On Interest-Bearing Central Bank Digital Currency with Heterogeneous Banks*

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September 2021

Abstract

We explore the implications of introducing an interest-bearing central bank digital currency (CBDC) through commercial banks that differ in size. Banks of heterogeneous sizes offer different convenience properties to depositors, which the CBDC adopts. The large bank gives depositors a higher convenience value and hence possesses market power. The interest rate on CBDC puts a lower bound on banks' deposit interest rates, which is particularly binding on the large bank. While a higher CBDC interest rate enhances monetary policy pass-through by raising deposit interest rates, it reduces the small bank's deposit market share and its lending volume. By contrast, a CBDC that delivers its own convenience value to users levels the playing field by shifting deposits and lending from the large bank to the small one, although it can enhance or reduce the transmission of monetary policy.

Keywords: central bank digital currency, interest on (excess) reserves, deposit interest rates, bank lending

JEL Classification Numbers: E42, G21, G28, L11, L15

*We thank Todd Keister for multiple discussions in the early stages of this work. For additional helpful discussions and comments, we also thank Yu An, David Andolfatto, Ben Bernanke, Darrell Duffie, Zhiguo He, Jiaqi Li, Debbie Lucas, Alessandro Rebucci, Daniel Sanches, Antoinette Schoar, Harald Uhlig, Yu Zhu, and Feng Zhu, as well as seminar participants at the Bank of Canada, Luohan Academy, MIT Sloan, Nova SBE, Microstructure Online Seminars Asia Pacific, the University of Hong Kong, the Canadian Economics Association, the JHU Carey Finance Conference and the Philadelphia Workshop on the Economics of Digital Currencies. Finally, we thank Jiaheng Yu for excellent research assistance.

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"If all a CBDC did was to substitute for cash – if it bore no interest and came without any of the extra services we get with bank accounts – people would probably still want to keep most of their money in commercial banks."
—Ben Broadbent, Deputy Governor of the Bank of England, in a 2016 speech

1 Introduction

A central bank digital currency (CBDC) "is a digital payment instrument, denominated in the national unit of account, that is a direct liability of the central bank" (BIS 2020). Since Ben Broadbent's remarks (in the epigraph), policymakers have expressed increasing interest in the possibility that a CBDC may pay a non-zero interest. In fact, the Committee on Payments and Market Infrastructures defines interest bearing as one of five key design features of CBDCs.[1]

In this paper, we propose a model of heterogeneous banks that we use to explore the implications of an interest-bearing CBDC. The CBDC is offered through the infrastructure of commercial banks and hence carries the same functionalities as regular bank deposits. Under this design, the interest paid on the CBDC puts a lower bound on the deposit interest rates of commercial banks. The model reveals how deposit interest rates and levels of deposits in the banking system depend on two key policy rates set by the central bank: the interest rate on reserves (IOR), which is paid to banks, and the interest on CBDC, which is paid to agents holding the CBDC.[2]

The commercial banking sector in the economy consists of a large bank and a small bank, both of which are strategic. The main distinction between them is that the large bank's deposit offers a higher convenience value than the small bank's deposit. For example, a large bank has a more expansive network of branches and ATMs. The difference in convenience value has a probability distribution across a continuum of agents who deposit in either bank. The higher convenience value of its deposit gives the large bank market power in the deposit market. Hence, the large bank offers a lower deposit interest rate than the small bank in equilibrium and yet has a larger market share.

The two commercial banks also lend to entrepreneurs who take on risky projects that differ in their quality (i.e., expected returns). When a loan is made by either bank, the bank creates a new deposit in the name of the entrepreneur as a new liability, which is exactly balanced by the new loan as a new asset. The entrepreneur immediately pays a randomly selected worker, and the deposit may flow out of the original bank to the other. For example, if the entrepreneur originally takes a loan from the small bank but the worker she hired has a high enough value for the convenience of the large bank, the deposit flows from the small bank to the large one, resulting in a flow of central bank reserves in the same direction.

[1] The other four features are 24/7 availability, anonymity, transfer mechanism, and limits or caps.

