Dynamic Information Asymmetry, Financing, and Investment Decisions*

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Abstract

We reexamine the classic yet static information asymmetry model of Myers and Majluf (1984) in a fully dynamic market. A firm has access to an investment project and can finance it by debt or equity. The market learns the quality of the firm over time by observing cash flows generated by the firm's assets in place. In the dynamic equilibrium, the firm optimally delays investment, but investment eventually takes place. In a "two-threshold" equilibrium, a high-quality firm invests only if the market's belief goes above an optimal upper threshold, while a low-quality firm invests if the market's belief goes above the upper threshold or below a lower threshold. However, a different "four-threshold" equilibrium can emerge if cash flows are sufficiently volatile. Relatively risky growth options are optimally financed with equity, whereas relatively safe projects are financed with debt, in line with stylized facts.

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1 Introduction

The classic paper by Myers and Majluf (1984) begins by stating the following problem: "Consider a firm that has assets in place and also a valuable real investment opportunity. However, it has to issue common shares to raise part or all of the cash required to undertake the investment project. If it does not launch the project promptly, the opportunity will evaporate." A major finding of Myers and Majluf (1984) is that adverse selection can cause a financial market breakdown because the market cannot ascertain the quality of a firm's assets in place and will only offer a price that a high-quality firm finds too low to accept.

This market breakdown occurs in Myers and Majluf (1984) largely because the firm's manager has a single opportunity to make the investment. In many practical applications, however, investment opportunities do not "evaporate" if not undertaken immediately. A firm usually has the option to delay investment and financing if the market conditions are unfavorable. In this paper, we reexamine the static problem of Myers and Majluf (1984) in a fully dynamic market, in which a firm can choose the timing of its project as well as the security it issues—debt or equity. The firm's quality, or "type," is determined by the average cash flow of its assets in place. The market observes cash flows generated by the firm's assets in place and learns over time about the quality of the firm. If the firm decides to invest and issue equity or debt, the market prices the offered security based on information revealed by past cash flows and the firm's actions. By delaying investment and financing, a high-quality firm faces the following trade-off. On the one hand, the market will eventually ascertain the true high quality of the firm, which minimizes the underpricing of its security. On the other hand, delay costs the firm the time value of the investment project. A low-quality firm faces a similar trade-off: by delaying, it loses the time value of the positive net present value (NPV) of the project but can potentially benefit from the overpricing of its security by pooling with a high-quality firm.

We present three main results in this paper. First, we characterize a strategic dynamic equilibrium of investment and financing in this market. Unlike in the static equilibrium of Myers and Majluf (1984), in our dynamic equilibrium, investment always takes place eventually, if not immediately, and the market does not break down. More specifically, as long as the market belief is not extremely high or extremely low, there is an initial, potentially protracted, period of inaction, when no investment or issuance takes place. If the market's belief about the firm's quality becomes sufficiently optimistic, both types invest. This upper belief threshold is effectively chosen by a high-quality firm. Intuitively, if the market assigns a high enough expectation of the value of the firm's assets in place, the underpricing of the

newly issued security is small, and the high-quality firm optimally invests immediately. A low-quality firm "imitates" a high-quality firm and invests as well. If the market's belief becomes sufficiently pessimistic, however, only a low-quality firm invests, thus revealing its type to the market. The lower belief threshold is effectively chosen by the low-quality firm such that it is indifferent between continuing to wait and investing immediately. Interestingly, a low-quality firm invests probabilistically, not deterministically, at the lower threshold; the investment probability is optimally chosen by the low-quality firm such that conditional on observing no investment at the lower threshold, the market belief "reflects" upward. Put together, the "two-threshold" equilibrium is characterized by three regions of the market's belief: an upper pooling region, a middle inaction region, and a lower separation region. The qualitative nature of this dynamic equilibrium holds for both debt and equity.

Our second main result is to analyze the firm's choice between debt and equity, two most commonly used securities. The most natural theoretical benchmark is the pecking order theory, as put forward by Myers (1984). The pecking order theory posits that debt is preferred over equity in the presence of asymmetric information because debt is less sensitive than equity to private information.

We find that the pecking order can be reversed if the new project is sufficiently risky. Intuitively, risky projects that resemble a "gamble"—that is, projects with a small probability of success but a large return conditional on success—are more likely to default if financed with debt. In default, the original shareholders incur losses on assets in place that implicitly serve as debt collateral. As the failure probability of the project increases, a high-quality firm finds it particularly costly to issue debt. Therefore, high-risk projects tend to be financed with equity, despite its dilutive nature. For relatively safe projects, debt is preferred to equity. Importantly, this result comes from information asymmetry alone and does not rely on deadweight costs of bankruptcy (which will make issuing debt even costlier) or any other debt-related frictions. Our results can help explain a stylized fact that high-growth risky projects (e.g., those financed by venture capital) tend to be financed by equity-like securities.

The combination of time-varying information asymmetry and security choice gives rise to a new type of "four-threshold" equilibrium, if the volatilities of cash flows are sufficiently high. The characterization of this four-threshold equilibrium is the third main result of our study. This equilibrium features five regions of the market's belief: two pooling regions (in which both types of firms invest), two inaction regions (in which both types of firms wait to invest), and one separation region (in which only the low-quality firm invests). Although one of the inaction regions is similar to the inaction region in the two-threshold equilibrium,

the other inaction region occurs for an entirely different reason. Because a high-quality firm can choose to finance a project with debt or equity, its static value function has a non-differentiable point, a kink. At this kink, the high-quality firm will never invest; it would rather delay and get a convex combination of its continuation values for issuing debt or equity. Therefore, a new inaction region emerges around this kink. On either side of this new inaction region are the two pooling regions, which have different securities in equilibrium.

We would like to emphasize that the four-threshold equilibrium distinguishes our analysis from that of Daley and Green (2012), who extend the classical lemon's problem of Akerlof (1970) to a fully dynamic setting and derive a two-threshold equilibrium. In their model, a seller wishes to sell an indivisible asset with unobservable quality, and buyers observe signals of the asset's quality over time. Their model is essentially equity-only. Relative to the results of Daley and Green (2012), we enrich the financing options of firms and characterize the dynamic trade-off between debt and equity. As a result, the four-threshold equilibrium emerges, and it is qualitatively different from the equilibrium of Daley and Green (2012).

Our results give rise to a number of empirical predictions. For example, security issuance is more likely when information asymmetry is less severe. This prediction is consistent with Korajczyk, Lucas, and McDonald (1991), who find that "equity issues cluster in the first half of the period between information releases . . . firms almost never issue equity just prior to an earnings release." Using analyst coverage as a proxy for information asymmetry, Chang, Dasgupta, and Hilary (2006) find that firms covered by fewer analysts issue equity less frequently.

A subtler but unique prediction of our model is that investment and issuance of debt or equity can happen after a series of *negative* news. After sufficient price declines, a low-quality firm has little chance of being mistaken for a high-quality firm; immediate issuance thus becomes optimal. This prediction is consistent with stylized facts about stock price behavior prior to equity issuance. For example, Korajczyk, Lucas, and McDonald (1990) find that while most equity issuances take place after large positive abnormal returns, 18% of issuances occur after share price declines relative to the market. A closely related empirical prediction is that if issuance follows a series of negative news, post-issue price declines further because issuance in this case is an additional signal of low firm quality.

Our results also predict that relatively safe projects are financed by debt, and relatively risky projects are financed by equity. Frank and Goyal (2003) document that small, high-growth firms tend to use equity financing instead of debt financing. This fact is often interpreted as evidence against the pecking order theory, or as evidence that information

asymmetry is unimportant for capital structure decisions. Our results, however, reveal that asymmetric information is perfectly consistent with, and may potentially explain, the predominant use of equity financing by small firms.

Our paper is related to several strands of the literature. Lucas and McDonald (1990) also model dynamic information asymmetry by considering its impact on equity financing decisions. In their model, information asymmetry is reset at fixed time intervals, so undervalued firms do not issue equity until complete resolution of information asymmetry. The immediate issuance by overvalued firms and the delayed issuance by undervalued firms jointly predict an increase in share prices prior to issuance. In our model, by contrast, information asymmetry is never completely resolved due to unpredictable shocks to cash flows. Thus, a high-quality firm may still issue in the presence of underpricing, while a low-quality firm may issue after a series of negative cash flow news.

Hennessy, Livdan, and Miranda (2010) develop a dynamic signaling model of investment and financing. In their paper, the firm's manager has superior information relative to the market, but this informational advantage is short lived. Markovian evolution of the firm's type together with short lived private information generate a time-invariant level of information asymmetry in their model. In contrast, our model features time-varying information asymmetry, stemming from persistent firm types and gradual information revelation. In our dynamic equilibrium, the ability of the market to obtain a more precise estimate of the firm's type over time is the driving force behind the firm's incentive to delay investment.

Investment delay in our model has an economic underpinning that is distinct from those in the real option literature, where investment delays are caused by the optimal timing of real option exercise. In a recent paper, Morellec and Schürhoff (2011) extend the Myers and Majluf (1984) model to a real option setting, in which the firm chooses the optimal investment timing, as well as the type of security to issue. They model the firm's revenue from the new project as the firm's unobservable type multiplied by a publicly observable cash flow process. The observable cash flow shocks in their model are independent of firm type and hence cannot reveal information of the firm's type over time. In our model, the market learns about the firm's type by observing its cash flows; the firm delays investment because learning reduces adverse selection. In addition, Morellec and Schürhoff (2011) model the impact of a positive deadweight cost of bankruptcy on the properties of a separating equilibrium, while we concentrate on the information asymmetry mechanism alone.

We are not the first to propose a model in which the pecking order is partially reversed. Fulghieri and Lukin (2001) show that debt might no longer be the preferred security if

investors endogenously acquire additional information about the issuer. Our results are driven by a different channel, namely the trade-off between underpricing due to asymmetric information and the risk of losing assets in place. A recent study by Fulghieri, Garcia, and Hackbarth (2013) shows that equity can dominate debt if both the asset in place and the growth option are subject to the type of asymmetric information that is similar to what we examine here. Pecking order fails because the probability distribution of the firm's value does not satisfy the "conditional stochastic dominance," under which debt is the optimal security (Nachman and Noe (1994)). Fulghieri, Garcia, and Hackbarth (2013) investigate the optimal security design problem under more general distributions of firm values, although information asymmetry in their model is not time-varying. In contrast, we focus on the choice between debt and equity and study time-varying asymmetric information and associated delays in investment.

Chakraborty and Yilmaz (2011) show that in some situations the adverse selection problem can be costlessly solved by the issuance of properly structured convertible debt. Their result requires that (1) information asymmetry should be sufficiently low at the time of maturity of the convertible debt, and (2) managers cannot benefit from the assets in place or the growth option before the debt matures. In our setup, the availability of cash flows prior to the resolution of market uncertainty limits the benefits of using convertible bonds. In this respect, our results complement those of Chakraborty and Yilmaz (2011).

The rest of the paper is organized as follows. In Section 2, we present the static model and equilibrium. In Section 3, we develop the dynamic model and study the dynamic equilibria. In Section 4, we explore the welfare and empirical implications of our results. We conclude in Section 5. All the proofs are in the Appendix.

2 Model Setup and Static Equilibrium

In this section, we develop and solve a static model of investment and financing decisions under asymmetric information. Closely related to that of Myers and Majluf (1984), our static model is simple yet sufficient to illustrate the intuition behind the main economic mechanisms. For this reason, we keep as many standard assumptions of Myers and Majluf (1984) as possible. In Section 3, we consider a fully dynamic model with asymmetric information and compare its implications with this static benchmark. A glossary of key model variables is also tabulated in the Appendix.

2.1 Model Setup

We start by introducing the economic agents, their strategy sets, and the model timeline.

2.1.1 The Firm and the Market

Consider an all-equity firm with pledgable assets in place that belong to one of two types θ, $\theta \in \{H, L\}$. The type is the private information of the firm's management, which we simply refer to as the "firm." As in Myers and Majluf (1984), the existing shareholders are passive, and there is no conflict of interest between existing shareholders and management. All parameters other than the firm's type are common knowledge. The firm's assets in place produce in expectation free cash flow μ_θ per unit of time, where $\mu_H > \mu_L \geq 0$. Thus, the firm is of high (low) quality if it is of the high (low) type. The cumulative cash flows of the type θ firm at time t, X_t^θ, follow:

$$dX_t^\theta = \mu_\theta dt + \sigma dB_t, \tag{1}$$

where μ_θ and σ are constants, and $B = (B_t, \mathcal{F}_t^B)_{t\geq 0}$ is a standard Brownian motion adapted to the natural filtration defined on a canonical probability space (Ω, $\mathcal{F}$, $\mathbf{Q}$). The firm's type θ and the Brownian motion B are independent.

In addition to its assets in place, the firm has a risky growth option, which consists of a monopoly access to a new production technology. At the time of investment, the firm pays a one-time cost of I and installs the new technology. If the new technology succeeds, then the expected free cash flow increases from μ_θ to $\mu_\theta + K$, where K is a positive constant that is independent of θ. If, however, the new technology fails, the expected free cash flow remains μ_θ. The success and failure of the new technology occur with respective probabilities γ and $1-\gamma$, independently of type θ and Brownian motion B. A type-independent new project provides a clear benchmark and allows us to focus on the asymmetric information of assets in place.[1,2] The constant risk-free rate is $r > 0$. The NPV of the new project is thus $\frac{k}{r} - I$, where $k \equiv \gamma K$. We assume that the NPV of the risky investment opportunity is positive:

[1] One could overlay asymmetric information about the new project by adjusting our model, but such an extension does not change our main results. For example, we can incorporate type-dependent K as follows. Denote the type-θ firm's additional cash flow by K_θ, conditional on success. As long as $\mu_H + K_H > \mu_L + K_L$, and γ is common knowledge, we can show that the same solution procedure carries through and the qualitative nature of the equilibrium does not change.

[2] Heider (2005) considers a static setting in which a firm chooses one project among many that differ in their risks and returns. His model does not have assets in place (that can be seized by debt holders) and does not consider dynamics.

$k > Ir$.

To make the firm's problem non-trivial, we assume that the cost I must be entirely financed by outside investors. Alternatively, we can interpret the cost I as the net funds that the firm must raise above and beyond its internal resources. In addition, the firm is unable to spin-off the growth option and finance it independently of the assets in place. These realistic assumptions on the firm are consistent with those of Myers and Majluf (1984).

Given that the mean cash flow of the firm is at least μ_L, any funding decision will involve the firm optimally pledging the assets in place and raising *secured, risk-free debt* with a face value of μ_L/r. The amount μ_L/r then effectively represents the financial resources available to the firm; regardless of adverse selection, it reduces the required funds I that must be raised from outside investors. We therefore can assume that $\mu_L = 0$. In other words, the mean cash flow μ_θ and the required investment I should be interpreted as those after the firm exhausts its capacity of issuing secured, risk-free debt. Any other debt issued afterwards is junior to the secured debt. For simplicity, we refer to the junior debt simply as "debt," bearing in mind that there could be senior debt, which is secured and risk-free but is irrelevant for our analysis of adverse selection.

There is a group of competitive outside investors, called the "market," that does not observe θ. The market has a prior belief $p_0 \in [0, 1]$ at time 0 that the firm is of type H, and $1 - p_0$ that the firm is of type L. The market and the firm are risk-neutral. There are no taxes or deadweight default costs in our model.

2.1.2 Timeline and Strategies

We now formally describe the timing of the static model and the strategies of the firm and the market.

1. The firm observes θ; the market does not.

2. The market quotes prices of debt and equity for the firm in anticipation of investment.[3] In return for providing the funds I, the market demands a fraction λ of the firm's total equity or a perpetual debt with coupon payment c.

3. Upon observing the quotes λ and c, the firm decides whether to invest or not and which security to use conditional on investment. The firm's strategy is a probability distribution over the set of feasible actions $a \in A = \{\emptyset, e, d\}$, conditional on quoted

[3]We restrict attention to debt and equity because they are the most commonly used securities in practice. The problem of optimal security design, albeit interesting, is not the objective of this study.

λ and c, where $a = \emptyset$ denotes no investment, $a = e$ denotes investment with equity financing, and $a = d$ denotes investment with debt financing.

A type θ firm chooses the probabilities of equity offering, $\pi^e(\theta)$, debt offering, $\pi^d(\theta)$, and no investment, $1 - \pi^e(\theta) - \pi^d(\theta)$.[4] For simplicity, a firm can finance the project by issuing equity only or debt only, but not a combination of the two.[5] If the firm decides not to invest at time 0, the game is over.

4. If the investment takes place, it is immediately revealed to be successful or unsuccessful. Since the firm's post-investment mean cash flow and coupon obligation (if debt is issued) are both constants, the firm defaults immediately if and only if the mean cash flow is lower than the coupon level.[6] If default occurs, the investors seize the firm's assets without incurring any deadweight cost, and the game ends.

We can write down the payoffs of the players as follows. The expected payoff E to the firm for passing on the investment opportunity, investing with equity financing, or investing with debt financing are, respectively,

$$E_\theta^\emptyset = \frac{\mu_\theta}{r}, \tag{2}$$

$$E_\theta^e(\lambda) = (1-\lambda)\frac{\mu_\theta + k}{r}, \tag{3}$$

$$E_\theta^d(c) = \gamma \max\left(\frac{\mu_\theta + K - c}{r}, 0\right) + (1-\gamma)\max\left(\frac{\mu_\theta - c}{r}, 0\right). \tag{4}$$

The $\max(\,\cdot\,)$ operator in $E_\theta^d(c)$ reflects the fact that the firm defaults if the post-investment mean cash flow is lower than the debt coupon.

[4] Probabilities of investment with a particular kind of financing depend on the observed market quotes λ and c, i.e., $\pi^e(\theta) = \pi^e(\theta; \lambda, c)$ and $\pi^d(\theta) = \pi^d(\theta; \lambda, c)$. For brevity, we suppress the function arguments (λ, c) of π^e and π^d.

[5] This assumption is without loss of generality in our setup. Allowing the market to quote prices for partial financing expands the space of deviations, but our equilibrium still survives.

[6] Strictly speaking, because the cash flows are risky, there is positive probability that the high type firm's cumulative cash flow in the small interval $(t, t + dt)$ falls below cdt, even if the project is successful. To deal with this technicality, we assume that the firm can access instantaneous (overnight) credit for this type of cash shortfalls. Importantly, this instantaneous credit is small in magnitude and cannot satisfy the investment need I.

Similarly, the expected payoff S to the market facing a type θ firm in cases of $a = e$ and $a = d$ are, respectively,

$$S_\theta^e(\lambda) = \lambda \frac{\mu_\theta + k}{r} - I, \tag{5}$$

$$S_\theta^d(c) = \gamma \min\left(\frac{c}{r}, \frac{\mu_\theta + K}{r}\right) + (1-\gamma)\min\left(\frac{c}{r}, \frac{\mu_\theta}{r}\right) - I. \tag{6}$$

Because the market does not observe θ, it has to infer the average quality of the firm attracted by the offer (λ, c), according to Bayes' rule. The probability of the firm being the high type, conditional on investment with debt (or equity) financing, is equal to:

$$q_0^i = \frac{p_0 \pi^i(H)}{p_0 \pi^i(H) + (1-p_0)\pi^i(L)}, \quad \text{if } \pi^i(H) + \pi^i(L) > 0, \quad \text{for } i = e, d. \tag{7}$$

In particular, if the firm uses deterministic strategies, (7) simplifies to:

$$q_0^i = \begin{cases} p_0, & \text{if } \pi^{i*}(H) = 1 \text{ and } \pi^{i*}(L) = 1, \\ 1, & \text{if } \pi^{i*}(H) = 1 \text{ and } \pi^{i*}(L) = 0, \\ 0, & \text{if } \pi^{i*}(H) = 0 \text{ and } \pi^{i*}(L) = 1. \end{cases} \quad \text{for } i \in \{e, d\}. \tag{8}$$

In other words, given λ and c, if a specific security (debt or equity) is selected by one type of firm but not the other, the market immediately and correctly separates the two types. If both types invest using the same security, the market keeps its prior.

