CCP Auction Design

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Abstract

Central counterparties (CCPs) are systemically important. When a clearing member defaults, the CCP sells the defaulted portfolio to surviving members in an auction, and losses, if any, are partly absorbed by a cash pool prefunded by the surviving members. We propose a tractable auction model that incorporates this salient feature. We find that "juniorization" – the CCP first uses prefunded cash of members who submit bad bids – increases the auction price. Aggressive juniorization can push the auction price above the fair value and almost eliminate the need to use prefunded resources. Nonetheless, juniorization generates heterogeneous impact on members of different sizes, and so does customer participation in the auction.

Keywords: central counterparty (CCP), auction, default management
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1 Introduction

A central counterparty (CCP) stands between a buyer and a seller in a derivatives trade or securities financing transaction, protecting the two parties from each other's default. Since the financial crisis of 2008-09 and the ensuing central clearing mandate for standardized over-the-counter derivatives, CCPs have grown substantially in size and scope and become systemically important.[1] As of the first half of 2020, about \$388 trillion notional amount of interest rate derivatives are cleared by CCPs, with a gross market value of about \$6 trillion (BIS, 2020).

A CCP may come under stress when a clearing member defaults, at which point the CCP inherits the defaulted portfolio and its directional exposure. If market prices continue to move against the defaulted portfolio, the CCP is obligated to make payments to the other side, though it no longer receives payment from the defaulted member. In a market turmoil that can push a clearing member into default, the CCP's directional exposures can quickly lead to large losses that threaten its own stability and the stability of the financial system.

As a key component of risk management, CCPs use auctions to sell the defaulted portfolio, therefore eliminating the directional exposure and stopping the losses (CPMI-IOSCO, 2012, 2020). The CCP's losses are covered by prefunded resources from the defaulting member, the CCP, and the surviving members, in this order. Importantly, resources that the CCP uses to pay the winners of the auction partly come from the bidders themselves. In this sense, CCP auctions are unique.

In this paper, we analyze CCP auction design that incorporates this important feature: bidders are also potential payers. In particular, we study the "juniorization" mechanism of surviving members' resources, known as guarantee funds.[2] The principle of juniorization is that members submitting bad bids will see their guarantee funds used ahead of, i.e., junior to, members submitting better bids. Building on Du and Zhu (2017), we model a CCP auction as a divisible, uniform-price auction, which tends to be used for large defaulted portfolios that few or no single bidders have the capacity to absorb.[3]

In our baseline model, only surviving members of the CCP can participate in the auction. The fundamental value of the auctioned portfolio is common knowledge, and bidders incur quadratic costs for holding inventories, which is equivalent to linearly decreasing marginal values in acquired

[1] In 2012, the Financial Stability Oversight Council of the United States designated eight systemically important financial market utilities: The Clearing House Payment Company, CLS Bank International, Chicago Mercantile Exchange, the Depositor Trust Company, Fixed Income Clearing Corporation, ICE Clear Credit, National Securities Clearing Corporation, the Options Clearing Corporation. Among the eight, five are CCPs (CME, FICC, ICE, NSCC, and OCC). See https://home.treasury.gov/system/files/261/here.pdf.

[2] The size of members' guarantee fund contributions is increasing in their required initial margin. Thus, members who have larger or riskier positions have larger guarantee fund contributions.

[3] In practice, CCP auctions are generally sealed-bid. Some CCPs such as ICE Credit use divisible auctions, whereas others such as Nasdaq clearing use indivisible ones.

quantity. The bidders submit demand schedules and pay for their allocations at the market clearing price, which can be negative. A very negative price means the CCP has to pay a substantial amount to the auction winners out of the prefunded resources.

We model the CCP's resources as two parts. The first part is the defaulters' and the CCP's resources, which is used first to absorb losses. If that is insufficient, the CCP uses the guarantee funds contributed by the surviving members. Surviving members have heterogeneous guarantee fund contributions that follow some probability distribution. To model juniorization, we assume that the use of a member's guarantee fund is linearly decreasing in the member's equilibrium winning amount of the defaulted portfolio, subject to the constraint that the used guarantee fund is positive and no more than the prefunded amount.[4]

With juniorization, a bid serves two purposes: making a profit on the defaulted portfolio and reducing the amount of one's own guarantee fund that the CCP uses to cover losses. We find that small members, i.e., those who contribute a low amount of guarantee funds, are barely affected by juniorization because they only need to purchase a small fraction of the defaulted portfolio to save all their guarantee funds. Large members, however, are highly incentivized by juniorization due to the large amount of guarantee funds at stake. Consequently, larger members are more aggressive bidders in the CCP auction and win a bigger fraction of the defaulted portfolio.

The key design variable is the degree of juniorization, modeled as how much guarantee funds a bidder can save by buying each additional unit of the defaulted portfolio. On top of the conventional motivation of making a profit, bidders in CCP auctions have an additional incentive to save their guarantee funds, and this incentive is increasing in the size of one's own guarantee funds at stake. We find that more aggressive juniorzation increases the large members' bids substantially and raises the price. The higher price, in turn, reduces the demand of small members who bid primarily to make a profit. As juniorization becomes sufficiently aggressive, the auction price rises above the fair market value, at which point the only reason a bidder trades at such an inflated price is to save its own guarantee fund. More starkly, it is possible to raise the auction price to such a high level that only a tiny amount of guarantee funds is used. It is not possible, however, to completely eliminate the use of survivors' guarantee funds because the threat of using their guarantee funds is the very reason that large members bid aggressively.

While juniorization raises the auction price effectively, it has heterogeneous impacts across

[4] In practice, CCPs tend to sort bidders into discrete tiers according to their bids. For instance, CME CDS/IRS Clear in its rulebook defines "non-qualifying bidders" as the ones that either submit no bids or their bids are worse than a pre-defined price range. If the loss from the auction dips into surviving members' guarantee fund contributions, the non-qualifying bidders' contributions will be used first, followed by the non-winning bidders', whereas the winning bidders' guarantee fund contributions have highest seniority. Because bids with higher prices are filled ahead of those with lower prices in an divisible auction, juniorization based on equilibrium allocation captures the core spirit of this design and retains analytical tractability.

members. At a high price, the allocation of the defaulted portfolio concentrates among the largest members and cause them to incur high inventory costs. The smallest members also suffer from more aggressive juniorization because the resulting high price prevents them from bidding profitably. It is the members who have intermediate range of guarantee fund contributions that benefit from juniorization. While unmodeled, it is reasonable to expect the CCP to prefer a high auction price due to the reputational benefit associated with a successful auction that leads to minimal use of guarantee funds.

In an extension of the baseline model, we study the auction participation of customers who have zero guarantee fund contributions. We show that including customers promotes competition and increases the auction price, which, in turn, reduces the use of guarantee funds and the losses of the largest clearing members. Small and intermediate-sized clearing members, whose guarantee funds are not used in equilibrium, tend to suffer from the participation of customers. All these effects are weaker when juniorization is more aggressive. If juniorization is sufficiently aggressive that the auction price is above the fair value, customers no longer bid and their participation does not affect auction outcomes.

