Design of CCP Default Management Auctions

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CCP recovery and resolution: Resources

1. Defaulter’s initial margin
2. Defaulter’s guarantee fund (g-fund)
3. CCP’s capital
4. Survivors’ g-fund
5. Survivors’ assessment

1—3 vs 4—5: Incentives are different! CCP can be quite creative in 4—5.

Source: Reserve Bank of Australia
CCP recovery and resolution: Procedure

- ISDA (2017): “Most importantly, successful CCP recovery or resolution must both: (1) allocate losses; and (2) rebalance the CCP’s book.”
- Step 1: Hedging the positions to slow down/stop further losses—similar to an auction, but facing the entire market and sometimes anonymous.
- Step 2: Auction off the defaulter’s position (including the hedges).
  - Case 1: The defaulter’s resource and CCP’s skin-in-the-game are sufficient.
  - Case 2: Use survivors’ g-fund (including assessment)—my focus today.
  - Case 3: G-fund is exhausted. Resort to more extreme method such as partial tear-ups or variation margin gain haircut.
The use of guarantee fund – The effect of juniorization

Dynamic considerations – Before and after the auction
A model of CCP default management auctions (1)

- The fundamental value of the auctioned portfolio is \( v \) per unit.
- The auctioned portfolio has size \( Q > 0 \).
- Auction is uniform price and fully divisible.
- Resources from the defaulter and the CCP sum up to \( M > 0 \).
- There are \( n \) strategic bidders (clearing members and customers)
- Bidder \( i \) already has inventory \( z_i \) of this portfolio. Denote \( Z = z_1 + z_2 + \cdots + z_n \).
- Bidder \( i \) has \( g_i \geq 0 \) guarantee fund (g-fund) at the CCP. Denote \( G = g_1 + g_2 + \cdots + g_n \).
- Denote the auction price by \( p \). Convention: \( p \) is how much the bidders pay the CCP, so \( p < 0 \) (CCP pays bidders) is the more interesting case.
A model of CCP default management auctions (2)

- Denote by $x_i$ the amount purchased by bidder $i$ in the auction. By definition, $x_1 + x_2 + \cdots + x_n = Q$. Bidder $i$ maximizes

$$\pi_i = (v - p)x_i - 0.5\lambda(z_i + x_i)^2 - \frac{T_i}{T_i}$$

where $\pi_i$ represents Profit, $0.5\lambda(z_i + x_i)^2$ represents Inventory cost, and $T_i$ represents the use of bidder $i$'s g-fund.

- Three cases:
  - $pQ + M \geq 0$: Zero use of (survivors') g-fund.
  - $pQ + M < 0$ but $pQ + M + G \geq 0$: G-fund is used but is not exhausted.
  - $pQ + M + G < 0$: G-fund is exhausted.

- Each bidder wishes to buy the portfolio cheap, but he also wants to minimize the use of his g-fund.
- CCP's design of $\{T_i\}$ will affect bidders' strategies.
Juniorization

- We focus on the case where g-fund is used but not exhausted, $-G < pQ + M < 0$.
- A bidder can easily avoid the penalty for not bidding enough by submitting bad bids.
- If a bidder puts in bad prices relative to peers by some metric, his guarantee fund is juniorized.
- To model juniorization, I assume CCP uses the rule:

$$T_i = -\frac{p^*Q + M}{\text{Shortfall}} \times \frac{(Ag_i - Cx_i(p^*))}{\text{Juniorization}}$$

$p^*$ is final auction price, and $A > 1$ and $C > 0$ are constants to be calibrated.

- Pro-rata means $A = 1$ and $C = 0$. $T_i = -(p^*Q + M) \times g_i/G$.
- But $T_1 + T_2 + \cdots + T_n + (pQ + M) = 0$ and $x_1 + x_2 + \cdots + x_n = Q$, so $A = 1 + \frac{CQ}{G}$. 
Suppose there are three bidders, with equal g-fund contribution.
Suppose at the final price $p^*$, they win 60%, 40% and 0% of the auction portfolio.
Normalize $Q = 1$. Suppose the shortfall $- (pQ + M)$ is 100 million.