The expected payoffs to the market, $S^e(\lambda)$ and $S^d(c)$, are then given by:

$$S^e(\lambda) = \begin{cases} q_0^e S_H^e(\lambda) + (1-q_0^e) S_L^e(\lambda), & \text{if } \pi^e(H) + \pi^e(L) > 0 \\ 0, & \text{if } \pi^e(H) + \pi^e(L) = 0, \end{cases} \tag{9}$$

$$S^d(c) = \begin{cases} q_0^d S_H^d(c) + (1-q_0^d) S_L^d(c), & \text{if } \pi^d(H) + \pi^d(L) > 0 \\ 0, & \text{if } \pi^d(H) + \pi^d(L) = 0. \end{cases} \tag{10}$$

Given the strategies and payoffs, a perfect Bayesian equilibrium of the static game consists of the strategy $(\pi^{d*}(\theta), \pi^{e*}(\theta))$ of the firm and quotes (c^*, λ^*) of the market such that: (i) the firm maximizes its payoff; (ii) the market breaks even in expectation, and the corresponding q_0^i is consistent with π^{i*} in the sense of (7); and (iii) there does not exist another pair of quotes $(\tilde{\lambda}, \tilde{c})$ that earns strictly positive profits for the market given π^{e*} and

π^{d*}.[7]

Without loss of generality, we impose the following tie-breaking rules in the static model. First, if the firm is indifferent between investing and not investing, the firm invests. Second, if the firm is indifferent between issuing debt or equity, the firm issues equity. Finally, if the market is indifferent between buying the firm's equity or debt and not buying it, the market buys it. Under the tie-breaking rules, it often suffices to write the parameter conditions using strict inequalities in our equilibrium characterization.

2.2 Economic Forces

In this section, we discuss the key trade-offs and economic mechanisms of the static game, which are also relevant for the dynamic model. The three possible actions of the firm, $A = \{\emptyset, e, d\}$, give rise to three different pairwise trade-offs: investment with equity financing versus no investment, investment with debt financing versus no investment, and investment with debt versus investment with equity. We start by analyzing these trade-offs one by one because it simplifies the expositions of our results later. For example, we find that the conventional pecking order can be reversed even in such a simplistic static environment, that is, equity financing can dominate debt financing under asymmetric information.

2.2.1 Investment with Equity Financing vs. No Investment

Upon equity offering, to break even on its investment, the competitive market would have to demand a fraction λ of the firm that satisfies:

$$I = \lambda \left(q_0^e \frac{\mu_H + k}{r} + (1 - q_0^e) \frac{\mu_L + k}{r} \right). \tag{11}$$

The right-hand side of (11) is the value of outside investors' equity. The value of λ is then:

$$\lambda(q_0^e) = \frac{Ir}{q_0^e(\mu_H + k) + (1 - q_0^e)(\mu_L + k)}. \tag{12}$$

A low type firm strictly prefers investing. Intuitively, its worst-case scenario is to invest in the positive-NPV project while revealing its type, and the best-case scenario is to

[7] Condition (iii) is a slight generalization of the "No Unrealized Deals" condition of Daley and Green (2012). Conditions (ii) and (iii) ensure that market offers are competitive.

be mistaken for a high type firm. Thus, for any belief q_0^e,

$$E_L^e(\lambda) > E_L^{\emptyset}. \tag{13}$$

For a high type firm, equity issuance involves a trade-off between the benefits of the positive-NPV project and the costs of subsidizing a low type firm. A high type firm strictly prefers equity financing (thus pooling with the low type) if and only if:

$$E_H^e(\lambda) > E_H^{\emptyset}. \tag{14}$$

By (12), (14) holds if and only if:

$$q_0^e \geq p_e \equiv \frac{Ir}{k}\frac{\mu_H + k}{\mu_H - \mu_L} - \frac{\mu_L + k}{\mu_H - \mu_L}. \tag{15}$$

It is easy to check that $p_e < 1$. The level of p_e thus establishes a belief threshold, with a high type firm always issuing equity if the market beliefs are sufficiently high.

Lemma 1. *For all market beliefs, a low type firm prefers investment with equity financing to no investment. A high type firm prefers investment with equity financing to no investment if and only if* $q_0^e \geq p_e$.

Lemma 1 restates a celebrated result of Myers and Majluf (1984) that, in the mold of Akerlof (1970), asymmetric information can prevent the market from providing financing for positive NPV projects. The economic thrust of the mechanism is that a decision to invest and issue equity can be interpreted by the market as a signal of the low quality of a firm's assets, which is immediately reflected in the share price. Low equity prices, in turn, deter a high type firm from investing in the first place.

2.2.2 Investment with Debt Financing vs. No Investment

We now turn to the comparison between investment with debt financing and no investment. Clearly, neither type defaults if the project is successful.[8] However, a low type firm always defaults if the project fails. Therefore, two scenarios can arise in the case of project failure: (i) only a low type firm defaults; and (ii) both types default. We consider both cases in turn.

[8]To see this, consider the debt coupon of Ir/γ. The market breaks even because its payoff is at least $\gamma \times Ir/(r\gamma) = I$. Because $k > Ir$, $Ir/\gamma < k/\gamma = K$, and no firm defaults upon project success.

Case 1. Suppose that only the low type defaults if the project fails. To break even the market has to demand a perpetuity with coupon c that satisfies:

$$I = q_0^d \frac{c}{r} + (1 - q_0^d)\gamma \frac{c}{r}, \tag{16}$$

$$\text{or} \quad c(q_0^d) = \frac{Ir}{1 - (1 - q_0^d)(1 - \gamma)}. \tag{17}$$

A high type firm does not default if and only if $\mu_H \geq c(q_0^d)$, or

$$q_0^d \geq p_r \equiv 1 - \frac{1}{1 - \gamma}\left(1 - \frac{Ir}{\mu_H}\right). \tag{18}$$

Thus, p_r is the belief threshold above which a high type firm does not default in this case.

Case 2. Suppose that both types of firms default if the project fails. To break even the market demands a perpetuity with coupon c that satisfies:

$$I = \gamma \frac{c}{r} + (1 - \gamma) q_0^d \frac{\mu_H}{r}, \tag{19}$$

$$\text{or} \quad c(q_0^d) = \frac{Ir - (1 - \gamma) q_0^d \mu_H}{\gamma}. \tag{20}$$

It is easy to show that the incentive condition for a high type firm to default is $q_0^d < p_r$.

Combining (17) and (20), the break-even coupon level is:

$$c(q_0^d) = \begin{cases} \frac{Ir}{1-(1-q_0^d)(1-\gamma)}, & \text{if } q_0^d > p_r, \\ \frac{Ir-(1-\gamma)q_0^d\mu_H}{\gamma}, & \text{if } q_0^d \leq p_r. \end{cases} \tag{21}$$

A low type firm always prefers investing with debt issuance to no investment. Upon project failure, it defaults and loses nothing, whereas project success brings not only the positive value of the project but also the benefits from a lower coupon due to overpriced debt. Formally,

$$E_L^d(c(q_0^d)) = \gamma \frac{K - c(q_0^d)}{r} > E_L^\emptyset = 0, \quad \forall\, q_0^d \in [0, 1]. \tag{22}$$

For a high type firm, debt issuance involves a trade-off between the benefits of the positive project NPV and the costs of subsidizing a low type firm. A high type firm strictly prefers debt financing (thus pooling with the low type) if and only if:

$$E_H^d(c(q_0^d)) \geq E_H^\emptyset. \tag{23}$$

By equations (2), (4), and (21), condition (23) holds if and only if:

$$q_0^d \geq p_d = \begin{cases} 1 - \frac{k - Ir}{k(1-\gamma)}, & \text{if } \mu_H \geq k; \\ 1 - \frac{k - Ir}{\mu_H(1-\gamma)}, & \text{if } \mu_H < k. \end{cases} \tag{24}$$

The level of p_d thus establishes a belief threshold, with a high type firm always issuing debt (rather than not investing) if the market belief is sufficiently high.

Lemma 2. *For all market beliefs, a low type firm prefers investment with debt financing to no investment. A high type firm prefers investment with debt financing to no investment if and only if* $q_0^d \geq p_d$.

Lemma 2 is similar to Lemma 1. In both cases, a low type firm always prefers investing, and a high type firm prefers investing only if the corresponding market belief is sufficiently high. This similarity stems from the absence of taxes and default costs in our model.

2.2.3 Equity Financing vs. Debt Financing

We now explore the decision to issue debt or equity. We focus on a high type firm because it determines the type of security issued in the equilibrium. Using (3) and (12), we can write the high type firm's value in the equity issuance case, E_H^e, as:

$$\begin{aligned} E_H^e((\lambda(q_0^e)) &= \big(1 - \lambda(q_0^e)\big)\frac{\mu_H + k}{r} \\ &= \underbrace{\frac{\mu_H}{r}}_{\text{PV assets in place}} + \underbrace{\left(\frac{k}{r} - I\right)}_{\text{NPV of the project}} - \underbrace{I\frac{(1-q_0^e)\mu_H}{q_0^e\mu_H + k}}_{\text{Loss due to underpricing}}. \end{aligned} \tag{25}$$

Similarly, using (4) and (21) we write the high type firm's value in the debt issuance case as:

$$\begin{aligned} E_H^d(c(q_0^d)) &= \gamma \max\left(\frac{\mu_H + K - c(q_0^d)}{r}, 0\right) + (1-\gamma)\max\left(\frac{\mu_H - c(q_0^d)}{r}, 0\right) \\ &= \underbrace{\frac{\mu_H}{r}}_{\text{PV assets in place}} + \underbrace{\left(\frac{k}{r} - I\right)}_{\text{NPV of the project}} - \underbrace{(1-\gamma)(1-q_0^d)\min\left(\frac{\mu_H}{r}, \frac{I}{1-(1-\gamma)(1-q_0^d)}\right)}_{\text{Loss due to underpricing and default}}, \end{aligned} \tag{26}$$

where the $\min(\cdot)$ operator reflects the fact that a high type firm may or may not default.

Hence, the comparison of E_H^d and E_H^e boils down to the comparison between losses from underpricing.

Note that, under debt financing, a high type firm's loss due to underpricing depends on the probability of success γ, but not on the expected payoff k. Under equity financing, the reverse is true. Intuitively, debt holders do not benefit from the upside of the project and are only concerned as to whether or not the coupon is paid in full. New equity holders, by contrast, face no default risk and only care about the NPV of the project. The comparison between E_H^d and E_H^e therefore eventually depends on the interplay between the success probability and the project NPV.

Lemma 3 below compares E_H^d and E_H^e under an additional conjecture, later verified in equilibrium, that the type of financing does not affect market beliefs, that is, $q_0^e = q_0^d = q_0 = p_0$. This comparison proves particularly useful for characterizing the equilibrium in Proposition 1.

Lemma 3. *Define $p_{d/e}$ as:*

$$p_{d/e} = \frac{1}{\mu_H}\left(\frac{Ir}{1-\gamma} - k\right). \tag{27}$$

1. *A high type firm prefers debt to equity if and only if either of the two following conditions holds:*

   (a) $q_0 < p_r$ *and* $q_0 < p_{d/e}$,

   (b) $q_0 > p_r$ *and* $\gamma > \frac{k}{k+\mu_H}$.

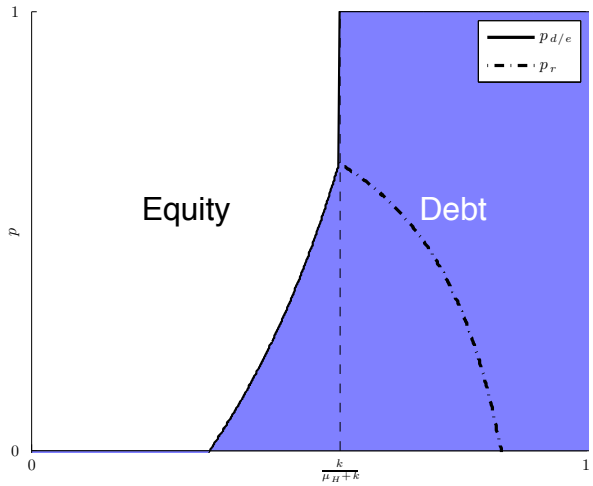
2. *A high type firm prefers equity to debt if and only if either of the two following conditions holds:*

   (a) $q_0 < p_r$ *and* $q_0 > p_{d/e}$,

   (b) $q_0 > p_r$ *and* $\gamma < \frac{k}{k+\mu_H}$.

Figure 1 shows a typical debt-versus-equity choice of a high type firm in the (γ, p_0) space, keeping the NPV of the project $k/r - I$ fixed. We can see that for a fixed p_0, a higher γ favors debt, whereas a lower γ favors equity. On the one hand, if the probability of success, γ, is low, the project is effectively a "gamble:" it is unlikely to succeed, but conditional on success, its return K is large. In this case, debt financing leads to a high probability of default, whereas equity financing assures that the firm keeps its assets in place. Even if the high type firm never defaults on its debt, debt is still inferior to equity because debt holders

Figure 1: Debt vs. Equity Regions



do not participate in the upside K. Equity therefore dominates debt for low values of γ. On the other hand, if γ is large, default and the subsequent loss of assets in place is less likely. Debt then allows a high type firm to lower its financing cost associated with adverse selection. Naturally, as the initial market belief increases, equity becomes less costly because of the lower adverse selection impact. We further illustrate the properties of this trade-off in Section 2.3, where we present the static equilibrium.

2.3 Static Equilibrium

Having analyzed the pairwise trade-offs, we are now ready to state the equilibrium. Proposition 1 characterizes the perfect Bayesian equilibrium of the static model.

Proposition 1. (Static Perfect Bayesian Equilibrium) *There exists a unique perfect Bayesian equilibrium with the following strategies:*

1. *Debt pooling:*

   (a) *If* $p_0 < p_r$, $p_0 < p_{d/e}$, *and* $p_0 > p_d$, *then both types of firms issue debt with coupon* $c^* = \frac{Ir-(1-\gamma)p_0\mu_H}{\gamma}$. *If the project fails, both types default.*

   (b) *If* $p_0 > p_r$, $\gamma > \frac{k}{k+\mu_H}$, *and* $p_0 > p_d$, *then both types of firms issue debt with coupon* $c^* = \frac{Ir}{1-(1-p_0)(1-\gamma)}$. *If the project fails, only a low type firm defaults.*

2. *Equity pooling: If either of the following two conditions is satisfied, both types of firms issue equity with* $\lambda^* = \frac{Ir}{p_0\mu_H+k}$*:*

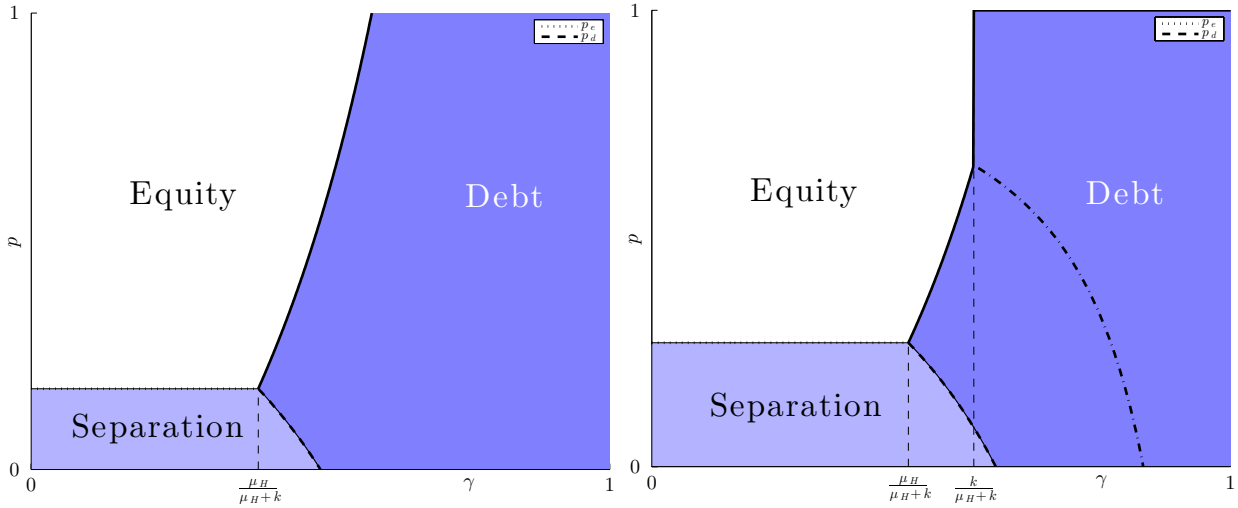
(a) $p_0 < p_r$, $p_0 > p_{d/e}$, *and* $p_0 > p_e$;

(b) $p_0 > p_r$, $\gamma < \frac{k}{k+\mu_H}$, *and* $p_0 > p_e$.

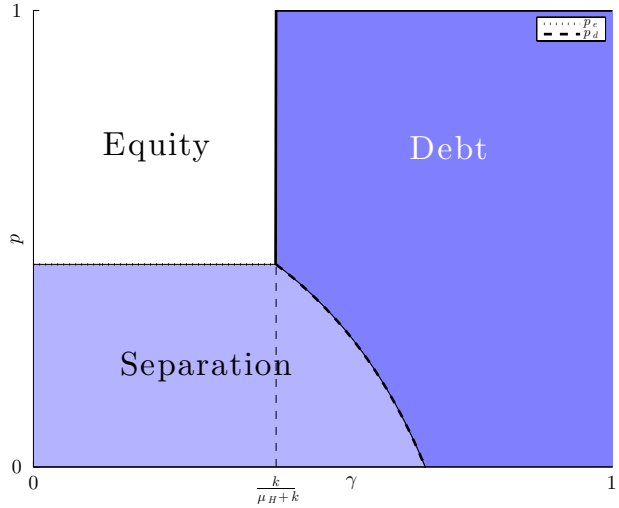
3. *Separation: If* $p_0 < \min(p_e, p_e)$, *only a low type firm invests by issuing equity with* $\lambda^* = \frac{Ir}{k}$.

In all cases, if the firm issues a security that is off-equilibrium, the market belief is that such deviation is done by a low type firm.

Figure 2: Static Investment and Financing Decisions



(a) $\mu_H < Ir < k$ (b) $Ir < \mu_H < k$



(c) $Ir < k < \mu_H$

Figure 2 illustrates the equilibrium strategies of Proposition 1 in the (γ, p_0) space, keeping the NPV of the project $k/r - I$ fixed. There are three cases, depending on the expected cash flow of a high type's assets in place, μ_H. In all cases, a high type firm prefers not to invest when both the market belief and new project's success probability are low. Equity is preferred when the market belief is sufficiently high but the project fails with a high probability. Debt is preferred if the project succeeds with a high probability. Graphically, part (b) of Figure 2 is obtained by "carving out" a separation region—a low belief p_0 and a low success probability γ—out of Figure 1. We emphasize that the high type firm is never able to separate itself from the low type in equilibrium. Intuitively, because there is no deadweight cost of default, the low type firm will always imitate the high type firm. There is no credible "signaling" that the high type can use to prevent the low type from imitating.

In addition to confirming the intuition of Myers and Majluf (1984) that adverse selection can prevent positive NPV investments, Proposition 1 also has a salient feature: the partial violation of the pecking order theory of Myers (1984). We demonstrate that even in a simple economic environment like ours, debt is not necessarily better than equity. Nachman and Noe (1994) characterize a sufficient condition, called "conditional stochastic dominance," under which debt is the optimal security design under asymmetric information. This condition is violated in our model.

Proposition 1 also informs the debt-versus-equity choices of firms with a large asset in place or a large growth option. For example, in Part 1(b) of Proposition 1, the condition $\gamma > k/(k + \mu_H)$ can be rewritten as $\mu_H > k(1 - \gamma)/\gamma$. That is, for fixed γ, if the asset in place, μ_H, is large relative to the expected project value k, then debt is preferred; otherwise equity is preferred.

3 Dynamic Investment and Financing

In this section, we extend the static framework of Section 2 to a fully fledged dynamic environment. The dynamic market introduces two critical changes. First, the firm's manager can flexibly decide the timing of investment: the investment option does not "evaporate" if not taken at date 0. Second, the market learns the true quality of the firm over time even in the absence of investment announcements. In postponing the investment, a high type firm therefore weighs the cost of losing cash flows by not exercising the profitable growth option immediately and the benefit of gaining from the market's upward belief update over

time. A low type firm, by contrast, faces the trade-off between investing immediately, which instantaneously increases its cash flows at the cost of revealing its type, and waiting for a high type firm to invest, which gives it an opportunity to pool. These intricate dynamic trade-offs significantly alter the equilibria of the model. A glossary of key model variables used in this section is also tabulated in the Appendix for the ease of reference.

3.1 Timing of the Dynamic Model

The firm can decide to invest at any time t, $t \geq 0$. The total amount of funds I required for investment must be raised at the same time.[9] The market's belief that it is facing a high (respectively, low) type firm at any time t, conditional on no investment up to time t, is given by p_t (respectively, $1 - p_t$), $0 \leq p_t \leq 1$. If the firm issues equity (debt) at time t, it does so at the quoted price λ_t (c_t). All players' decisions at time t are conditioned on the history of cash flows, market offers, and market beliefs available up to time t. Formally, the history of these variables is adapted to the filtration $\mathcal{G}_t$, generated by the family of random variables $\{X_s, p_s, \lambda_s, c_s;\ s \leq t\}$.