[2] Since the financial crisis of 2008-09, interest on excess reserves has become the Federal Reserve's main policy tool to adjust interest rate. We do not model reserve requirements (there currently are none in the United States) and hence we are effectively talking about excess reserves throughout the analysis. For simplicity, however, we will just refer to excess reserves as reserves.

Because the large bank has an inherent advantage in the deposit market, in equilibrium, the large bank's newly created deposit in the lending process has a high probability of staying at the large bank. Thus, by lending out each dollar, not only does the large bank make a profit on the loan, it also earns the interest on reserves with a high probability on the newly created dollar of deposit. By contrast, the small bank is likely to lose the newly created deposit and the associated interest on the accompanying reserves. Therefore, holding fixed the quality of the entrepreneur's project, the large bank earns a higher total profit by lending. Put differently, the large bank is willing to lend to lower-quality projects due to the higher expected profit of earning interest on reserves. This is a mechanism in which a deposit market advantage translates into a lending market advantage.

Before assessing the impact of CBDC we demonstrate that our model with heterogeneous banks captures key features of US deposit markets. We vary the IOR rate and examine its impact on the deposit market interest rates. There are two cases. In the first case, the zero lower bound on the deposit interest rate binds. When the IOR rate is sufficiently low, the equilibrium deposit interest rates of the two banks are necessarily close to each other because both rates are between zero and the IOR rate. Due to the convenience value of the large bank's deposit, the large bank pays depositors a zero interest rate and still retains the lion's share of the deposit market. When the lower bound is binding, an increase in the IOR rate leads to a higher deposit interest rate at the small bank and increases its market share. But because the large bank's deposit interest rate remains at zero and it still dominates the deposit market, the average deposit interest rate only rises by a small fraction of the IOR rate.

Figure 1 shows the relationship between the interest rate paid on non-jumbo (i.e., less than $100K) deposits by U.S. commercial banks and the IOR rate since May 2009. As predicted by the model, the deposit interest rate does not respond to the increases in the IOR rate that began in December of 2015. When questioned about this fact, Federal Reserve Chair Jerome Powell said that deposit interest rates respond to changes in the IOR rate with a lag. This seems to hold true in Figure 1, in the sense that deposit interest rates fell slowly after the crisis in 2009, and rose slowly after lift-off began. The delayed response, however, does not explain why the deposit interest rate remained much lower than the IOR rate after it had time to catch up. Our model explains this fact without resorting to common frictions such as search costs in the deposit market. Instead, it is the depositors' value for convenience that breaks the price competition and gives the large bank market power.

The second case of the equilibrium is when the zero lower bound on deposit interest rates does not bind. This happens when the IOR rate is sufficiently high. Both banks' deposit interest rates are "interior" and have a constant spread between them, which is determined by the distribution of convenience value among depositors. In this case, a change in the IOR rate translates into the same change in deposit interest rates, but does not affect the banks' market shares in the deposit market. As an illustration of the second case, Figure 2 shows that before the 2008-09 crisis, consumer deposit interest rates tended to move proportionally with the Fed Funds rate (the relevant benchmark before IOR was introduced in 2009) when the Fed Funds rate was above approximately 3 percent. As in the post-crisis era, changes in