Our results shed lights on recent market and regulatory debate around the design of CCP auctions. A case in mind is Nasdaq Clearing. In September 2018, a single (albeit large) trader's default loss wipes away two-thirds of surviving members' guarantee funds in Nasdaq Clearing (Financial Times, 2018). Out of more than 100 clearing members, Nasdaq Clearing only invited five to the auction. The restriction to a small number of participants could have contributed to the large default loss (Faruqui et al., 2018). Our results also suggest that Nasdaq Clearing could have used juniorization to promote more competitive bidding.

To the best of our knowledge, the only other papers that study CCP auctions are Ferrara, Li and Marszalec (2020) and Oleschak (2019). Both papers apply the standard model of private-value indivisible auctions to the CCP setting. Ferrara, Li and Marszalec (2020) introduce a reduced-form penalty for submitting bad bids, but the penalty is independent of the default losses of the CCP. Oleschak (2019) finds that the CCP's loss allocation method does not affect the CCP's losses or the bidders'. This is because in his model, loss allocations are not constrained by the prefunded amount of guarantee funds. Our model incorporates key institutional details of CCP auctions and derive novel results, in particular, the effect of juniorization on prices, bidding strategies, and profits across hetergeneous bidders.

Our results contribute to the broad literature on central clearing. In particular, the strategic incentives of clearing members to bid in CCP auctions can be combined with other known aspects of the economics of central clearing, including netting efficiency (Duffie and Zhu, 2011), risk sharing and risk management (Haene and Sturm, 2009; Biais, Heider and Hoerova, 2016; Raykov, 2017), loss allocation rules (Elliott, 2013; Nahai-Williamson et al., 2013; Wang, Capponi and Zhang, 2021), and the choice between for-profit and utilities (Huang, 2019), among others. Menkveld and Vuillemey (2020) provide an in-depth review of the literature on central clearing.

2 A Model of CCP Default Management Auctions

A CCP inherits a portfolio from its defaulting members and need to sell it in an auction. The portfolio has size $Q > 0$. The auction is uniform price and fully divisible. We assume that the auction is held immediately after default, and the mark-to-market value of the portfolio is zero. Let p denote the per unit price of the portfolio paid by the bidders. Typically, $p < 0$, so $-pQ > 0$ is the loss from the auction that the CCP has to pay from the prefunded resources.

The CCP has three types of prefunded resource to absorb the loss: (i) the defaulting members' initial margin and guarantee funds; (ii) the CCP's skin-in-the-game; and (iii) the surviving member's guarantee funds. Resources (i) and (ii) add up to $M > 0$, whereas resource (iii) is $G > 0$.

For analytical simplicity, we assume that there is a continuum of infinitesimal surviving members of mass one.[5] Member i contributes $g_i \geq 0$ into the guarantee fund, where g_i has the probability distribution function F with support $[0, \infty)$. Hence, the total guarantee fund is the integral of all individual guarantee fund contributions, i.e., $G \equiv \int g dF(g)$.

The fundamental value of the defaulted portfolio is v per unit, reflecting the bidders' view that the value of the defaulted portfolio may continue to decline ($v < 0$) or rebound ($v > 0$) after the auction.[6]

The distribution function $F(\cdot)$, as well as v, Q, M and G, is common knowledge to all bidders.

Once the auction produces a price p for the defaulted portfolio, there are three scenarios:

I. $pQ + M \geq 0$: The defaulters' and CCP's resources are sufficient to cover the auction losses, and there is no need to use the guarantee fund.

II. $pQ + M < 0 \leq pQ + M + G$: The defaulters' and CCP's resources are insufficient, but the guarantee fund is sufficient to cover the losses.

III. $pQ + M + G < 0$: The resources from the defaulters, the CCP, and the surviving clearing members are insufficient to cover losses, and the auction "fails." The CCP resorts to

[5] In practice, of course, there are finitely many members, and some are very large. The main advantage of using infinitesimal members is that they have zero price impact and hence do not wish to affect the auction price. This simplification shortens the analytical solutions dramatically. One can extend this model to allow for finitely many clearing members and positive price impact.

[6] In reality, the fundamental value of the defaulted portfolio is not necessarily common knowledge to all bidders and the CCP. But modeling such asymmetric information makes the model intractable quickly. This paper proposes a prototype model in which the novelty comes from heterogeneous guarantee fund contributions and associated incentives, while keeping all other dimensions as simple as possible.

more extreme methods of default management such as variation margin gains hair-cutting (VMGH) and partial tear-ups, among others. If these mechanisms are not credible or their consequences are not deemed to be acceptable, the official sector may step in and resolve the CCP.

In this paper, we focus on Scenarios I and II. Because Scenario I is the best outcome and is relatively straightforward, most of our modeling effort is devoted to Scenario II. Scenario III threatens the viability of the CCP itself and will likely involve the official sector and its political constraints, which are beyond the scope of our simple model.

Now we turn to the auction mechanism and the preference of bidders. In the main model, the auction is open only to clearing members. In the auction, each bidder i submits a demand curve $x_i(p)$, that is, at the price p bidder i is willing to purchase quantity $x_i(p)$. As in practice, bidders cannot release additional supply in the auction, so $x_i(p) \geq 0$. The market-clearing condition is

$$\int_i x_i(p)di = Q. \tag{1}$$

As described before, CCPs typically adopt a "juniorization" mechanism to incentivize bids. If a clearing member fails to submit bids or submits "bad" bids relative to peers, its guarantee fund contribution is "juniorized," that is, used before the guarantee fund contribution of other members who submit better bids. To model juniorization, we directly specify the use of bidder i's guarantee fund to be

$$T_i = \max\left[\frac{-(pQ+M)}{A}g_i - cx_i, 0\right], \tag{2}$$

where $x_i \geq 0$ is the equilibrium allocation to bidder i, and $A > 0$ and $c \geq 0$ are constants to be determined by the CCP. Once they are set, A and c are common knowledge. All else equal, a bidder who wins more in the auction sees a less amount of her guarantee funds used by the CCP. As it becomes clear shortly, A is chosen to balance the CCP's budget, so it is a function of c. This leaves c to be the only design variable. A special case is the "pro-rata" use of guarantee funds: $c = 0$, $A = G$, and $T_i = -(pQ + M)\frac{g_i}{G}$, where the transfer T_i is proportional to guarantee fund contribution g_i.

Finally, each bidder i maximizes the risk-adjusted profit

$$\pi_i \equiv (v-p)x_i - 0.5\lambda x_i^2 - T_i. \tag{3}$$

The first term $(v - p)x_i$ is the profit made for purchasing the portfolio at the price p, but with fair value v; the second term $-0.5\lambda x_i^2$ is the inventory cost of acquiring a part of the defaulted portfolio in the auction, where $\lambda > 0$ is a constant; and the last term $-T_i$, given in (2), represents the use of bidder i's guarantee fund.

3 Equilibrium and Properties

In this section, we analyze the equilibrium bidding strategies. For simplicity, we omit the subscript i whenever there is no potential confusion.

3.1 Equilibrium for Scenario I

The first and simplest scenario is if the defaulters' resource M is sufficient to cover the CCP's losses, that is, $p^*Q + M \geq 0$. In this case, survivors' guarantee funds are not used at all, i.e., $T_i = 0$. Everyone bids

$$x(p) = \frac{1}{\lambda}(v - p). \tag{4}$$

And the equilibrium price satisfies $\frac{1}{\lambda}(v - p) = Q$, or

$$p^* = v - \lambda Q. \tag{5}$$

The condition for obtaining this case of equilibrium is that $p^*Q + M = (v - \lambda Q)Q + M \geq 0$.

