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Strategic Trading When Central Bank Intervention Is Predictable

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Market prices are noisy signals of economic fundamentals. In a two-period model, we show that if the central bank uses market prices as guidance for intervention, large strategic investors who benefit from high prices would depress market prices to induce a market-supportive intervention. Stronger anticipated interventions lead to deeper price depressions preintervention and sharper price reversals post-intervention. The central bank intervention harms strategic investors even though it is the investors who tried to mislead the central bank. The model predicts a V-shaped price pattern around central bank interventions, consistent with recent evidence. (*JEL* G14, G18)

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Central bank interventions have a material impact on financial markets. A hike or cut of the short-term risk-free interest rate affects the discount rate of all assets. To combat the financial crisis of 2008–2009 and its aftermath, major central banks not only cut short-term interest rates to zero or negative territories but also conducted unprecedented asset purchases, including government securities, mortgage-backed securities, corporate bonds, and even equities.[1] These actions were aimed to offset the worsening market conditions and stimulate the economy. When central banks were still debating whether their balance sheets with trillions of dollars of assets should be unwound, the COVID-19 pandemic plunged financial markets and the global economy into

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[1] Government securities are purchased by all major central banks during their quantitative easing operations, including the Federal Reserve (Fed), the European Central Bank (ECB), the Bank of England (BoE), and the Bank of Japan (BoJ). Mortgage-backed security (MBS) purchases were conducted by the Fed and the ECB. Corporate bond purchases were conducted by the ECB and the BoE. The BoJ has purchased equity exchange-traded funds.

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another crisis. Once again, major central banks immediately cut short-term interest rates and have restarted aggressive asset purchases to stabilize the market.

In this paper, we study how strategic investors exploit central bank interventions that aim to mitigate perceived deterioration of economic fundamentals as inferred from asset price declines. We show that if the central bank is anticipated to support the market following a price decline, then strategic investors will produce such a price decline to induce the intervention. Although the central bank anticipates such "manipulation," this strategic interaction still produces unintended consequences.

We consider a two-period economy with a single risky asset, say the stock market. The final payoff of the risky asset, which we refer to as the economic fundamental, is observed by a unit mass of risk-averse competitive investors who also receive endowment shocks. In period 1 of the economy, the competitive informed investors submit an endogenous demand schedule that balances profit-making and hedging motives to maximize their expected utility.

There are also $J \geq 1$ risk-neutral strategic investors, and investor j starts with λ_j units of the risky asset. The total holding $\Lambda \equiv \sum_j \lambda_j$ is assumed to be common knowledge, but each individual λ_j could be private information. The strategic investors are "large" in the sense that they take into account the price impact of their trades. Strategic investors do not observe economic fundamentals. Facing the unobserved demand curve of the informed investors, each strategic investor j chooses an endogenous sale amount x_j, where a negative x_j means a purchase. Investor j's sale x_j is determined in period 1 and is unobservable to anyone else, including the central bank. The equilibrium price p_1 equates the demand of the informed investors and the supply $\sum_j x_j$ of strategic uninformed investors. Because the price p_1 mixes informed investors' information and their endowment shocks, p_1 is a noisy signal of the economic fundamental.

The central bank does not observe the economic fundamental, either, but it infers the fundamental from the price p_1 of the risky asset. The central bank is assumed to have an inherent preference to support asset prices if the perceived economic fundamental is weak and to reduce asset prices if the perceived economic fundamental is strong. We refer to the central bank's action broadly as "intervention," which in practical terms include changes of the short-term interest rate and more unconventional policies, such as asset purchases or sales. Intervention takes place at the very beginning of period 2, which is, of course, after p_1 is realized.

In period 2, the final realized asset price p_2 is the sum of the economic fundamental plus the effect of central bank intervention. Investors' asset holdings are evaluated at p_2. There is no further action.

Before proceeding to the equilibrium characterization, it is useful to further motivate two features of the model. The first is the central bank's preference.

The literature has shown that central banks respond to stock prices. For example, Rigobon and Sack (2003) find that "an unexpected increase in the S&P 500 index by 5 percent increases the federal funds rate expected after the next FOMC meeting by about 14 basis points." Bjornland and Leitemo (2009) find that a real stock price increase of 1% leads to an increase in the Federal Reserve's interest rate close to 4 bps. Cieslak and Vissing-Jorgensen (2020) find that "[a] 10% stock market decline predicts a reduction in the FFR [federal funds rate] target of 32 bps at the next meeting and 127 bps after one year."

The second feature of our model is the focus on the strategic action of large, uninformed investors. Central banks predominantly intervene at the market level, such as by reducing the risk-free interest rate or by conducting large-scale asset purchases. It is well known that the returns of stocks, bonds, and other major asset classes are difficult to predict at short horizons. Indeed, the majority of investors are mutual funds, pension funds, and other institutions that hold diversified portfolios to earn long-term risk premiums, rather than predicting the short-term asset returns. These investors are large and strategic.

Compared to a hypothetical world in which the central bank does not intervene, we find that strategic investors who have (approximately) above-average initial holdings engage in "excessive sell-off" of the risky asset in period 1, in order to "persuade" the central bank to intervene and support asset prices. Intuitively, because the asset price is only a noisy signal of economic fundamentals, a larger sale amount may be read by the central bank as a weak fundamental, leading to interventions that are more supportive of asset prices. The resultant higher prices benefit those investors with large asset holdings. Because of their excessive sell-off, in equilibrium, the expected preintervention price $E(p_1)$ is below the expected post-intervention price $E(p_2)$. Therefore, strategic investors with large initial holdings suffer from selling the risky assets at too low a price. In contrast, strategic investors who have below-average initial asset holdings purchase the asset at p_1 as it is a profit opportunity. In equilibrium, large sellers dominate small buyers, so the preintervention net order flow is still a sale.

In equilibrium, the central bank correctly anticipates the net sale from strategic investors and forms the correct expectation of fundamentals. But why would large strategic investors remain sellers despite the cost and their inability to mislead the central bank? The reason is that the central bank cannot observe off-equilibrium deviations. If the large investors refrained from selling, or if they sold too little, the smaller-than-expected sale amount would lead to a higher preintervention market price, reducing the central bank's supportive intervention that these investors seek in the first place. In other words, large strategic investors sell despite the cost because not selling is even more costly. This nature of the equilibrium is reminiscent of the signal-jamming model of Stein (1989). The large strategic investors become the victim of their own strategic behavior.

The assumption that each strategic investor's sale amount x_j is unobservable by the central bank is, therefore, important for the equilibrium. Unobservable trade is a realistic assumption for several reasons. First, in practice, large investors only disclose their positions infrequently. For example, institutional investors in the United States with assets above $100 million need to file their asset holdings to the SEC only every quarter (Form 13 F, within 45 days of quarter-ends), and short positions are not reported. In addition, institutional investors may request further delays in disclosure, subject to SEC approval.[2] Agarwal, Jiang, Tang, and Yang (2013) find that these "confidential holdings" of institutional investors, in particular hedge funds, earn abnormal returns. Understandably, a complete disclosure of holdings and trading records can greatly reduce investors' alpha (if any)[3] and expose them to other predatory traders, such as "front-runners." Even passive investors would suffer from temporary price impact caused by their trade disclosure if it is in real time. Second, the incentive of any specific investor to disclose its trades is greatly reduced if all others have disclosed theirs and given the central bank most of the information to tell signal from noise in the price. This is the familiar free-riding problem. Finally, because of the risks associated with cyberattacks and inadvertent data breaches, investors may not be fully comfortable with disclosing too many details even if the disclosure is made only to the government.[4] For all these reasons, our assumption that x_j^* is private information seems reasonable. If these trades are partially private, the same qualitative results would still go through.

Our results predict that preintervention asset prices tend to drop "too much" relative to what is justified by economic fundamentals, followed by a reversal post-intervention. The more aggressive is the anticipated central bank's intervention, the larger is strategic investor's net sale preintervention, the lower is the preintervention asset price, and the stronger is the price reversal post-intervention. In addition, strategic investors trade against each other preintervention. Large ones sell to induce a favorable intervention, whereas small ones buy to benefit from a temporarily depressed price. Both empirical predictions can be tested in the data.

[2] When a request is pending approval from the SEC, the institutional investor does *not* need to disclose.