- Bidder 1’s g-fund use: $\frac{100}{G} \times (A \frac{G}{3} - 0.6C)$
- Bidder 2’s g-fund use: $\frac{100}{G} \times (A \frac{G}{3} - 0.4C)$
- Bidder 3’s g-fund use: $\frac{100}{G} \times A \frac{G}{3}$

Similar to ranking by prices
In my view, ranking by quantity at the equilibrium price is slightly better than ranking by non-equilibrium prices.
Juniorization: Bidding strategy

- Each bidder’s optimal demand curve (implemented by limit orders) is
  \[ x_i(p) = \frac{n - 2}{\lambda(n - 1)} \left( v - p - \lambda z_i + \left( -\frac{pQ + M}{G} \right) C \right) + \frac{Q}{n - 1} g_i. \]
  where \( C \) is the juniorization sensitivity, to be determined.

- Low inventory or high g-fund encourages bidding (also true for pro-rata).

- Assuming all bidders purchase positive amounts, the final auction price \( p^* \) is
  \[ p^* = v - \frac{Z + Q}{n} + \left( -\frac{p^*Q + M}{G} \right) C \]
  where \( p^* \) is the competitive price, and \( C \) is the juniorization.

- Conditional on a positive shortfall, juniorization increases bids and the price.
Juniorization: Incentives

- \( T_i = -\frac{p^*Q + M}{G} \times (A g_i - C x_i(p^*)) \). We need \( 0 \leq T_i \leq g_i \) for all \( x_i(p^*) \in [0, Q] \).

- \( T_i \geq 0 \) part: \( A g_i - C Q = \left(1 + \frac{C Q}{G}\right)g_i - C Q > 0 \), so \( C \) needs to be small enough:
  \[
  C \leq \min_i \left\{ \frac{g_i}{Q \left(1 - \frac{g_i}{G}\right)} \right\} \Rightarrow \text{maximum is } \frac{G}{(n - 1)Q}
  \]

- \( T_i \leq g_i \) part: We want \(-\frac{p^*Q + M}{G}A g_i \leq g_i\).
  \[
  A = \left(1 + \frac{C Q}{G}\right) \leq \frac{G}{-(p^*Q + M)}, \quad C \leq \frac{G}{Q} \times \frac{p^*Q + M + G}{-(p^*Q + M)}
  \]

- If the total g-fund \( G \) is sufficient, the condition on \( g_i \) is more likely binding.
Juniorization: Bidders’ profits

• Somewhat surprisingly, juniorization (in this model) does not affect the equilibrium allocations or the profits of bidders.

\[ x_i(p^*) = \frac{n - 2}{n - 1} \left( \frac{Z}{n} - z_i \right) + \frac{Q}{n - 1} \left( \frac{n - 2}{n} + \frac{g_i}{Q} \right). \]

\[ \pi_i = \frac{\lambda(Z + Q)}{n} x_i(p^*) - 0.5\lambda(z_i + x_i(p^*))^2 + \frac{p^c Q + M}{G}. \]

• Intuition: Since everyone bids more by the same amount, there is no change in allocation. And the cost of paying a higher price is exactly offset by a lower use of g-fund.

• Bidder \( i \) buying a positive amount means \( z_i < \frac{Z + Q}{n} + \frac{g_i}{G} \frac{Q}{n - 2}. \)
Juniorization: Summary

• If the price is low enough that g-fund is used (but not exhausted), juniorization can increase the auction price, implying less use of g-fund.
• But the net effects on allocations and bidder profits could be neutral.
• The incentive and higher price brought by juniorization are limited by the lowest g-fund among all bidders.
Juniorization: Questions & discussion

- Since bidding incentives depend on g-funds at stake, should customers be charged g-fund to participate in bidding?
- Do clearing members have incentives to let in their customers?
- If customers do not wish to put in g-fund, does an aggressive enough juniorization schedule effectively limit participation to clearing members?
- If juniorization is so effective that only a tiny amount of g-fund ends up being used, does the CCP want to fill in a bit more capital to avoid using g-fund altogether?
Juniorization vs competitive equilibrium

- Juniorization of g-fund does not deliver efficient allocations.
- In principle, one can achieve the competitive equilibrium and efficient allocations using the “mechanism design” approach.

The use of g-fund is $T_i^c = -x_i(p^*)p^* + \frac{n-1}{\lambda} [(p^*)^2 - (p^c)^2] - \frac{M}{n}$.