We assume that the cash flows $\{X_t\}$ from assets in place are paid out to the existing shareholders immediately and cannot be accumulated inside the firm. If the firm were able to accumulate cash over time, one would expect the high type firm to have a higher incentive to delay investment, as cash accumulation reduces the required amount of external funds. Therefore, abstracting away from this additional delay incentive is likely to understate the effect of dynamic asymmetric information on investment and financing behavior.[10]

The timing of the dynamic game is as follows:

1. Nature draws the firm's type $\theta \in \{H, L\}$, according to the probability distribution given by the prior $(p_0, 1 - p_0)$, independently from B. The firm observes θ but the market does not.

2. At every moment of time t, the market quotes the prices of debt and equity for the firm in anticipation of immediate investment. In return for funds I, the market demands a fraction λ_t of the firm's total equity or a perpetual debt with coupon payment c_t,

[9] The dynamic equilibrium survives if we allow for several stages of fund raising, because a high type firm finds it optimal to raise the full amount in one stage and a low type firm would be forced to mimic.

[10] Although this assumption is restrictive, a dynamic problem with two state variables—market belief and inside cash—is technically hard to solve explicitly. To illustrate the intuition, however, we provide an explicitly solved two-period model with information asymmetry and cash accumulation in the Online Appendix of this paper.

conditional on the observed history $\mathcal{G}_{t-}$.[11] (Without cash accumulation, the required investment I does not change over time.)

3. The firm's action is defined by a pair of processes $(\pi_t^e(\theta), \pi_t^d(\theta))$.[12] Define

$$\pi_t(\theta) = \pi_t^d(\theta) + \pi_t^e(\theta) \leq 1, \quad \forall\, t \geq 0, \tag{28}$$

where $\pi_t(\theta)$ is the cumulative probability of issuance up to time t, with $\pi_t^d(\theta)$ and $\pi_t^e(\theta)$ being its debt and equity components. As an example, consider the following "pure" strategy used by a high type firm: issue debt with probability 1 as soon as the market's belief p_t hits an upper threshold $\bar{p}$ and never issue equity. The corresponding $(\pi^d(H), \pi^e(H))$ is:

$$\pi_t^d(H) = \begin{cases} 0, & \text{if } \sup\limits_{s \leq t} p_s < \bar{p} \\ 1, & \text{if } \sup\limits_{s \leq t} p_s \geq \bar{p} \end{cases} \qquad \pi_t^e(H) \equiv 0. \tag{29}$$

The general definition of strategies allows the firm to mix every instance between issuing equity, issuing debt, and waiting with "probabilities" $\frac{d\pi_t^e(\theta)}{1-\pi_{t-}(\theta)}$, $\frac{d\pi_t^d(\theta)}{1-\pi_{t-}(\theta)}$, and $1 - \frac{d\pi_t^e(\theta)+d\pi_t^d(\theta)}{1-\pi_{t-}(\theta)}$, respectively.[13] The firm may choose not to invest at all, that is, $\mathbf{P}\left(\lim\limits_{t\to\infty} \pi_t(\theta) = 1\right)$ could be less than 1.

4. If the investment takes place, it is immediately revealed to be successful or unsuccessful. Since the firm's post-investment mean cash flow and coupon obligation (if debt is issued) are both constants, the firm defaults immediately if and only if the mean cash flow is lower than the coupon level. If default occurs, the investors seize the firm's assets without incurring any deadweight loss and the game ends.

[11] As standard, $\mathcal{G}_{t-} = \cup_{s<t}\mathcal{G}_s$.

[12] More rigorously, $\pi^d(\theta)$ and $\pi^e(\theta)$ are cádlág (right continuous with existing left limits) non-decreasing processes adapted to $(\mathcal{G}_t)_{t\geq 0}$ that satisfy $0 \leq \pi_t^d(\theta) + \pi_t^e(\theta) \leq 1$ for all $t \geq 0$. We allow for both flow and jumps in the firm's distribution of investment decisions.

[13] As standard, $\pi_{t-}(\theta)$ stands for the left limit $\lim_{s\uparrow t} \pi_s(\theta)$. The formal definition of $d\pi_t^i(\theta)$ for $i \in \{d, e\}$ is omitted for brevity. We use this notation as a shorthand for two important scenarios: (i) if there is a discrete jump in the probability of investment, then $d\pi_t^i(\theta) = \pi_t^i(\theta) - \pi_{t-}^i(\theta)$; and (ii) if only a high (low) type firm invests with a positive probability, the ratio $d\pi_t^i(H)/d\pi_t^i(L)$ equals to $+\infty$ (0), in which case the exact meaning of $d\pi_t^i(L)$ ($d\pi_t^i(H)$) is irrelevant.

If the firm invests at time t, the payoff to the firm's existing shareholders is:

$$E^e_\theta(\lambda_t) = (1-\lambda_t)\frac{\mu_\theta + k}{r}, \tag{30}$$

$$E^d_\theta(c_t) = \gamma \max\left(\frac{\mu_\theta + K - c_t}{r}, 0\right) + (1-\gamma)\max\left(\frac{\mu_\theta - c_t}{r}, 0\right), \tag{31}$$

and the payoff to the market facing a type θ firm is given by:

$$S^e_\theta(\lambda_t) = \lambda_t \frac{\mu_\theta + k}{r} - I. \tag{32}$$

$$S^d_\theta(c_t) = \gamma \min\left(\frac{c_t}{r}, \frac{\mu_\theta + K}{r}\right) + (1-\gamma)\min\left(\frac{c_t}{r}, \frac{\mu_\theta}{r}\right) - I. \tag{33}$$

Because the market does not observe θ, it has to infer the average quality of the firm attracted by the offer λ_t (or c_t), according to Bayes' rule. The probability of the firm being of high type conditional on investment with debt (or equity) financing is determined from:

$$\frac{q^i_t}{1-q^i_t} = \frac{p_0}{1-p_0} \cdot \frac{\varphi^H_t(X_t)}{\varphi^L_t(X_t)} \cdot \frac{d\pi^i_t(H)}{d\pi^i_t(L)} \quad \text{when} \quad d\pi^i_t(H) + d\pi^i_t(L) > 0, \tag{34}$$

where $\varphi^\theta_t(\cdot)$ is the probability density function of the normal $\mathcal{N}(\mu_\theta t, \sigma^2 t)$ random variable.

The expected payoffs to the market, $S^e(\lambda_t)$ and $S^d(c_t)$, are then given by:

$$S^e(\lambda_t) = [q^e_t S^e_H(\lambda_t) + (1-q^e_t)S^e_L(\lambda_t)] \cdot \mathbb{1}(d\pi^e_t(H) + d\pi^e_t(L) > 0), \tag{35}$$

$$S^d(c_t) = [q^d_t S^d_H(c_t) + (1-q^d_t)S^d_L(c_t)] \cdot \mathbb{1}(d\pi^d_t(H) + d\pi^d_t(L) > 0). \tag{36}$$

3.2 Equilibrium of the Dynamic Model

The nature of dynamic equilibrium depends on the primitive model parameters $(\mu_H, I, r, k, \gamma, \sigma)$ and the structure of the corresponding static equilibrium. For simplicity of exposition, we enumerate various parameter cases below. Each case represents a distinct economic scenario. Definition 6 in the Appendix enumerates these cases in terms of the primitive parameters.

Definition 1. *We define the parameter regions as follows.*

***Case I**: Primitive model parameters satisfy one of the restrictions 1(a), 1(b′), 2(a), 2(b′), or 3(a) of Definition 6. These restrictions are generally satisfied for low and intermediate values of the new project's success probability γ, with the additional restriction that volatility σ is small when γ takes intermediate values.*

Case II: *Primitive model parameters satisfy restriction 1(c) of Definition 6. This condition is satisfied when the project is sufficiently safe but the debt of a high type firm is risky regardless of market beliefs.*

Case III: *Primitive model parameters satisfy restriction 2(c′) or 3(b) of Definition 6. These restrictions are generally satisfied when the project is sufficiently safe, σ is small, and a high type firm never defaults on its debt when the market is sufficiently optimistic.*

Case IV: *Primitive model parameters satisfy restriction 1(b″) or 2(b″) of Definition 6. These conditions are generally satisfied for intermediate values of γ and high values of σ.*

Case V: *Primitive model parameters satisfy restriction 2(c″) of Definition 6. These restrictions are generally satisfied when the project is sufficiently safe, σ is large, and a high type firm never defaults on its debt when the market is sufficiently optimistic.*

Cases I–III give rise to a two-threshold dynamic equilibrium described in Proposition 2. Cases IV and V give rise to a four-threshold dynamic equilibrium characterized in Proposition 4. We start by defining a perfect Bayesian equilibrium in the dynamic model.

Definition 2. *A perfect Bayesian equilibrium of the dynamic model consists of a pair of strategies $(\pi^{d*}(\theta), \pi^{e*}(\theta))$ of the firm and (λ^*, c^*) of the market, and the market belief p, such that:*

1. *For each type θ, the strategy $(\pi^{d*}(\theta), \pi^{e*}(\theta))$ maximizes the value of the firm's original shareholders, given the market's strategy (λ^*, c^*) and beliefs p:*

$$(\pi^d(\theta), \pi^e(\theta)) \in \arg\max_{(\pi^d, \pi^e)} \mathbf{E}\left[\int_0^\infty \left(\int_0^t e^{-ru} dX_u\right) d(\pi_t^e + \pi_t^d) + \int_0^\infty e^{-rt}\left(E_\theta^e(\lambda_t)\, d\pi_t^e + E_\theta^d(c_t)\, d\pi_t^d\right)\right]. \tag{37}$$

2. *Conditional on the firm's investment at time t, the market earns zero profits:*

$$S^e(\lambda_t) = 0 \qquad S^d(c_t) = 0, \tag{38}$$

where the probabilities q_t^e and q_t^d are consistent with $\pi^{e*}(\theta)$ and $\pi^{d*}(\theta)$ in the sense of (34).

3. *There does not exist another pair of quotes $(\tilde{\lambda}, \tilde{c})$ adapted to $\mathcal{G}_{t-}$ that earns strictly positive profits for the market given $\pi^{e*}(\theta)$ and $\pi^{d*}(\theta)$.*

4. *The market belief p is consistent with $(\pi^{d*}(\theta), \pi^{e*}(\theta))$, i.e., it satisfies Bayes' rule along the equilibrium path:*

$$\frac{p_t}{1-p_t} = \frac{p_0}{1-p_0} \cdot \frac{\varphi_t^H(X_t)}{\varphi_t^L(X_t)} \cdot \frac{1-\pi_{t-}^*(H)}{1-\pi_{t-}^*(L)}, \tag{39}$$

where $\varphi_t^\theta(\cdot)$ is the probability density function of the normal $\mathcal{N}(\mu_\theta t, \sigma^2 t)$ random variable.

The firm's value in (37) is decomposed into two components. The first component is the present value of cash flows generated by assets in place that the firm receives until the exercise of the growth option. The second component is the discounted value of the firm's (old) equity after new funds are raised and the growth option is exercised. The sum of these two components is integrated over the equilibrium strategy (π^{e*}, π^{d*}) that determines the timing of the investment and the type of security used.

To explore the evolution of the market's belief, note that as long as no investment has occurred, the market revises its belief based on two sources of information: publicly available cash flows and equilibrium investment strategies. The former gives rise to the non-strategic component of the belief process, while the latter gives rise to the strategic (signaling) one. To disentangle these two components, as well as to simplify exposition, we introduce an auxiliary belief process P_t, which is defined as:

$$\frac{P_t}{1-P_t} = \frac{p_0}{1-p_0} \cdot \frac{\varphi_t^H(X_t)}{\varphi_t^L(X_t)}. \tag{40}$$

The process P_t encapsulates the probability of facing a high type firm, conditional *only* on the observed cash flows up to time t. In other words, P_t represents the non-strategic component of the belief process.

To demonstrate the effect of the second, strategic, source of information, define z_t as $z_t = \ln\left(\frac{p_t}{1-p_t}\right)$ and the corresponding auxiliary process Z_t as $Z_t = \ln\left(\frac{P_t}{1-P_t}\right)$. Then, equations (39) and (40) can be rewritten as:

$$z_t = Z_t + \ln\left(\frac{1-\pi_{t-}^*(H)}{1-\pi_{t-}^*(L)}\right), \tag{41}$$

$$Z_t = \ln\left(\frac{p_0}{1-p_0}\right) + \frac{\mu_H - \mu_L}{\sigma^2}\left(X_t - \frac{\mu_H + \mu_L}{2}t\right). \tag{42}$$

The second term in (41) captures the pure signaling effect on the market's belief. Obviously,

$Z_t = z_t$ as long as there is no investment (i.e., if $\pi_t(H) = \pi_t(L) = 0$).

As Z_t is monotone in P_t and z_t is monotone in p_t, we can also refer to Z_t and z_t as the market's "belief" processes. The market's belief process Z_t does not depend on the firm's type. The firm, however, knows its type, and the manager observes the "true" belief process Z_t, denoted Z_t^θ. Substituting (1) into (42), we can write:

$$dZ_t^H = \frac{h^2}{2}dt + hdB_t, \tag{43}$$

$$dZ_t^L = -\frac{h^2}{2}dt + hdB_t, \tag{44}$$

where $h = (\mu_H - \mu_L)/\sigma$. Path by path, Z_t coincides with Z_t^H if the firm is of the high type, while it coincides with Z_t^L if the firm is of the low type. At any instance s, the market's conditional distribution of future realizations of Z_t, $t > s$, is given by (42), the high type's corresponding distribution of Z_t^H is given by (43), and the low type's conditional distribution of Z_t^L is given by (44). This difference in the assessment of future conditional distributions is *the* essential source of *dynamic* information asymmetry.

The advantage of using Z_t is that it is the only state variable that affects decision-making. Note that (42) establishes the linear relation between three potential state variables: time t, cumulative cash flow X_t, and belief Z_t. Cash flow X_t and time t affect strategies only through their linear combination $X_t - (\mu_H + \mu_L)t/2$. Thus, the dependence of Z_t on X_t and t is implicit.

Following Daley and Green (2012), we introduce belief processes and a strategy profile $\Xi^i(\underline{z}, \bar{z})$, where $(\underline{z}, \bar{z})$, $\underline{z} < \bar{z}$ is a pair of threshold beliefs and $i \in \{d, e\}$ denotes whether the two types of firms pool at the upper threshold $\bar{z}$ with debt (superscript "d") or with equity (superscript "e"), whenever pooling occurs.

Definition 3. *For $i \in \{d, e\}$ and for each pair of real numbers $(\underline{z}, \bar{z})$, where $\underline{z} < \bar{z}$, let $\Xi^i(\underline{z}, \bar{z})$ be a tuple of strategy profiles and belief processes such that:*

1. *The market's belief, conditional on not observing issuance up to time t, is $p_t = e^{z_t}/(1 + e^{z_t})$.*

2. *If $i = d$ (i.e., pooling with debt), then the strategies of a type θ firm are given by:*

$$\pi_t^d(H) = \mathbb{1}(z_t \geq \bar{z}), \qquad \pi_t^e(H) \equiv 0; \tag{45}$$

$$\pi_t^d(L) = (1 - \pi_t^e(L)) \cdot \mathbb{1}(z_t \geq \bar{z}), \qquad \pi_t^e(L) = 1 - e^{-Y_t}, \tag{46}$$

where $\mathbb{1}(\cdot)$ *is the indicator function and*

$$Y_t = \max\left(\underline{z} - \inf_{s\le t} Z_s, 0\right). \tag{47}$$

3. *If* $i = e$ *(i.e., pooling with equity), then the strategies of a type* θ *firm are given by:*

$$\pi_t^e(H) = \mathbb{1}(z_t \ge \bar{z}), \qquad \pi_t^d(H) \equiv 0; \tag{48}$$

$$\pi_t^e(L) = \begin{cases} 1, & \text{if } \sup_{s\le t} z_s \ge \bar{z}; \\ 1 - e^{-Y_t}, & \text{if } \sup_{s\le t} z_s < \bar{z}; \end{cases} \qquad \pi_t^d(L) \equiv 0. \tag{49}$$

4. *The market offers at time* t *correspond to the beliefs:*

$$q_t^i = p_t \cdot \mathbb{1}(z_t \ge \bar{z}), \qquad q_t^{-i} \equiv 0, \tag{50}$$

where notation "$-i$*" refers to the security other than* i*.*

In other words, this strategy is a "two-threshold strategy." If the market belief process z_t reaches or goes above an upper threshold $\bar{z}$, both types of firms instantaneously pool with debt or pool with equity with probability one. If z_t reaches or goes below a lower threshold $\underline{z}$, a high type firm does not issue anything, but the low type issues equity with some positive probability. Recall that according to our tie-breaking rules, a low type firm separates by equity. The equilibrium of Proposition 2 has this structure.

To concentrate on the cases that are most interesting economically, in the rest of the paper we assume that $p_e > 0$ and $p_d > 0$. This assumption guarantees that immediate investment at time zero by both firm types is not an equilibrium, and delay is most valuable for the high type firm.

Proposition 2 states a stationary perfect Bayesian equilibrium of the dynamic model.

Proposition 2. (Two-Threshold Perfect Bayesian Equilibrium)

1. ***Equity pooling equilibrium.*** *If the primitive model parameters are as in Case I of Definition 1, then there exists a unique pair* $(\underline{z}^*, \bar{z}^*)$*,* $\underline{z}^* < \bar{z}^*$*, such that* $\Xi^e(\underline{z}^*, \bar{z}^*)$ *is a perfect Bayesian equilibrium with:*

$$\lambda^*(z_t) = \begin{cases} \frac{Ir}{\mu_H + k - \mu_H/(1+e^{z_t})}, & \text{if } z_t \ge \bar{z}^*; \\ \frac{Ir}{k}, & \text{if } z_t < \bar{z}^*; \end{cases} \qquad c^*(z_t) = \frac{Ir}{\gamma}. \tag{51}$$

In this equilibrium, debt is never issued and $\bar{z}^* > z_{d/e}$.

2. ***Debt pooling equilibrium in which both types can default.*** *If the primitive model parameters are as in Case II of Definition 1, then there exists a unique pair* $(\underline{z}^*, \bar{z}^*)$, $\underline{z}^* < \bar{z}^*$, *such that* $\Xi^d(\underline{z}^*, \bar{z}^*)$ *is a perfect Bayesian equilibrium with:*

$$c^*(z_t) = \begin{cases} \frac{Ir-(1-\gamma)e^{z_t}\mu_H/(1+e^{z_t})}{\gamma}, & \text{if } z_t \geq \bar{z}^*; \\ \frac{Ir}{\gamma}, & \text{if } z_t < \bar{z}^*; \end{cases} \qquad \lambda^*(z_t) = \frac{Ir}{k}. \tag{52}$$

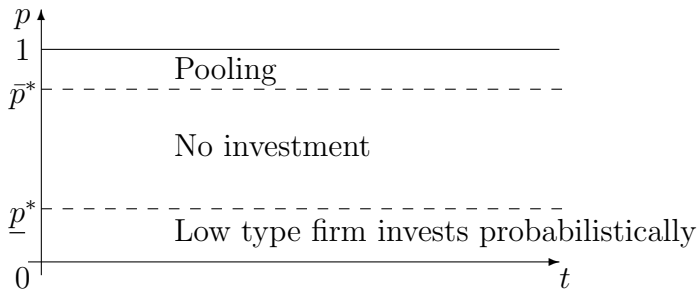
In this equilibrium, if the project fails, both types of firms default.