[3] Here, "alpha" refers to the excess returns of the security selection, as in the vast literature on the cross-section of stock returns. Investors who have positive alphas for picking securities still may be uninformed when it comes to predicting market returns. Indeed, many investors that are supposed to have positive alphas in security selection try to hedge out market exposure.

[4] For example, the original version of the CFTC's Reg AT (Regulation Automated Trading) in 2015 required that the source code of trading algorithms be made available to the regulator if requested. This part of the proposed rule met strong opposition on the ground of data protection concerns. The CFTC eventually loosened this requirement. For more details, see https://www.sidley.com/en/insights/newsupdates/2016/11/cftc-approves-supplemental.

Another unintended consequence of central bank intervention is its impact on asset volatility. In the equilibrium, the variances of p_1 and p_2 are both U-shaped functions of the aggressiveness of central bank intervention. Compared to a world without intervention, a moderate central bank intervention reduces asset price volatility, but a sufficiently aggressive intervention ends up increasing asset price volatility. As long as the intervention offsets shocks to perceived fundamentals no more than one-for-one, intervention reduces asset price volatility, as intended.

Our results contribute to the literature on the role of asset prices in setting monetary policy. Bernanke and Gertler (1999) argue that "[monetary] policy should not respond to changes in asset prices, except insofar as they signal changes in expected inflation." Fuhrer and Tootell (2008) find that the Federal Reserve's rate decisions responded to equity markets only through their impact on the forecasts of inflation and output gap. Cieslak and Vissing-Jorgensen (2020) find that negative stock returns predict changes in the Federal Reserve's target interest rates "mostly due to their strong correlation with downgrades to the Fed's growth expectations and the Fed's assessment of current economic growth." Yet, we show that even when asset prices are used as monetary policy input only for their information content about economic fundamentals, central banks should nonetheless consider the possibility that strategic players in the market may artificially depress asset prices to induce favorable interventions. Although the central bank in our model is not misled by the market at all, in reality the central bank can at most guess it correctly on average. Errors in inference are unavoidable if any of the common-knowledge assumptions about preferences and model parameters is violated or if a group of market participants coordinate ("collude") implicitly to influence the central bank's action.

At the theoretical level, our paper is most related to the literature on feedback effects. The key idea of this literature is that financial markets affect the real economy by discovering important information. For this reason, the vast majority of papers in this literature focus on the strategic behavior of informed investors and how they affect the actions of firms' managers and governments through price discovery. Papers focusing on informed investors include Bond, Goldstein, and Prescott (2010), Khanna and Mathews (2012), Bond and Goldstein (2015), Edmans, Goldstein, and Jiang (2015), and Boleslavsky, Kelly, and Taylor (2017), among others. In a step toward broadening the applicable scope of feedback theories, we show that uninformed investors also strategically exploit the feedback effect.

Three theoretical papers are closest to ours. Goldstein and Guembel (2008) study the strategic behavior of a large, uninformed investor in a feedback effect model, where the uninformed investor short sells the stock so that the depressed price may misguide the firm manager to take suboptimal investment actions. The critical difference of our model is that the uninformed investors actually benefit from a higher market valuation, and yet they depress asset prices to induce favorable central bank interventions and profit from a higher valuation of their remaining assets. In Benhabib, Liu, and Wang (2019), the investors and the firms are informed in different ways, and they learn from each other through a combination of prices and disclosure. In their model, however, all traders are small and exhibit no strategic behavior. In contrast, the strategic behavior of large investors in our model is the key driver of asset price dynamics around interventions. Ahnert, Machado, and Pereira (2020) study the strategic behavior of a large, informed trader who derives a private benefit from government intervention. They find that the informed trader refrains from trading or even sells shares of the firm in the good state if it derives a high enough benefit from interventions. However, as an extension of Kyle (1985), their model generates prices that are martingales without predictable returns. In contrast, the key empirical prediction of our model is a V-shaped price pattern around the central bank intervention, which is supported by evidence (e.g., Lucca and Moench 2015; Hu, Pan, Wang, and Zhu 2021).

1. Model

There are two periods, $t=1,2$. For simplicity, we assume zero time discounting. The traded risky asset has a fundamental value of

$$v = \theta + \varepsilon, \tag{1}$$

where $\theta \sim N(0, \tau_\theta^{-1})$ and $\varepsilon \sim N(0, \tau_\epsilon^{-1})$ are independent. There are three types of players in the model: a unit mass of infinitesimal informed investors who also receive endowment shocks, $J \geq 1$ large strategic uninformed investors, and a central bank. The informed investors and uninformed investors act simultaneously in period 1. The central bank acts in period 2, after observing the period-1 market-clearing price.

1.1 Informed investors

Each informed investor perfectly observes θ. No one in the economy observes ε. In addition, each informed investor has a nontradable endowment that pays $-u\varepsilon$ in period 2, where $u \sim N(0, \tau_u^{-1})$ is independent of θ and ε, and only informed investors observe u. Given the continuum of informed investors, it is without loss of generality to focus on one investor. Denote by D the demand of the risky asset by a generic informed investor. Her wealth at period 2 is

$$w_2^i = D(p_2 - p_1) - u\varepsilon, \tag{2}$$

where p_2 and p_1 are the prices at $t=2$ and $t=1$ respectively.

Each informed investor has a constant-absolute-risk-aversion (CARA) utility functions over wealth:

$$-\exp(-w_2^i/\rho), \tag{3}$$

where $1/\rho > 0$ is the risk aversion coefficient. She chooses the trading strategy in period 1, represented by a demand curve $D(p_1, \theta, u)$, to maximize the expected utility

$$E[-\exp(-w_2^i/\rho)|p_1, \theta, u]. \tag{4}$$

Here, the investor can condition on p_1 because the demand curve allows the investor to choose the optimal demand at each price. This CARA setting is a standard way to generate an endogenous demand curve. There are other ways to do this.

1.2 Strategic uninformed investors

The second type of investors are risk-neutral, strategic, and uninformed investors. There are $J \geq 1$ of them, and investor j is endowed with $\lambda_j > 0$ shares of the risky asset at the beginning of period 1. The total asset holding of all uninformed investors is a commonly known constant

$$\Lambda \equiv \sum_{j=1}^{J} \lambda_j > 0. \tag{5}$$

Each uninformed investor chooses an endogenous and unrestricted amount x_j to sell in period 1 at the market-clearing price p_1, taking into account the price impact. A negative sale is a purchase. As discussed in the introduction, the sale amount x_j is unobservable to anyone else, including the central bank. The remaining amount $\lambda_j - x_j$ would be evaluated at the period-2 price p_2. The total wealth of uninformed investor j in period 2 is

$$w_{2,j}^u \equiv x_j p_1 + (\lambda_j - x_j) p_2. \tag{6}$$

Each uninformed investor chooses x_j to maximize her expected profit, where the expectation is unconditional because we assumed, for simplicity, that each uninformed investor uses a market order that does not condition on the price p_1.[5]

1.3 Central bank

The final player in the model is the central bank. The central bank is assumed to intervene in the market in a direction opposite to the perceived shocks in economic fundamentals. Specifically, denoting by z the central bank's action after observing p_1, we assume that the central bank's maximization problem is

[5] In Section 4, we allow uninformed investors to submit demand schedules, and the qualitative results do not change.

$$\max_z E\left[z(\alpha_0 - \alpha_1 v) - \frac{c}{2}z^2|p_1\right], \tag{7}$$

where α_0 (no sign restriction), $\alpha_1 > 0$, and $c > 0$ are commonly known constants. Here, the marginal benefit of intervention $E(\alpha_0 - \alpha_1 v|p_1)$ is higher if the perceived economic fundamental $E(v|p_1)$ is lower, and the term $\frac{c}{2}z^2$ represents increasing marginal costs of central bank intervention (see also Bond, Goldstein, and Prescott 2010; Bond and Goldstein 2015). The central bank intervention z directly affects the period-2 price in a simple additive way:

$$p_2 = v + bz = \theta + \varepsilon + bz, \tag{8}$$

where $b > 0$ represents the effectiveness of the central bank intervention, and the exact size of b depend on the applications. For example, Bernanke and Kuttner (2005) find that "a hypothetical unanticipated 25-basis-point cut in the Federal funds rate target is associated with about a 1% increase in broad stock indexes." In this context, the intervention z is the reduction in the target interest rate and b is approximately $1\%/0.25\% = 4$. In a different context of asset purchases, D'Amico and King (2013) find that during the large-scale asset purchases of 2009, every \$10 billion of Treasuries bought is associated with a 7-bp increase in the price of the purchased Treasury security. Here, z is the amount of asset purchase in \$billions and b is approximately $0.07\%/10 = 0.007\%$.

