Online Appendix for
"Dynamic Information Asymmetry, Financing, and Investment Decisions"

Not for Publication

Ilya A. Strebulaev[*] **Haoxiang Zhu**[†] **Pavel Zryumov**[‡]

January 21, 2014

1 A Two-Period Model with Cash Accumulation

In Strebulaev, Zhu, and Zryumov (2014, SZZ), we have assumed that all the cash generated by the firm prior to investment is paid out to existing shareholders. In this section we will relax this assumption and allow firm to accumulate cash; the cash can be used to finance the new project later. For tractability, we do not attempt to solve cash accumulation in our original dynamic model; rather, we solved a simplified, two-period version.

More specifically, we keep the continuous-time cumulative cash flow process X_t^θ but only allow two investment dates, $t = 0$ and $t = T > 0$, where T is a parameter. At time 0, the firm starts with zero cash reserves and decides whether to undertake the project immediately or delay the decision until $t = T$. If the investment opportunity is not taken at time 0, existing shareholders of the firm receive the flow δdX_t of dividends up to time $t = T$, whereas the remaining cash flow $(1 - \delta)dX_T$ is kept within the firm as cash reserves, where $\delta \in [0, 1]$ is a parameter that can be interpreted as the firm's pre-existing dividend

[*]Graduate School of Business, Stanford University and NBER. Email: istrebulaev@stanford.edu.
[†]MIT Sloan School of Management. Email: zhuh@mit.edu.
[‡]Graduate School of Business, Stanford University. Email: zryumov_pavel@gsb.stanford.edu.

policy that cannot be easily changed.[1] The model of SZZ has $\delta = 1$. We assume that cash accumulated within the firm receives zero return, so the firm's cash position at $t = T$ is $(1-\delta)X_T$. At time $t = T$, the cash reserves are either used to partially finance the project or returned to existing shareholders if the firm decides to forgo the investment opportunity.

We solve the two-period model by backward induction. We first characterize the equilibrium at $t = T$ using the following lemma, which is similar to Proposition 1 of SZZ.

Lemma 1. *If cash reserves R at time $t = T$ exceed I, then both firms invest using cash only. Otherwise, the equilibrium strategies of the firm and market are described by the Proposition 1 of SZZ with the adjusted thresholds:*

$$p_e = \frac{\mu_H r(I - R^+) - k(k - Ir)}{\mu_H(k - R^+ r)}, \tag{1}$$

$$p_d = 1 - \frac{k - Ir}{(1-\gamma)\min(\mu_H, k - R^+ r)}, \tag{2}$$

$$p_r = 1 - \frac{1}{1-\gamma}\left(1 - \frac{r(I - R^+)}{\mu_H}\right), \tag{3}$$

where $R^+ = \max(R, 0)$.

Proof. The result for $R \geq I$ is obvious. If $R \leq 0$, the reserves can not be used to finance the project. Thus, the equilibrium coincides with the one described in Proposition 1 of SZZ.

If $0 < R < I$ we get

$$\lambda = \frac{r(I-R)}{q^e \mu_H + k} \quad \text{and} \quad c = \begin{cases} \frac{r(I-R)}{1-(1-\gamma)(1-q^d)}, & \text{if } p \geq p_r; \\ \frac{r(I-R)-(1-\gamma)q^e\mu_H}{\gamma}, & \text{if } p < p_r. \end{cases} \tag{4}$$

Substituting the above expression into the value function of the existing shareholders,

[1] As in the main text of SZZ, the Brownian motion of the cumulative cash flows implies that $X_{t+\delta} - X_t$ can be negative over any time interval $(t, t+\delta)$ with positive probability. In that case, we effectively assume that the existing shareholders contribute a fraction δ of the cash shortfall, and the firm covers the rest (with existing cash reserves or overnight bank credit). Again, this is a convenient modeling shortcut and does not affect the qualitative nature of our results.

we find that:

$$E_H^{\emptyset}(R) = \frac{\mu_H}{r} + R, \tag{5}$$

$$E_H^{e}(R, q^e) = \frac{\mu_H}{r} + \frac{k}{r} - I - (1-q^e)\frac{\mu_H(I-R^+)}{q^e\mu_H + k} + R, \tag{6}$$

$$E_H^{d}(R, q^d) = \frac{\mu_H}{r} + \frac{k}{r} - I - (1-\gamma)(1-q^d)\min\left(\frac{\mu_H}{r}, \frac{I-R^+}{1-(1-\gamma)(1-q^d)}\right) + R. \tag{7}$$

Comparing these value functions yields the thresholds p_e, p_d, and $p_{d/e}$. Since a low type firm always prefer mimicking a high type firm, the lemma holds. ■

Now, we study the strategy at $t = 0$. The following proposition characterizes the investment and financing strategies of the firms in the two-period model for a suitable region of model parameters.