3. ***Debt pooling equilibrium in which only the low type can default.*** *If the primitive model parameters are as in Case III of Definition 1, then there exists a unique pair* $(\underline{z}^*, \bar{z}^*)$, $\underline{z}^* < \bar{z}^*$, *such that* $\Xi^d(\underline{z}^*, \bar{z}^*)$ *is a perfect Bayesian equilibrium with:*

$$c^*(z_t) = \begin{cases} \frac{Ir}{1-(1-\gamma)/(1+e^{z_t})}, & \text{if } z_t \geq \bar{z}^*; \\ \frac{Ir}{\gamma}, & \text{if } z_t < \bar{z}^*; \end{cases} \qquad \lambda^*(z_t) = \frac{Ir}{k}. \tag{53}$$

In this equilibrium, if the project fails, only a low type firm defaults (i.e., $\bar{z}^* > z_r$*).*

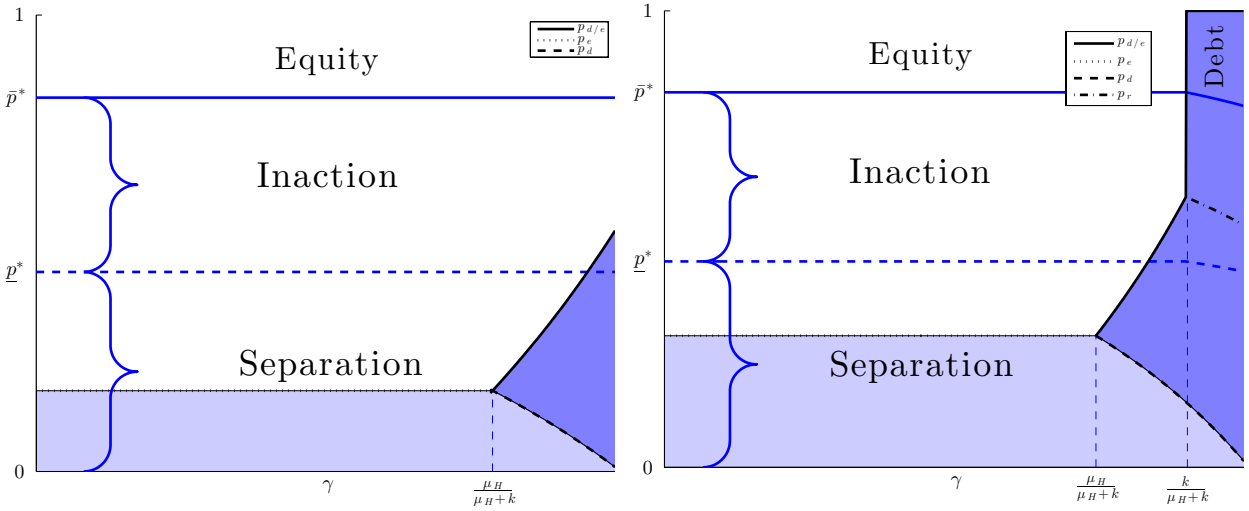
Figure 3: Equilibrium Beliefs with Reflecting Barrier at $\underline{p}^*$



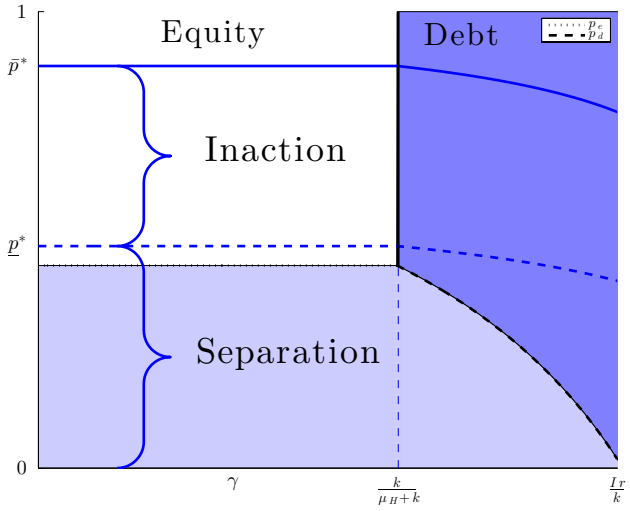
The equilibrium described in Proposition 2 is a partial pooling one. (In Section 3.4 we characterize economically reasonable selection criteria for uniqueness.) Figure 3 shows the generic equilibrium regions that depend on the market's belief, p.[14] Figure 4 illustrates

[14]We can also use the original belief variable p to describe the equilibrium regions because there is a one-to-one correspondence between the thresholds $(\underline{z}^*, \bar{z}^*)$ and the thresholds $(\underline{p}^*, \bar{p}^*)$.

Figure 4: Dynamic Equilibrium Strategies. The figure overlays the dynamic strategies on top of the static strategies plotted in Figure 2. The curly brackets indicate the dynamic inaction region and the dynamic separation region; the region above the dynamic inaction region is the dynamic pooling region (with debt or equity). The figure is plotted for parameters satisfying $p_d > 0$ and $p_e > 0$ (i.e., truncating parameters with high γ in the (γ, p_0) space).



(a) $\mu_H < Ir < k$ (b) $Ir < \mu_H < k$



(c) $Ir < k < \mu_H$

the dynamic strategies and the behaviors of the thresholds $(\bar{p}^*, \underline{p}^*)$, where the three subplots correspond one-to-one to Figure 2 of the static model.

When the market's belief reaches the upper threshold $\bar{p}^*$ (i.e., when outside investors become sufficiently optimistic about the firm's quality), both types invest by issuing equity

or debt. The upper barrier $\bar{p}^*$ is essentially chosen by a high type firm to maximize its value; a low type firm always imitates the high type. The trade-off between equity and debt, conditional on investment, is similar to that in the static setting. For a fixed NPV, a higher probability of success γ favors debt, whereas a lower γ favors equity. The debt coupon also depends on whether a high type firm defaults upon project failure. Holding the NPV fixed, conditional on equity issuance, the upper threshold $\bar{p}^*$ does not vary with γ; however, conditional on debt issuance, $\bar{p}^*$ varies with γ.

When the market's belief reaches $\underline{p}^*$, only a low type firm invests. By (46) and (49), it does so probabilistically. The lower barrier is chosen so that a low type firm is indifferent between investing now (and thus revealing its type) and postponing investing with the hope that positive shocks would lead the market's belief to hit the upper boundary $\bar{p}^*$ in the future. The equilibrium rate of mixing by a low type firm at the lower boundary forces the beliefs to be *reflecting*. That is, conditional on not observing investment at $\underline{p}^*$, the market's belief immediately adjusts upwards, because a high type firm would never invest at the low threshold. By (46) and (49), we can verify that in the equilibrium, z_t is indeed the reflected version of Z_t at $\underline{z}^*$:

$$z_t = Z_t + Y_t = Z_t + \max\left(\underline{z}^* - \inf_{s \le t} Z_s, 0\right). \tag{54}$$

For all the market belief levels between the two thresholds, $\bar{p}^*$ and $\underline{p}^*$, there is a region of optimal inaction in which both firm types postpone their decisions: a high type firm expects the market's belief to become more optimistic, which reduces underpricing, while a low type firm speculates on positive shocks to its cash flows and higher overpricing of its (yet to be issued) securities. This region of inaction is a new feature of the dynamic financing problem and resembles the optimal economic behavior in many real option contexts, in which firms face transaction costs of adjustment (e.g., see Stokey (2009)). The key difference from the real option models is that delay in our models comes entirely from time-varying information asymmetry.

The equilibrium thresholds $(\underline{z}^*, \bar{z}^*)$ can be shown to be unique, but their expressions are not in closed form. Nonetheless, the model is tractable enough to sign the comparative statics with respect to the volatility level, σ. Volatility is an important parameter for the dynamic model because it controls the accuracy of cash flow information and hence the firm's incentive to delay.

Proposition 3. (Behavior of Thresholds $\bar{p}^*$ and $\underline{p}^*$)

1. *As $\sigma \to 0$ and if the primitive model parameters are as in Proposition 2, then the*

equilibrium thresholds $\bar{p}^* \to 1$ *and* $\underline{p}^* \to 1/2$.

2. *As* $\sigma \to +\infty$*:*

   (a) *If the primitive model parameters are as described in Part 1 of Proposition 2, then the equilibrium thresholds* $\bar{p}^* \to p_e$ *and* $\underline{p}^* \to p_e$.

   (b) *If the primitive model parameters are as described in Parts 2 and 3 of Proposition 2, then the equilibrium thresholds* $\bar{p}^* \to p_d$ *and* $\underline{p}^* \to p_d$.

As discussed in Section 2, the thresholds p_e and p_d are strategically chosen by a high type firm on the basis of the trade-off between investing in a positive NPV project and reducing underpricing caused by asymmetric information about the assets in place. In the dynamic setup, a high type firm can reduce the underpricing of assets by waiting and investing in the positive NPV project at a later date. As $\sigma \to \infty$, the realized cash flows become increasingly uninformative, and investors can no longer learn the quality of assets over time. Without learning, the dynamic environment is very similar to the static one of Section 2, and the dynamic thresholds, $\bar{p}^*$ and $\underline{p}^*$, converge to the static threshold, p_e or p_d.

Conversely, as the cash flows become infinitely informative (i.e., as $\sigma \to 0$), the true type of the firm is revealed immediately. A high type firm sets the highest upper threshold (i.e., $\bar{p}^* \to 1$) to avoid underpricing. The less obvious limiting behavior of $\underline{p}^*$ is the result of two countervailing forces. On the one hand, a low type firm wishes to decrease its separation threshold because pooling becomes more attractive. On the other hand, it wishes to increase its separation threshold because a high type firm's behavior implies a longer waiting time. In equilibrium, these two forces exactly offset each other, making the lower threshold $\underline{p}^*$ converge to $1/2$.

3.3 Four-Threshold Equilibrium

In this subsection, we characterize a dynamic equilibrium for Cases IV–V of Definition 1. In these regions of the model parameters, the volatility σ is "high" and the dynamic equilibrium has a different structure: instead of two thresholds, it has four thresholds.

Before a formal analysis, it is useful to discuss intuitively why the two-threshold equilibrium no longer applies for sufficiently high volatility. Consider, for example, Case I of Definition 1, and increase σ. As σ increases, the benefits of waiting for a high type firm deteriorate, because the cash flows become less informative. As the noisiness of cash flows increases, the dynamic payoff $V_H(z)$ from following $\Xi^e(\underline{z}^*, \bar{z}^*)$ of Proposition 2 decreases;

thus, so does the upper threshold, $\bar{z}^*$. In the limiting case, as $\sigma \to \infty$, there is no incentive to delay, so the region of investment should be exactly the same as in the static equilibrium. It is tempting to conclude, by the argument of continuity, that the dynamic equilibrium should be similar to the static one, at least qualitatively. This conclusion, however, does not hold. Lemma 4 shows that there is an important discontinuity point within the static investment region, at which a high type firm always delays investment for any finite σ.

Lemma 4. *Suppose that market beliefs and the low type firm's strategy are given by $\Xi^e(\underline{z}, \bar{z})$ for some thresholds $(\underline{z}, \bar{z})$. If equity is the equilibrium security in the pooling region, a high type firm never invests at $z_t = z_{d/e}$. If debt is the equilibrium security in the pooling region, a high type firm never invests at $z_t = z_r$.*

The technical reason behind Lemma 4 is that a high type firm's static value function has a kink at $z_t = z_{d/e}$ (the threshold between risky debt and equity) and at $z_t = z_r$ (the threshold between relatively risky debt and relatively safe debt). At $z_t = z_{d/e}$, for example, a high type firm prefers to wait and get an average of $E_H^e(z_{d/e}+\epsilon)$ and $E_H^d(z_{d/e}-\epsilon)$, rather than collect $E_H^e(z_{d/e}) = E_H^d(z_{d/e})$ immediately. Therefore, regardless of σ, there exists an inaction region around $z_{d/e}$, as long as issuing equity is the equilibrium strategy for z_t sufficiently above $z_{d/e}$. Similarly, a high type firm does not invest at $z_t = z_r$ as long as issuing debt on which a high type firm never defaults is the equilibrium strategy for the values of z_t sufficiently above z_r.

What does Lemma 4 say about the structure of the dynamic equilibrium? Intuitively, it implies that for sufficiently high σ, $p = p_{d/e}$ (or $p = p_r$) can separate the investment regions into two, generating a new inaction region around $p = p_{d/e}$ (or $p = p_r$). This gives rise to a four-threshold equilibrium, as characterized in Proposition 4.

Proposition 4. (Four-Threshold Perfect Bayesian Equilibrium)

1. ***Debt and Equity Pooling.*** *If the primitive model parameters are as in Case IV of Definition 1, there exists a unique four-tuple $(\underline{z}^*, z_l^*, z_h^*, \bar{z}^*)$, where $\underline{z}^* < z_l^* < z_h^* < \bar{z}^*$, such that the following is a perfect Bayesian equilibrium:*

   (a) The strategies of a type θ firm are given by:

$$\pi_t^d(H) = \mathbb{1}(z_t \in [z_l^*, z_h^*]), \qquad \pi_t^e(H) = \mathbb{1}(z_t \geq \bar{z}^*); \tag{55}$$

$$\pi_t^d(L) = (1 - \pi_{t-}^e(L)) \cdot \mathbb{1}(z_t \in [z_l^*, z_h^*]), \quad \pi_t^e(L) = \begin{cases} 1, & \text{if } z_t \geq \bar{z}^*; \\ 1 - e^{-Y_t}, & \text{if } z_t < \bar{z}^*; \end{cases} \tag{56}$$

where $Y_t = \max\left(\underline{z}^* - \inf_{s\le t} Z_s, 0\right)$.

(b) The market demands a share $\lambda^*(z_t)$ *of the firm's equity or a perpetuity with coupon* $c^*(z_t)$*, where* $\lambda^*(z_t)$ *and* $c^*(z_t)$ *are given by:*

$$\lambda^*(z_t) = \begin{cases} \frac{Ir}{\mu_H + k - \mu_H/(1+e^{z_t})}, & \text{if } z_t \ge \bar{z}^*; \\ Ir/k, & \text{if } z_t < \bar{z}^*; \end{cases} \quad c^*(z_t) = \begin{cases} \frac{Ir - (1-\gamma)e^{z_t}\mu_H/(1+e^{z_t})}{\gamma}, & \text{if } z_t \in [z_l^*, z_h^*]; \\ Ir/\gamma, & \text{if } z_t \notin [z_l^*, z_h^*]; \end{cases} \tag{57}$$

with corresponding beliefs:

$$q_t^e = p_t \cdot \mathbb{1}(z_t \ge \bar{z}^*), \qquad q_t^d = p_t \cdot \mathbb{1}(z_t \in [z_l^*, z_h^*]). \tag{58}$$

In this equilibrium, $z_h^* < z_{d/e} < \bar{z}^*$.

2. ***Debt Pooling.*** *If the primitive model parameters are as in Case V of Definition 1, there exists a unique four-tuple* $(\underline{z}^*, z_l^*, z_h^*, \bar{z}^*)$*, where* $\underline{z}^* < z_l^* < z_h^* < \bar{z}^*$*, such that the following is a perfect Bayesian equilibrium:*

(a) The strategies of a type θ *firm are given by:*

$$\pi_t^d(H) = \mathbb{1}(z_t \in [z_l^*, z_h^*] \text{ or } z_t \ge \bar{z}^*), \qquad \pi_t^e(H) \equiv 0; \tag{59}$$

$$\pi_t^d(L) = (1 - \pi_{t-}^e(L)) \cdot \mathbb{1}(z_t \in [z_l^*, z_h^*] \text{ or } z_t \ge \bar{z}^*), \quad \pi_t^e(L) = 1 - e^{-Y_t}, \quad \text{if } z_t < \bar{z}^*; \tag{60}$$

where $Y_t = \max\left(\underline{z}^* - \inf_{s\le t} Z_s, 0\right)$.

(b) The market demands a share $\lambda^*(z_t)$ *of firm's equity or a perpetuity with coupon* $c^*(z_t)$*:*

$$\lambda^*(z_t) = Ir/k, \quad c^*(z_t) = \begin{cases} \frac{Ir}{1-(1-\gamma)/(1+e^{z_t})}, & \text{if } z_t \ge \bar{z}^*; \\ \frac{Ir - (1-\gamma)e^{z_t}\mu_H/(1+e^{z_t})}{\gamma}, & \text{if } z_t \in [z_l^*, z_h^*]; \\ Ir/\gamma, & \text{otherwise ;} \end{cases} \tag{61}$$

with corresponding beliefs

$$q_t^e \equiv 0, \qquad q_t^d = p_t \cdot \mathbb{1}(z_t \in [z_l^*, z_h^*] \text{ or } z_t \ge \bar{z}^*). \tag{62}$$

In this equilibrium, $z_h^* < z_r < \bar{z}^*$.

The equilibrium described in Proposition 4 has two pooling regions, two inaction regions, and one separating region. The reasons for waiting in the two inaction regions are entirely different. In the lower inaction region, $z_t \in (\underline{z}^*, z_l^*)$, a high type firm waits to avoid heavy underpricing. It anticipates good news and plans to invest later at more favorable terms. Waiting in the upper inaction region, $z_t \in (z_h^*, \bar{z}^*)$, arises from the kink of the static value function at $z_{d/e}$ (or z_r); in this region, a high type firm essentially "gambles" and waits for news. Good news results in lower underpricing and issuance of equity or debt, while bad news results in debt issuance under less favorable terms. Note that if debt is issued in the upper region, a high type firm never defaults on it; however, if debt is issued in the lower region, a high type firm defaults on it upon project failure. Therefore, the securities issued in the two pooling regions are different as well.

Figure 5 illustrates the equilibrium strategies of Proposition 4. In Panel (a), the upper pooling region has equity, while the lower pooling region has (risky) debt. For a sufficiently low σ, the equilibrium is given by Proposition 2, whereas for a sufficiently high σ, the equilibrium is given by Proposition 4.[15] A particular feature of Figure 5 is that once σ crosses the threshold $\hat{\sigma}$, the lower pooling region with debt appears "discontinuously," expanding from an empty set to covering a positive measure of beliefs. Associated with it is the discrete downward "jump" of the lower threshold $\underline{z}^*$. These discontinuities have simple intuition. Once a high type firm starts waiting, a low type firm does as well. If the current market belief p_t is below this new lower pooling region, the upper pooling region is never reached in equilibrium, which implies that the expected waiting time to pooling is reduced discontinuously. A strictly shorter waiting time to pooling reduces the incentives of a low type firm to separate, implying a discontinuous downward adjustment of $\underline{z}^*$. Panel (b), in which the upper pooling region has relatively safe debt and the lower pooling region has relatively risky debt, has the same intuition.

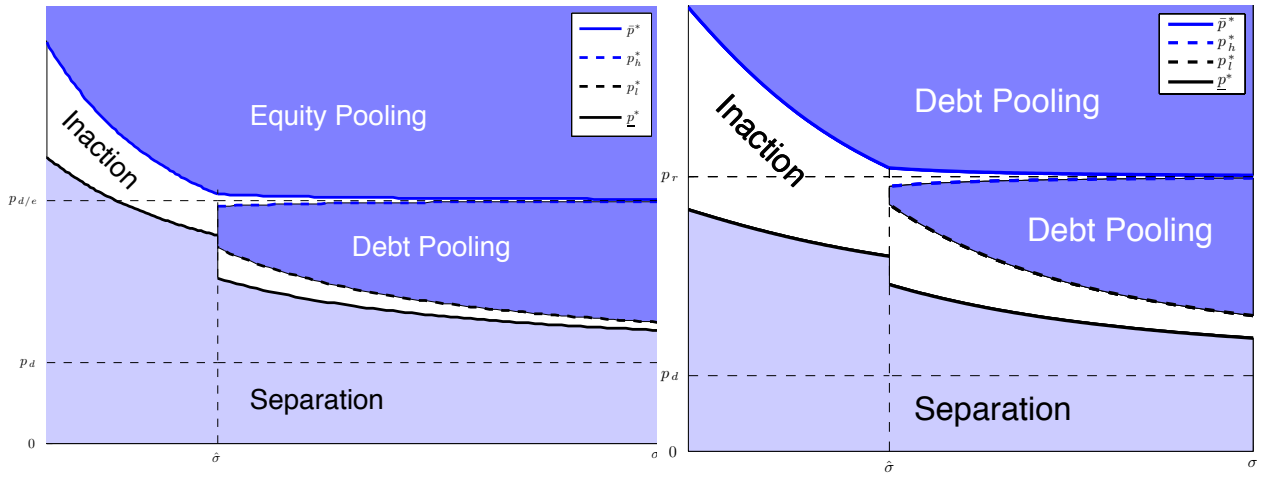
Proposition 5. (Behavior of Thresholds $\bar{p}^*$, p_h^*, p_l^* and $\underline{p}^*$) *As* $\sigma \to +\infty$*:*

1. *If the primitive model parameters are as in Part 1 of Proposition 4, then the equilibrium thresholds* $\bar{p}^*$ *and* p_h^* *converge to* $p_{d/e}$*, whereas* p_l^* *and* $\underline{p}^*$ *converge to* p_d*.*

2. *If the primitive model parameters are as in Part 2 of Proposition 4, then the equilibrium thresholds* $\bar{p}^*$ *and* p_h^* *converge to* p_r*, whereas* p_l^* *and* $\underline{p}^*$ *converge to* p_d*.*

[15] While the proof for the equilibrium for intermediary ranges of σ is not yet obtained, numerical solutions show that the two regions are connected.

Figure 5: Equilibrium Thresholds for Different σ. To the left of the cutoff volatility $\hat{\sigma}$, we plot the two-threshold equilibrium of Proposition 2. To the right of $\hat{\sigma}$, we plot the four-threshold equilibrium of Proposition 4.



(a) Part 1 of Proposition 4

(b) Part 2 of Proposition 4

Similar to the results of Proposition 3, as $\sigma \to \infty$, the cash flows become increasingly uninformative, and the incentives to wait become weaker, leading to the collapse of both waiting regions.