3.2 Equilibrium for Scenario II: Pro-rata allocation

Before analyzing juniorization, it is useful to first examine the special case of pro-rata allocation of losses, i.e., $c = 0$ and $A = G$. The pro-rata use of guarantee funds means $T_i = \frac{-(pQ+M)}{G} g_i$, and thus $\int_i T_i = -(pQ + M)$, i.e., budget balance is satisfied.

Because the guarantee fund use T_i is independent of x_i, the bidding strategy is the same as Scenario I: $x = \frac{v-p}{\lambda}$. And the equilibrium price is $p^* = v - \lambda Q$.

To ensure that this equilibrium indeed belongs to Scenario II, we need $p^*Q + M < 0 \leq p^*Q + M + G$, or $(v - \lambda Q)Q + M < 0 \leq (v - \lambda Q)Q + M + G$.

3.3 Equilibrium for Scenario II: Juniorization with $p \leq v$

Now let's turn to the key part of the analysis with juniorization, i.e., $c > 0$. Important for the equilibrium characterization of juniorization is the fact that T has a kink at $x = \frac{-(pQ+M)}{A} g/c$. Therefore, a bidder's profit π is differentiable in x in the interval $\left[0, \frac{-(pQ+M)}{A} g/c\right)$ and $\left(\frac{-(pQ+M)}{A} g/c, \infty\right)$, but not at the kink.

We start by conjecturing that there are three possible cases of equilibrium:

1. $x < \frac{-(pQ+M)}{A} g/c$, and $T > 0$;
2. $x = \frac{-(pQ+M)}{A} g/c$, and $T = 0$; and
3. $x > \frac{-(pQ+M)}{A} g/c$, and $T = 0$.

Case 1 and Case 3 are "interior" cases in the sense that if a bidder changes her demand slightly, she remains in the same case. But Case 2 is at the kink because a slight decrease in x pushes a bidder into Case 1 and a slight increase in x pushes the bidder into Case 3. Therefore, the first-order conditions of Case 1 and Case 3 are equalities, whereas the first-order condition of Case 2 should be inequalities.

We further conjecture that there are two thresholds, g_L and $g_H \geq g_L$, such that Case 1 applies if $g > g_H$, Case 2 applies if $g \in [g_L, g_H]$, and Case 3 applies if $g < g_L$.

The first-order conditions of Case 1 and Case 3 are, respectively,

$$v - p - \lambda x_1 + c = 0; \tag{6}$$

$$v - p - \lambda x_3 = 0. \tag{7}$$

The corresponding demand functions are

$$x_1(p) = \frac{1}{\lambda}(v - p + c); \tag{8}$$

$$x_3(p) = \frac{1}{\lambda}(v - p). \tag{9}$$

Obviously, for x_3 to be positive, we need $p \leq v$. We will look for equilibrium with $p \leq v$ in this subsection. The case for $p > v$ is deferred until the next subsection.

In Case 2, $x_2 = \frac{-(pQ+M)}{A} g/c$ is conjectured to be optimal. An increase of x_2 pushes a bidder into Case 3 (where juniorization is ineffective), and a decrease of x_2 pushes the bidder into Case 1 (where juniorization is effective). Thus, for $g \in (g_L, g_H)$, we have

$$v - p - \lambda \frac{-(pQ + M)}{A} \frac{g}{c} + c > 0, \tag{10}$$

$$v - p - \lambda \frac{-(pQ + M)}{A} \frac{g}{c} < 0. \tag{11}$$

At the equilibrium price p^*, the boundary conditions are

$$v - p^* - \lambda \frac{-(p^* Q + M)}{A} \frac{g_H}{c} + c = 0, \tag{12}$$

$$v - p^* - \lambda \frac{-(p^* Q + M)}{A} \frac{g_L}{c} = 0. \tag{13}$$

The market clearing condition is

$$
\begin{aligned}
Q &= \int_0^{g_L} x_3 f(g)dg + \int_{g_L}^{g_H} x_2 f(g)dg + \int_{g_H}^{\infty} x_1 f(g)dg, \\
&= F(g_L)\frac{1}{\lambda}(v-p^*) + \int_{g_L}^{g_H} \frac{-(p^*Q+M)}{A}\frac{g}{c} f(g)dg + (1-F(g_H))\frac{1}{\lambda}(v-p^*+c).
\end{aligned} \tag{14}
$$

The budget balance constraint is

$$
p^*Q + M + \int_{g_H}^{\infty}\left[\frac{-(p^*Q+M)}{A}g - c\frac{v-p^*+c}{\lambda}\right] f(g)dg = 0, \tag{15}
$$

where the integral starts with g_H because only bidders with guarantee funds at least g_H lose part of it. Substituting (12), or $v - p^* + c = \lambda\frac{-(p^*Q+M)}{A}\frac{g_H}{c}$, into (15) and simplifying, we get

$$
A = \int_{g_H}^{\infty} (g - g_H) f(g)dg, \tag{16}
$$

which depends on c only through g_H. From (12) and (13), we get

$$
\frac{-(p^*Q+M)}{A}(g_H - g_L) = \frac{c^2}{\lambda}. \tag{17}
$$

Substitute the above expression into (14), we get an alternative expression of the market-clearing condition

$$
Q = F(g_L)\frac{1}{\lambda}(v-p^*) + \frac{c}{\lambda}\frac{1}{g_H - g_L}\int_{g_L}^{g_H} g f(g)dg + (1-F(g_H))\frac{1}{\lambda}(v-p^*+c). \tag{18}
$$

The equilibrium price can be rewritten as

$$
p^* = v - \frac{\lambda Q - \frac{c}{g_H - g_L}\int_{g_L}^{g_H} g f(g)dg - c(1-F(g_H))}{F(g_L) + 1 - F(g_H)}. \tag{19}
$$

Before characterizing the equilibrium, it is useful to characterize the solutions p^*, g_H, and g_L.

Lemma 1. *For a fixed $c > 0$, let g_H, g_L, and p^* solve (12), (13), and (19), where the constant A is given by (16). As $c \to 0$, we have*

$$
g_H \to 0, \quad g_L \to 0, \quad A \to G, \quad \frac{d(g_H - g_L)}{dc} \to 0, \quad p^* \to v - \lambda Q. \tag{20}
$$

Proof. By (17), we have $\lim_{c\to 0}(g_H - g_L) = 0$. In the numerator of the expression of p^* in (19), we can use the intermediate value theorem to write $\int_{g_L}^{g_H} gf(g)dg = (g_H - g_L)g'f(g')$ for some $g' \in [g_L, g_H]$. Then the term $\frac{c}{g_H - g_L}\int_{g_L}^{g_H} gf(g)dg$ simplifies to $cg'f(g')$, which converges to zero as $c \to 0$. Thus, $p^* \to v - \lambda Q$ as $c \to 0$.

Since $g_H - g_L \to 0$, either both converge to the same finite value or both diverge. If both diverge to infinity, then (12) and (13) cannot hold, as A is bounded above by G. So they must converge to a finite value, implying that A has a well-defined limit that is positive. Now taking the limit on both sides of (13), we have $\lambda Q - \lambda \frac{-((v-\lambda Q)Q+M)}{\lim_{c\to 0} A} \lim_{c\to 0} \frac{g_L}{c} = 0$. Since $\lim_{c\to 0} A > 0$, we have $g_L \to 0$, which implies $g_H \to 0$ and $A \to G$.