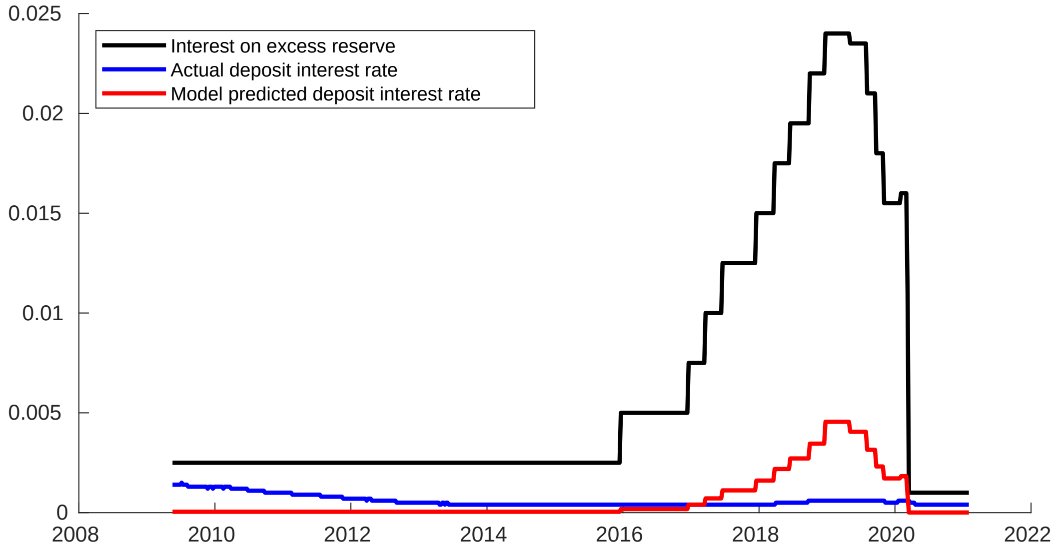

Figure 1: Actual U.S. deposit interest rates from May 18, 2009 to February 1, 2021. Weekly deposit interest rates for amount less than $100,000 are obtained from FDIC through FRED. During this period, interest on reserves is taken to be the interest on excess reserves (IOER).

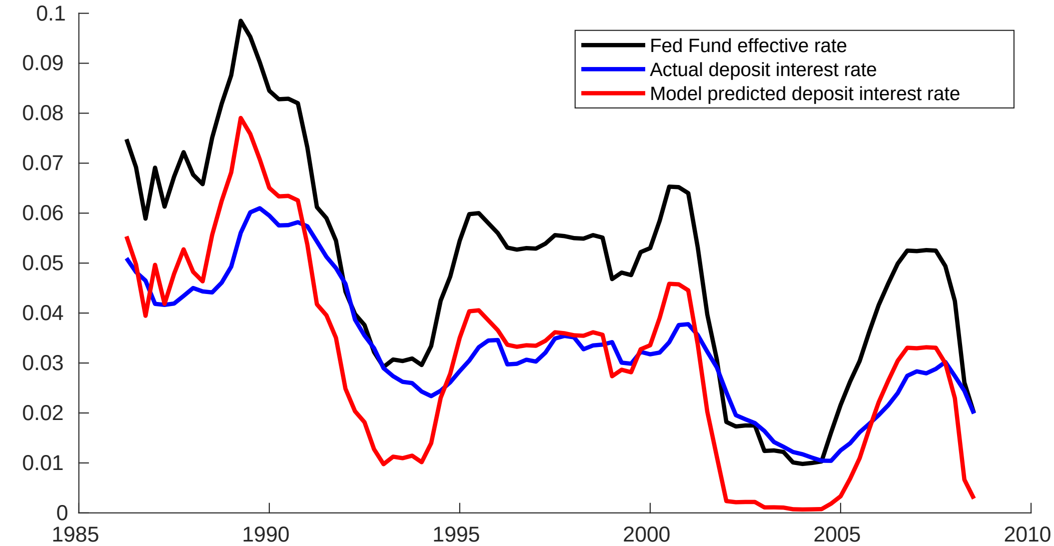

Figure 2: Actual U.S. deposit interest rates from 1986Q1 to 2008Q2. Domestic deposit interest rates are quarterly, calculated from call reports, as total interest expense on domestic deposit divided by total domestic deposit, multiplied by 4. During this period, interest on reserves is taken to be the actual Federal Funds rate.

the deposit interest rate appear to lag behind changes in the fed funds rate.

For completeness, we also provide model predictions for how equilibria in the lending market change as f varies. However, we have so far not attempted to confront the performance of these predictions with the data.

We begin our analysis of CBDC by assuming that the CBDC is offered through commercial banks, and that CBDC deposits inherit the convenience properties of the host institution. This makes each bank's deposit accounts perfect substitutes for the CBDC accounts offered at their respective institution, and hence the CBDC interest rate becomes a lower bound on the deposit interest rate at each bank. The two cases of equilibrium again apply here analogously, and only the case where the lower bound is binding is relevant for our discussion (in the other case, the CBDC has no impact because its interest rate is too low).