3.4 Equilibrium Selection

Dynamic games typically have multiple equilibria. In our model, this multiplicity problem is amplified by the presence of asymmetric information; therefore, characterizing all possible perfect Bayesian equilibria is infeasible. Instead, in what follows, we describe two restrictions—*stationarity* and *belief monotonicity*—on strategies and off-equilibrium beliefs and show that equilibrium established by Proposition 2 is unique given such refinement.

The first restriction, stationarity, is dictated by tractability. Due to the lack of mathematical apparatus to deal with arbitrary path-dependent strategies, we restrict attention to stationary strategies only.[16,17] In the current setting, this restriction is natural because

[16] Although we confine the strategies used in the construction of equilibrium to be stationary, we place no restrictions on the set of deviations. In particular, equilibria derived in Propositions 2 and 4 survive against arbitrary non-stationary deviations.

[17] Because the main objective of our analysis is to compare the static market with a dynamic one, focusing only on stationary strategies is likely to understate the importance of dynamics in determining investment and capital formation. Nonetheless, as we have demonstrated in Propositions 2 and 4, our dynamic equilibrium with stationary strategies already has substantially different implications from its static counterpart; allowing

the dynamics of our model are driven by the time-homogeneous Markov cash flow process $\{X_t\}_{t\geq 0}$.

Definition 4. (Stationary Equilibrium) *An equilibrium is stationary if:*

(i) The market belief p is a time-homogeneous Markov process; and

(ii) The market pricing rules λ^* and c^* are functions of p_t only.

Stationarity substantially shrinks the set of possible equilibria by placing restrictions on strategies along the equilibrium path. Off the equilibrium path, however, stationarity has limited power and does not eliminate economically unreasonable beliefs. For example, if the market "threatens" the firm that it believes the firm is of the low type with probability 1 if there is no issuance at $t = 0$, we are back to the static equilibrium. Such beliefs, however, fail the following intuitive forward induction reasoning: if the market does not observe equity issuance at $t = 0$, this deviation is more likely to be caused by a high type firm (because a low type firm always prefers to invest under any belief). This reasoning suggests that the market's conditional probability that the firm is of the high type, p_t, should increase right after $t = 0$, if there is no issuance. This, in turn, creates incentives for a low type firm to delay investment.

To exclude such economically unreasonable equilibria, we require beliefs to be *monotone*. This refinement is a natural extension of the Divinity refinement (Banks and Sobel (1987)) to the continuous-time setting and has been used by Daley and Green (2012) and Gul and Pesendorfer (2012).

Definition 5. (Monotone Beliefs) *A belief process p is monotone if for all $t > s$:*

$$p_t \geq \frac{\varphi^H_{t-s}(X_t - X_s)p_s}{\varphi^H_{t-s}(X_t - X_s)p_s + \varphi^L_{t-s}(X_t - X_s)(1 - p_s)}. \tag{63}$$

Notice that if we replace p with P in (63), it would hold with equality. In other words, the belief monotonicity condition requires the market belief p_t to be at least as high as a non-strategic posterior P_t, given that they start from the same point, $P_s = p_s$, in the absence of issuance between s and t. Hence, the investment delay is interpreted as a signal, which is more likely to be sent by a high type firm.

Stationarity, together with competitiveness of the market, implies that at any moment in time only one of the two offers (λ_t, c_t) from the market is relevant: either the offer selected

nonstationary, time-dependent strategies is likely to make the difference starker.

by a high type firm (and hence also by a low type firm) or the equity offer selected only by a low type firm. The former case corresponds to a pooling equilibrium, while the latter corresponds to the separating equilibrium. Such behavior implies that (63) is satisfied along the equilibrium path. The monotone-beliefs refinement requires that this condition hold off the equilibrium path as well.

Both the four-threshold and two-threshold equilibria introduced earlier are stationary and satisfy belief monotonicity. Proposition 6 shows the uniqueness of equilibrium in Proposition 2, given the two refinements discussed above.

Proposition 6. (Equilibrium Uniqueness) *If primitive model parameters are as stated in Proposition 2, then the equilibrium of Proposition 2 is the unique stationary perfect Bayesian equilibrium with monotone belief process p.*

Thus, the equilibrium derived in Proposition 2 is the only reasonable equilibrium within a broad class of equilibria that admit only stationary strategies.[18] Instead of proving this result directly, we refer readers to Daley and Green (2012)'s Theorem 5.1. Using essentially the same technique, one could show that the uniqueness of the equilibrium in their model implies the uniqueness of the equilibrium for parametric cases of Proposition 2. While the uniqueness proof is similar, we note again that the equilibrium of Daley and Green (2012) is essentially equity-only and does not consider debt.

3.5 Extension: Stochastic Decay of the Investment Opportunity

So far in our analysis, either the project evaporates immediately if not taken at time zero, or it does not evaporate at all. In reality, however, the firm cannot delay the investment decision indefinitely. The project could become unavailable or infeasible for a wide variety of reasons, including technology becoming obsolete, competitors gaining a first-mover advantage, or regulatory intervention. In this section, we model the rate of decay of the investment opportunity and study its effect on the dynamic equilibrium.

Suppose that the investment option evaporates with time-invariant intensity, $\delta > 0$. That is, conditional on no investment up to time t, the project evaporates with probability δdt in the next instance of time, dt; it remains available at time $t + dt$ with probability $1 - \delta dt$. The static model of Section 2 corresponds to one extreme case of $\delta = +\infty$ and the dynamic model of this section so far corresponds to the other extreme case, $\delta = 0$.

[18] Proposition 6 also holds for a more general definition of strategies that includes off-equilibrium behavior. The resulting equilibrium is equivalent to the one described in Proposition 2 on the equilibrium path.

Proposition 7 characterizes the dynamic equilibrium with a stochastic decay.

Proposition 7. ***(Stochastic Decay)*** *Given the primitive model parameters and $\delta > 0$, define:*

$$\tilde{\sigma} = \sigma\sqrt{1 + \frac{\delta}{r}}. \tag{64}$$

Then, the dynamic equilibrium with (σ, δ) is equivalent to the dynamic equilibrium with $\tilde{\sigma}$ and $\delta = 0$ characterized in Propositions 2 and 4.

The introduction of a positive rate of decay reduces the incentives to delay investment for both types of firms. Intuitively, an increase in δ has a similar effect on the equilibrium strategies as an increase in σ does, since the latter makes it more difficult for the market to separate the two firm types based on the observed cash flows, which reduces the option value to wait. Proposition 7 makes this intuition precise and shows that by properly adjusting the volatility of the cash flows, one can compute the equilibrium with a stochastic decay based on the equilibrium with an infinitely lived project. In light of this result, the comparative statics with respect to δ have the same sign as those with respect to σ.

4 Discussion

In this section, we explore the economic implications of the dynamic equilibrium and compare them to the implications of the static equilibrium of Myers and Majluf (1984). We first examine the effect of dynamics on social welfare. Then, we discuss the empirical implications of our results, including firm market values, investment delay, and security choices, among others.

4.1 Firm Values and Social Welfare

In the static market, adverse selection can lead to a welfare loss because a high type firm forgoes a positive-NPV project because of the low market prices of its securities. In the dynamic market, the project is eventually implemented, but the welfare loss comes from the endogenous delay by both types of firms. In this subsection, we explore the net welfare effect of delayed investment.

As a first step, we analyze the private benefits and costs of the two types of firms. For the purpose of this section, we let $E_\theta(p)$ and $V_\theta(p)$ be the values of a type θ firm (i.e., its existing shareholders), given the belief p, in the static and dynamic models, respectively.

The dynamic value function V_θ is given by Proposition 2 or Proposition 4, depending on the primitive model parameters. Proposition 8 compares firm values in both environments.

Proposition 8. (Static and Dynamic Firm Values) *The values of the firm's existing shareholders, $E_\theta(p)$ and $V_\theta(p)$, satisfy the following:*

1. *For a high type firm:*
   (a) $V_H(p) = E_H(p)$ *in the dynamic pooling region;*
   (b) $V_H(p) > E_H(p)$ *in the dynamic inaction region.*
2. *For a low type firm:*
   (a) $V_L(p) = E_L(p)$ *in the dynamic pooling region and in the intersection between the dynamic and static separating regions;*
   (b) $V_L(p) < E_L(p)$ *in the intersection between the dynamic inaction region and the static pooling region;*
   (c) $V_L(p) > E_L(p)$ *in the intersection between the dynamic inaction region and the static separating region.*

A high type firm never prefers the static world, because it gains from an option to delay. It is a high type firm that effectively chooses the lower boundary of the dynamic pooling region. The value of waiting is zero if market belief is very high, in which case immediate investment is optimal anyway. The low type firm's preference is slightly subtler. If the market belief is very high or very low, the dynamic equilibrium dictates immediate investment, either by pooling or separation. In this case, investment delay has zero value. If delay allows a low type firm to avoid separation, delay has a positive value. If, however, delay prevents immediate pooling by a low type firm with the high type, then the option to delay harms a low type firm. The last case occurs under an intermediate belief of the market, which is arguably the most interesting case. Therefore, the net welfare implications of options to delay become ambiguous, as delay benefits the high type firm and harms the low type firm.

To study the net effect of the delay option, we let E and V be the unconditional firm values in the static and the dynamic economies, respectively. That is, $E(p) = pE_H(p) + (1-p)E_L(p)$ and $V(p) = pV_H(p) + (1-p)V_L(p)$. These values summarize the social welfare, and a comparison between them enables us to explore welfare implications. Proposition 9 shows that adding dynamics does not always lead to a higher social welfare.

Proposition 9. (Static and Dynamic Social Welfare) *The unconditional firm values, $E(p)$ and $V(p)$, satisfy the following:*

1. *$V = E$ in the dynamic pooling regions;*
2. *$V > E$ in the static separation region;*
3. *$V < E$ in the intersection between the dynamic inaction region and the static pooling region.*

In the separation region of the static world, a high type firm never invests. Thus, allowing delay leads to eventual investment and improves welfare. In the intersection between the dynamic inaction region and the static pooling region, immediate investment is first best. Allowing delay in this case reduces welfare because of the lost time value of money. In other words, the delay is inefficient from the social planner's point of view.

The result that delay can lead to a welfare loss may hinge, of course, on the specific economic structure we consider. For example, one may reasonably suspect that high-quality firms in practice would scale down investments if they are perceived as low quality and suffer from underpricing. If delay improves market belief and leads to a larger positive-NPV project, welfare may be improved. We leave this and other alternative specifications to future research.

4.2 Empirical Implications

Our model has a number of empirical implications.

Pecking Order and Project Risk. Both our static and dynamic equilibria predict that conditional on the same NPV, a safer project (i.e., more likely to succeed) is more likely to be financed by debt, while a riskier one is more likely to be financed by equity. This result is not driven by risk aversion (all agents in our model are risk-neutral) or default costs (default incurs no deadweight costs in our model). Instead, it stems from the trade-off between underpricing of securities and the loss of assets in place in default. Even though debt is less sensitive to private information, a high probability of default can force a high type firm to issue equity and a low type firm to follow.

Investment and Negative News. Our model predicts that investments can take place even after a series of bad news. Such behavior arises either because of the separation by the

low type firm, or if the lower pooling region is reached (see Proposition 4). The two scenarios are governed by different mechanisms and produce distinct implications for equity prices.

If a low type firm invests at the lower threshold $\underline{z}^*$, it does so because of dim prospects of pooling with the high type firm and issuing overpriced securities to finance investments. Thus, investment is financed by new equity issuance, revealing the type of the firm to the market. Therefore, the theory predicts a negative impact of secondary equity offerings on the equity price above and beyond the informational content of the unfavorable news.

If the lower pooling region (z_l^*, z_h^*) is reached from above, both firm types invest using debt financing. Such investment behavior provides no additional information to the market because both the high type firm and the low type firm invest at that point. Therefore, our model predicts that new debt issuance after bad news has no discernible impact on the equity price above and beyond the informational content of the negative news before investments.

Investment and Positive News. As positive news arrives over time, the market becomes increasingly optimistic about the firm's type and improves its financing terms (lower interest rates for debt and higher equity price). With cheaper financing, one may expect the firm to undertake more investments. However, this is not always the case in our model. Conditional on the same pool of investment opportunities, a firm with more volatile cash flows is less likely to invest in response to an increase in market beliefs as compared to a less volatile firm. This difference in behavior stems from the presence of the upper inaction region for some primitive model parameters and a high σ, which leads to delayed investment (Proposition 4).

Security Choice and Announcement Returns. Our results predict that the abnormal stock returns upon debt issuance should be zero. This is because debt is a pooling security in our equilibrium. By contrast, equity issuance should be followed by negative abnormal stock returns because the low-type firm separates by equity. Although separation by equity is imposed by the tie-breaking rule in our model, this choice is robust to the introduction of distress cost.

Quality of Information and Security Choice. We also predict that an increase in the quality of the information available to the market weakly increases the probability of equity issuance and weakly decreases the probability of debt issuance. In our model, as cash flows become increasingly more informative, the four-threshold equilibrium no longer exists. Because the lower pooling region in the four-threshold equilibrium always has debt issuance,

a higher informativeness of cash flows tilts the equilibrium towards more equity issuance and less debt issuance in expectation.

5 Conclusion

In this paper, we reexamine the classic yet static model of Myers and Majluf (1984) in a dynamic environment. Firms can optimally delay investment decisions in an attempt to signal their quality, while the market learns about firms' qualities by observing the noisy cash flows generated from their assets in place. In a two-threshold equilibrium, a high-quality firm optimally delays investment, balancing the trade-off between the underpricing of its securities and the lost time value of project NPV. A low-quality firm imitates the high quality firm as much as possible, but upon a series of sufficiently bad cash flows, the low-quality firm invests probabilistically and separates itself. Unlike in the static equilibrium of Myers and Majluf (1984), in our model, investment eventually takes place.

We also characterize firms' optimal choice between debt and equity in this dynamic model, building on the pecking order theory of Myers (1984). We show that if the new project can fail with a sufficiently high probability, a high-quality firm can have a higher cost of losing the existing assets upon debt default than the underpricing of its equity. Therefore, for a given project NPV, relatively safe projects favor debt, whereas relatively risky projects favor equity. This partial violation of the pecking order is consistent with empirical evidence that small, high-growth firms prefer equity over debt.

The combination of security choices and the option to delay leads to a new type of four-threshold equilibrium, which features two pooling regions with different securities, two inaction regions, and one separation region. The choice between debt and equity introduces a kink in the static value function. At this kink, investing immediately is never optimal, and the firm would rather "wait and see" before deciding the security type. Therefore, the four-threshold equilibrium is qualitatively different from our two-threshold equilibrium with a single security and Daley and Green (2012)'s (essentially) equity-only equilibrium.

Appendix

A List of Model Variables

Variable	Description
	Variables introduced in Section 2 (static model)
$\theta \in \{H, L\}$	Type of the firm
$(X_t^\theta)_{t\geq 0}$	Cumulative cash flow process of type θ firm
μ_θ	Drift of the cash flow process X^θ
σ	Instantaneous volatility of X^θ
K	Increase in instantaneous cash flow rate if new production technology succeeds
γ	Success probability of the new production technology
$k = \gamma K$	Expected increase in instantaneous cash flow rate due to new technology
r	Risk-free rate
I	Cost of adopting new technology
p_0	Prior probability of $\theta = H$
λ	Fraction of the company demanded by the market
c	Perpetuity coupon demanded by the market
$\pi^e(\theta) = \pi^e(\theta; \lambda, c)$	Probability of equity issuance by type θ firm given market offers (λ, c)
$\pi^d(\theta) = \pi^d(\theta; \lambda, c)$	Probability of debt issuance by type θ firm given market offers (λ, c)
$E_\theta^\emptyset$	Expected equity value of old shareholders for not making investment
$E_\theta^e(\lambda)$	Expected equity value of old shareholders for investing with equity financing
$E_\theta^e(c)$	Expected equity value of old shareholders for investing with debt financing
$S^e(\lambda, c)$	Expected payoff to the market from buying equity given offers (λ, c)
$S^d(\lambda, c)$	Expected payoff to the market from buying debt given offers (λ, c)
q_0^e	Market belief about the quality of the type issuing equity at time 0
q_0^d	Market belief about the quality of the type issuing debt at time 0
$\lambda(q_0^e)$	A break-even equity offer λ given the belief q_0^e
$c(q_0^d)$	A break-even debt offer c given the belief q_0^d
p_e	Belief threshold solving $E_H^e(\lambda(p_e)) = E_H^\emptyset$
p_d	Belief threshold solving $E_H^d(c(p_e)) = E_H^\emptyset$
$p_{d/e}$	Belief threshold solving $E_H^e(\lambda(p_{d/e})) = E_H^d(c(p_{d/e}))$
p_r	Belief threshold solving $c(p_r) = \mu_H$

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Variable	Description
Variables introduced in Sections 3 and 4 (dynamic model)	
p_t	Market belief that it is facing a high type firm conditional on not observing investment up to and including time t
λ_t	Fraction of the company demanded by the market at time t
c_t	Perpetuity coupon demanded by the market at time t
$\pi_t^e(\theta)$	Cumulative probability of issuing equity by type θ firm up to and including time t
$\pi_t^d(\theta)$	Cumulative probability of issuing debt by type θ firm up to and including time t
$\pi_t(\theta)$	Cumulative probability of investing by type θ firm up to and including time t
q_t^e	Market belief about the quality of the type issuing equity at time t
q_t^d	Market belief about the quality of the type issuing debt at time t
P_t	Probability of facing a high type firm conditional on prior p_0 and history of cash flows $(X_s)_{s\leq t}$
z_t	Log-likelihood corresponding to p_t
Z_t	Log-likelihood corresponding to P_t
$\Xi^e(\underline{z}, \bar{z})$	Two-threshold strategy profile with equity pooling at $\bar{z}$
$\Xi^d(\underline{z}, \bar{z})$	Two-threshold strategy profile with debt pooling at $\bar{z}$
$\bar{z}^*$, $\bar{p}^*$	Upper pooling threshold in the dynamic equilibrium of Propositions 2 and 4
$\underline{z}^*$, $\underline{p}^*$	Lower reflecting threshold in the dynamic equilibrium of Propositions 2 and 4
z_h^*, p_h^*	Upper threshold of the lower pooling region of Proposition 4
z_l^*, p_l^*	Lower threshold of the lower pooling region of Proposition 4
δ	Intensity of the project decay
$V_\theta(p)$	Equilibrium payoff of a type θ firm in the dynamic model corresponding to Propositions 2 and 4
$E_\theta(p)$	Equilibrium payoff of a type θ firm in the static model corresponding to Proposition 1
$V(p)$	Social welfare in the dynamic model corresponding to Propositions 2 and 4
$E(p)$	Social welfare in the static model corresponding to Proposition 1

B Proofs and Details

B.1 Proof of Lemma 3

Consider first the case where the min(·) in (26) is equal to $\frac{\mu_H}{r}$, i.e. $q_0 < p_r$. The comparison between (25) and (26) boils down to:

$$E_H^e(\lambda(q_0)) \quad vs. \quad E_H^d(C(q_0)) \tag{B1}$$

$$(1-q_0)(1-\gamma)\frac{\mu_H}{r} \quad vs. \quad (1-q_0)I\frac{\mu_H}{q_0\mu_H+k} \tag{B2}$$

$$\frac{1-\gamma}{r} \quad vs. \quad \frac{I}{q_0\mu_H+k} \tag{B3}$$

$$q_0\mu_H+k \quad vs. \quad \frac{Ir}{1-\gamma} \tag{B4}$$

$$q_0 \quad vs. \quad \frac{1}{\mu_H}\left(\frac{Ir}{1-\gamma}-k\right) \equiv p_{d/e}. \tag{B5}$$

Thus, if q_0 is above $p_{d/e}$, then equity dominates risky debt for the high type firm; if $q_0 < p_{d/e}$, risky debt dominates equity. This concludes the proof of statements 1(a) and 2(a) of Lemma 3.

Consider the situation when the high type firm's debt is riskless (i.e., $q_0 > p_r$). Then, the trade-off is:

$$E_H^e(\lambda(q_0)) \quad vs. \quad E_H^d(C(q_0)) \tag{B6}$$

$$(1-q_0)(1-\gamma)\frac{I}{1-(1-\gamma)(1-q_0)} \quad vs. \quad (1-q_0)I\frac{\mu_H}{q_0\mu_H+k} \tag{B7}$$

$$\frac{1-\gamma}{1-(1-\gamma)(1-q_0)} \quad vs. \quad \frac{\mu_H}{q_0\mu_H+k} \tag{B8}$$

$$\frac{k}{k+\mu_H} \quad vs. \quad \gamma. \tag{B9}$$

It turns our that the comparison between riskless debt and equity for the high type firm does not depend on the level of market belief, q_0. Debt dominates equity if $\gamma > \frac{k}{k+\mu_H}$, and equity dominates debt if $\gamma < \frac{k}{k+\mu_H}$. ■

B.2 Proof of Proposition 1

First, one can easily strengthen the results of Lemmas 1 and 2 and conclude that:

$$E_L^e(\lambda(q_0^e)) \geq \frac{\mu_L}{r}+\left(\frac{k}{r}-I\right),\ \forall\, q_0^e \in [0,1] \quad \text{and} \quad E_L^d(C(q_0^d)) \geq \frac{\mu_L}{r}+\left(\frac{k}{r}-I\right),\ \forall\, q_0^d \in [0,1]. \tag{B10}$$

The above inequalities imply that the low type firm is always willing to pool with the

high type firm on any kind of security rather than separate itself regardless of the market beliefs. Therefore, in the most efficient equilibrium, pooling will occur as soon as the high type firm prefers investment with any kind of financing to passing on the investment. The type of security issued in a pooling equilibrium will be determined by the high type firm.