Finally, taking the derivative with respect to c on both sides of (17), we have

$$\frac{d}{dc}\left(\frac{-(p^*Q+M)}{A}\right)(g_H - g_L) + \frac{-(p^*Q+M)}{A}\frac{d(g_H - g_L)}{dc} = \frac{2c}{\lambda}. \tag{21}$$

Taking the limit on both sides as $c \to 0$ and using $g_H - g_L \to 0$, we have $\frac{d(g_H - g_L)}{dc} \to 0$ as well. ■

The above result establishes a form of continuity in Scenario II. As juniorization becomes weaker and weaker, the solution converges to the pro-rata case. Now we are ready to characterize the equilibrium with juniorization.

Proposition 1. *Suppose that $-G < (v - \lambda Q)Q + M < 0$. There exists some $c_1 > 0$ such that for all $c < c_1$, the following is an equilibrium:*

Let g_H, g_L, and p^* solve (12), (13), and (19), where the constant A is given by (16). Each bidder uses the strategy

$$x(p) = \begin{cases} \frac{1}{\lambda}(v - p + c), & \text{if } g > g_H \\ \frac{-(pQ+M)}{A}\frac{g}{c}, & \text{if } g \in [g_L, g_H] \\ \frac{1}{\lambda}(v - p), & \text{if } g < g_L \end{cases}. \tag{22}$$

In this equilibrium, the auction price satisfies $p^* < v$, $-G < p^*Q + M < 0$ and $0 \le T_i \le g_i$.

Proof. The equilibrium strategy is already derived in the discussion proceeding this proposition. To see the final part, note that as $c \to 0$, $A \to G$ and $p^* \to v - \lambda Q$. Combining these with the assumption that $-G < (v - \lambda Q)Q + M < 0$, we see that by continuity, $p^* < v$ and $-G < p^*Q + M < 0$ for any sufficiently small c. Also by continuity, $p^*Q + M + A \ge 0$, so $T_i \in [0, g_i]$. ■

Note that the condition $p^* < v$ guarantees that bidders in case 3 (small guarantee funds, buying $(v-p)/\lambda$) acquire positive quantities, and the condition $p^*Q + M < 0$ guarantees that bidders in case 2 (buying $\frac{-(pQ+M)}{A}\frac{g}{c}$) also acquire positive quantities. Under these conditions, bidders in case 1 always acquire positive quantities as $x_1 = x_3 + c/\lambda > 0$.

With the two results, we can ask whether juniorization is effective in increasing the auction price, relative to pro-rata use of the guarantee fund. The next proposition shows that $dp^*/dc > 0$ when c is sufficiently small, that is, pro-rata is dominated by a mild use of juniorization.

Proposition 2. *In the equilibrium of Proposition 1, there exists some $c_2 > 0$ such that $dp^*/dc > 0$ for $c < c_2$.*

Proof. For simplicity of notation, we can write $p^* = v - \frac{B_1}{B_2}$, where B_1 and B_2 are the numerator and denominator in the (19). As shown in the previous proposition, $B_1 \to \lambda Q$ and $B_2 \to 1$ as $c \to 0$. Again, by the intermediate value theorem, write $\int_{g_L}^{g_H} g f(g) dg = (g_H - g_L) g' f(g')$ for some $g' \in [g_L, g_H]$. The derivatives are

$$\frac{dB_1}{dc} = -g' f(g') - (1 - F(g_H)) + c f(g_H) \frac{dg_H}{dc} \to -1, \tag{23}$$

$$\frac{dB_2}{dc} = f(g_L) \frac{dg_L}{dc} - f(g_H) \frac{dg_H}{dc} \to -f(0) \frac{d(g_H - g_L)}{dc} \to 0. \tag{24}$$

Hence, as $c \to 0$,

$$\frac{dp^*}{dc} \to -\frac{(-1)(1) - (\lambda Q)(0)}{1^2} = 1. \tag{25}$$

By continuity, $dp^*/dc > 0$ in an open neighborhood of $c = 0$ as well. ■

The intuition of Proposition 2 could be seen as follows. For any sufficiently small c, we know that g_L and g_H are both close to zero, i.e., the vast majority of clearing members are in Case 1. Their demand function is $x_i(p) = (v - p + c)/\lambda$. A higher c directly translates into a higher demand from almost all members, hence a higher price.[7]

The main conclusion from Proposition 2 is that switching from pro-rata to (mild) juniorization increases the equilibrium price. If price maximization is part of the CCP's objective, this result is consistent with the practice of juniorization used by CCPs.

A numerical procedure for finding equilibrium with $p^* < v$. While the analytical characterization in Proposition 1 requires c be sufficiently close to 0, it is possible to numerically search for an equilibrium for any $c > 0$. Specifically, for $c > 0$, let g_H, g_L, and p^* solve (12), (13), and (19), where the constant A is given by (16). If $p^* < v$ and $-A \leq p^* Q + M < 0$, then it is an equilibrium that each bidder uses the same bidding function as in Proposition 1. In this equilibrium, the auction price is p^*, and $0 \leq T_i \leq g_i$.

[7] This logic may or may not work if c is already high and g_H is sufficiently far from zero. Any further increase in c raises the demand from Case-1 members (demand $(v - p + c)/\lambda$) but reduces the demand from Case-2 members (demand $\frac{-(pQ+M)}{A} g_i/c$). The net impact is not obvious.

3.4 Equilibrium for Scenario II: Juniorization with $p \geq v$

So far, we have focused on equilibrium in which $p^* < v$ and even bidders whose guarantee funds are very close to zero win a positive amount of the auctioned portfolio. Now, we look for an equilibrium in which $p^* \geq v$.

With $p^* \geq v$, only Case 1 and Case 2 apply, since the Case-3 demand $(v - p)/\lambda$ hits the floor of zero. The first-order condition for Case 1 remains (6). Bidders in Case 2 still bid $-\frac{-(pQ+M)}{A} g/c$. At the equilibrium price, the boundary condition is still (12). The budget balance constraint remains unchanged, which means that A is still (16).

The market clearing condition, however, changes to

$$Q = \int_0^{g_H} x_2 f(g)dg + \int_{g_H}^{\infty} x_1 f(g)dg = \int_0^{g_H} \frac{-(p^*Q + M)}{A} \frac{g}{c} f(g)dg + (1 - F(g_H))\frac{1}{\lambda}(v - p^* + c),$$
$$= \frac{v - p^* + c}{\lambda}\left(\frac{1}{g_H}\int_0^{g_H} g f(g)dg + (1 - F(g_H))\right). \tag{26}$$

The equilibrium price is

$$p^* = v + c - \frac{\lambda Q}{\frac{1}{g_H}\int_0^{g_H} g f(g)dg + (1 - F(g_H))} = v + c - \lambda Q \frac{g_H}{G - A}. \tag{27}$$

Substituting (12) into the above equation, we have

$$\frac{-(p^*Q + M)}{A} = \frac{cQ}{G - A} \tag{28}$$

Thus, $-(p^*Q + M) > 0$ because $0 \leq A \leq G$, $c > 0$ and $Q > 0$. In other words, if $p^* \geq v$, then at this price, the CCP will need to use the guarantee fund.