- But $T_i^c$ requires “too much” knowledge by the CCP before the auction, in particular $\lambda$ and $p^c$. And the conditions for $x_i(p^*) > 0$ and $T_i \in [0, g_i]$ are more stringent than those for juniorization. See accompanying notes for full comparison.
- Bottom line: Juniorization seems a good mechanism (albeit imperfect).
Outline

• The use of guarantee fund – The effect of juniorization

➢ Dynamic considerations – Before and after the auction
Pre-auction hedging

- CCPs hedge the most important risks of the auctioned portfolio before the auction.
- Pre-auction hedging vs auction:

<table>
<thead>
<tr>
<th>Pre-auction hedging</th>
<th>Auction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use defaulter’s and CCP’s resources</td>
<td>Could dip into g-fund</td>
</tr>
<tr>
<td>Anonymous or not</td>
<td>Not anonymous</td>
</tr>
<tr>
<td>Facing the entire market</td>
<td>Facing mostly clearing members (customers need approval)</td>
</tr>
<tr>
<td>Potentially hedge multiple risks</td>
<td>Sell vertical slice of the same portfolio</td>
</tr>
</tbody>
</table>
Do hedging and auction conflict?

- The hedging CCP is competing against its future self, the auctioning CCP.

<table>
<thead>
<tr>
<th>Hedging (price $p^h$)</th>
<th>Auction (price $p^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bidder $i$ starts with $w_i$</td>
<td>Bidder $i$ starts with $z_i = w_i + y_i$</td>
</tr>
<tr>
<td>Acquires $y_i$</td>
<td>Acquires $x_i$</td>
</tr>
</tbody>
</table>

- Recall $\pi_i$ is bidder $i$’s profits in the auction stage, taking $z_i$ as given.
- Bidder $i$’s total profit in the two stages is $\Pi_i = (v - p^h)y_i + \pi_i$. Fixing $G$ and $Z$:

  $$\frac{d\Pi_i}{dy_i} = v - p^h + \frac{d\pi_i}{dz_i}, \quad \frac{d^2\Pi_i}{dy_idg_i} = \frac{d^2\pi_i}{dz_idg_i} = -\frac{\lambda Q}{nG} < 0.$$  

- Every additional unit of g-fund decreases a bidder’s willingness to pay during the hedging stage by $\lambda Q/(nG)$, assuming that g-fund is used but not exhausted.
Liquidity during hedging vs auction

- If clearing members correctly anticipate the auction and juniorization, they may not be willing to provide sufficient liquidity during the hedging stage.
- Worse, they may even sell to get to an advantageous position for the auction.
- CCPs should recognize clearing members’ purchase in the hedge stage in the juniorization schedule (CCPs know the identities)—to encourage early “bids.”
- Who are in the best position to provide liquidity during the hedging stage? Those with low \( g_i \), i.e., small clearing members or customers, and those with negative \( z_i \), i.e., those with positive mark-to-market value on the auctioned portfolio. They need to be involved and encouraged to participate.
- In terms of incentives, it seems clearing members and CCP would be more willing to involve customers in the hedging stage than the auction stage.
Post-auction liquidation

• Unless bidders would like to buy anyway, they are likely to liquidate some of their purchases after the auction.
• This creates a “crowded trade” scenario—multiple auction winners could be liquidating the same portfolio! This is particularly risky if bidders are “forced” to purchase the portfolio due to juniorization.
• Because crowded trades are riskier if they are more crowded, there is an argument for size priority at the same price.
  • Example: The auction price is −$100,000 per 1%. At this price, prioritize bids with larger quantities. (Bids with strictly better prices are filled fully.)
Final thoughts

• My talk today focuses on the middle ground case in which the g-fund is used but not exhausted.
• What if g-fund is not used at all? In this case, wider participation is usually better for efficiency and is in the CCP’s interest.
• What if g-fund is exhausted? More extreme methods like partial tear-ups actually encourage participation in the auction, especially from the in-the-money side. One can also model this formally.
  • Settle-to-market (STM) vs collateral-to-market (CTM): STM slightly weakens the “threat” of tear-ups because the lost variation margin is only for one day.
Incentives are critical in CCP auction design.

During the hedging stage, the CCP should:
- Count clearing members’ liquidity provision during the hedging stage toward the juniorization schedule in the auction stage.
- Invite broad participation (including customers).

During the auction stage:
- Allow bids to be submitted conditional on the use of g-fund. Because incentives depend on g-fund use, this reduces guesswork and makes bidding easier.
- The juniorization schedule increases the auction price, but it also requires careful calibration to keep incentives aligned. The lowest g-fund could be the binding factor.
- What are the incentives to involve customers in the auction?