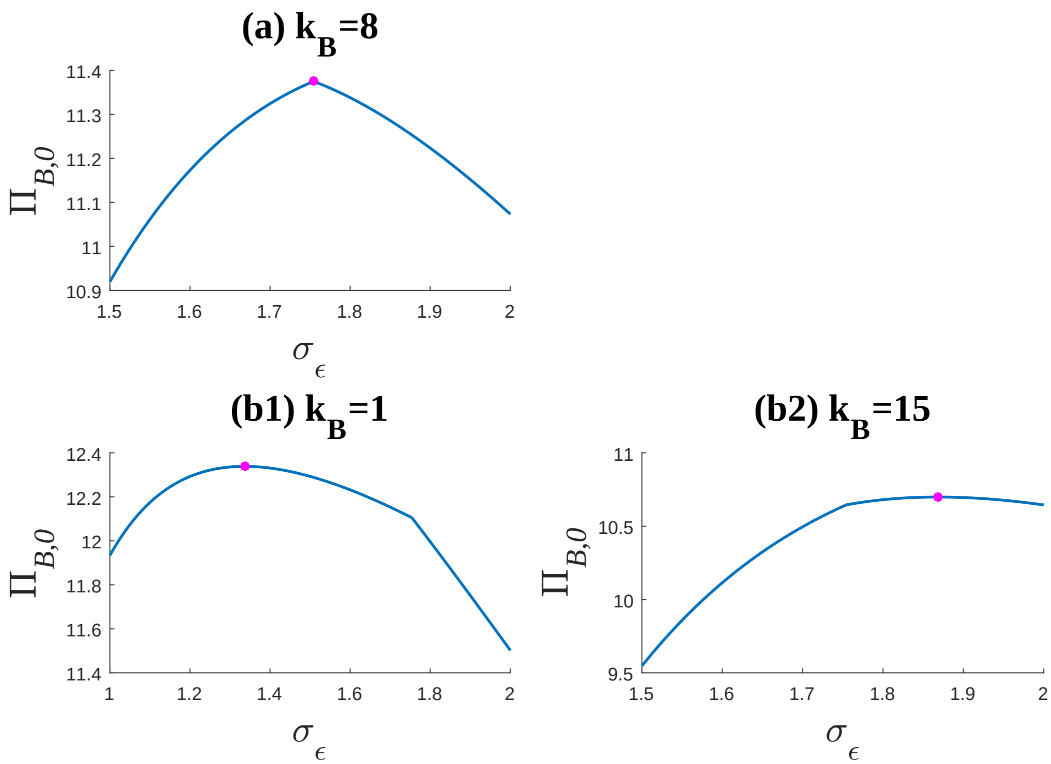
where

$$k_B > 0, k_F > 0, h_B > 1 \text{ and } h_F > 1.$$

We will conduct comparative statics with respect to parameter k_B, which is taken as a proxy for the cost of acquiring order flow information. A larger k_B means a higher cost.

In order to gain better intuition of the comparative statics, it is useful to first illustrate the kink in $\Pi_{B,0}$. Figure 1 plots the profit function $\Pi_{B,0}$ against σ_ε, for $k_B \in \{1, 8, 15\}$, in the three panels. In each panel, ϕ is set to its equilibrium value corresponding to the particular k_B and does not vary with σ_ε, since this value of ϕ is the belief of the back-runner at the information-acquisition stage. But for each σ_ε, other equilibrium variables in periods 1 and 2

Figure 1: Illustration of Possible Kink in $\Pi_{B,0}$



This figure plots $\Pi_{B,0}$ against σ_ε for three values of k_B. In each panel, ϕ is set to its equilibrium value corresponding to the particular k_B and does not vary with σ_ε. For each σ_ε, other equilibrium variables in periods 1 and 2 are optimized to this particular σ_ε. The red dot is the global maximum. Other parameters: $\sigma_u = 10, \Sigma_0 = 100, k_F = 1$, and $h_F = h_B = 2$.

are determined according to Propositions 1 and 2 of Yang and Zhu (2017) for this particular σ_ε (and the fixed equilibrium value of ϕ), because at the information-acquisition stage, the back-runner takes into account how the fundamental investor and the market maker react in future periods. As in earlier figures, we set $\sigma_u = 10$ and $\Sigma_0 = 100$. We also set $k_F = 1$ and $h_F = h_B = 2$.