Proposition 1. *For some model parameters $(\mu_H, I, r, k, \gamma, \sigma, \delta, T)$ that have a positive measure,*[2] *there exists a pair of thresholds $(\underline{p}^*, \bar{p}^*)$, where $\bar{p}^* > \underline{p}$, such that the following strategies constitute an equilibrium:*

1. *At time $t = 0$:*
   - *(i) If $p_0 \geq \bar{p}^*$, then both types of firms invest at $t = 0$ using the security specified by Proposition 1 of SZZ;*
   - *(ii) If $p_0 \in [\underline{p}^*, \bar{p}^*)$ then both types of firms delay investment until time $t = T$;*
   - *(iii) If $p_0 < \underline{p}^*$, then a high type firm delays investment, whereas a low type firm invests at $t = 0$ with probability $\alpha = p_0(1-\underline{p}^*)/[\underline{p}^*(1-p_0)] < 1$ and delays investment with probability $1-\alpha$.*

   Moreover, if any issuance at time $t = 0$ is off the equilibrium path, the market believes that the deviating firm is of the low type with probability 1. If any non-issuance at time $t = 0$ is off the equilibrium path, the market keeps its prior p_0.

2. *At time $t = T$, the firm plays a static equilibrium similar to Proposition 1 of SZZ, with suitably adjusted thresholds p_e, p_d, p_r, and $p_{d/e}$. The market belief p_T is determined by*

$$\ln\left(\frac{p_T}{1-p_T}\right) = \ln\left(\frac{p_{0+}}{1-p_{0+}}\right) + \frac{\mu_H}{\sigma^2}\left(X_T - \frac{\mu_H}{2}T\right), \tag{8}$$

[2] These model parameters should satisfy conditions (*) and (**), spelled out in the proof of this proposition. We have verified numerically that the strategies specified in this proposition work for parameters in Cases I–III in Definition 1 of SZZ and small T.

where $p_{0+} = \max(p_0, \underline{p}^*)$.

Proof. Part 2 of Proposition 1 simply follows from Lemma 1.

We now characterize the equilibrium strategies at time $t = 0$. Define

$$E_H(R, p) = \max(E_H^{\emptyset}(R), E_H^e(R, p), E_H^d(R, p))$$

and

$$E_H^{invest}(R, p) = \max(E_H^e(R, p), E_H^d(R, p)).$$

Part 1(i). If a high type firm invests at $t = 0$ and a low type firm imitates, the high type firm will receive $E_H^{invest}(0, p_0)$ right away. Delay of investment brings the expected payoff

$$\mathbf{E}\left[\delta \int_0^T e^{-rt} dX_t + e^{-rT} E_H((1-\delta)X_T, p_T)\right] = \delta\frac{\mu_H}{r}(1-e^{-rT}) + e^{-rT}\,\mathbf{E}\left[E_H\big((1-\delta)X_T, p_T\big)\right]. \tag{9}$$

Clearly, if

$$p_0 \in \mathcal{I}_H \equiv \left\{p \,:\, E_H^{invest}(0, p) > \delta\frac{\mu_H}{r}(1 - e^{-rT}) + e^{-rT}\,\mathbf{E}\left[E_H\big((1-\delta)X_T, p_T\big)|p_0 = p\right]\right\}, \tag{10}$$

the high type firm prefers pooling with the low type at time $t = 0$ to delaying investment until time T. Clearly, if $p \to 1$, investment financed by the better of debt and equity brings the high type firm a payoff that is close to the first best, $(\mu_H + k)/r - I$. But for all possible belief p_T,

$$\begin{aligned}&\delta\frac{\mu_H}{r}(1 - e^{-rT}) + e^{-rT}\,\mathbf{E}\left[E_H\big((1-\delta)X_T, p_T\big)|p_0 = p\right]\\ \le\, &\delta\frac{\mu_H}{r}(1 - e^{-rT}) + e^{-rT}\left(\frac{\mu_H}{r} + \frac{k}{r} - I\right) < \frac{\mu_H}{r} + \frac{k}{r} - I.\end{aligned}$$