In practical terms, we may interpret $v = \theta + \varepsilon$ as the "no-intervention" price, and the intervention z could represent a change in the central bank interest rate or government spending that add on top of the no-intervention price.

While this simplistic formulation of the central bank's preference is reduced form, it is not difficult to map it to reality. For example, an equity market crash during the COVID-19 pandemic contains bearish information about corporate earnings, investments, consumer demand, and employment. By directly cutting the benchmark interest rate (e.g., the Fed Fund rate) or conducting asset purchases (e.g., Treasuries, MBS, and corporate bonds), the central bank aims to reverse these negative impacts. Caballero and Simsek (2020) show theoretically that interest rate cuts and asset purchases can effectively mitigate the adverse economic consequences of large supply shocks. The term $E[z(\alpha_0 - \alpha_1 v)|p_1]$ represents how much the central bank "values" an action that offsets a perceived deterioration in fundamentals. The term $\frac{c}{2}z^2$ represents the costs of intervention already taken into account by the central bank, such as potentially higher inflation, moral hazard, and the unequal benefits of asset appreciation accrued to different demographic groups.

The mitigation of perceived shocks to economic fundamentals is not the only purpose of central bank intervention in practice. Pasquariello (2018) and Pasquariello, Roush, and Vega (2020) study central bank intervention when the central bank tries to achieve a nonpublic "price target," while minimizing trading losses against informed investors. This different objective, however, is outside the scope of this paper.

2. Equilibrium

In this section, we characterize a linear equilibrium of the following form:

$$\text{Central bank's intervention} : \quad z(p_1) = k_0 - k_1 p_1, \tag{9}$$

$$\text{Informed investors' demand} : \quad D(p_1, \theta, u) = u + \delta\theta + \gamma_0 - \gamma_1 p_1, \tag{10}$$

$$\text{Uninformed investor } j\text{'s sale} : \quad x_j^*, \text{for } j = 1, \ldots, J, \tag{11}$$

where $(k_0, k_1, \delta, \gamma_0, \gamma_1, \{x_j^*\}_{j=1}^J)$ are endogenous constants. As usual, each player takes as given other players' strategies when maximizing her own conditional expected utility.

2.1 Central bank

The central bank chooses z to maximize $E\left[z(\alpha_0 - \alpha_1 v) - \frac{c}{2} z^2 | p_1\right]$. Given this preference, the central bank's optimal intervention is

$$z^* = \frac{\alpha_0 - \alpha_1 E(\theta | p_1)}{c} = A_0 - A_1 E(\theta | p_1), \tag{12}$$

where

$$A_0 \equiv \frac{\alpha_0}{c} \text{and} A_1 \equiv \frac{\alpha_1}{c}. \tag{13}$$

By $E(\theta) = 0$, we have $E[z(p_1)] = A_0 - A_1 E[E(\theta|p_1)] = A_0$. Thus, the parameter A_0 represents the "dovish" ($A_0 > 0$), "neutral" ($A_0 = 0$), or "hawkish" ($A_0 < 0$) leaning of the central bank. The parameter A_1 represents the sensitivity of the central bank's action to the perceived economic fundamental, which we also refer to as the aggressiveness of the central bank intervention. To solve the central bank's strategy, we need to solve the inference problem $E(\theta|p_1)$. Define the aggregate selling by the strategic uninformed investors as

$$X^* \equiv \sum_{j=1}^{J} x_j^*. \tag{14}$$

In equilibrium, X^* will be correctly anticipated by the central bank. Using the informed demand function (10) and the market-clearing condition $X^* = D(p_1, \theta, u)$, the central bank infers the following signal from the price:

$$s_p \equiv \frac{1}{\delta}(\gamma_1 p_1 - \gamma_0 + X^*) = \theta + \frac{1}{\delta} u. \tag{15}$$

By the usual projection theorem for the normal distribution, we obtain the following optimal intervention strategy of the central bank:

$$z(p_1) = A_0 + \frac{A_1\delta\tau_u(\gamma_0 - X^*)}{\tau_\theta + \delta^2\tau_u} - \frac{A_1\delta\tau_u\gamma_1}{\tau_\theta + \delta^2\tau_u}p_1. \tag{16}$$

Comparing (16) with the conjectured intervention strategy (9), we have

$$k_0 = A_0 + \frac{A_1\delta\tau_u(\gamma_0 - X^*)}{\tau_\theta + \delta^2\tau_u}, \tag{17}$$

$$k_1 = \frac{A_1\delta\tau_u\gamma_1}{\tau_\theta + \delta^2\tau_u}. \tag{18}$$

2.2 Informed investors

A representative informed investor chooses D to maximize

$$E[-\exp(-w_2^i/\rho)|\theta, u, p_1], \tag{19}$$

where

$$w_2^i = D(p_2 - p_1) - u\varepsilon. \tag{20}$$

In solving the above problem, we obtain the following demand function:

$$D(p_1, \theta, u) = u + \rho\tau_\varepsilon(\theta + bz - p_1). \tag{21}$$

Inserting the conjectured intervention rule (9) into (21), we can compute

$$D(p_1, \theta, u) = u + \delta\theta + \delta bk_0 - \delta(bk_1 + 1)p_1. \tag{22}$$

Comparing (22) with the conjectured trading strategy (10), we have

$$\delta = \rho\tau_\varepsilon, \tag{23}$$

$$\gamma_0 = \delta bk_0, \tag{24}$$

$$\gamma_1 = \delta(bk_1 + 1). \tag{25}$$

2.3 Strategic uninformed investors

Without loss of generality, let us consider uninformed investor j. She takes as given central bank's intervention strategy (9), informed investors' demand function (10), and other uninformed investors' selling $x_{j'}^*$, to maximize her expected profits, $E(w_{2,j}^u)$, where $w_{2,j}^u = x_jp_1 + (\lambda_j - x_j)p_2$.

Using the informed demand function (10) and the market-clearing condition $X^* = D(p_1, \theta, u)$, investor j understands that

$$p_1 = \frac{1}{\gamma_1}(\gamma_0 + \delta\theta + u - x_j - \textstyle\sum_{j'=j} x_{j'}^*). \tag{26}$$

This equation implies that investor j may have an incentive to increase x_j to reduce p_1, which in turn increases the central bank's intervention $z(p_1)$. In addition, investor j fully takes into account the price impact of her sales, $\frac{\partial p_1}{\partial x_j} = -\frac{1}{\gamma_1}$. By $p_2 = \theta + bz + \varepsilon$, the central bank's intervention rule (9), and investor j's perceived pricing function (26), we can compute

$$E(w^u_{2,j}) = -\frac{1+bk_1}{\gamma_1}x_j^2 + \left[\frac{\lambda_j bk_1}{\gamma_1} + \frac{1+bk_1}{\gamma_1}(\gamma_0 - \sum\nolimits_{j'=j}x^*_{j'}) - bk_0\right]x_j \\ +\lambda_j\left(bk_0 - bk_1\frac{\gamma_0 - \sum_{j'=j}x^*_{j'}}{\gamma_1}\right). \tag{27}$$

By (23) and (25), the second-order coefficient in (27) is $-(1+bk_1)/\gamma_1 = -1/(\rho\tau_\varepsilon) < 0$. Thus, the optimal sale x_j^* of investor j is given by the usual first-order condition, which gives

$$x_j^* = \frac{\lambda_j bk_1 + (1+bk_1)(\gamma_0 - \sum_{j'=j}x^*_{j'}) - \gamma_1 bk_0}{2(1+bk_1)}. \tag{28}$$

From the system of Equations (17), (18), (23), (24), (25), and (28), we can solve k_0, k_1, δ, γ_0, γ_1, and $\{x_j^*\}_{j=1}^J$. The following proposition presents the solution.