In Panel (a), where $k_B = 8$, the optimal σ_ε occurs exactly at the kink. In Panels (b1) and (b2), where $k_B = 1$ and $k_B = 15$ respectively, the optimal values of σ_ε are found in the smooth regions. Intuitively, if the information-acquisition cost k_B is very high or very low, the unconstrained optimal σ_ε—the solution without considering the equilibrium switch—is sufficiently far away from the threshold $(\sqrt{\sqrt{17-4}/2})\sigma_u$, so the switch in equilibrium does not bind, as in Panels (b1) and (b2). If, however, k_B takes an intermediate value, the nature of equilibrium depends heavily on σ_ε. In the mixed strategy region of Panel (a), i.e. if $\sigma_\varepsilon < (\sqrt{\sqrt{17}-4}/2)\sigma_u$, the back-runner prefers to acquire less precise information because the fundamental investor injects noise anyway; but in the pure strategy region of Panel (a), i.e. if $\sigma_\varepsilon \geq (\sqrt{\sqrt{17}-4}/2)\sigma_u$, the back-runner prefers more precise information because the fundamental investor does not inject any noise. The result is that the unique maximum of $\Pi_{B,0}$ is obtained when σ_ε is exactly at the threshold $(\sqrt{\sqrt{17}-4}/2)\sigma_u$. As we see shortly, this corner solution leads to the stickiness in the responses of equilibrium outcomes to changes in k_B.

Now we proceed with describing the comparatives statics. The variables of interest include:

- Equilibrium values of $\phi, \sigma_\varepsilon, \sigma_z, \beta_{v,1}, \beta_{v,2}, \beta_{x_1}, \beta_{y_1}, \delta_s, \delta_{y_1}, \lambda_1$ and λ_2;
- Equilibrium profits: $\Pi_{F,0}$ and $\Pi_{B,0}$, and the expected cost of noise traders $\Pi_{F,0} + \Pi_{B,0}$;
- Expected price discovery: $\phi\Sigma_1 + (1-\phi)\,\Sigma_0$ for period 1 and $\phi\Sigma_2 + (1-\phi)\,\Sigma_0$ for period 2;
- Expected illiquidity: $\phi\lambda_1 + (1-\phi)\,0 = \phi\lambda_1$ for period 1 and $\phi\lambda_2 + (1-\phi)\,0 = \phi\lambda_2$ for period 2.