Proposition 3. *Suppose $vQ + M < 0$. There exist some $c_4 > c_3 > 0$ such that for any $c \in [c_3, c_4]$, the following is an equilibrium:*

Let g_H and p^* solve (12) and (27), where the constant A is given by (16). Each bidder uses the strategy

$$x(p) = \begin{cases} \frac{1}{\lambda}(v - p + c), & \text{if } g > g_H \\ \frac{-(pQ+M)}{A}\frac{g}{c}, & \text{if } g \leq g_H \end{cases}. \tag{29}$$

In this equilibrium, the price $p^* > v$ and $0 \leq T_i \leq g_i$. Moreover, p^* is increasing in c.

Proof. The full proof is in the Appendix, but we outline the main steps here. The equilibrium strategy is already derived in the discussion proceeding this proposition. We first show that $dp^*/dc > 0$ if the equilibrium exists. To see that, we take the total derivatives of (16), (28) and (27). Next, to prove the existence of this equilibrium, we establish the continuity between the equilibrium in Proposition 1 and the equilibrium in this proposition around the price $p^* = v$. Finally, we show that when $c \in [c_3, c_4]$, $-(p^*Q + M)/A \leq 1$ so that $T_i \leq g_i$ for all x_i. ■

The above results show that a large c can increase the equilibrium price above the fundamental value v. If the CCP's objective is to increase the auction price, it has a strong incentive to set a high c. We stress, however, that c cannot be too high, for otherwise $p^*Q + M$ would become positive and juniorization would not be effective.

A numerical procedure for finding equilibrium with $p^* \geq v$. The analytical characterization of the equilibrium in Proposition 3 requires c be in a particular range. But as before, it is possible to numerically search an equilibrium with $p^* \geq v$. Suppose $vQ + M < 0$. For any $c > 0$, let g_H and p^* solve (12) and (27). If the resulting p^* satisfies $p^* \in [v, -M/Q)$, then the equilibrium bidding strategy is given in (29). Compared to the analytical characterization, the numerical procedure involves verifying $p^* \in [v, -M/Q)$ ex post.

Our last analytical result is the impact of juniorization c on the members' profits.

Proposition 4. *In an equilibrium with $p^* \geq v$, as c increases, the profit of a member with $g_i \leq g_H$ decreases, the profit of a member with $g_i \in (g_H, \frac{G}{G-A}g_H)$ increases, and the profit of a member with $g_i \geq \frac{G}{G-A}g_H$ decreases.*

Proof. When $p^* \geq v$ and $g_i \leq g_H$, by Proposition 3, one has $x_i = \frac{Q}{G-A}g_i$ and $\pi_i = (v - p^*)x_i - \frac{\lambda}{2}x_i^2$. Furthermore, from the proof of Proposition 3, one has $dp^*/dc \in (0, 1)$. Suppressing p^* as p and taking the total derivative of x_i, one has $dx_i = Qg_i d\frac{1}{G-A} = \frac{Qg_i}{(G-A)^2}dA$. Thus, dx_i/dc can be written as $\frac{dx_i}{dc} = \frac{dx_i}{dA}\frac{dA}{dg_H}\frac{dg_H}{dc} = -\frac{Qg_i}{cG}\left(\frac{dp}{dc} + \frac{A}{G-A}\right) < 0$. Taking the total derivative of π_i, one has $d\pi_i = (v - p - \lambda x_i)dx_i - x_i dp$. Thus,

$$
\begin{aligned}
\frac{d\pi_i}{dc} &= \frac{dp}{dc}\left(\frac{\lambda Qg_i - cG}{cG}\frac{Qg_i}{G-A} + \frac{Qg_i}{G} - \frac{Qg_i}{G}\frac{\lambda Qg_H}{c(G-A)}\right) + \frac{Qg_i}{cG}\frac{A}{G-A}\left(c - \frac{\lambda Qg_H}{G-A} + \lambda\frac{Qg_i}{G-A}\right); \\
&< \frac{Qg_i}{G}\left(\frac{\lambda Qg_i - cG}{c(G-A)} + 1 - \frac{\lambda Qg_H}{c(G-A)} + \frac{A}{G-A}\left(1 - \frac{\lambda Qg_H}{c(G-A)} + \lambda\frac{Qg_i}{c(G-A)}\right)\right); \\
&= \frac{Qg_i}{G}\frac{G}{G-A}\frac{\lambda Q(g_i - g_H)}{c(G-A)} < 0;
\end{aligned}
\tag{30}
$$

where the first inequality comes from $\frac{dp}{dc} \in (0, 1)$ and the second one comes from $g_i \leq g_H$. Thus, for a member with $g_i \leq g_H$, his profit decreases in c.

When $p > v$ and $g_i > g_H$, x_i and π_i can be written as $x_i = \frac{Qg_H}{G-A}$ and $\pi_i = \frac{\lambda}{2}x_i^2 - \frac{cQ}{G-A}g_i$. Thus, one has $dx_i = (dc - dp)/\lambda$. Similarly, one has $d\pi_i = \lambda x_i dx_i - Qg_i d\frac{c}{G-A} = \frac{Qg_H}{G-A}(dc - dp) -$

$\frac{Qg_i}{G-A}\left(dc + \frac{c}{G-A}dA\right)$. Replacing $dA = -\frac{(G-A)^2}{cG}\left(dp + \frac{A}{G-A}dc\right)$, one has

$$\frac{d\pi_i}{dc} = \frac{Qg_H}{G-A}\left(1 - \frac{dp}{dc}\right) - Qg_i\left(\frac{1}{G-A} - \frac{1}{G}\frac{dp}{dc} - \frac{A}{G(G-A)}\right) = \left(\frac{Qg_H}{G-A} - \frac{Qg_i}{G}\right)\left(1 - \frac{dp}{dc}\right). \quad (31)$$

As $dp/dc \in (0, 1)$ when $p \geq v$, for $g_i \in (g_H, \frac{G}{G-A}g_H)$, $d\pi/dc > 0$; for $g_i \geq \frac{G}{G-A}g_H$, $d\pi/dc < 0$. ■

Proposition 4 shows that, in the case of $p^* > v$, members with very small or very large guarantee funds suffer from more aggressive juniorization, while members with medium size guarantee funds benefit from it. The intuition is the following. More aggressive juniorization raises the price, which leads to a loss to those members who purchase at this inflated price, but it also reduces the shortfall $-(p^*Q + M)$, which means a smaller amount of the guarantee funds is used. For the largest clearing members, who have the highest demand, the loss from buying at inflated prices dominates. For members with intermediate guarantee fund contribution, the benefit of a smaller shortfall dominates. For the smallest members who bid only to save their guarantee funds, it turns out that the cost of buying at a higher price dominates.

3.5 A numerical illustration of the impact of juniorization c

To summarize, we have shown, analytically, that if c is sufficiently small, the equilibrium price p^* is below v and is increasing in c. If c is sufficiently large, the equilibrium price p^* is above v and is also increasing in c; moreover, increasing c has heterogeneous impacts on the cross section of clearing members. For intermediary values of c, we have a numerical procedure to find the equilibrium in both cases ($p^* < v$ and $p^* \geq v$). While our analytical characterization does not preclude multiple equilibria, numerical calculations have produced a unique equilibrium for each parameter set.