Proposition 1. Suppose that $\tau_\theta + (1-bA_1)\delta^2\tau_u \neq 0$. There exists a unique linear equilibrium with the strategies given by Equations (9)–(11), where

$$k_0 = \frac{A_0(J+1)(\tau_\theta+\delta^2\tau_u)^2 - b\Lambda A_1^2\delta^3\tau_u^2}{(J+1)(\tau_\theta+\delta^2\tau_u)\Big(\tau_\theta+(1-bA_1)\delta^2\tau_u\Big)}, \tag{29}$$

$$k_1 = \frac{A_1\delta^2\tau_u}{\tau_\theta+(1-bA_1)\delta^2\tau_u}, \tag{30}$$

$$\delta = \rho\tau_\varepsilon, \tag{31}$$

$$\gamma_0 = \delta b\frac{A_0(J+1)(\tau_\theta+\delta^2\tau_u)^2 - b\Lambda A_1^2\delta^3\tau_u^2}{(J+1)(\tau_\theta+\delta^2\tau_u)\Big(\tau_\theta+(1-bA_1)\delta^2\tau_u\Big)}, \tag{32}$$

$$\gamma_1 = \frac{\delta(\tau_\theta+\delta^2\tau_u)}{\tau_\theta+(1-bA_1)\delta^2\tau_u}, \tag{33}$$

$$x_j^* = \frac{bA_1\delta^2\tau_u}{\tau_\theta+\delta^2\tau_u}\left(\lambda_j - \frac{\Lambda}{J+1}\right), \forall j. \tag{34}$$

The equilibrium total amount of sales by strategic uninformed investors is

$$X^* \equiv \sum\nolimits_{j=1}^J x_j^* = \frac{bA_1\delta^2\tau_u}{\tau_\theta+\delta^2\tau_u}\frac{\Lambda}{J+1}. \tag{35}$$

We will defer the discussions of the equilibrium outcomes to the next section, but let us briefly discuss two features that already stand out from the characterization of equilibrium itself. The first is the strategic trading behavior of uninformed investors. The expression of x_j^* reveals that uninformed investors with relatively large (small) initial asset positions submit sell (buy) orders in period 1. Intuitively, uninformed investors with large initial asset holdings sell to depress p_1 and draw out a favorable central bank intervention because supportive interventions increase the value of their remaining assets in period 2. But, as we see in the next section, their selling pressure will reduce the period-1 price, so that investors with small initial asset holdings find it attractive to buy. The aggregate sale amount by strategic uninformed investors, X^*, is always positive and decreases at the rate of $1/(J+1)$.

Although the central bank can correctly anticipate the aggregate sales X^* on equilibrium paths, it cannot observe the off-equilibrium deviations. Thus, given the central bank's belief about X^*, refraining from selling would give the central bank an incorrect bullish signal about fundamentals and reduce its supportive intervention, which would be self-defeating.

The second feature of the equilibrium is that the demand curves of the informed investors become upward sloping if $\gamma_1 < 0$ or if $\tau_\theta + (1 - bA_1)\delta^2\tau_u < 0$. (Note that an upward-sloping demand curve does not violate the second-order condition.) This situation happens if $bA_1 > 1 + \tau_\theta/(\delta^2\tau_u)$, that is, if the central bank intervention is sufficiently aggressive. Moreover, whenever $\gamma_1 < 0$, $k_1 < 0$ as well, so the central bank becomes a "momentum trader": its action goes in the same direction of the period-1 price. To gain some intuition about what "aggressive" intervention means here, we write

$$E(p_2|p_1) = E(\theta|p_1) + b(A_0 - A_1E(\theta|p_1)) = bA_0 + (1 - bA_1)E(\theta|p_1). \tag{36}$$

If $bA_1 < 1$, the central bank's intervention merely mitigates the perceived shock to fundamentals given p_1. If $bA_1 = 1$, the central bank's intervention exactly offsets it. But if $bA_1 > 1$, the central bank's intervention will more than reverse the perceived shock in fundamentals; that is, a lower perceived fundamental leads to a *higher* asset price. In this last situation, $\gamma_1 < 0$ and $k_1 < 0$ become possible. Intuitively, because the central bank intervention will more than offset the perceived shocks in fundamentals, a lower $E(\theta|p_1)$ is in fact "good news" for asset prices. Informed investors, who observe θ, send a demand curve that is increasing in price p_1; equivalently, the price p_1 in (26) is decreasing in fundamentals. Hence, the central bank's intervention $z(p_1)$ becomes increasing in p_1 as well (i.e., k_1 is negative).

2.4 Comparison with benchmarks

Before analyzing the implications of the equilibrium, we compare this equilibrium with two benchmarks. The first benchmark is that there is no central bank intervention, that is, $b=0$. The second is that there are infinitely many uninformed investors, that is, $J \to \infty$. The corollary below summarizes both benchmarks.

Corollary 1. (a) Suppose $b=0$ in the equilibrium of Proposition 1. Then:

$$\gamma_0 = 0, \gamma_1 = \delta = \rho\tau_\varepsilon, \text{and } x_j^* = 0 \text{ for all } j. \tag{37}$$

(b) Fix Λ and let $J \to \infty$ in the equilibrium of Proposition 1. Then

$$k_0 \to \frac{A_0(\tau_\theta + \delta^2\tau_u)}{\tau_\theta + \delta^2\tau_u - b\delta^2\tau_u A_1}, \tag{38}$$

$$k_1 = \frac{A_1\delta^2\tau_u}{\tau_\theta + (1 - bA_1)\delta^2\tau_u}, \tag{39}$$

$$\gamma_0 \to \frac{b\delta A_0(\tau_\theta + \delta^2\tau_u)}{\tau_\theta + (1 - bA_1)\delta^2\tau_u}, \tag{40}$$

$$\gamma_1 = \frac{\delta(\tau_\theta + \delta^2\tau_u)}{\tau_\theta + (1 - bA_1)\delta^2\tau_u}, \tag{41}$$

$$x_j^* \to \frac{bA_1\delta^2\tau_u}{\tau_\theta + \delta^2\tau_u} \lim_{J\to\infty} \lambda_j, \tag{42}$$

$$X^* \equiv \sum\nolimits_{j=1}^{J} x_j^* \to 0. \tag{43}$$

In the benchmark without central bank intervention, strategic uninformed investors do not trade in period 1 at all. They can affect the price but doing so has no benefit. Obviously, the aggregate order flow from uninformed investors is also zero.

In the benchmark with infinitely many uninformed investors, the uninformed investors' aggregate order flow X^* converges to zero as well. But orders from a given individual uninformed investors need not vanish as $J \to \infty$. For example, suppose that $\Lambda = 1$, $\lambda_1 = 0.1$ and, for $j > 1$, $\lambda_j = 0.9/(J-1)$. As $J \to \infty$, the first investor always retains 10% of total asset supply, whereas the asset held by each of the $J-1$ investors diminishes to zero. In equilibrium, as J becomes sufficiently large, investor 1 sells (because $\lambda_1 > \Lambda/(J+1)$) and other investors buy (because $\lambda_j < \Lambda/(J+1)$ for $j \neq 1$), leading to a zero net order flow from uninformed investors in the limit. These limits are easily verified in Proposition 1.

We can make the second benchmark yet more special by requiring that each $\lambda_j \to 0$ as $J \to \infty$. A common way of doing so is to impose symmetry:

$\lambda_j = \Lambda/J$ for all j. In this case, all uninformed investors become "small" and each $x_j^* \to 0$ as well.

These two benchmarks reveal that the presence of a central bank intervention (i.e., $b > 0$) is a necessary condition for the strategic sales of assets by *any* uninformed investor. In addition, as the number of investors increases, those that remain large in the limit (with $\lim_{J\to\infty} \lambda_j > 0$) still make strategic sales, although the aggregate sale amount converges to zero. Theoretically, the distinction of the two benchmarks can be seen in the trading behavior: all else equal, the benchmark equilibrium with many uninformed investors (and the central bank) would still see active transactions among the investors, whereas the benchmark without a central bank intervention would not.

3. Properties of the Equilibrium

In this section, we discuss market outcomes and investor profits in the equilibrium of Proposition 1.

3.1 Market outcomes

We start with the market outcomes, including the mean and variance of the prices in the two periods (p_1 and p_2) and the return between the two periods ($p_2 - p_1$).