Because of (B10) and the Bayesian updating rule (7), both q_0^e and q_0^d cannot be larger than p_0. And if the high type firm is investing using a pure strategy, the belief on the corresponding security should be exactly p_0.

A high type firm prefers investment with debt financing if debt dominates equity (Lemma 3) and $p_0 > p_d$. Investment with equity financing is preferred if equity dominates debt (Lemma 3) and $p_0 > p_e$. Finally, the high type firm does not invest if $p_0 < \min(p_e, p_d)$. Thus, the low type firm is forced to separate. ■

B.3 Restatement of Parameter Regions of Definition 1

Definition 6 restates Definition 1 by enumerating the primitive parameter cases.

Definition 6. *There exist two functions* $\sigma_1(\mu_H, I, r, k, \gamma) : \mathbb{R}^5 \to \mathbb{R}_+$ *and* $\sigma_2(\mu_H, I, r, k, \gamma) : \mathbb{R}^5 \to \mathbb{R}_+$*, where* $\sigma_1(\cdot) \leq \sigma_2(\cdot)$*, which are defined in the proofs of Propositions 2 and 4, respectively. We consider the following regions of the primitive model parameters* $(\mu_H, I, r, k, \gamma, \sigma)$*:*

Case $1 : \mu_H < Ir$	*Case* $2 : Ir < \mu_H < k$	*Case* $3 : k < \mu_H$
(a) $\gamma < \frac{\mu_H}{\mu_H+k}$	(a) $\gamma < \frac{\mu_H}{\mu_H+k}$	(a) $\gamma < \frac{k}{\mu_H+k}$
(b') $\frac{\mu_H}{\mu_H+k} < \gamma < \frac{\mu_H+k-Ir}{\mu_H+k}$ *and* $\sigma < \sigma_1$	(b') $\frac{\mu_H}{\mu_H+k} < \gamma < \frac{k}{\mu_H+k}$ *and* $\sigma < \sigma_1$	(b) $\gamma > \frac{k}{\mu_H+k}$
(b'') $\frac{\mu_H}{\mu_H+k} < \gamma < \frac{\mu_H+k-Ir}{\mu_H+k}$ *and* $\sigma > \sigma_2$	(b'') $\frac{\mu_H}{\mu_H+k} < \gamma < \frac{k}{\mu_H+k}$ *and* $\sigma > \sigma_2$	
(c) $\gamma > \frac{\mu_H+k-Ir}{\mu_H+k}$	(c') $\gamma > \frac{k}{\mu_H+k}$ *and* $\sigma < \sigma_1$	
	(c'') $\gamma > \frac{k}{\mu_H+k}$ *and* $\sigma > \sigma_2$	

As in the static model, we write the parameter cases in Definition 6 using strict inequalities; if the primitive model parameters lie on the boundary of one of the cases, the same tie-breaking rules outlined in Section 2 lead to the equilibrium outcome.

B.4 Proof of Proposition 2

The proof of Proposition 2 can be summarized in four steps. First, we show that, given the reflecting market belief process with any lower boundary $\underline{z}$, there exists a well-defined function $f(\underline{z})$, such that the barrier policy $\bar{z} = f(\underline{z})$ is optimal for a high type firm. Then, given any upper boundary $\bar{z}$, there exists a well-defined function $g(\bar{z})$, such that a low type firm is indifferent between revealing itself at the lower boundary, $\underline{z} = g(\bar{z})$, and waiting to pool. Third, we show that there exists a unique fixed point. Fourth, we verify that the threshold strategy obtained this way is optimal.

Let the total dynamic value from following $\Xi^i(\underline{z}, \bar{z})$ starting from $z_0 = z$ of type θ be $V_\theta(z)$. In the inaction region, $V_\theta(z)$ satisfies:

$$\frac{h^2}{2}V'_H(z) + \frac{h^2}{2}V''_H(z) - rV_H(z) + \mu_H = 0, \tag{B11}$$

$$-\frac{h^2}{2}V'_L(z) + \frac{h^2}{2}V''_L(z) - rV_L(z) + \mu_L = 0. \tag{B12}$$

In Part I to Part IV below, we prove the results for cases of Definition 6 that do not restrict σ, namely Cases $1(a)$, $2(a)$, and $3(a)$ of for $i = e$ and Cases $1(c)$ and $3(b)$ for $i = d$. In Part V, we prove the results for Cases $1(b')$, $2(b')$, and $2(c')$ of Definition 6.

Part I. Given the lower reflecting barrier of beliefs at $\underline{z}$, it is well known (e.g., Harrison (1985)) that the optimal issuance decision by a high type firm is of the threshold type, and the threshold $\bar{z}$ should satisfy the following boundary conditions:

$$V_H(\bar{z}) = E^i_H(\bar{z}), \tag{B13}$$

$$V'_H(\bar{z}) = E^{i\prime}_H(\bar{z}), \tag{B14}$$

$$V'_H(\underline{z}) = 0, \tag{B15}$$

where $E^e_H(z) = E^e_H\left(\lambda\left(\frac{e^z}{1+e^z}\right)\right)$ as defined in (25) and $E^e_H(z) = E^e_H\left(C\left(\frac{e^z}{1+e^z}\right)\right)$ from (26).

Equation (B11) has a general solution of the form:

$$V_H(z) = C_1 e^{u_1 z} + C_2 e^{u_2 z} + \frac{\mu_H}{r}, \tag{B16}$$

where C_1 and C_2 are constants, and u_1 and u_2 are, respectively, the positive and negative roots of the characteristic equation:

$$\frac{h^2}{2}u^2 + \frac{h^2}{2}u - r = 0, \tag{B17}$$

and their values are:

$$u_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{2r}{h^2}}. \tag{B18}$$

Using (B16), we can rewrite (B13), (B14), and (B15) as:

$$C_1 e^{u_1 \bar{z}} + C_2 e^{u_2 \bar{z}} + \frac{\mu_H}{r} = E^i_H(\bar{z}), \tag{B19}$$

$$C_1 u_1 e^{u_1 \bar{z}} + C_2 u_2 e^{u_2 \bar{z}} = E^{i\prime}_H(\bar{z}), \tag{B20}$$

$$C_1 u_1 e^{u_1 \underline{z}} + C_2 u_2 e^{u_2 \underline{z}} = 0. \tag{B21}$$

Solving (B19) and (B20) for C_1 and C_2, and substituting the result into (B21), we

obtain:

$$e^{(u_1-u_2)\underline{z}} = -\frac{u_2}{u_1}\frac{u_1(E_H^i(\bar{z})-\mu_H/r)-E_H^{i\prime}(\bar{z})}{E_H^{i\prime}(\bar{z})-u_2(E_H^i(\bar{z})-\mu_H/r)}\cdot e^{(u_1-u_2)\bar{z}}. \tag{B22}$$

As we prove below, the right-hand side (RHS) of (B22) is strictly increasing in $\bar{z}$. Together with the limiting values and the implicit function theorem, this would imply the existence of a continuous and strictly increasing function, $f(\underline{z})$.

The partial derivative of the RHS of (B22) w.r.t. $\bar{z}$ is equal to (up to a positive multiplier):

$$\begin{aligned}
&(u_1-u_2)\frac{u_1(E_H^i(\bar{z})-\mu_H/r)-E_H^{i\prime}(\bar{z})}{E_H^{i\prime}(\bar{z})-u_2(E_H^i(\bar{z})-\mu_H/r)}\\
&+\frac{(u_1E_H^{i\prime}(\bar{z})-E_H^{i\prime\prime}(\bar{z})(E_H^{i\prime}(\bar{z})-u_2(E_H^i(\bar{z})-\mu_H/r))}{(E_H^{i\prime}(\bar{z})-u_2(E_H^i(\bar{z})-\mu_H/r))^2}\\
&-\frac{(u_1(E_H^i(\bar{z})-\mu_H/r)-E_H^{i\prime}(\bar{z}))(E_H^{i\prime\prime}(\bar{z})-u_2E_H^{i\prime}(\bar{z}))}{(E_H^{i\prime}(\bar{z})-u_2(E_H^i(\bar{z})-\mu_H/r))^2}.
\end{aligned} \tag{B23}$$

The second term of (B23) is positive because $E_H^{i\prime}>0$, $E_H^{i\prime\prime}\leq 0$ (which can be seen by differentiating equations (25)–(26)), $u_2<0$, and $E_H^i(\bar{z})-\mu_H/r>0$. For all $\bar{z}>\bar{z}_{min}$, where $\bar{z}_{min}$ is a unique root of:

$$u_1(E_H^i(\bar{z}_{min})-\mu_H/r)-E_H^{i\prime}(\bar{z}_{min})=0. \tag{B24}$$

The sign of the sum of the first and the third terms of (B23) is determined by the sign of

$$(u_1-u_2)[E_H^{i\prime}-u_2(E_H^i-\mu_H/r)]-[E_H^{i\prime\prime}-u_2E_H^{i\prime}]=u_1E_h^{i\prime}-u_2(u_1-u_2)(E_H^i-\mu_H/r)-E_H^{i\prime\prime}>0. \tag{B25}$$

As $\bar{z}\to+\infty$, the RHS of (B22) also approaches $+\infty$.

Hence, for any $\underline{z}$ there exists a well-defined, smooth, and strictly increasing best response function $\bar{z}=f(\underline{z})$.

Part II. Next, we solve the problem for a low type firm. Given $\bar{z}$ and a reflecting belief process at $\underline{z}$, there exists a unique $\underline{z}$, such that a low type firm is exactly indifferent at the lower boundary. Therefore, it randomizes to sustain the reflecting beliefs. Similar to the above, (B12) has a general solution:

$$V_L(z)=D_1e^{v_1z}+D_2e^{v_2z}+\frac{\mu_L}{r}, \tag{B26}$$

where D_1 and D_2 are constants, and v_1 and v_2 are, respectively, the positive and negative roots of the characteristic equation:

$$\frac{h^2}{2}v^2-\frac{h^2}{2}v-r=0, \tag{B27}$$

and their values are:

$$v_{1,2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{2r}{h^2}}. \tag{B28}$$

The value function has to satisfy (see Harrison (1985)) three conditions: indifference and reflecting at the lower boundary, and value matching at the upper boundary:

$$V_L(\underline{z}) = \frac{\mu_L + k}{r} - I, \tag{B29}$$

$$V_L'(\underline{z}) = 0, \tag{B30}$$

$$V_L(\bar{z}) = E_L^i(\bar{z}). \tag{B31}$$

Using (B26), we can re-write (B29) and (B30) as:

$$D_1 e^{v_1 \underline{z}} + D_2 e^{v_2 \underline{z}} + \frac{\mu_L}{r} = \frac{\mu_L + k}{r} - I, \tag{B32}$$

$$D_1 v_1 e^{v_1 \underline{z}} + D_2 v_2 e^{v_2 \underline{z}} = 0. \tag{B33}$$

Solving (B32) and (B33) for D_1 and D_2, we can write D_1 and D_2 in terms of $\underline{z}$:

$$\begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = \frac{1}{v_1 - v_2} \begin{pmatrix} -v_2 e^{(v_2 - 1)\underline{z}} \\ v_1 e^{(v_1 - 1)\underline{z}} \end{pmatrix} \left(\frac{k}{r} - I \right). \tag{B34}$$

Substituting (B34) into (B31), we obtain:

$$\frac{1}{v_1 - v_2} \left(\frac{k}{r} - I \right) \left(v_1 e^{(v_1 - 1)\underline{z} + v_2 \bar{z}} - v_2 e^{(v_2 - 1)\underline{z} + v_1 \bar{z}} \right) = E_L^i(\bar{z}) - \frac{\mu_L}{r}. \tag{B35}$$

It suffices to show that for any given $\bar{z}$ there exists a unique $\underline{z}$ such that (B35) holds. The RHS of (B35) is not a function of $\underline{z}$, while for the left-hand side (LHS):

$$\frac{d\ LHS}{d\underline{z}} = \frac{1}{v_1 - v_2} \left(\frac{k}{r} - I \right) v_1 v_2 \left(e^{v_1(\bar{z} - \underline{z})} - e^{v_2(\bar{z} - \underline{z})} \right) < 0, \quad \forall \underline{z} \leq \bar{z}. \tag{B36}$$

Because $v_1 + v_2 = 1$, as $\underline{z} \to \bar{z}$, $LHS \to \frac{k}{r} - I < RHS$, and as $\underline{z} \to -\infty$, $LHS \to \infty$. Thus, by the intermediate value theorem and monotonicity, we have a unique solution $\underline{z} = g(\bar{z}) < \bar{z}$.

Part III. Re-writing (B35), using $\underline{z} = f^{-1}(\bar{z})$, we obtain:

$$\frac{1}{v_1 - v_2} \left(\frac{k}{r} - I \right) \left(v_1 \left(\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}} \right)^{v_2} - v_2 \left(\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}} \right)^{v_1} \right) - \left(E_L^i(\bar{z}) - \frac{\mu_L}{r} \right) = 0. \tag{B37}$$

Any root of this equation greater than $\bar{z}_{min}$ (defined in (B24)) gives rise to a pair $(\bar{z}, \underline{z})$ that characterizes an equilibrium. Here, $\bar{z} \geq \bar{z}_{min}$ is needed, because both sides of (B22)

must be positive.

Notice that, as $\bar{z} \to \bar{z}_{min}+$, the LHS of (B37) approaches $+\infty$, because $f^{-1}(\bar{z})$, which is a positive multiple of the log of the RHS of (B22), approaches $-\infty$, and $v_1 > 0 > v_2$.

By (B22), $\bar{z} - \underline{z} \to 0$ as $\bar{z} \to \infty$. Thus, as $\bar{z} \to +\infty$, the limit of the LHS of (B37) is:

$$\left(\frac{k}{r} - I\right) - \left(E_L^i(\bar{z}) - \frac{\mu_L}{r}\right) < 0. \tag{B38}$$

The partial derivative of the LHS of (B37) is equal to:

$$\frac{v_1 v_2}{v_1 - v_2}\left(\frac{k}{r} - I\right)\left[\left(\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}}\right)^{v_2-1} - \left(\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}}\right)^{v_1-1}\right] \cdot \frac{d}{d\bar{z}}\left(\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}}\right) - E_L^{i\prime}(\bar{z}). \tag{B39}$$

We already know that $E_L^{i\prime} > 0$, $v_1 > 0 > v_2$, and $\frac{k}{r} - I > 0$. Differentiating $\frac{e^{\bar{z}}}{e^{\bar{z}}}$ from (B22), we find that $\frac{d}{d\bar{z}}\left(\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}}\right) < 0$. Finally, $\left(\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}}\right)^{v_2-1} < \left(\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}}\right)^{v_1-1}$, because $\frac{e^{\bar{z}}}{e^{f^{-1}(\bar{z})}} > 1$.

We have proved that the whole expression in (B39) is negative. Thus, the LHS of (B37) is strictly decreasing from $+\infty$ at $\bar{z}_{min}+$ to a negative value at $+\infty$. Therefore, there exists a unique $\bar{z}^* > \bar{z}_{min}$ that solves (B37). The other boundary $\underline{z}^*$ is given by $f^{-1}(\bar{z}^*)$ or, equivalently, by $g(\bar{z}^*)$.

Part IV. We have shown that the pair $(\underline{z}^*, \bar{z}^*)$ constitutes a unique equilibrium, if we allow deviations within the class of two threshold (lower reflective and upper pooling, respectively) strategies. However, it does not account for arbitrary deviations that are allowed by the definition of $\pi(\theta)$. We now verify the sub-optimality of arbitrary deviations by considering a modified version of our game. Define $V_\theta^*(z)$ as the value function of the original game:

$$V_\theta^*(z) = \sup_{(\pi^d, \pi^e)} \mathbf{E}\left[\int_0^\infty \left(\int_0^t e^{-ru} dX_u\right) d(\pi_t^e + \pi_t^d) + \int_0^\infty e^{-rt}\left(E_\theta^e(\lambda^*(q_t^e))\, d\pi_t^e + E_\theta^d(c^*(q_t^d))\, d\pi_t^d\right)\right]. \tag{B40}$$

Consider the following modification: at any moment the firm can be sold to an outsider for $V_\theta(z)$, where V_θ is the expected payoff to a type θ player when the parties play $\Xi^i(\underline{z}^*, \bar{z}^*)$ strategies.

Define $\tilde{V}_\theta(z)$ as be the value function of the modified game:

$$\begin{aligned}\tilde{V}_\theta(z) = \sup_{(\pi^d, \pi^e)} \mathbf{E}\Bigg[&\int_0^\infty \left(\int_0^t e^{-ru} dX_u\right) d(\pi_t^e + \pi_t^d) \\ &+ \int_0^\infty e^{-rt}\left(\max(V_\theta(z_t), E_\theta^e(\lambda^*(q_t^e)))\, d\pi_t^e + \max(V_\theta(z_t), E_\theta^d(c^*(q_t^d)))\, d\pi_t^d\right)\Bigg].\end{aligned} \tag{B41}$$

Clearly, $\tilde{V}_\theta(z) \geq V_\theta^*(z) \geq V_\theta(z)$. In addition, $V_\theta(z_t) \geq E_\theta^i(z_t)$, which can be easily seen from equations (B13)–(B15) and (B29)–(B31), as well as debt coupon c^* and equity share λ^* defined in (51)–(53). Moreover, in Cases 1(a), 2(a), 3(a), 1(c) or 3(b) of Definition 6,

$E_H^i(z_t) \geq E_H^{-i}(z_t)$, where $-i$ denotes the security other than the equilibrium one.[19] Hence, $V_H(z_t) \geq E_H^{-i}(z_t)$.

For the low type firm, $E_L^{-i}(\cdot) \equiv \frac{\mu_L}{r} + \frac{k}{r} - I$ because using the off-equilibrium security $-i$ implies separation. Thus, $E_L^{-i}(\cdot) < V_L(z_t)$ and

$$V_\theta(z_t) \geq \max\left(E_\theta^e(\lambda^*(q_t^e)), E_\theta^d(c^*(q_t^d))\right). \tag{B42}$$

Therefore:

$$\tilde{V}_\theta(z) = \sup_{\pi(\theta)} \mathbf{E}\left[\int_0^\infty \left(\int_0^t e^{-ru} dX_u + e^{-rt} V_\theta(z_t)\right) d\pi_t(\theta)\right]. \tag{B43}$$

In this case, there is no need for mixed strategies, because every player is facing a simple optimal stopping problem. Thus, we can re-write $\tilde{V}$ as:

$$\begin{aligned} \tilde{V}_\theta(z) &= \sup_\tau \mathbf{E}\left[\int_0^\tau e^{-ru} dX_u + e^{-r\tau} V_\theta(z_\tau)\right] \\ &= \sup_\tau \mathbf{E}\left[(1 - e^{-r\tau})\frac{\mu_\theta}{r} + e^{-r\tau} V_\theta(z_\tau)\right] \\ &= \sup_\tau \mathbf{E}\, f_\theta(\tau, z_\tau). \end{aligned} \tag{B44}$$

Because V_θ is in $\mathbb{C}^2$, except for the two points $\{\underline{z}^*, \bar{z}^*\}$, we can use Itô's lemma and write:

$$df_\theta(t, z_t) = re^{-rt}\frac{\mu_\theta}{r} dt - re^{-rt} V_\theta(z_t) dt + e^{-rt} dV_\theta(z_t) \tag{B45}$$

$$= e^{-rt}\left(\mu_\theta - rV_\theta(z_t) + \operatorname{sgn}(2\mu_\theta - \mu_H - \mu_L)\frac{h^2}{2} V'_\theta(z_t) + \frac{h^2}{2} V''_\theta(z_t)\right) dt \tag{B46}$$

$$+ he^{-rt} V'_\theta(z_t) dB_t + e^{-rt} V'_\theta(\underline{z}) dY_t \tag{B47}$$

$$= \Gamma_\theta V_\theta(z_t) dt + he^{-rt} V'_\theta(z_t) dB_t + e^{-rt} V'_\theta(\underline{z}) dY_t, \tag{B48}$$

where $Y_t = \max(\underline{z}^* - \inf_{s \leq t} Z_t, 0)$ and Γ_θ is a second order differential operator that corresponds to equations (B11)–(B12).