To analyze the impact of an interest-bearing CBDC, we consider the impact of increasing the CBDC interest rate while holding the IOR rate fixed. This analysis mimics the type of testing that was done with Overnight Reverse Repurchase rates when that facility was first introduced by the Federal Reserve (see the discussion in section 5). When the lower bound on the large bank's deposit interest rate is binding, i.e., the large bank's deposit interest rate is equal to the CBDC interest rate, an increase in the CBDC interest rate increases the large bank's market share in the deposit market. Under certain conditions, a higher CBDC interest rate also increases the small bank's deposit interest rate.[3]

Turning to the lending market, under the same condition that induces a positive response of the small bank's deposit interest rate to increases in the CBDC interest rate, the small bank's loan quality threshold increase and its loan volume decreases in the CBDC interest rate. On the other hand, because a higher CBDC interest rate increases the large bank's deposit market share but reduces its interest rate spread between IOR and the deposit interest rate, the CBDC interest rate has an ambiguous impact on the large bank's loan market outcomes. The fact that the loan market is affected at all by the CBDC interest rate, while holding the IOR rate fixed, is not mechanical. While bank lending in our model is not restricted by the amount of deposits or reserves, it is affected by the opportunity cost of capital, which, in turn, depends on the probability that a lent dollar returns to the lending bank and earns interest on reserves. It is through the CBDC's impact on deposit market shares that the CBDC interest rate affects lending.

Our final theoretical exploration is an alternative CBDC design that focuses on other attributes that can complement a CBDC's interest-bearing property, or that may make a CBDC attractive even if it is non-interest bearing. This exploration is consistent with the Banking for All Act put forward in the U.S. Congress, which argues that a CBDC should provide a number of auxiliary services including debit cards, online account access, automatic bill-pay and mobile banking. These features (in particular mobile banking which could give access to a variety of platforms that customers of a particular bank might otherwise not have

[3]In general, because the large bank's deposit interest rate increases with the CBDC interest rate one-for-one when the lower bound in binding, the small bank could either increase its own rate to compete on market share or reduce its own rate to maximize profit margin on its limited set of depositors. Under the parameter conditions that we characterize, the small bank increases the deposit interest rate to compete.

access to) could result in a CBDC with its own convenience value.[4] Although the convenient CBDC is also offered through commercial banks in our model, customers of a commercial bank now enjoys the maximum of two convenience values: that of the commercial bank deposit and that of the CBDC.

The decision to embed a CBDC with its own convenience properties has significant impacts on overall market outcomes, but also on market composition. This is because a convenient CBDC weakens the market power of the large bank by narrowing the convenience gap between the two banks. For example, by hosting a convenient CBDC, a small community bank partially "catches up" with large global banks in offering payment functionalities. The most immediate implication is that a convenient CBDC results in a lower deposit rate at the small bank, because the small bank does not have to compensate depositors as much for foregoing the large banks convenience. At the same time, deposit interest rates at the large bank remain unchanged (if constrained) or increase (if unconstrained). In the range of low convenience values, where the large bank deposit rates are non-responsive, increasing convenience weakens the transmission of monetary policy through IOR to the deposit market. However, once the convenience value of the CBDC reaches a certain level that the large bank's interest rate is no longer at the lower bound, a further increase in CBDC convenience increases monetary policy transmission by increasing the weighted average deposit interest rate.

Interestingly, the overall impact of increasing CBDC convenience (and the corresponding changes in equilibrium deposit rates) is that the small bank and the large bank start to converge in deposit interest rates, deposit market shares, loan quality thresholds, and loan volume. In other words, a convenient CBDC "levels the playing field" for bank competition by chipping away the convenience advantage of the large bank and making the two banks more similar. Monetary policy transmission and leveling the playing field are consistent with each other if the CBDC convenience value is sufficiently high that the large bank is no longer at the lower bound.