Proposition 2. In the equilibrium of Proposition 1, the unconditional means and variances of prices and returns are

$$E(p_1) = b\left[A_0 - \frac{\Lambda\delta\tau_u A_1}{(J+1)(\tau_\theta + \delta^2\tau_u)}\right], \tag{44}$$

$$Var(p_1) = \frac{[\tau_\theta + (1 - bA_1)\delta^2\tau_u]^2}{(\tau_\theta + \delta^2\tau_u)\delta^2\tau_u\tau_\theta}, \tag{45}$$

$$E(p_2) = A_0 b, \tag{46}$$

$$Var(p_2) = \frac{1}{\tau_\varepsilon} + \frac{\tau_\theta + \delta^2\tau_u(1 - bA_1)^2}{\tau_\theta(\tau_\theta + \delta^2\tau_u)}, \tag{47}$$

$$E(p_2 - p_1) = \frac{b\Lambda A_1\delta\tau_u}{(J+1)(\tau_\theta + \delta^2\tau_u)}, \tag{48}$$

$$Var(p_2 - p_1) = \frac{1}{\tau_\varepsilon} + \frac{1}{\delta^2\tau_u}. \tag{49}$$

The comparative statics of the market outcomes are summarized in the following proposition.

Proposition 3. Suppose that $b > 0$ and $A_1 > 0$ (but A_0 has unrestricted signs). The comparative statics of the equilibrium in Proposition 1 are summarized in the following table:

Outcome of interest	$\frac{\partial}{\partial b}$	$\frac{\partial}{\partial A_0}$	$\frac{\partial}{\partial A_1}$	$\frac{\partial}{\partial \Lambda}$	$\frac{\partial}{\partial J}$
X^*	> 0	0	> 0	> 0	< 0
$x_j^*, \forall j$	> 0 iff $\lambda_j > \frac{\Lambda}{J+1}$	0	> 0 iff $\lambda_j > \frac{\Lambda}{J+1}$	> 0 iff $\frac{\partial \lambda_j}{\partial \Lambda} > \frac{1}{J+1}$	> 0 iff $\frac{\partial \lambda_j}{\partial J} + \frac{\Lambda^2}{(J+1)^2} > 0$
$E(p_1)$	> 0 iff $A_0(J+1)$ $(\tau_\theta + \delta^2 \tau_u)$ $> \Lambda \delta \tau_u A_1$	> 0	< 0	< 0	> 0
$Var(p_1)$	< 0 iff $\tau_\theta +$ $(1 - bA_1)\delta^2 \tau_u > 0$	0	< 0 iff $\tau_\theta +$ $(1 - bA_1)\delta^2 \tau_u > 0$	0	0
$E(p_2)$	A_0	> 0	0	0	0
$Var(p_2)$	> 0 iff $bA_1 > 1$	0	> 0 iff $bA_1 > 1$	0	0
$E(p_2 - p_1)$	> 0	0	> 0	> 0	< 0
$Var(p_2 - p_1)$	0	0	0	0	0

Let us now discuss the intuition of Proposition 3. Our primary variable of interest is A_1 (column 4 of Proposition 3), the aggressiveness of the central bank intervention. A more aggressive central bank intervention, A_1, leads to a larger total sale amount, X^*, by strategic investors, in an attempt to "persuade" the central bank to support the market. Again, although the equilibrium sale amount is anticipated, the central bank cannot observe the off-equilibrium deviation of strategic investors. Depending on the distribution of initial asset holdings $\{\lambda_j\}$ across investors, each individual x_j^* may be positive or negative (see Proposition 1), but a larger A_1 always make the dispersion of $\{x_j^*\}$ more pronounced. Overall, the net sale by strategic investors depresses the expected period-1 price $E(p_1)$, compared to a world without central bank intervention. But the strategic net sale of large investors and the anticipated central bank intervention offset each other perfectly, leading to a constant expected period-2 price $E(p_2)$. Consequently, a higher A_1 increases the price reversal $E(p_2 - p_1)$. The model thus predicts that, around predictable central bank interventions, asset prices exhibit a V-shaped pattern, and this pattern is more pronounced the more aggressive is the intervention.

Implications for asset volatility are more nuanced. Presumably, central bank interventions aim to reduce asset price volatility. But the expression of $\partial Var(p_2)/\partial A_1$ reveals that sufficiently aggressive intervention increases asset price volatility. Specifically, intervention minimizes $Var(p_2)$ if $bA_1 = 1$; that is, intervention aims to offset the change in economic fundamental one-for-one. A further increase in the aggressiveness of intervention, however, may increase volatility. Indeed, if $bA_1 > 2$, $Var(p_2)$ is higher under intervention than under no intervention.

A similar logic applies to preintervention variance $Var(p_1)$. If $\tau_\theta + (1 - bA_1)\delta^2\tau_u > 0$, $Var(p_1)$ is decreasing in the aggressiveness A_1 of intervention. Otherwise, $Var(p_1)$ is increasing in A_1. Recall that $\tau_\theta + (1 - bA_1)\delta^2\tau_u < 0$ also implies that the investors' demand curve is upward-sloping in price p_1 and the central bank intervenes in the same direction of market prices shocks.

The comparative statics with respect to b is very similar to those with respect to A_1. This is because the total impact of intervention is $bz(p_1) = b(A_0 - A_1 E(\theta|p_1))$, so bA_1 often shows up as a pair. Yet, b and A_1 do not have fully identical effects because of A_0. If $A_0 = 0$, columns 2 and 4 would be identical. Indeed, the only difference between columns 2 and 4 of Proposition 3 is $E(p_1)$ and $E(p_2)$. Even there, under a market-neutral central bank ($A_0 = 0$), we have $\partial E(p_1)/\partial b < 0$ and $\partial E(p_2)/\partial b = 0$, just like $\partial E(p_1)/\partial A_1 < 0$ and $\partial E(p_2)/\partial A_1 = 0$. The ex ante bias of the central bank A_0 directly affects the prices $E(p_1)$ and $E(p_2)$, but it does not affect asset volatility. Finally, a larger supply Λ increases the net sale X^*, reduces $E(p_1)$, and increases the price reversal $E(p_2 - p_1)$, whereas the number of large investors, J, generally has the opposite effects.

3.2 Investor profits

We now analyze the impact of the central bank intervention on uninformed investors' profits and informed investors' (risk-adjusted) profits. We start with the strategic uninformed investors. Investor j's profit is

$$\begin{aligned} U^*_{u,j} &\equiv E[(x^*_j p_1 + (1 - x^*_j)p_2] \\ &= \lambda_j bA_0 - \left(\lambda_j - \frac{\Lambda}{J+1}\right)\frac{A_1^2 b^2 \Lambda \delta^3 \tau_u^2}{(J+1)(\tau_\theta + \delta^2\tau_u)^2}. \end{aligned} \tag{50}$$

The aggregate profit of all strategic uninformed investors is:

$$U^*_u \equiv \sum\nolimits_{j=1}^{J} U^*_{u,j} = bA_0\Lambda - \frac{A_1^2 b^2 \Lambda^2 \delta^3 \tau_u^2}{(J+1)^2(\tau_\theta + \delta^2\tau_u)^2}. \tag{51}$$

Next, let's consider the informed investors. The ex ante unconditional utility of the unit mass of investors is $E[-e^{-\frac{1}{\rho}(D(p_1,\theta,u)(p_2-p_1)-u\varepsilon)}]$. We can use the certainty equivalent to represent their utility, or risk-adjusted profit:

$$U^*_i \equiv -\rho \ln\left(-E\left[-e^{-\frac{1}{\rho}\left(D(p_1,\theta,u)(p_2-p_1)-u\varepsilon\right)}\right]\right). \tag{52}$$

Direct computation gives

$$U_i^* = \frac{\rho}{2}\ln\left(\frac{\delta\rho\tau_u - 1}{\delta\rho\tau_u}\right) + \frac{A_1^2 b^2 \Lambda^2 \delta^3 \tau_u^2}{2(J+1)^2(\tau_\theta + \delta^2\tau_u)^2}. \tag{53}$$

For the above to be well-defined, we require

$$\delta\rho\tau_u \equiv \rho^2\tau_\varepsilon\tau_u > 1. \tag{54}$$

Otherwise, U_i^* diverges. This kind of condition is standard in the literature (e.g., Vayanos and Wang 2012).

The next proposition summarizes the comparative statics of investor profits.