Thus, by continuity, we let $\bar{p}^* < 1$ be the smallest number such that if $p_0 \in (\bar{p}^*, 1]$, the high type firm prefers pooling to delaying. This means $\mathcal{I}_H$ contains the interval $(\bar{p}^*, 1]$. The appropriate security used in this case would be determined as in Proposition 1 of SZZ.

Furthermore, our numerical calculations indicate that $\mathcal{I}_H$ is an interval for a wide range of primitive model parameters in Cases I-III of Definition 1 of SZZ and for a sufficiently small

T. In what follows, we work under the condition:

$$\mathcal{I}_H = (\bar{p}^*, 1]. \tag{*}$$

That is, the high type firm's incentive to wait can be summarized by a single threshold $\bar{p}^*$. Notice that $1 \in \mathcal{I}_H$ and $0 \notin \mathcal{I}_H$ as long as p_e and p_d are positive, regardless of condition (*).

Part 2(iii). Denote by $E_L(R, p)$ the pay-off that low type firm receives in the equilibrium of the Lemma 1. Low type firm prefers revealing her type at time $t = 0$ to delaying investment, given the market belief p_0, if and only if

$$p_0 \in \mathcal{I}_L \equiv \left\{ p \,:\, \frac{k}{r} - I > e^{-rT}\,\mathbf{E}\left[E_L\big((1-\delta)X_T, p_T\big)|p_0 = p\right]\right\}. \tag{11}$$

Because $\mathbf{E}\left[E_L\big((1-\delta)X_T, p_T\big)|p_0 = p\right]$ is increasing in p, it is clear that $\mathcal{I}_L = [0, \underline{p}^*)$ for some $\underline{p}^*$.

In the rest of the proof, we work under the condition:

$$\mathcal{I}_L \cap \mathcal{I}_H = \emptyset. \tag{**}$$

We verify numerically that condition (*) holds for a wide range of primitive model parameters in Cases I-III of Definition 1 of SZZ and for a sufficiently small T.

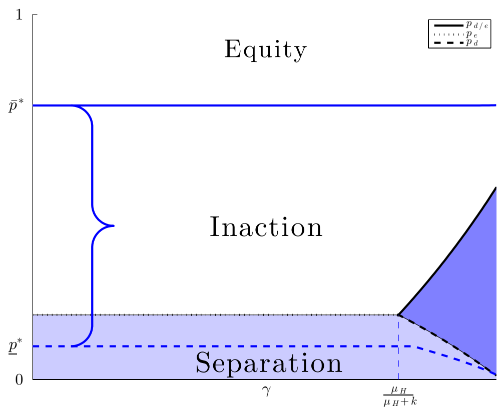
If $p_0 < \underline{p}^*$, then only a low type firm invests with probability α such that conditional on no investment at time $t = 0$, the low type firm's continuation payoff is equal to the payoff for revealing her type, i.e., $p_{0+} = \underline{p}^*$. Bayes' rule pins down α by direct calculation.

Part 3(ii). If $p_0 \in [\underline{p}^*, \bar{p}^*)$, both types of firms prefer to delay investment because $p_0 \notin \mathcal{I}_H$ and $p_0 \notin \mathcal{I}_L$. ■

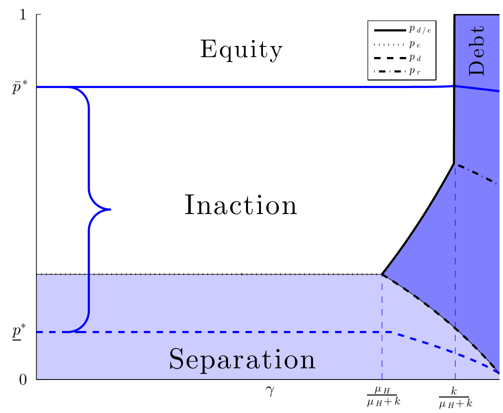
Figure 1 illustrates the equilibrium strategies at $t = 0$ as specified in Proposition 1.