Since $V'_\theta(\underline{z}^*) = 0$, the last term in (B48) disappears; and because $V'_\theta(z)$ is bounded, the second term is a martingale, and $\Gamma_\theta V_\theta(z) \leq 0$.[20] Using the optional sampling theorem,[21] we conclude that:

$$\mathbf{E}\, f_\theta(\tau_b, z_{\tau_b}) \leq f_\theta(0, z_0) \tag{B49}$$

for all bounded stopping times τ_b. Because every other stopping time τ can be obtained as

[19] For example, for Case 1(a) of Definition 6, $E_H^e(z_t) > E_H^d(z_t)$ for $z_t \geq z_e$. Other cases are similar.

[20] For $z \in (\underline{z}^*, \bar{z}^*)$, $\Gamma_\theta V_\theta(z) = 0$ by (B11)–(B12). For $z > \bar{z}^*$, it can be shown that $\Gamma_\theta V_\theta(z) = \Gamma_\theta E_\theta^i(z_t) < 0$.

[21] See, for example, Peskir and Shiryaev (2006), Theorem 3.2.A.

a limit of $\tau_b = \min(\tau, b)$ when $b \to \infty$, we conclude that:

$$\tilde{V}_\theta(z) \leq f_\theta(0, z) = V_\theta(z). \tag{B50}$$

Therefore:

$$\tilde{V}_\theta(z) = V_\theta^*(z) = V_\theta(z), \tag{B51}$$

which implies that V_θ is a maximal attainable equilibrium payoff in the original game. Because this payoff is guaranteed when players stick to the two threshold strategies $(\underline{z}^*, \bar{z}^*)$, the latter constitutes an equilibrium.

Part V. In the previous four steps we have verified the equilibrium for Cases 1(a), 2(a), 3(a), 1(c), and 3(b) of Definition 6. In order to complete the proof, we now consider parametric cases $1(b')$, $2(b')$ and $2(c')$. We will use the following result from Proposition 3: as $\sigma \to 0$, $\bar{z}^* \to +\infty$ and $\inf_z V_H(z) \to \frac{\mu_H}{r} + \frac{k}{r} - I$.

For Cases $1(b')$ and $2(b')$ of Definition 6, $E_H^e(z_t) > E_H^d(z_t)$ for $z_t > z_{d/e}$. Therefore, there exists $\sigma_1 = \sigma_1(\mu_H, I, r, k, \gamma) > 0$ such that for any $\sigma < \sigma_1$, the upper threshold $\bar{z}^*$ is above $z_{d/e}$ and $V_H(z) > E_H^d(z_{d/e}) \forall z$. Then, $V_\theta(z_t) > E_\theta^e(z_t)$ for all z_t as guaranteed by (B13)–(B15) and (B29)–(B31), as well as the debt coupon c^* and equity share λ^* defined in (51)–(53). $V_H(z_t) > E_H^e(z_t) > E_H^d(z_t)$ for $z_t > z_{d/e}$ and $V_H(z_t) > E_H^d(z_{d/e}) \geq E_H^d(z_t)$ for $z_t \leq z_{d/e}$. Thus, inequality (B42) still holds and the argument in Parts I–IV goes through.

For Case $2(c')$ of Definition 6, $E_H^d(z_t) \geq E_H^e(z_t)$ for all z_t and debt becomes riskless for the high type when $z_t > z_r$. Therefore, there exists $\sigma_1 = \sigma_1(\mu_H, I, r, k, \gamma) > 0$ such that for any $\sigma < \sigma_1$ the upper threshold $\bar{z}^*$ is above z_r and $V_H(z) > E_H^d(z_r)$ for all z. $V_L(z_t) \geq \max(E_L^e(\lambda^*(q_t^e)), E_L^d(c^*(q_t^d)))$ for all z_t as guaranteed by (B13)–(B15) and (B29)–(B31), as well as the debt coupon c^* and the equity share λ^* defined in (51)–(53). $V_H(z_t) \geq E_H^d(z_t)$ for $z_t > z_r$ for the same reasons, while $V_H(z_t) > E_H^d(z_r) \geq E_H^d(z_t)$ for $z_t \leq z_r$ because of the choice of σ. Thus, inequality (B42) still holds and the argument in Parts I–IV goes through. ■

B.5 Proof of Proposition 3

1. $\sigma \to 0$. Consider first parametric Cases $1(a), 1(b'), 2(a), 2(b')$, and $3(a)$ of Definition 6. Recall that $\lambda(p) = \frac{Ir}{p\mu_H + k}$. Hence:

$$\frac{d}{dp}\lambda(p) = -\frac{Ir\mu_H}{(p\mu_H + k)^2} = -\lambda^2(p)\frac{\mu_H}{Ir}, \tag{B52}$$

and

$$\frac{d}{dz}\lambda(p(z)) = -\lambda^2(p(z))\frac{\mu_H}{Ir}\frac{e^z}{(1+e^z)^2}. \tag{B53}$$

Because $E_H^e(\lambda(z)) = \Big(1 - \lambda(p(z))\Big)\dfrac{\mu_H + k}{r}$, we can write:

$$\frac{d}{dz}E_H^e(\lambda(p(z))) = \lambda^2 \frac{\mu_H(\mu_H + k)}{Ir^2} p(1-p). \tag{B54}$$

Using (B22), we can write:

$$e^{(u_1-u_2)(\underline{z}-\bar{z})} = -\frac{u_2}{u_1}\frac{u_1(E_H^e(\bar{z}) - \mu_H/r) - E_H^{P'}(\bar{z})}{E_H^{e\prime}(\bar{z}) - u_2(E_H^e(\bar{z}) - \mu_H/r)}. \tag{B55}$$

Recall that $\bar{z}^* > \bar{z}_{min}$, where $\bar{z}_{min}$ is the unique root of (B24). It is easy to see that $\bar{z}_{min} \to +\infty$ as $\sigma \to 0$. Thus, $\bar{z}^*$ converges to $+\infty$ (i.e., $\bar{p}^*$ converges to 1). Plugging the limit of $\bar{z}^*$ back yields:

$$E_H^e(\bar{z}^*) \to \frac{\mu_H + k}{r} - I \quad \text{and} \quad \frac{d}{dz}E_H^e(\bar{z}^*) \to 0+. \tag{B56}$$

Part II of Proposition 2 establishes that $\underline{z}^* = g(\bar{z}^*)$ for some continuous function g. Therefore, the lower threshold $\underline{z}^*$ also converges. This implies that the limit $\lim(\bar{z}^* - \underline{z}^*)$ exists. Therefore, we can expand (B35):

$$\begin{aligned}
&\frac{1}{v_1 - v_2}\left(\frac{k}{r} - I\right)\left(v_1 e^{(v_1-1)\underline{z}^* + v_2\bar{z}^*} - v_2 e^{(v_2-1)\underline{z}^* + v_1\bar{z}^*}\right) - E_L^e(\bar{z}^*) \\
&\sim \left(\frac{k}{r} - I\right)\left(e^{-v_2(\underline{z}^* - \bar{z}^*)} - v_2 e^{-(\underline{z}^* - \bar{z}^*)}\right) - E_L^e(+\infty) \\
&= \left(\frac{k}{r} - I\right)\left(e^{u_1(\underline{z}^* - \bar{z}^*)} + u_1 e^{-(\underline{z}^* - \bar{z}^*)}\right) - E_L^e(+\infty),
\end{aligned} \tag{B57}$$

where the notation "$\sim$" means that the expressions on both sides of it have the same limit. If $\lim(\bar{z}^* - \underline{z}^*)$ were finite, then the last line of equation (B57) would converge to:

$$-E_L^e(+\infty) \neq 0,$$

contradicting that (B57) should be converging to the 0 because (B35) holds for all σ. Thus, $\lim(\bar{z}^* - \underline{z}^*) = +\infty$.

Expanding the RHS of (B22), we obtain:

$$\begin{aligned} e^{(u_1-u_2)(\underline{z}^*-\bar{z}^*)} &\sim \frac{\frac{k}{r}-I-\frac{\frac{d}{dz}E_H^e(\bar{z}^*)}{u_1}}{\frac{k}{r}-I} \\ &= 1-\frac{\frac{d}{dz}E_H^e(\bar{z}^*)}{u_1\left(\frac{k}{r}-I\right)} \\ &= 1-\frac{\lambda^2\mu_H(\mu_H+k)}{u_1 Ir^2\left(\frac{k}{r}-I\right)}\frac{e^{\bar{z}^*}}{(1+e^{\bar{z}^*})^2} \\ &\sim 1-\frac{I\mu_H}{(\mu_H+k)\left(\frac{k}{r}-I\right)}\cdot\frac{1}{u_1 e^{\bar{z}^*}}. \end{aligned} \tag{B58}$$

Because $\lim e^{(u_1-u_2)(\underline{z}^*-\bar{z}^*)}=0$, it must be the case that:

$$\lim_{\sigma\downarrow 0} u_1 e^{\bar{z}^*} = \frac{I\mu_H}{(\mu_H+k)\left(\frac{k}{r}-I\right)}. \tag{B59}$$

This fact in turn implies that $\lim u_1\bar{z}^* = \lim\left(u_1 e^{\bar{z}^*}\cdot\frac{\bar{z}^*}{e^{\bar{z}^*}}\right)=0$. Hence, $\lim u_1\underline{z}^*=0$ as well. Plugging it back to (B57), we solve for $\lim e^{\underline{z}^*}$:

$$\begin{aligned} \lim_{\sigma\downarrow 0}\left[\left(\frac{k}{r}-I\right)\left(e^{u_1(\underline{z}^*-\bar{z}^*)}+u_1 e^{-(\underline{z}^*-\bar{z}^*)}\right)-\frac{k}{r}+I\frac{k}{\mu_H+k}\right]=0 \\ \left(\frac{k}{r}-I\right)\left(1+\frac{\lim u_1 e^{\bar{z}^*}}{\lim e^{\underline{z}^*}}\right)-\frac{k}{r}+I\frac{k}{\mu_H+k}=0 \\ \left(\frac{k}{r}-I\right)\left(1+\frac{I\mu_H}{(\mu_H+k)\left(\frac{k}{r}-I\right)}\cdot\frac{1}{\lim e^{\underline{z}^*}}\right)-\frac{k}{r}+I\frac{k}{\mu_H+k}=0 \end{aligned} \tag{B60}$$

and find that $\lim e^{\underline{z}^*}=1$. Thus, $\underline{z}^*\to 0$ and $\underline{p}^*\to\frac{1}{2}$ as $\sigma\downarrow 0$.

For the parametric Case 1(c) of Definition 6, one should repeat the argument above with minor modifications. First, we conclude that $\bar{z}^*>z_{min}\to+\infty$. Since $-E_L^d(+\infty)\neq 0$, there exists a $\lim(\bar{z}^*-\underline{z}^*)$. Using the same technique as above, we conclude that $\lim u_1 e^{\bar{z}^*}=\mu_H(1-\gamma)/(k-Ir)$ since $E_H^{d\prime}(\bar{z}^*)\sim\mu_H(1-\gamma)/(re^{\bar{z}^*})$. Plugging everything back to (B57), we find that $\lim e^{\underline{z}^*}=1$. Thus, $\underline{p}^*\to 1/2$.

For the parametric Cases 2(c') and 3(b), one can follow the proof for the parametric Case 1(c) with only one modification: since the coupon for the high type is riskless, we have $E_H^{d\prime}(\bar{z}^*)\sim I(1-\gamma)/e^{\bar{z}^*}$.

2. $\sigma\to\infty$**.** First, consider $\sigma\to\infty$. It is easy to see that both thresholds converge to the same limit. If this were not true, then there exists some $z\in(\underline{z}^*,\bar{z}^*)$ for all σ. For this z, equation (B12) implies that $E_L(z)\to 0$, which cannot happen, because $E_L(z)\geq\frac{k}{r}-I>0$. Given that $\lim_{\sigma\to\infty}\bar{p}^*=\lim_{\sigma\to\infty}\underline{p}^*$, the dynamic problem for a high type firm reduces to the

static one: either to invest right away, or not to invest at all. Therefore, the static threshold p_d (or p_d depending on the primitive model parameters) is the limit of $\bar{p}^*$ and $\underline{p}^*$. ■

B.6 Proof of Lemma 4

We now prove that the high type firm never invests at $z_{d/e}$ if equity is the equilibrium security above $z_{d/e}$. The case of never investing at $z_t = z_r$ is similar and is omitted.

If the low type firm invests at $z_{d/e}$, then it is optimal for the high type firm to wait because the beliefs instantaneously move up and give him a higher payoff. Therefore, it is sufficient to consider the case in which there exists an ϵ-neighborhood of $z_{d/e}$ such that the low type firm will wait if the high type firm does so.

Since the low type firm always mimics issuance of the high type firm, the payoff upon investment of the high type firm is $\max\{E_H^e(z), E_H^d(z)\}$. In the ϵ-neighborhood of $z_{d/e}$, z_t satisfies:

$$dz_t = \frac{h^2}{2}dt + hdB_t. \tag{B61}$$

Applying the Itô-Tanaka lemma to $e^{-rt}g(z_t)$, where g is any piecewise-C^2 function with a discontinuity of the first derivative at $z_{d/e}$, one gets:

$$d\Big(e^{-rt}g(z_t)\Big) = e^{-rt}\left(\frac{h^2}{2}g'(z_t) + \frac{h^2}{2}g''(z_t) - rg(z_t)\right)dt + hg'(z_t)dB_t + \frac{1}{2}h^2\Delta e^{-rt}dL_t(z_{d/e}), \tag{B62}$$

where $\Delta = f'(z_{d/e}+) - f'(z_{d/e}-)$ and $L_t(z_{d/e})$ is a local time[22] of the process $(z_t)_{t\geq 0}$ at $z_{d/e}$.

Define $\tau = \inf\{t > 0 : |z_t - z_{d/e}| \geq \epsilon\}$, then:

$$\begin{aligned} E_{z_{d/e}}e^{-r\tau}g(z_\tau) = g(z_{d/e}) &+ E_{z_{d/e}}\left[\int_0^\tau e^{-rt}\left(\frac{h^2}{2}g'(z_t) + \frac{h^2}{2}g''(z_t) - rg(z_t)\right)dt\right] \\ &+ \frac{1}{2}h^2\Delta E_{z_{d/e}}\left[\int_0^\tau e^{-rt}dL_t(z_{d/e})\right]. \end{aligned} \tag{B63}$$

First, we put $g(z) = (z - z_{d/e})^2$ and $r = 0$ and simplify (B63) to:

$$\epsilon^2 = E_{z_{d/e}}\left[\int_0^\tau \left(h^2(z_t - z_{d/e}) + h^2\right)dt\right] \tag{B64}$$

$$\epsilon^2 - h^2E_{z_{d/e}}\tau = h^2E_{z_{d/e}}\left[\int_0^\tau (z_t - z_{d/e})dt\right] \tag{B65}$$

$$\left|\epsilon^2 - h^2E_{z_{d/e}}\tau\right| \leq h^2E_{z_{d/e}}\left[\int_0^\tau |z_t - z_{d/e}|dt\right] \leq h^2\epsilon E_{z_{d/e}}\tau. \tag{B66}$$

[22] The local time of process $(z_t)_{t\geq 0}$ at point z can be defined as $L_t(z) = \lim_{\epsilon\downarrow 0}\frac{1}{2\epsilon}\int_0^t \mathbb{1}(|z_s - z| < \epsilon)ds$. See, for example, Karatzas and Shreve (1991) Chapter 3.6.

Hence, as $\varepsilon \to 0$,

$$E_{z_{d/e}}\tau \sim \frac{\epsilon^2}{h^2}. \tag{B67}$$

Now take (B63) and put $g(z) = |z - z_{d/e}|$. Then,

$$\begin{aligned} E_{z_{d/e}} e^{-r\tau}|z_\tau - z_{d/e}| = E_{z_{d/e}} &\left[\int_0^\tau e^{-rt}\left(-r|z_t - z_{d/e}| - \frac{h^2}{2} sgn(z_t - z_{d/e})\right) dt\right] \\ &+ h^2 E_{z_{d/e}}\left[\int_0^\tau e^{-rt} dL_t(z_{d/e})\right]. \end{aligned} \tag{B68}$$

Solving for the term with a local time yields:

$$h^2 E_{z_{d/e}}\left[\int_0^\tau e^{-rt} dL_t(z_{d/e})\right] = E_{z_{d/e}} e^{-r\tau}|z_\tau - z_{d/e}| - E_{z_{d/e}}\left[\int_0^\tau e^{-rt}\left(-r|z_t - z_{d/e}| - \frac{h^2}{2} sgn(z_t - z_{d/e})\right) dt\right] \tag{B69}$$

$$h^2 E_{z_{d/e}}\left[\int_0^\tau e^{-rt} dL_t(z_{d/e})\right] \leq \epsilon E_{z_{d/e}} e^{-r\tau} + \epsilon E_{z_{d/e}}(1 - e^{-r\tau}) - \frac{h^2}{2} E_{z_{d/e}}\left[\int_0^\tau e^{-rt} sgn(z_t - z_{d/e}) dt\right] \tag{B70}$$

$$\left|h^2 E_{z_{d/e}}\left[\int_0^\tau e^{-rt} dL_t(z_{d/e})\right] - \epsilon\right| \leq \frac{h^2}{2} E_{z_{d/e}}\left[\int_0^\tau e^{-rt}|sgn(z_t - z_{d/e})| dt\right] \leq \frac{h^2}{2r} E_{z_{d/e}}(1 - e^{-r\tau}) \tag{B71}$$

$$\left|h^2 E_{z_{d/e}}\left[\int_0^\tau e^{-rt} dL_t(z_{d/e})\right] - \epsilon\right| \leq \frac{h^2}{2} E_{z_{d/e}}\tau, \tag{B72}$$

which implies that as $\epsilon \to 0$,

$$E_{z_{d/e}}\left[\int_0^\tau e^{-rt} dL_t(z_{d/e})\right] \sim \frac{\epsilon}{h^2}. \tag{B73}$$

Finally, we take (B63) and put $g(z) = \max\{E_H^e(z), E_H^d(z)\}$. Rewrite (B63) as:

$$\begin{aligned} E_{z_{d/e}} e^{-r\tau} g(z_\tau) - g(z_{d/e}) = &E_{z_{d/e}}\left[\int_0^\tau e^{-rt}\left(\frac{h^2}{2} g'(z_t) + \frac{h^2}{2} g''(z_t) - r g(z_t)\right) dt\right] \\ &+ \frac{1}{2} h^2 \Delta E_{z_{d/e}}\left[\int_0^\tau e^{-rt} dL_t(z_{d/e})\right]. \end{aligned} \tag{B74}$$

Divide both sides by ϵ and take a limit. The first term on the RHS converges to 0 because:

$$\left|\frac{h^2}{2} g'(z_t) + \frac{h^2}{2} g''(z_t) - r g(z_t)\right| \leq \sup_{|z - z_{d/e}| < \epsilon}\left|\frac{h^2}{2} g'(z) + \frac{h^2}{2} g''(z) - r g(z)\right| < M \tag{B75}$$

for some constant M and (B67).

The non-zero part of the limit comes from the second term of the RHS by (B73). Eventually we get:

$$\lim_{\epsilon \downarrow 0} \frac{E_{z_{d/e}} e^{-r\tau} g(z_\tau) - g(z_{d/e})}{\epsilon} = \frac{\Delta}{2} > 0. \tag{B76}$$

Hence, for small ϵ, the payoff from waiting, $E_{z_{d/e}} e^{-r\tau} g(z_\tau)$, is higher than the one from immediate pooling, $g(z_{d/e})$. ■

B.7 Proof of Proposition 4

We provide a detailed proof of Proposition 4 for parametric cases $1(b'')$ and $2(b'')$. The proof for the case $2(c'')$ is almost identical, with a minor adjustment that we outline at the end of this proof.