To further illustrate the results and the intuition, we now conduct a numerical exercise by setting the model parameters to plausible magnitude in the interest rate swaps market. The CFTC (2019) finds that as of September 5, 2018, LCH has a guarantee fund of 6.6 billion USD and a skin-in-the-game of 56 million USD. The CFTC stress testing results show that in the stress scenario of Lehman's default on 15 September, 2008, the shortfall in excess of the defaulter's initial margin in LCH would be 0.31 billion USD.

Based on these statistics, we set $v = -0.31$, $M = 0.056$, and $G = 6.6$, all with the unit of billions of USD. (The \$0.31 billion shortfall is before the use of the defaulter's guarantee fund contribution, so the mapping to the model is not exact.) We normalize $Q = 1$. These parameters already imply that $vQ + M < 0$, i.e., the defaulter's and CCP's resources are insufficient to cover the losses. The last parameter, λ, does not have a direct empirical counterpart, so we provisionally set it to be 0.31, so that the loss beyond initial margin would double if the defaulted portfolio is sold without juniorization (i.e., $v - \lambda Q = -0.62$).

For simplicity, we will use the exponential distribution of default fund contributions:

$$f(g) = \frac{1}{\beta} e^{-\frac{g}{\beta}}, \quad \text{if } g \geq 0, \tag{32}$$

where β is the mean of the distribution and is equal to G as the mass of clearing members is normalized to be one.

In the equilibrium case with $p^* < v$, the equilibrium price can be written as

$$p^* = v - \frac{\lambda Q - e^{-\frac{g_H}{\beta}} c - \frac{c}{g_H - g_L}\left((\beta + g_L)e^{-\frac{g_L}{\beta}} - (\beta + g_H)e^{-\frac{g_H}{\beta}}\right)}{1 - e^{-\frac{g_L}{\beta}} + e^{-\frac{g_H}{\beta}}}. \tag{33}$$

In the equilibrium case with $p^* \geq v$, the equilibrium price can be written as

$$p^* = v + c - \frac{\lambda Q g_H}{\beta(1 - e^{-\frac{g_H}{\beta}})}. \tag{34}$$

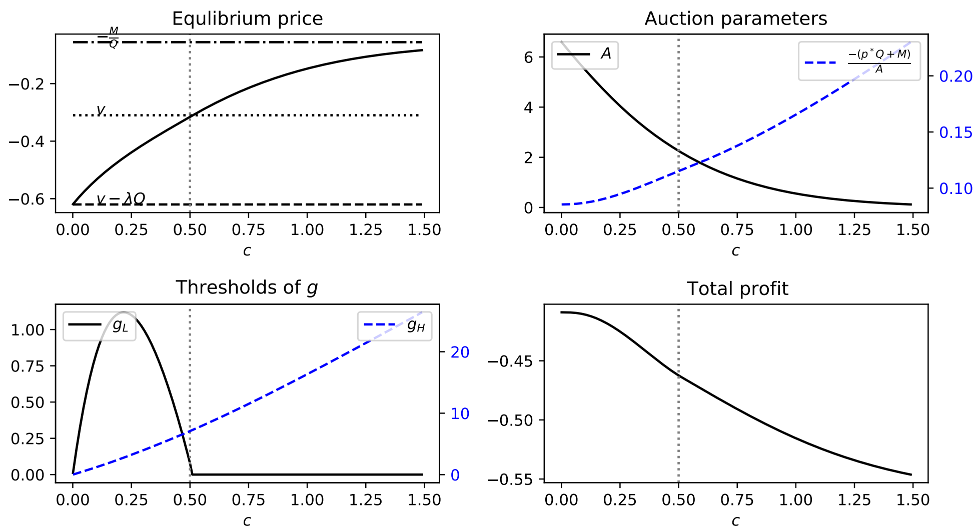
Figure 1 shows the comparative statics of market outcomes with respect to c, the aggressiveness of juniorization, as well as the equilibrium allocations and profits as functions of g. In the first and second rows, the vertical line indicates the threshold value of c (= 0.5 in this example) at which $p^* = v$. As characterized in Propositions 2 and 3, p^* is monotone increasing in c, and it is reaching $-M/Q$ if c is sufficiently high (top left plot). Consistent with Lemma 1, when c is close to zero, A is close to G and p^* is close to $v - \lambda Q$ (top plots). When $p^* > v$, the Case-3 demand hits the floor so that g_L hits zero (bottom left plot). Except for a small region, the aggregate profit of all members is generally decreasing in c, suggesting worsening allocative efficiency associated with juniorization (bottom right plot).

Figure 2 shows that more aggressive juniorization (e.g., a higher c) raises the auction price and reduces the demand from smaller members (top left plot). As a result, as shown in Proposition 4, profits of members with very small or very large guarantee funds decrease in c, while members with guarantee fund size in between have higher profits when c is larger (top right plot).

We further decompose the profit π_i into the auction profit $(v-p)x_i - 0.5\lambda x_i^2$ and the lost guarantee fund $-T_i$. A higher c raises the auction price and reduces the profits of members with sufficiently small guarantee funds and sufficiently large guarantee funds (bottom left plot). On the other hand, by raising the price a higher c strictly reduces the use of large members' guarantee funds, whereas the use of small members' remains zero (bottom right plot). Combining the two components, the smallest and largest members suffer from a higher c, but members with medium-size guarantee funds benefit from it.

Figure 1: Comparative statics w.r.t. c

The plots show comparative statics with respect to c. The base case is $\lambda = 0.31, Q = 1, M = 0.056, v = -0.31, G(= \beta) = 6.6$. The vertical line is the threshold of $c(= 0.5)$ below which p^* is smaller than v.



4 Customer participation

In the baseline model, only members of the clearinghouse can participate in the auction. Now, we turn to the case in which customers, of mass $\mu > 0$, can also participate. Customers contribute zero guarantee funds. A customer's risk-adjusted profit is similar to that of a member (3), except that the customer does not have the term of guarantee fund usage:

$$\pi_c = (v - p)x_c - 0.5\lambda x_c^2. \tag{35}$$

The customers' first-order condition gives rise the following demand function

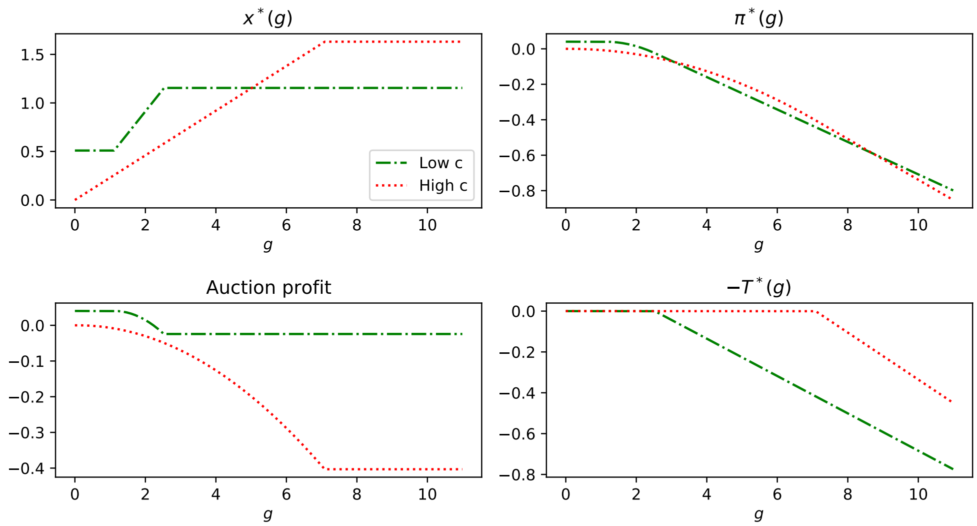
$$x_c(p) = \frac{1}{\lambda}(v - p), \tag{36}$$

which is the same as that of a member in Case 3. Apparently, the customers will only buy a positive amount of the portfolio when $p < v$.