Overall, our analysis fleshes out the implications of introducing an interest-bearing CBDC to U.S. financial markets, and relates this analysis to other design features that the affect convenience of holding CBDC accounts. A key takeaway of our theoretical analysis is that the design of an interest-bearing CBDC has important implications not only on the aggregate deposit and lending markets, but also the distributional effect across large and small banks. For example, in our model where the CBDC adopts the convenience of commercial bank deposits, setting the CBDC interest rate too close to the IOR tends to further concentrate deposits in large banks. Building a CBDC with its own convenience can reduce disparities in deposit and lending markets, although the convenience value needs to be sufficiently high to avoid weakening monetary policy transmission.

[4] In China, the central bank has experimented with its own CBDC mobile App in collaboration with state-owned banks.

2 Literature

Our work builds on previous literature that has modelled deposit and lending markets in the current regime of large excess reserves. In Martin, McAndrews and Skeie (2013), a loan is made if its return exceeds the marginal opportunity cost of reserves, which can be either the federal funds rate or the IOR rate, depending on the regime. Our model differs in that we have multiple banks and hence lent money may return to the same bank as new deposits. Hence, our opportunity cost of lending is lower. Nevertheless, we share the conclusion that the aggregate level of bank reserves does not determine the level of bank lending.

Our results on how changes in the federal funds rate affect deposit markets are connected to the literature on monetary policy transmission. Drechsler et al. (2017) provide a model in which increases in the federal funds rate give banks more market power in setting deposit rates. As a result, increases in the federal funds rate lead to an increase in the spread between the federal funds rate and deposit interest rates. Our model provides the same conclusion at lower levels of the federal funds rate. In our case, the increase in spread is amplified by the fact that in the constrained solution of our model with heterogeneous banks, the deposit interest rates at the large bank are completely non-responsive to changes in the federal funds rate.[5] Drechsler et al. (2017) predict that a contraction in deposit supply induced by an increase in the federal funds rate will cause a reduction in lending. This does not occur in our model, since, in our model with large excess reserves, loans are not tied to deposit levels. Nevertheless, we also find that loan volumes decrease in response to an increase in the federal funds rate because of increased opportunity costs of lending.

There is now a growing literature that seeks to examine the impact of CBDC on deposit and lending markets. The conclusions vary and depend upon the level of competition, the interest rate on the CBDC, and other features (e.g., liquidity properties of CBDC and reserve requirements). Keister and Sanches (2020) consider a competitive banking environment in which deposit interest rates are determined jointly by the transactions demand for deposits and the supply of investment projects. If the CBDC serves as a substitute for bank deposits, then its introduction causes deposit interest rates to rise, and levels of deposits and bank lending to fall.

In contrast, if banks have market power in the deposit market, the introduction of a CBDC does not disintermediate banks, as banks can prevent consumers from holding the CBDC by matching its interest rate. This lowers their profit margin, but does not lower the level of deposits, and may even increase it. This is true in the model proposed by Andolfatto (2020), where the bank is a monopolist. In that work, an interest bearing CBDC causes deposit interest rates to raise and the level of deposits to increase. Likewise, in that work, banks have monopoly power in the lending market, and, as in Martin, McAndrews and Skeie (2013), lending is not tied directly to the level of deposits, Hence, a CBDC does not impact the interest rate on bank lending or the level of investment.

Chiu et al. (2019) also consider banks with market power and show that an interest-

[5] The "stickiness" of deposit rates in the United State is also documented in Driscoll and Judson (2013), who provide evidence that deposit interest rates respond less to increases in the federal funds rate than they do to decreases.

bearing CBDC can lead to more, fewer or no change in deposits, depending on the level of the CBDC interest rate. In an intermediate range of rates, the CBDC impacts the deposit market in a manner similar to Andolfatto (2020) in that banks offer higher deposit interest rates and increase deposits. Since, similar to Keister and Sanches (2020), lending is tied to the level of deposits, adding the CBDC results in increased lending.

Our work is closest to Andolfatto (2020). We do not specify the overlapping generations framework that he uses to make money essential. However, like Andolfatto, in our model, reserves are abundant, lending is determined by a performance threshold, and banks have monopoly power in lending market. Hence, lending is determined not by deposit levels, but instead by the opportunity cost of funds. In our model, this opportunity cost is lower than the IOR rate, since we allow for the realistic feature that reserves come back to the lending bank with a probability that depends on the deposit market share.