Proposition 4. In the equilibrium of Proposition 1, the profit $U_{u,j}^*$ (i.e., expected profit) of large uninformed investor j and their aggregate profit U_u^* are respectively given by (50) and (51); and assuming $\rho^2\tau_\varepsilon\tau_u > 1$, the certainty equivalent U_i^* of informed investors' utility is given by (53). Suppose that $\Lambda > 0$, $b > 0$ and $A_1 > 0$ (but A_0 has unrestricted signs). Then,

$$\frac{\partial U_{u,j}^*}{\partial b} > 0 \Longleftrightarrow \lambda_j A_0 > 2\left(\lambda_j - \frac{\Lambda}{J+1}\right)\frac{A_1^2 b\Lambda\delta^3\tau_u^2}{(J+1)(\tau_\theta + \delta^2\tau_u)^2}; \tag{55}$$

$$\frac{\partial U_{u,j}^*}{\partial A_0} = \lambda_j b; \tag{56}$$

$$\frac{\partial U_{u,j}^*}{\partial A_1} = -\frac{2A_1 b^2\Lambda\delta^3\tau_u^2\left(\lambda_j - \frac{\Lambda}{J+1}\right)}{(J+1)(\tau_\theta + \delta^2\tau_u)^2} < 0 \Longleftrightarrow \lambda_j > \frac{\Lambda}{J+1}; \tag{57}$$

$$\frac{\partial U_u^*}{\partial b} > 0 \Longleftrightarrow A_0 > \frac{2A_1^2 b\Lambda\delta^3\tau_u^2}{(J+1)^2(\tau_\theta + \delta^2\tau_u)^2}; \tag{58}$$

$$\frac{\partial U_u^*}{\partial A_0} = b\Lambda > 0; \tag{59}$$

$$\frac{\partial U_u^*}{\partial A_1} = -\frac{2A_1 b^2\Lambda^2\delta^3\tau_u^2}{(J+1)^2(\tau_\theta + \delta^2\tau_u)^2} < 0; \tag{60}$$

$$\frac{\partial U_i^*}{\partial b} = \frac{A_1^2 b\Lambda^2\delta^3\tau_u^2}{(J+1)^2(\tau_\theta + \delta^2\tau_u)^2} > 0; \tag{61}$$

$$\frac{\partial U_i^*}{\partial A_0} = 0; \tag{62}$$

$$\frac{\partial U_i^*}{\partial A_1} = \frac{A_1 b^2\Lambda^2\delta^3\tau_u^2}{(J+1)^2(\tau_\theta + \delta^2\tau_u)^2} > 0. \tag{63}$$

We discuss the intuition of Proposition 4 under the natural special case of a market-neutral central bank $A_0 = 0$. This way, zero unconditional "windfall" gains or losses are caused by the central bank's intervention. As before, we can interpret the absence of the central bank's intervention as $b = 0$. From the comparative statics of U_u^* with respect to b, strategic uninformed investors are collectively harmed by the presence of intervention. This is because a potential central bank intervention creates the incentive for the uninformed investors to collectively sell in period 1 at too low a price: $E(p_1) < E(p_2)$. But the impacts on individual uninformed investors are heterogeneous (see $\partial U_{u,j}^* / \partial b$). Those with sufficiently large initial holdings are sellers in period 1 and hence are worse off when the central bank intervenes. Others are net buyers in period 1 and hence are better off. Likewise, a more aggressive central bank intervention (a larger A_1) harms uninformed investors with large initial asset holdings and benefit those with small initial asset holdings (see $\partial U_{u,j}^* / \partial A_1$). The net effect remains negative ($\partial U_u^* / \partial A_1 < 0$).

The above result on uninformed investors suggests that, if our model were a complete characterization of the world, the large, uninformed investors ought to refrain from selling and disclose to the central bank as such. The central bank, then, would not misinterpret a smaller-than-expected sale amount as a positive fundamental shock. In other words, full, credible, and continuous disclosures of asset positions and trades of large investors should solve the signal-jamming problem faced by the central bank. Yet, as discussed in the introduction, institutional investors in reality have more severe concerns about the complete transparency of their asset holdings than convincing the central bank that they did not attempt to manipulate its belief. For instance, the leakage of valuable proprietary investment strategies inferred from transaction history can greatly reduce the alpha of the institutional investor (see Yang and Zhu 2020). Even if an investor is uninformed, a full disclosure of trading patterns may also lead to "front-running" concerns of liquidity-driven orders. Moreover, even confidential disclosures to regulators are subject to inadvertent data breaches. See also Stein (1989) for additional discussions of why full transparency of actions is not fully realistic in all situations.

In contrast, the presence of a central bank intervention (b) benefits the informed investors, and so does an increase in the aggressiveness of the intervention (A_1). The intuition is the standard one. The presence of a central bank intervention and its aggressiveness both increase the net sale by uninformed investors at too low a price, so informed investors can benefit from trading against them.

4. Price-Contingent Orders

So far, we have assumed that the strategic uninformed investors submit market orders. In this section, we consider a variant of the model in which uninformed investors submit demand schedules. For simplicity, we consider a single uninformed investor (i.e., $J=1$). Our main results of the previous sections are robust to this variation in the model.

We still consider linear equilibria. The central bank's intervention rule and the informed investors' demand function are still given by (9) and (10), respectively. But the strategy of the uninformed investor changes from (11) to the following:

$$x(p_1) = h_0 + h_1 p_1, \tag{64}$$

where h_0 and h_1 are endogenous constants. The derivation of the equilibrium is similar. The main difference is that the uninformed investor now needs to extract information from the market-clearing price p_1 because her transaction can now condition on the price. The derivations have been relegated to the appendix. We report the equilibrium strategies and outcomes in the following propositions.

Proposition 5. Suppose that $\tau_\theta + 2(1 - bA_1)\delta^2\tau_u \neq 0$. There exists a unique linear equilibrium with the strategies given by Equations (9), (10), and (64), where

$$k_0 = \frac{A_0(\tau_\theta + 2\delta^2\tau_u)^2 - 2\Lambda b A_1^2 \delta^3 \tau_u^2}{(\tau_\theta + 2\delta^2\tau_u)\left(\tau_\theta + 2(1 - bA_1)\delta^2\tau_u\right)}, \tag{65}$$

$$k_1 = \frac{2A_1\delta^2\tau_u}{\tau_\theta + 2(1 - bA_1)\delta^2\tau_u}, \tag{66}$$

$$\delta = \rho\tau_\varepsilon, \tag{67}$$

$$\gamma_0 = \delta b \frac{A_0(\tau_\theta + 2\delta^2\tau_u)^2 - 2\Lambda b A_1^2 \delta^3 \tau_u^2}{(\tau_\theta + 2\delta^2\tau_u)\left(\tau_\theta + 2(1 - bA_1)\delta^2\tau_u\right)}, \tag{68}$$

$$\gamma_1 = \frac{\delta(\tau_\theta + 2\delta^2\tau_u)}{\tau_\theta + 2(1 - bA_1)\delta^2\tau_u}, \tag{69}$$

$$h_0 = -\frac{b\delta}{\tau_\theta + 2\delta^2\tau_u} \frac{\tau_\theta A_0(\tau_\theta + 2\delta^2\tau_u) - 2\Lambda A_1 \delta\tau_u(\tau_\theta + \delta^2\tau_u - bA_1\delta^2\tau_u)}{\tau_\theta + 2(1 - bA_1)\delta^2\tau_u}, \tag{70}$$

$$h_1 = \frac{\tau_\theta\delta}{\tau_\theta + 2(1 - bA_1)\delta^2\tau_u}. \tag{71}$$

Proposition 6. In the equilibrium of Proposition 5, we have

$$E[x(p_1)] = \frac{bA_1\Lambda\delta^2\tau_u}{\tau_\theta + 2\delta^2\tau_u}, \tag{72}$$

$$Var[x(p_1)] = \frac{\tau_\theta}{4\tau_u(\tau_\theta + \delta^2\tau_u)}, \tag{73}$$

$$E(p_1) = b\left(A_0 - \frac{\Lambda A_1 \delta\tau_u}{\tau_\theta + 2\delta^2\tau_u}\right), \tag{74}$$

$$Var(p_1) = \frac{(\tau_\theta + 2\delta^2\tau_u - 2bA_1\delta^2\tau_u)^2}{4(\tau_\theta + \delta^2\tau_u)\tau_u\tau_\theta\delta^2}, \tag{75}$$

$$E(p_2) = A_0 b, \tag{76}$$

$$Var(p_2) = \frac{\tau_\theta + \delta^2\tau_u(1 - bA_1)^2}{\tau_\theta(\tau_\theta + \delta^2\tau_u)} + \frac{1}{\tau_\varepsilon}, \tag{77}$$

$$E(p_2 - p_1) = \frac{\Lambda b A_1 \delta\tau_u}{\tau_\theta + 2\delta^2\tau_u}, \tag{78}$$

$$Var(p_2 - p_1) = \frac{\tau_\theta + 4\delta^2\tau_u}{4\delta^2\tau_u(\tau_\theta + \delta^2\tau_u)} + \frac{1}{\tau_\varepsilon}. \tag{79}$$