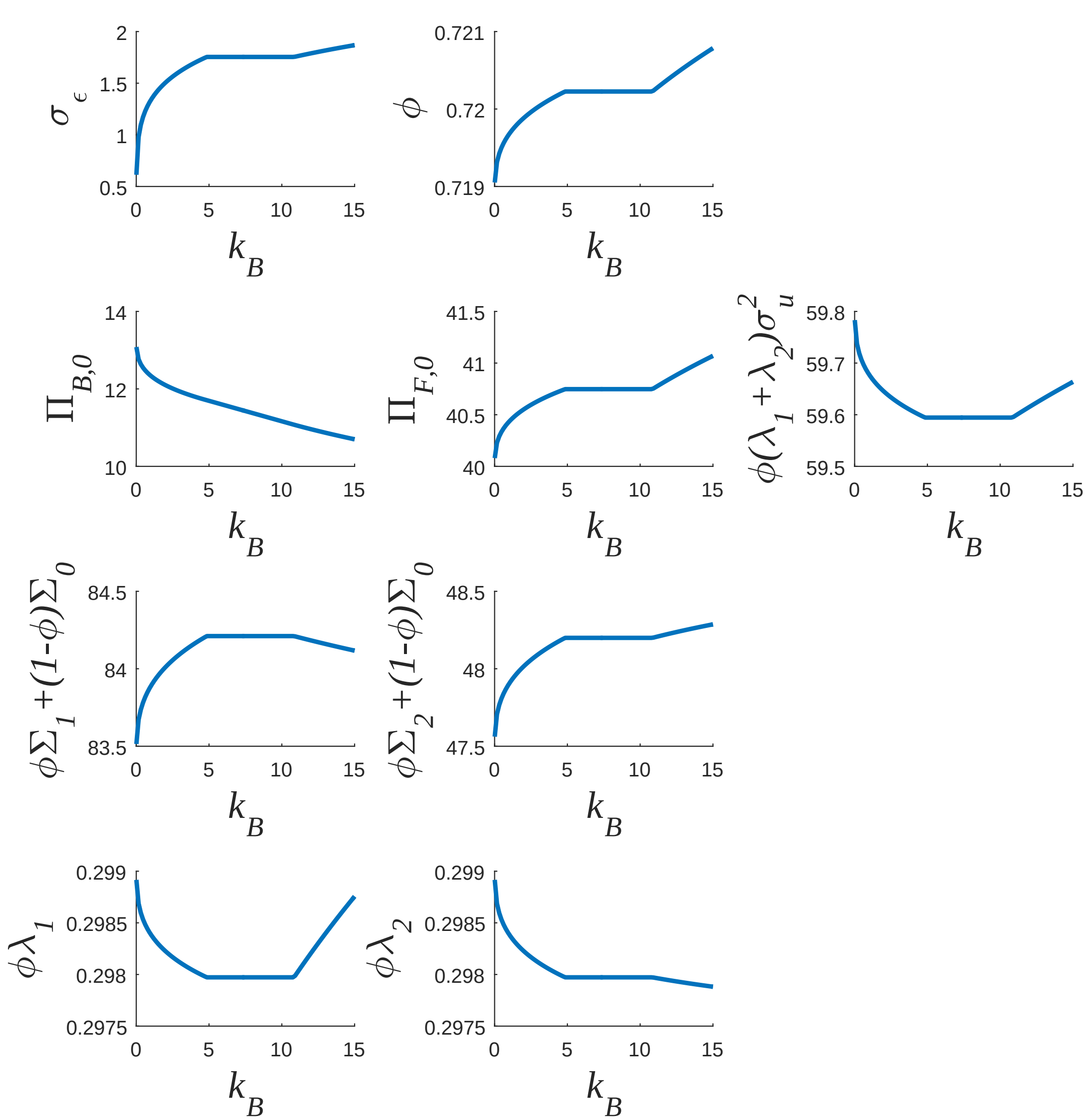
where λ_1 and λ_2 are are Kyle's lambdas, and Σ_1 and Σ_2 are conditional variances of v, all defined in Yang and Zhu (2017).

Figure 2 plots the implications of changes in information acquisition cost k_B for information-acquisition decisions, profits of various groups of traders, price discovery, and market illiquidity.

An interesting and salient pattern is that all but one of these variables are entirely irresponsive to changes in k_B when k_B is in an intermediate range. As discussed earlier, in this range, the optimal σ_ε is always equal to $(\sqrt{\sqrt{17}-4}/2)\sigma_u$ regardless of k_B. As a result, the equilibrium has zero sensitivity to k_B, leading to the flat parts of equilibrium variables.

Moreover, we observe that a lower k_B weakly reduces ϕ, which implies that technology improvement in processing order flow information reduces investment in fundamental information (top row of Figure 2). A lower cost of acquiring order flow information leads to a higher profit of the back-runner but a lower profit of the fundamental investor. The loss of noise traders, the period-1 price discovery, and the period-1 market liquidity are all non-monotone in k_B. This last result mirrors the patterns in Figures 3 and 4 of Yang and Zhu (2017) that these variables are also non-monotone in σ_ε.

Figure 2: Equilibrium Strategies and Implications of Information Acquisition



This figure plots the equilibrium levels of information acquisition, expected profits of various parties, price discovery, and market illiquidity, as functions of k_B. Other parameters: $\sigma_u = 10, \Sigma_0 = 100, k_F = 1$, and $h_F = h_B = 2$.

Overall, results in Yang and Zhu (2017) are robust to information acquisition. A unique and novel prediction with information acquisition is that equilibrium outcomes can be insensitive to the cost of order flow information. This insensitivity is the consequence of the switch between a pure strategy equilibrium and a mixed strategy one.

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