For Scenario I, i.e., $p^*Q + M > 0$, the equilibrium price should satisfy $(v - p^*)(1 + \mu)/\lambda = Q$,

Figure 2: Equilibrium allocations and profits for bidders

The plots report the equilibrium allocations and profits as functions of g. The base case is $\lambda = 0.31, Q = 1, M = 0.056, v = -0.31, G(= \beta) = 6.6$. "High c" produces a price $p^* > v$ and "Low c" produces a price $p^* < v$.



which is

$$p^* = v - \frac{\lambda Q}{1+\mu}. \tag{37}$$

As a result, increasing the mass of participating customers pushes up the equilibrium price. The condition for obtaining this case of equilibrium is that $\left(v - \frac{\lambda Q}{1+\mu}\right) Q + M > 0$, which is also less stringent when the mass of customers is larger.

For Scenario II, the special case of the pro-rata use of guarantee funds (i.e., $c = 0$ and $A = G$) leads to the same equilibrium price in (37). Juniorization (i.e., $c > 0$) can give rise to equilibrium outcomes with $p^* < v$ or those with $p^* \geq v$. When $p^* < v$, customers actively participate in the auction and buy a positive amount of the portfolio. The market clearing condition (14) becomes

$$\begin{aligned} Q &= \mu x_c + \int_0^{g_L} x_3 f(g)dg + \int_{g_L}^{g_H} x_2 f(g)dg + \int_{g_H}^{\infty} x_1 f(g)dg, \\ &= (\mu + F(g_L))\frac{1}{\lambda}(v - p^*) + \int_{g_L}^{g_H} \frac{-(p^*Q + M)}{A}\frac{g}{c} f(g)dg + (1 - F(g_H))\frac{1}{\lambda}(v - p^* + c). \end{aligned} \tag{38}$$

The equilibrium price, thus, can be rewritten as

$$p^* = v - \frac{\lambda Q - \frac{c}{g_H - g_L} \int_{g_L}^{g_H} g f(g) dg - c(1 - F(g_H))}{\mu + F(g_L) + 1 - F(g_H)}. \tag{39}$$

Similar to the proof for Lemma 1, we can establish a form of continuity because $p^* \to v - \frac{\lambda Q}{1+\mu}$ when $c \to 0$. Given $p^* < v$, a wider customer participation (i.e., larger μ) increases the auction price. Proposition 5 characterizes the equilibrium with active customer participation and juniorization.

Proposition 5. *Suppose that $-G < (v - (\lambda Q)/(1 + \mu))Q + M < 0$. There exists some $c'_1 > 0$ such that for all $c < c'_1$, the following is an equilibrium: Let g_H, g_L, and p^* solve (12), (13), and (39), where the constant A is given by (16). Each customer uses the strategy $x_c(p) = (v - p)/\lambda$ and each member uses the strategy in (22).*

In this equilibrium, the auction price satisfies $p^* < v$, $-G < p^* Q + M < 0$ and $0 \leq T_i \leq g_i$. There exists some $c'_2 > 0$ such that $dp^*/dc > 0$ and $d\pi^*_c/dc < 0$ for $c < c'_2$. Furthermore, p^* increases in μ.

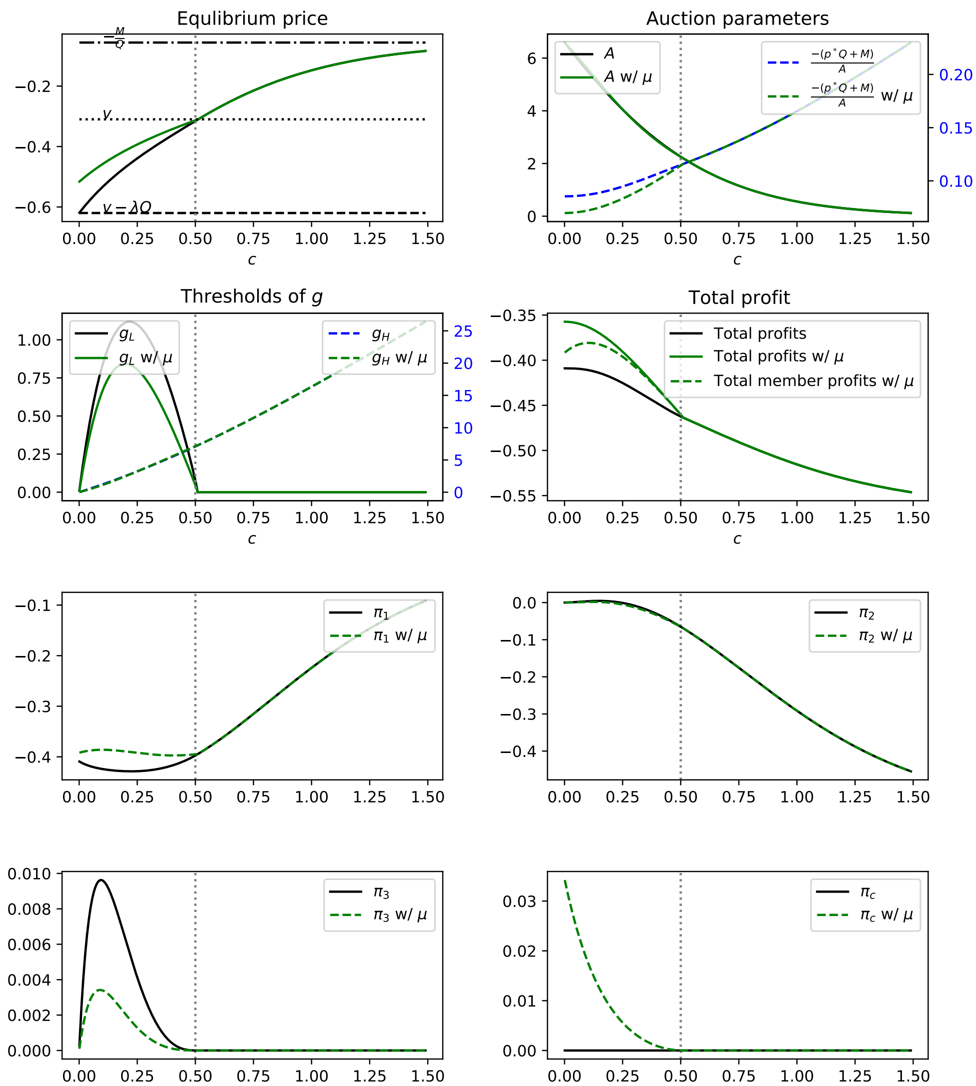
Proof. As a customer's strategy is the same as that of a small member in Case 3, customers' participation is equivalent to an increase of the mass of these small members. Thus, the addition of customers does not change the bidding strategy for the members. The equilibrium outcomes are not affected materially, except replacing $F(g_L)$ in (19) with $(\mu + F(g_L))$ in (39). The proofs of Proposition 1 and 2 also carry through given the continuity discussed previously. The part about $d\pi^*_c/dc$ comes from the fact that $\pi^*_c = (v - p^*)^2/(2\lambda)$. ■

When the juniorization scheme results in $p^* \geq v$, the equilibrium outcomes are not affected by the customers as they remain inactive and buys nothing in equilibrium.