In addition, suppose that $b > 0$ and $A_1 > 0$ (but A_0 has unrestricted signs). The comparative statics of the model are summarized in the following table:

The qualitative results in Propositions 5 and 6 are similar to those in Propositions 1, 2, and 3. In particular, even if the uninformed investor can use a demand schedule, she is still a net seller in expectation (since $E[x(p_1)] > 0$). On some equilibrium paths, the uninformed investor may

Outcome of interest	$\frac{\partial}{\partial b}$	$\frac{\partial}{\partial A_0}$	$\frac{\partial}{\partial A_1}$	$\frac{\partial}{\partial \Lambda}$
$E[x(p_1)]$	> 0	0	> 0	> 0
$E(p_1)$	> 0 iff $A_0(\tau_\theta + 2\delta^2\tau_u) > \Lambda A_1\delta\tau_u$	> 0	< 0	< 0
$Var(p_1)$	< 0 iff $\tau_\theta + 2\delta^2\tau_u > 2bA_1\delta^2\tau_u$	0	< 0 iff $\tau_\theta + 2\delta^2\tau_u > 2bA_1\delta^2\tau_u$	0
$E(p_2)$	A_0	> 0	0	0
$Var(p_2)$	> 0 iff $bA_1 > 1$	0	> 0 iff $bA_1 > 1$	0
$E(p_2 - p_1)$	> 0	0	> 0	> 0
$Var(p_2 - p_1)$	0	0	0	0

end up being a buyer if p_1 is very low because of the informed investors' hedging demand. The use of demand schedule also allows the uninformed investor to better optimize against her price impact, so $E[x(p_1)]$ is not identical to X^* in Proposition 1 for $J = 1$. Despite these differences, the uninformed investor still expects to sell ahead of the central bank intervention.

All other results in in Propositions 5 and 6 are also similar to their counterparts in the main model. A V-shaped price pattern remains in the two periods

($E(p_2 - p_1) > 0$), and the price reversal is stronger if the central bank intervention is more aggressive. If the central bank does not intervene ($b = 0$), the unconditional asset return $E(p_2 - p_1)$ would be zero. If the central bank intervention becomes sufficiently aggressive, the demand schedule of informed investors again becomes upward-sloping in price ($\gamma_1 < 0$) and the central bank intervenes in the same direction as recent price movement ($k_1 < 0$). Aggressive intervention may increase price volatility as well.

The two models, however, are not entirely identical. In particular, the use of price-contingent orders leads to a deeper price depression in period 1 and a larger price reversal in period 2. Plugging in $J = 1$ into the baseline model yields $x^* = X^* = \frac{bA_1\Lambda\delta^2\tau_u}{2(\tau_\theta+\delta^2\tau_u)}$, whereas in the price-contingent model with a single uninformed investor, $E[x(p_1)] = \frac{bA_1\Lambda\delta^2\tau_u}{\tau_\theta+2\delta^2\tau_u}$. The expected sale amount is, therefore, greater if price-contingent orders are used. The intuition is that price-contingent orders give uninformed investors flexibility to adjust their demand according to the price; as a result, they can afford to send more aggressive orders on average. A similar comparison reveals that the ability to use price-contingent orders reduces $E(p_1)$ and increases the unconditional price reversal $E(p_2 - p_1)$. Although the uninformed investors in the price-contingent model generally trade to provide liquidity to the informed investors, the expected sale $E[x(p_1)]$ is positive only if the central bank intervenes. Combining the implications of the two models, we see that the essential and robust implication of central bank intervention is the price depression and reversal, rather than the trading activity (positive or negative) of uninformed investors before the intervention.

5. Concluding Remarks: Empirical and Policy Implications

The model generates at least two empirical predictions. First, there is a V-shaped price pattern around the central bank's intervention. Second, before the intervention, investors with higher-than-average holdings sell risky assets, and investors with lower-than-average holdings buy risky assets.

The first prediction about the V-shaped price pattern is consistent with evidence on equity markets. Lucca and Moench (2015) find that U.S. equity prices tend to increase during the 24-hour window before scheduled FOMC announcements. Hu, Pan, Wang, and Zhu (2021) find, additionally, a gradual drop in equity prices a few days before FOMC announcements, in particular before those associated with large preannouncement return or a large accumulation of VIX days before. In other words, there is a salient V-shaped price pattern around the most important FOMC announcements, which is consistent with our prediction.[6] That said, ours is not the only model that can generate a V-shaped price pattern. Models of Ai and Bansal (2018), Wachter

[6] For our purposes here, it is not critical whether equity returns are realized shortly before or after the FOMC announcement itself. Krueger and Kuttner (1996) show that the Fed Funds futures market is accurate in predicting the rate decisions of the FOMC before the official announcement.

and Zhu (2019), and Hu, Pan, Wang, and Zhu (2021) can all potentially explain the large equity return around FOMC announcements. The critical difference is that in all those other theories, the central bank's announcement (or intervention in our terminology) carries material information, so the high equity return is essentially a risk premium. In our model, the central bank's intervention is entirely nondiscretionary and does not carry incremental information beyond the preintervention price p_1. Yet, despite the predictability of intervention given p_1, there is still a positive expected return $E(p_2 - p_1)$. Because strategic investors are all risk-neutral in our model, this positive return is not a risk premium. Instead, it is entirely driven by the strategic interactions between the central bank and market participants.

It is probably the second prediction on trading behavior that can better distinguish our theory from others. In the risk-based explanations of equity returns around central bank intervention, trading plays a negligible role. In ours, strategic trading is what generates the V-shaped price pattern: large investors sell and small investors buy preintervention. Here, "large" means that the institution's wealth is highly sensitive to asset prices. In this sense, financial intermediaries whose profits depend heavily on the stock market performance are considered "large," even if they hold little inventory of stocks on average. Testing this prediction, however, requires more granular data that contain the transaction records of investors, preferably at the daily or higher frequency. The Abel Noser (ANcerno) data set is a possible testing ground, with the caveat that institutions self-select into this data set. Alternatively, supervisory data available to regulators are also suitable for this test.

Our results highlight the unintended consequences of central bank interventions if the central bank relies on noisy asset prices for its intervention decisions. In an ideal world, the central bank would use real-time economic data to assess economic fundamentals rather than inferring it from noisy prices that are influenced by strategic trading. But macroeconomic data in reality are released infrequently and with significant delays. A potential way forward for the central bank in the digital age is to exploit low-latency economic data that are already available on Big Tech platforms, such as Amazon, Google, and the Alibaba Group. The COVID-19 crisis has, if anything, expedited the adoption of technology and the transition of economic activities online. Although not all economic activities can be digitized, a real-time economic indicator based on partial data would already be helpful because it is the growth rate, not the absolute value, that matters the most. Perhaps this silver lining of the COVID-19 crisis will afford the central bank more accurate information about economic fundamentals when the next crisis comes.

Appendix: Proofs

Proof of Proposition 1 Parameter δ is explicitly given by (23) in terms of exogenous deep parameters. So, we use Equations (17), (18), (24), (25), and (28) to compute the remaining $J+4$ parameters, $(k_0, k_1, \gamma_0, \gamma_1, \{x_j^*\}_{j=1}^J)$. By Equations (18) and (25), we can compute the expressions of k_1 and γ_1 in Equations (30) and (33), respectively. Using Equations (24) and (25) to replace the expressions of γ_0 and γ_1 in (28), we can obtain

$$x_j^* = \frac{\lambda_j b k_1}{2(bk_1+1)} - \frac{\sum_{j'=j} x_{j'}^*}{2} = \frac{\lambda_j b k_1}{2(bk_1+1)} - \frac{\sum_j x_j^* - x_j^*}{2}, \forall j, \tag{A1}$$

which further implies

$$x_j^* = \frac{\lambda_j b k_1}{bk_1+1} - \sum_j x_j^*, \forall j. \tag{A2}$$

Summing (A2) across j gives

$$\sum_j x_j^* = \frac{bk_1}{bk_1+1} \frac{\sum_j \lambda_j}{J+1}. \tag{A3}$$

Inserting the expression of k_1 in (30) into (A3), we obtain the expression of $X^* \equiv \sum_{j=1}^J x_j^*$ in (14). Inserting (14) into (A2), we obtain the expression of x_j^* in (34). Finally, after computing X^*, we use Equations (17) and (24) to pin down the expressions of k_0 and γ_0 in Equations (17) and (24), respectively. QED.