Our goal is to show that for all σ greater than some σ_2, there exists a quadruple of thresholds $(\underline{z}^*(\sigma), z_l^*(\sigma), z_h^*(\sigma), \bar{z}^*(\sigma))$, where $\underline{z}^*(\sigma) < z_l^*(\sigma) < z_h^*(\sigma) < \bar{z}^*(\sigma))$, such that the strategies defined in Proposition 4 constitute an equilibrium. The dynamic value functions from following the four-thresholds strategies, $G_H(\cdot)$ and $G_L(\cdot)$, should satisfy:

$$\frac{h^2}{2r} G_H''(z) + \frac{h^2}{2r} G_H'(z) - \left(G_H(z) - \frac{\mu_H}{r} \right) = 0 \tag{B77}$$

$$\frac{h^2}{2r} G_L''(z) - \frac{h^2}{2r} G_L'(z) - G_L(z) = 0 \tag{B78}$$

in the inaction regions $z \in (\underline{z}^*(\sigma), z_l^*(\sigma))$ and $z \in (z_h^*(\sigma), \bar{z}^*(\sigma))$ with boundary conditions:

$$G_H(z_h^*(\sigma)) = E_H^d(z_h^*(\sigma)), \tag{B79}$$

$$G_H'(z_h^*(\sigma)) = E_H^{d\prime}(z_h^*(\sigma)), \tag{B80}$$

$$G_H(\bar{z}^*(\sigma)) = E_H^e(\bar{z}^*(\sigma)), \tag{B81}$$

$$G_H'(\bar{z}^*(\sigma)) = E_H^{e\prime}(\bar{z}^*(\sigma)), \tag{B82}$$

and

$$G_L(\underline{z}^*(\sigma)) = \frac{k}{r} - I, \tag{B83}$$

$$G_L'(\underline{z}^*(\sigma)) = 0, \tag{B84}$$

$$G_H'(\underline{z}^*(\sigma)) = 0, \tag{B85}$$

$$G_H(z_l^*(\sigma)) = E_H^d(z_l^*(\sigma)), \tag{B86}$$

$$G_H'(z_l^*(\sigma)) = E_H^{d\prime}(z_l^*(\sigma)), \tag{B87}$$

$$G_L(z_l^*(\sigma)) = E_L^d(z_l^*(\sigma)). \tag{B88}$$

The dynamic value function of the low type firm also satisfies two value matching conditions:

$$G_L(z_h^*(\sigma)) = E_L^d(z_h^*(\sigma)), \tag{B89}$$

$$G_L(\bar{z}^*(\sigma)) = E_L^e(\bar{z}^*(\sigma)). \tag{B90}$$

Note that we use the notation "G" instead of "V" to distinguish the four-threshold equilibrium from the two-threshold equilibrium of Proposition 2.

The proof proceeds as follows. First, we construct thresholds $(\underline{z}^*(\sigma), z_l^*(\sigma), z_h^*(\sigma), \bar{z}^*(\sigma))$ and the corresponding value functions G_H and G_L that satisfy all equations above. Then, we verify that the strategies corresponding to the constructed thresholds constitute an equilibrium.

Existence and Uniqueness of $z_h^*(\sigma)$ and $\bar{z}^*(\sigma)$. We first show the following lemma.

Lemma 5. *There exists some $\hat{\sigma} > 0$ such that the dynamic value function in Proposition 2, $V_H(\cdot, \hat{\sigma})$, is tangent to $E_H^d(\cdot)$ at a unique point, $z_h^*(\hat{\sigma}) < z_{d/e}$.*

Proof. First, for any $\sigma > 0$, Proposition 2 guarantees the existence of thresholds $\underline{z}^*(\sigma) < \bar{z}^*(\sigma)$ such the dynamic payoff $V_H(\cdot, \sigma)$ from following $\Xi^e(\underline{z}^*(\sigma), \bar{z}^*(\sigma))$ satisfies:

$$\frac{h^2}{2r} V_H''(z,\sigma) + \frac{h^2}{2r} V_H'(z,\sigma) - \left(V_H(z,\sigma) - \frac{\mu_H}{r}\right) = 0 \quad z \in (\underline{z}^*(\sigma), \bar{z}^*(\sigma)) \tag{B91}$$

with boundary conditions:

$$V_H(\bar{z}^*(\sigma), \sigma) = E_H^e(\bar{z}^*(\sigma)), \quad V_H'(\bar{z}^*(\sigma), \sigma) = E_H^{e\prime}(\bar{z}^*(\sigma)). \tag{B92}$$

Note that $V_H(\cdot, \sigma)$ coincides with equilibrium payoff $V_H^*(\cdot, \sigma)$ only for $\sigma < \sigma_1$. However, $V_H(\cdot, \sigma)$ and the thresholds are still well defined for $\sigma > \sigma_1$ because Parts I-III of the proof of Proposition 2 go through.

If $\sigma < \sigma_1$, $V_H(\cdot, \sigma)$ is higher than $E_H^d(\cdot)$ because debt pooling is not optimal. Recall that $\bar{z}^*(\sigma) \to z_e$ as $\sigma \to +\infty$, and that $z_e < z_{d/e}$ because the model parameters lie in Cases 1(b'') or 2(b''). We increase σ until $\bar{z}^*(\tilde{\sigma}) = z_{d/e}$ at some $\tilde{\sigma} > \sigma_1$. Since $V_H'(\bar{z}^*(\tilde{\sigma}), \tilde{\sigma}) = E_H^{e\prime}(\bar{z}^*(\tilde{\sigma})) = E_H^{e\prime}(z_{d/e}) > E_H^{d\prime}(z_{d/e})$ and $V_H(\cdot, \tilde{\sigma}) \in \mathbb{C}^1$, there exists $\tilde{z} < z_{d/e}$ such that $V_H(z, \tilde{\sigma}) < E_H^d(z)$ for all $z \in (\tilde{z}, z_{d/e})$. The mapping $\sigma \to V_H(z; \sigma)$ is continuous for all z; therefore, there exists $\hat{\sigma} < \tilde{\sigma}$ such that $V_H(\cdot, \hat{\sigma})$ intersects E_H^d for the first time in the region $z < z_{d/e}$. This intersection has to be a tangent one with the point of tangency $z_h^*(\hat{\sigma})$.

Notice that if $\sigma = \hat{\sigma}$, the dynamic value function $V_H(\,\cdot\,; \hat{\sigma})$ satisfies (B77) with boundary conditions (B79)–(B82). ■

We define a function $F(z, \sigma, z_h)$ for $z_h < z_{d/e} < z$ as follows:

$$\begin{aligned} F(z,\sigma,z_h) = \frac{1}{u_1 - u_2} \Bigg[& e^{u_1(z-z_h)} \left(E_H^{d\prime}(z_h) - u_2 \left(E_H^d(z_h) - \frac{\mu_H}{r} \right) \right) \\ & + e^{u_2(z-z_h)} \left(u_1 \left(E_H^d(z_h) - \frac{\mu_H}{r} \right) - E_H^{d\prime}(z_h) \right) \Bigg] + \frac{\mu_H}{r}, \end{aligned} \tag{B93}$$

where $u_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{2r}{h^2}}$.

It is easy to check that $F(\cdot, \sigma, z_h^*(\hat{\sigma}))$ satisfies the differential equation (B77) and boundary conditions (B79) and (B80) for all σ. If $\sigma = \hat{\sigma}$, F also satisfies (B81) and (B82), where $\bar{z}^*(\hat{\sigma})$ is given by Proposition 2.

As $\sigma \to \infty$, u_1 becomes arbitrarily large (and positive), and

$$F(z, \sigma, z_h^*(\hat{\sigma})) \sim \frac{1}{2} e^{u_1(z - z_h^*(\hat{\sigma}))} \left(E_H^d(z_h^*(\hat{\sigma})) - \frac{\mu_H}{r} \right). \tag{B94}$$

Thus, there exists $\sigma_M = \sigma_M(\mu_h, I, r, k, \gamma) > 0$ such that:

$$\frac{\partial}{\partial \sigma} F(z, \sigma, z_h^*(\hat{\sigma})) > 0, \quad \forall z > z_{d/e}. \tag{B95}$$

Next, we compute the partial derivative of F with respect to the "starting point" z_h:

$$\frac{\partial}{\partial z_h} F(z, \sigma, z_h) = \frac{1}{u_1 - u_2} \left[E_H^{d\prime\prime}(z_h) + E_H^{d\prime}(z_h) + u_1 u_2 \left(E_h^d(z_h) - \frac{\mu_H}{r} \right) \right] \cdot \left[e^{u_1(z - z_h)} - e^{u_2(z - z_h)} \right]. \tag{B96}$$

Expression in the second pair of square brackets is positive since $z_h < z_{z/e} < z$ and $u_2 < 0 < u_1$. As $\sigma \to \infty$, $u_1 \to \infty$ and $u_2 \to -\infty$. Expression in the first pair of square brackets becomes negative as soon as σ becomes larger than some threshold, $\sigma_N = \sigma_N(\mu_H, I, r, k, \gamma) > 0$.

Now put $\sigma_2 = \max(\sigma_M, \sigma_N)$ and consider $\sigma > \sigma_2$. For all $z_{d/e} < z \leq \bar{z}^*(\hat{\sigma})$:

$$F(z, \sigma, z_h^*(\hat{\sigma})) > V_H(z; \hat{\sigma}) \geq E_H^e(z). \tag{B97}$$

We start increasing z_h away from $z_h^*(\hat{\sigma})$. With an increase in z_h, the value of $F(\cdot, \sigma, z_h)$ is monotonically decreasing for all $z_{d/e} < z \leq \bar{z}^*(\hat{\sigma})$. If $z_h = z_{d/e}$, then:

$$\frac{\partial}{\partial z} F(z, \sigma, z_{d/e}) \bigg|_{z = z_{d/e}} = E_H^{d\prime}(z_{d/e}) < E_H^{e\prime}(z_{d/e}). \tag{B98}$$

Thus, there exists some $\tilde{z} > z_{d/e}$ such that $F(z, \sigma, z_{d/e}) < E_H^e(z)$ for all $z_{d/e} < z < \tilde{z}$. Since F is continuous in z_h, there exists some $z_h^*(\sigma)$ between $z_h(\hat{\sigma})$ and $z_{d/e}$ such that F intersects E_H^e for the first time in the region $z_{d/e} < z \leq \bar{z}^*(\hat{\sigma})$; at this intersection, F is tangent to E_H^e. We denote this point of tangency by $\bar{z}^*(\sigma)$.

Note that $F(\cdot, \sigma, z_h^*(\sigma))$ satisfies differential equation (B77) with boundary conditions (B79)–(B82). Therefore, we let $G_H(z) = F(z, \sigma, z_h^*(\sigma))$ for $z \in (z_h^*(\sigma), \bar{z}^*(\sigma))$. Given the thresholds, G_L is defined as a unique solution of the ODE (B78) with boundary conditions (B89) and (B90).

The uniqueness of thresholds $\bar{z}^*(\sigma)$ and $z_h^*(\sigma)$ follows from the strict monotonicity of F in z_h. Recall that by construction $z_h^*(\hat{\sigma}) \leq z_h^*(\sigma) < z_{d/e} < \bar{z}^*(\sigma) < \bar{z}^*(\hat{\sigma})$.

Existence and Uniqueness of $\underline{z}^*(\sigma)$ and $z_l^*(\sigma)$. Using Parts I–III of the proof of Proposition 2 with $i = d$, we conclude that differential equations (B77) and (B78) with boundary conditions (B83)-(B88) have a unique solution; thresholds $\underline{z}^*(\sigma)$ and $z_l^*(\sigma)$ are uniquely defined. This is done by a simple relabeling of $\bar{z}^*$ from the proof of Proposition 2 to $z_l^*(\sigma)$. As $\sigma \to +\infty$, $z_l^*(\sigma) \to z_d$ and $z_h^*(\sigma) \to z_{d/e}$ according to Proposition 5. For parameters in case $1(b'')$ and $2(b'')$, $z_d < z_{d/e}$; hence, for sufficiently high σ, $z_l^*(\sigma) < z_h^*(\sigma)$.

Verification. With the quadruple of thresholds $(\underline{z}^*(\sigma), z_l^*(\sigma), z_h^*(\sigma), \bar{z}^*(\sigma))$ the dynamic value functions G_H and G_L from following strategies described in Proposition 4 clearly satisfy equations (B77)–(B90), together with:

$$G_\theta(z) = E_H^e(z) \quad z > \bar{z}^*(\sigma), \tag{B99}$$

$$G_\theta(z) = E_H^d(z) \quad z \in (z_l^*(\sigma), z_h^*(\sigma)), \tag{B100}$$

$$G_H(z) = V_H(\underline{z}^*) \quad z < \underline{z}^*(\sigma), \tag{B101}$$

$$G_L(z) = \frac{k}{r} - I \quad z < \underline{z}^*(\sigma). \tag{B102}$$

Now we will proceed exactly as in Part IV of the proof of Proposition 2. The first thing we need to verify is:

$$G_H(z_t) \geq \max(E_H^e(\lambda^*(q_t^e)), E_H^d(c^*(q_t^d))). \tag{B103}$$

The dynamic value G_H is greater than $E_H^d(c^*)$ for $z < z_l^*(\sigma)$ by the arguments presented in the proof of Proposition 2. Also, $G_H(z_t) = E_H^d(c^*(q_t^d))$ for $z_t \in [z_l^*(\sigma), z_h^*(\sigma)]$ and $G_H(z_t) = E_H^e(\lambda^*(q_t^e))$ for $z_t \geq \bar{z}^*(\sigma)$. By the construction of $z_h^*(\sigma)$, $G_H(z) > E_H^e(\lambda^*(q_t^e))$ for $z_t \in [z_{d/e}, \bar{z}^*(\sigma)]$. Finally, $G_H(z) > E_H^d(c^*(q^d))$ because G_H is convex and E_H^d is concave in $(z_h^*(\sigma), \bar{z}^*(\sigma))$ and the two functions are tangent at $z_h^*(\sigma)$. Therefore, inequality (B103) indeed holds.

Now we check that

$$G_L(z_t) \geq \max(E_L^e(\lambda^*(q_t^e)), E_L^d(c^*(q_t^d))). \tag{B104}$$

Notice that $G_L(z) = E_L^e(\lambda^*(q_t^e))$ for $z_t > \bar{z}^*(\sigma)$ and $G_L(z_t) = E_H^d(c^*(q_t^d))$ for $z_t \in [z_l^*(\sigma), z_h^*(\sigma)]$. Outside of these regions, $\max(E_L^e(\lambda^*(q_t^e)), E_L^d(c^*(q_t^d))) = \frac{k}{r} - I = G_L(\underline{z}^*(\sigma))$. But G_L is increasing in z_t; hence, $G_L(z_t) \geq \max(E_L^e(\lambda^*(q_t^e)), E_L^d(c^*(q_t^d)))$ in the two inaction regions, i.e., $z_t \in (\underline{z}^*(\sigma), z_l^*(\sigma))$ and $z_t \in (z_h^*(\sigma), \bar{z}^*(\sigma))$.

Using the same argument as in the proof of Proposition 2, we conclude that:

$$\tilde{G}_\theta(z) = \sup_\tau \mathbf{E}\left[\int_0^\tau e^{-ru} dX_u + e^{-r\tau} G_\theta(z_\tau)\right]. \tag{B105}$$

The final step is to confirm that:

$$\Gamma_\theta G_\theta(z) \leq 0 \quad \forall\, z. \tag{B106}$$

It is easy to check that $\Gamma_L E_L^e(z)$ and $\Gamma_L E_L^d(z)$ are always negative. Thus, (B106) holds for $\theta = L$.

As for the high type firm, (B106) holds for all $z < z_h^*(\sigma)$, as in the proof of Proposition 2. It also holds with equality if $z \in (z_h^*(s), \bar{z}^*(\sigma))$. Since G_H is tangent to E_H^e from above at $z = \bar{z}^*(\sigma)$, $\Gamma_H E_H^e(\bar{z}^*(\sigma)) < 0 = \Gamma_H G_H(\bar{z}^*(\sigma)-)$. Furthermore, $\Gamma_H E_H^e(z)$ is monotonically decreasing, which implies that $\Gamma_H E_H^e(z) < 0$ for $z > \bar{z}^*(\sigma)$ as well.

With (B106) satisfied, we conclude similarly to the proof of Proposition 2 that:

$$\tilde{G}_\theta(z) = G_\theta(z) = V_\theta^*(z) \quad \forall\, z, \tag{B107}$$

and that no firm type can improve its payoff by any deviation. This completes the proof for Case 1(b'') and 2(b'').

Case 2($\mathbf{c}''$). The whole structure of the proof remains the same as that of Cases 1(b'') and 2(b''), with the only difference being that the kink in the static value function of the high type firm arises not because of a jump in derivatives from $E_H^{d\prime}(z_{d/e})$ to $E_H^{e\prime}(z_{d/e})$ but from switching the coupon from risky $E_H^{d\prime}(z_r-)$ to riskless $E_H^{d\prime}(z_r+)$. The exact proof above follows by relabeling $z_{d/e}$ to z_r and E_H^e to E_H^d.

■

B.8 Proof of Proposition 5

Recall that the thresholds z_l^* and $\underline{z}^*$ are those discussed in Proposition 2, thus their limiting behavior is described in Proposition 3.

As for z_h^* and $\bar{z}^*$, we will prove the statement only for part 1. The proof for part 2 is analogous.

First, note that $(\bar{z}^* - z_h^*) \to 0$ as $\sigma \to \infty$. If it were not the case, there would exist a non-empty interval $(a, b) \in (z_h^*, \bar{z}^*)$ for all σ. Recall that G_L satisfies (B78) in (a, b). Thus, $G_L(z) \to 0$ for $z \in (a, b)$, which contradicts $G_L(z) \geq \frac{k}{r} - I > 0$. But by construction, $z_h^* < z_{d/e} < \bar{z}^*$. Because $(\bar{z}^* - z_h^*) \to 0$, both z_h^* and $\bar{z}^*$ converge to $z_{d/e}$ as $\sigma \to +\infty$. ■

B.9 Proof of Proposition 7

With the positive rate of decay δ, the dynamic value function of the high type firm satisfies:

$$0 = \mu_H - rV_H(z) + \frac{h^2}{2}V_H'(z) + \frac{h^2}{2}V_H''(z) + \delta\left(\frac{\mu_H}{r} - V_H(z)\right) \tag{B108}$$

in the inaction region. The last term in the equation above reflects the possibility of the investment opportunity evaporating in the next instance of time, which results in the loss of the option to invest later, $V_H(z)$, and a gain of the NPV of the assets in place, μ_H/r.

Rearranging yields:

$$rV_H(z) = \mu_H + \frac{h^2 r}{2(r+\delta)}V_H'(z) + \frac{h^2 r}{2(r+\delta)}V_H''(z). \tag{B109}$$

Recall that $h = \mu_H/\sigma$ and define $\tilde{\sigma} = \sigma\sqrt{1+\delta/r}$ and $\tilde{h} = \mu_H/\tilde{\sigma}$. This relabeling of

parameters reduces the equation above to a familiar equation:

$$rV_H(z) = \mu_H + \frac{\tilde{h}^2}{2}V'_H(z) + \frac{\tilde{h}^2}{2}V''_H(z), \tag{B110}$$

which establishes the equivalence since the positive rate of decay, δ, does not affect the values conditional on investment (E^e_H and E^d_H). An analogous argument holds for the low type firm as well. ■

B.10 Proof of Proposition 8

1. Part (a) holds trivially because (i) the dynamic pooling region is always inside the static pooling one and (ii) the dynamic equilibrium strategy prescribes the firm to invest right away in the dynamic pooling region, yielding the static pooling payoff.

Part (b) follows from the proofs of Propositions 2 and 4, which guarantee that $V^*_H(z) > \max\left(E^e_H(z), E^d_H(z)\right)$ and $G_H(z) > \max\left(E^e_H(z), E^d_H(z)\right)$ inside the inaction regions. The last comparison is between E^D_H and the value of foregoing the investment opportunity $E^\emptyset_H$. Finally, notice that:

$$E^S_H(z) = \max\left(E^e_H(z), E^d_H(z), E^\emptyset_H\right). \tag{B111}$$

2. Part (a) holds in the dynamic pooling region because of the same argument as Part 1(a). Moreover, in the intersection between the static and dynamic separating regions, both E^D_L and E^S_L are equal to $\frac{k}{r} - I$ and thus equal to each other.

Recall that $\Gamma_L E^i_L(z) < 0$ for all z and $i = \{e, d\}$, which implies that when the market updates its beliefs based only on the history of cash flows but not the equilibrium strategies, the low type firm would prefer immediate pooling, yielding the static payoff E^S_L. Part (b) follows immediately since in the intersection between the dynamic inaction and static pooling regions, the low type is forced to wait in the dynamic environment instead of immediate pooling.

Dynamic value functions V^*_L and G^*_L are strictly increasing in market belief z_t if $z_t > \underline{z}^*$ and $V^*_L(\underline{z}^*) = G^*_L(\underline{z}^*) = \frac{k}{r} - I$. Part (c) follows from the fact that in the intersection between the dynamic inaction and static separation regions, $E^S_L = \frac{k}{r} - I$. ■

B.11 Proof of Proposition 9

1. Part 1 follows directly from Parts 1(a) and 2(a) of Proposition 8.

2. The static separating region can intersect either the dynamic separating region or the dynamic inaction region. In the former case, the low type firm is indifferent but the high type firm is strictly better off in the dynamic environment. In the latter case, both types are strictly better off, as shown in Parts 1(b) and 2(c) of Proposition 8. ■

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