Figure 3 illustrates the comparative statics of various market outcomes with respect to c in the model with customer participation. If $p^* < v$, customer participation increases the auction price. It increases the total profit of all clearing members, as well as giving rise to a positive customer profit. That said, not all clearing members are affected the same. Somewhat surprisingly, the profits of the largest clearing members (π_1) increase. This is because only the largest clearing members' guarantee funds are used in equilibrium, and customer participation reduces the amount of guarantee fund needed. Profits of clearing members who do not lose guarantee funds in equilibrium (i.e., π_2 and π_3) decline with customer participation because customers directly compete with them. Customers are obviously better off if they can participate. All these effects become more muted as juniorization becomes more aggressive. Once juniorization pushes p^* above v, customer participation has no impact on the equilibrium outcome.

Figure 3: Comparative statics w.r.t. c with customers

The plots show comparative statics with respect to c with customers. The base case is $\lambda = 0.31, Q = 1, M = 0.056, v = -0.31, G(= \beta) = 6.6, \mu = 0.5$. The vertical line is the threshold of $c(= 0.5)$ below which p^* is smaller than v. Green line indicates the case with customer participation.



5 Concluding Remarks

This paper proposes a tractable modeling framework for CCP auctions, with the unique feature that some financial resources that the CCP uses to pay auction winners come from the bidders themselves. We find that the juniorization of guarantee funds is an effective mechanism to encourage aggressive bidding and raise the auction price. As juniorization becomes sufficiently aggressive, the price can become so high that only a tiny amount of survivors' guarantee funds is used.

The model framework of this paper can be adapted to analyze other dimensions of CCP auctions. For example, one could incorporate heterogeneous inventory costs $\{\lambda_i\}$, assuming they are common knowledge. In that setup, $1/\lambda$ in the market clearing condition becomes an integral of $1/\lambda_i$. This extension involves additional complexity because clearing members would be sorted by the combination of $\{g_i\}$ and $\{\lambda_i\}$.

Another potential dimension is to model the end of the waterfall. Under variation margin gain hair-cutting or partial tear-up, the in-the-money side does not receive the full value. To the extent that some surviving members are on the opposite side of the defaulted portfolio, the potential to lose some of the mark-to-market profit adds another layer of heterogeneous bidding incentives.

A third possibility is to incorporate the pre-auction hedging by the CCP, which may be conducted secretly. This calls for a dynamic model: a hedging stage conducted in the open market, followed by an auction stage that we already modeled. In this case, the juniorization of guarantee funds should probably recognize the liquidity provision of surviving members in the hedging stage.

Appendix

Proof of Proposition 3. First, we show $dp^*/dc > 0$ if the equilibrium exists. Taking the total derivatives of (16), (28) and (27), we gets

$$\frac{dp^*}{dc} = \frac{cG(G-A)(1-F(g_H)) - \lambda QA\int_0^{g_H} gf(g)dg}{(G-A)\left(cG(1-F(g_H)) + \lambda Q\int_0^{g_H} gf(g)dg\right)}$$

where the denominator is positive; and it is larger than the numerator as $G > 0$. Thus, $dp^*/dc < 1$. For a $\hat{c}$ that the equilibrium of Proposition 3 exists, there is a set of $\{\hat{A}, \hat{g_H}, \hat{p^*}\}$ that satisfies (16), (12) and (27). Substituting $\hat{c} = (\hat{p^*} - v) + \lambda Q\frac{\hat{g_H}}{G-\hat{A}}$ and using $\hat{p^*} - v \geq 0$, the numerator of $\frac{dp}{dc}$ is

$$\hat{c}G(G-\hat{A})(1-F(\hat{g_H})) - \lambda Q\hat{A}\int_0^{\hat{g_H}} gf(g)dg \geq \lambda Q\left(G\hat{g_H}(1-F(\hat{g_H})) - \hat{A}\int_0^{\hat{g_H}} gf(g)dg\right). \tag{A1}$$

Let n_1 denote $\hat{g_H}(1-F(\hat{g_H}))$ and n_2 denote $\int_0^{\hat{g_H}} gf(g)dg$. Note that $n_1 < \int_{\hat{g_H}}^{\infty} gf(g)dg$, which means $G > n_1 + n_2$. The above expression can be written as

$$\begin{aligned}
&\hat{c}G(G-\hat{A})(1-F(\hat{g_H})) - \lambda Q\hat{A}\int_0^{\hat{g_H}} gf(g)dg \geq \lambda Q\left(Gn_1 - \hat{A}n_2\right) = \lambda Q\left(Gn_1 - (G-n_1-n_2)n_2\right), \\
&= \lambda Q\left((n_1-n_2)G + (n_1+n_2)n_2\right) > \lambda Q\left((n_1-n_2)(n_1+n_2) + (n_1+n_2)n_2\right) = \lambda Q\left(n_1(n_1+n_2)\right) > 0.
\end{aligned} \tag{A2}$$

Thus, $\frac{dp}{dc} > 0$ for any c in the equilibrium of Proposition 3.

Next, we establish the continuity between the equilibrium in Proposition 1 and that in this proposition around $p^* = v$. Rewriting (19) as

$$p^* = v + c - \frac{(\lambda Q + cF(g_L))g_H}{G - A - \int_0^{g_L} gf(g)dg + g_H F(g_L)} \tag{A3}$$

and substituting $g_L = 0$, p^* is the same as that in (27). Moreover, in this special case when $p^* = v$, one can rewrite (12) and (27) as $c = \lambda\frac{-(vQ+M)}{A}\frac{g_H}{c}$ and $c = \frac{\lambda Q}{G-A}g_H$. Thus, g_H can be solved by $\frac{g_H A}{(G-A)^2} = \frac{-(vQ+M)}{\lambda Q^2}$, and there exists a c_3 such that when $c > c_3$, p^* is larger than v.

Finally, for $T_i \leq g_i$ to hold for all x_i, we need to ensure there exists some $c_4 > c_3$ such that when $c \leq c_4$, $\frac{-(p^*Q+M)}{A} \leq 1$. To see that, take the total derivatives of (16), (28) and (27), we have $\frac{dA}{dg_H} < 0$ and $\frac{dg_H}{dc} = \left(\frac{dp}{dc} + \frac{A}{G-A}\right)\frac{(G-A)^2}{cG(1-F(g_H))} > 0$. Thus, we have $dA/dc < 0$.

By (28), one can solve explicitly A when $c = c_3$. Let A_3 denote $A|_{c=c_3}$, which can be written as $G/(\frac{c_3 Q}{-(vQ+M)}+1)$. Let c_4 denote $(G-A_3)/Q$, which can be written explicitly as $c_3 G/(c_3 Q - (vQ+M))$. When $c_3 \leq c \leq c_4$, one can show that $\frac{-(p^*Q+M)}{A} = \frac{cQ}{G-A} \leq \frac{G-A_3}{G-A} \leq 1$, in which the first inequality comes from $c \leq c_4$ and the second one comes from $dA/dc < 0$ and $c \geq c_3$.

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