Proof of Corollary 1 Part (a) follows from directly setting $b=0$ in Proposition 1. Part (b) follows from letting $J \to \infty$ in Proposition 1. QED.

Proof of Propositions 2, 3, and 4 The results can be derived by direct computation of taking derivatives. QED.

Proof of Proposition 5 The central bank's intervention rule, the informed investors' demand function, and the uninformed investor's demand function are given by (9), (10), and (64), respectively. We will examine each player's optimization problem to figure out their respective implied optimal strategies to form the fixed point problem in terms of $(k_0, k_1, \delta, \gamma_0, \gamma_1, h_0, h_1)$.

Central bank

The central bank's problem still implies the optimal intervention rule given by (12). But now the central bank needs to update $E(\theta|p_1)$ in (12) differently, as the bank understands that the uninformed investor submits demand schedule (64). Specifically, by the demand functions $D(p_1, u, \theta)$ in (10) and $x(p_1)$ in (64), as well as the market-clearing condition, $D(p_1, u, \theta) = x(p_1)$, the central bank reads the price as the following signal:

$$\widehat{s}_p = \frac{(\gamma_1 + h_1)p_1 - \gamma_0 + h_0}{\delta} = \theta + \frac{1}{\delta}u. \tag{A4}$$

Using signal $\widehat{s}_p$ to update $E(\theta|p_1)$ in (12), we get the following implied intervention strategy:

$$z(p_1) = A_0 + \frac{A_1\delta\tau_u(\gamma_0 - h_0)}{\tau_\theta + \delta^2\tau_u} - \frac{A_1\delta\tau_u(\gamma_1 + h_1)}{\tau_\theta + \delta^2\tau_u}p_1. \tag{A5}$$

Comparing (A5) with the conjectured intervention strategy (9), we have

$$k_0 = A_0 + \frac{A_1\delta\tau_u(\gamma_0 - h_0)}{\tau_\theta + \delta^2\tau_u}, \tag{A6}$$

$$k_1 = \frac{A_1\delta\tau_u(\gamma_1 + h_1)}{\tau_\theta + \delta^2\tau_u}. \tag{A7}$$

Informed investors

Their decision problem does not change. Specifically, informed investors do not infer information from the price, so that the change in uninformed investor's strategy does not directly affect the informed investors' decision problem. Informed investors only need to consider how the central bank's intervention affects period-2 price p_2, but the central bank's intervention rule is still given by (9) and its form remains unchanged to informed investors. As a result, the implied optimal demand by informed investors is still given by Equation (22). Comparing (22) with the conjectured trading strategy (10), we still obtain Equations (23)–(25).

Large, uninformed investor

The large uninformed investor's problem changes to $\max_{x(p_1)} E[w_2^u]$, where

$$w_2^u = xp_1 + (\Lambda - x)p_2 = xp_1 + (\Lambda - x)[\theta + \varepsilon + b(k_0 - k_1 p_1)]. \tag{A8}$$

In this maximization problem, the uninformed investor needs to take into account her price impact and extract information from the price. Following Kyle (1989), the uninformed investor's problem is equivalent to choosing x against her residual supply function. Specifically, by demand function $D(p_1, u, \theta)$ in (10) and the market-clearing condition, $D(p_1, u, \theta) = x(p_1)$, the uninformed investor understands that

$$p_1 = \frac{\gamma_0 + \delta\theta + u - x}{\gamma_1}. \tag{A9}$$

This price function delivers the uninformed investor's price impact,

$$\frac{\partial p_1}{\partial x} = -\frac{1}{\gamma_1}. \tag{A10}$$

In addition, understanding the price function (A9), the uninformed investor can obtain signal $\delta\theta + u$ from the price p_1, which further implies

$$E(\theta|p_1) = \frac{\delta\tau_u(\gamma_1 p_1 - \gamma_0 + x)}{\tau_\theta + \delta^2\tau_u}. \tag{A11}$$

Taking derivative of the objective function (A8) with respect to x and using the price impact (A10), we get the following first-order equation:

$$E\left(\frac{\partial w_2^u}{\partial x}|p_1\right) = p_1 - \frac{x}{\gamma_1} - E(\theta|p_1) - b(k_0 - k_1 p_1) + (\Lambda - x)b\frac{k_1}{\gamma_1} = 0 \tag{A12}$$

Replacing $E(\theta|p_1)$ with (A11) in (A12), we obtain the implied demand schedule by the uninformed investor as follows:

$$\begin{aligned} x = &\frac{b\Lambda\tau_\theta k_1 - b\gamma_1\tau_\theta k_0 + \delta\gamma_0\gamma_1\tau_u + b\Lambda\delta^2\tau_u k_1 - b\delta^2\gamma_1\tau_u k_0}{\tau_\theta + \delta^2\tau_u + b\tau_\theta k_1 + \delta\gamma_1\tau_u + b\delta^2\tau_u k_1} \\ &+ \frac{\gamma_1(\tau_\theta + \delta^2\tau_u + b\tau_\theta k_1 - \delta\gamma_1\tau_u + b\delta^2\tau_u k_1)}{\tau_\theta + \delta^2\tau_u + b\tau_\theta k_1 + \delta\gamma_1\tau_u + b\delta^2\tau_u k_1} p_1. \end{aligned} \tag{A13}$$

Comparing (A13) with the conjectured trading strategy (64), we have

$$h_0 = \frac{b\Lambda\tau_\theta k_1 - b\gamma_1\tau_\theta k_0 + \delta\gamma_0\gamma_1\tau_u + b\Lambda\delta^2\tau_u k_1 - b\delta^2\gamma_1\tau_u k_0}{\tau_\theta + \delta^2\tau_u + b\tau_\theta k_1 + \delta\gamma_1\tau_u + b\delta^2\tau_u k_1}, \tag{A14}$$

$$h_1 = \frac{\gamma_1(\tau_\theta + \delta^2\tau_u + b\tau_\theta k_1 - \delta\gamma_1\tau_u + b\delta^2\tau_u k_1)}{\tau_\theta + \delta^2\tau_u + b\tau_\theta k_1 + \delta\gamma_1\tau_u + b\delta^2\tau_u k_1}. \tag{A15}$$

Solve the unknowns $(k_0, k_1, \delta, \gamma_0, \gamma_1, h_0, h_1)$

Equations (A6), (A7), (23), (24), (25), (A14), and (A15) for the system of seven unknowns $(k_0, k_1, \delta, \gamma_0, \gamma_1, h_0, h_1)$. Again, variable δ is directly given by (23). So, we will use the remaining 6 equations to figure out $(k_0, k_1, \gamma_0, \gamma_1, h_0, h_1)$. The idea of solving this system is to first use (A7), (25), and (A15) to compute (k_1, γ_1, h_1) and then to use the remaining three equations to compute (k_0, γ_0, h_0). Inserting (25) into (A15), we have

$$h_1 = \frac{\tau_\theta\delta(bk_1 + 1)}{\tau_\theta + 2\delta^2\tau_u}. \tag{A16}$$

Plugging (25) and (A16) into (A7), we obtain a single equation in terms of k_1. Solving this equation delivers the expression of k_1 in Proposition 5. Inserting the expression of k_1 into (25) and (A16), we obtain the expressions of γ_1 and h_1 in Proposition 5. Inserting (24) and the expressions of (25) and (A16) into (A14), we have

$$h_0 = \frac{b\delta(-\tau_\theta^2 k_0 - 2\delta^2\tau_u\tau_\theta k_0 + 2\Lambda\delta^3\tau_u^2 A_1 + 2\Lambda\delta\tau_u\tau_\theta A_1)}{(\tau_\theta + 2\delta^2\tau_u)^2}. \tag{A17}$$

Plugging (24) and (A17) into (A6), we can compute k_0. Inserting the expression of k_0 into (24) and (A17), we obtain the expressions of γ_0 and h_0 in Proposition 5. QED.

Proof of Proposition 6. The results follow from direct computation. QED.

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