Figure 2 illustrates the effect of accumulating cash within the firm for the equilibrium strategies of Proposition 1. Starting at $\delta = 1$, which corresponds to the main model of SZZ, a decrease in δ increases the expected amount of cash for a high type firm at time T, which reduces the expected funds that firm has to raise from the market at time T. This, in turn, reduces the firm's cost of adverse selection, making delay more attractive for the high type firm. A low type firm imitates a higher type firm and is also more likely to delay. Overall, a lower δ leads to a wider inaction region but a smaller pooling region and a smaller separation region. The comparative statics suggest that if cash accumulation were allowed, the effect of delay associated with adverse selection would be even stronger.

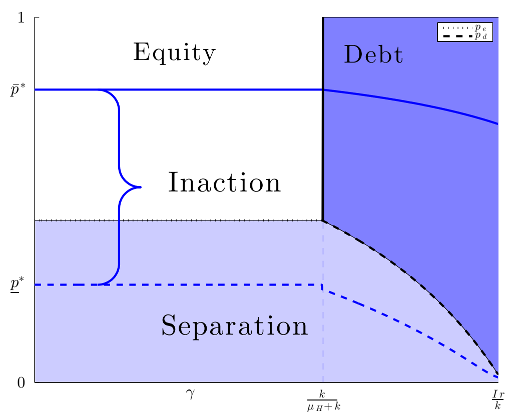
Figure 1: Equilibrium strategies at $t = 0$. The figure overlays equilibrium strategies of the two-period model on top of the static strategies plotted in Figure 2 of SZZ. The figure is plotted for parameters satisfying $p_d > 0$ and $p_e > 0$ (i.e., truncating parameters with high γ in the (γ, p_0) space).



(a) $\mu_H < Ir < k$

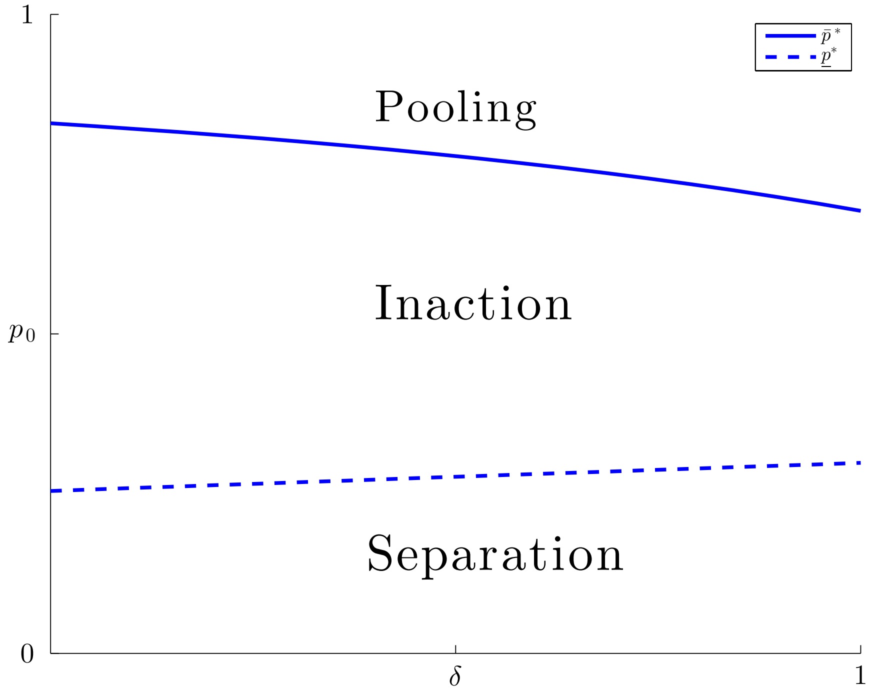


(b) $Ir < \mu_H < k$



(c) $Ir < k < \mu_H$

Figure 2: Equilibrium thresholds at $t = 0$ for different dividend payout rates δ.



References

Strebulaev, Ilya A., Haoxiang Zhu, and Pavel Zryumov, 2014, Dynamic information asymmetry, financing, and investment decisions, Working paper.