The main difference between our model and those of the above CBDC papers is that we have heterogeneous banks. This allows us to look at the differential impact of the CBDC interest rate on the competitive landscape of banks in the deposit market and the loan market. The predictions regarding the large and small banks' responses in deposit interest rates and market shares can be tested in the data. Moreover, our heterogeneous bank setting also implies that the design details of the CBDC matters for its impact. In particular, depositors in our model put an extra convenience value on the large bank's deposit, so the CBDC cannot be a perfect substitute for large bank deposit and small bank deposit simultaneously. The degree to which the CBDC intends to compete with the large bank, the small bank, or a mix of the two is an active design choice.

The impact of adding a CBDC can be richer in the presence of other frictions. For example, in a model with real goods and competitive banks, Piazzesi and Schneider (2020) find that the introduction of CBDC is beneficial if all payments are made through deposits and the central bank has a lower cost in offering deposits. However, they also find that the CBDC can be harmful if the payer prefers to use a commercial bank credit line, but the receiver prefers central bank money. In the latter case, a transaction leads to a flow of funds from commercial banks to the central bank; if the liquid assets required to back up this flow are sufficiently costly for the paying bank, the CBDC increases overall costs.

A CBDC may be welfare increasing but not Pareto improving in the presence of network externalities. Agur et al. (2019) consider an environment where households suffer disutility from using a payment instrument that is not commonly used. They examine trade-offs faced by the central bank in preserving variety in payment instruments and show that mitigating the adverse effects of CBDC on financial intermediation is harder to overcome with a non-interest-bearing CBDC.

Fernández-Villaverde et al. (2020) extend the analysis of CBDC to a Diamond and Dybvig (1983) environment in which banks are prone to bank runs. In this setting, the fact that the central bank may offer more rigid deposit contracts allows it to prevent runs. Since commercial banks cannot commit to the same contract, the central bank becomes a deposit monopolist. Provided the central bank does not exploit this monopoly power, the first-best amount of maturity transformation in the economy is still achieved.

Brunnermeier and Niepelt (2019) and Fernández-Villaverde et al. (2020) derive conditions under which the addition of a CBDC does not affect equilibrium outcomes. Key to their result is the central bank's active role in providing funding to commercial banks in order to neutralize the CBDC's impact on their deposits.

3 Model and Equilibrium

3.1 Setup

The economy has a large bank (L) and a small bank (S). There are $X = X_S + X_L$ reserves in the banking system, where X_S denotes the reserve holding of the small bank and X_L denotes the reserve holding of the large bank.[6] For simplicity, the banks start off with the only assets being reserves, balanced by exactly the same amounts of deposits. Following Martin, McAndrews and Skeie (2013), we assume that the level of reserves X is exogenously determined by the central bank and is assumed to be sufficiently high that the demand curve for reserves is flat at the exogenously determined interest rate f, which denotes the interest rate the central bank pays on reserves (IOR).

In addition to the interest on reserve f that is paid only to the commercial banks, the central bank also offers an interest-bearing CBDC that pays an interest rate $s \geq 0$. The CBDC is available to any customer of the commercial banks and is offered through them. In the baseline model considered here, we assume that the CBDC has no inherent convenience value of its own, but instead inherits the convenience value of the deposits at the commercial bank that offers the CBDC. Therefore, the CBDC interest rate s becomes a lower bound on commercial bank deposit interest rates.

There is a unit mass of agents. Each agent potentially plays three roles in the model:

- Entrepreneur: agent $i \in [0, 1]$ is endowed with a project of quality (i.e., success probability) q_i that has a cumulative distribution function Q. Project i requires \$1 of investment and pays $A > 1$ with probability q_i and zero with probability of $1 - q_i$, where A is a commonly known constant. The expected payoff per dollar invested is thus $q_i A$. Agent i can only borrow from the bank where she keeps her deposit (the "relationship" bank).

- Worker: with some probability, an agent is randomly matched with an entrepreneur who receives financing from a bank. The first agent plays the role of worker (contractor) and gets paid the full \$1 to work on the project. An entrepreneur who receives financing can be a worker for another entrepreneur.

- Depositor: If an agent receives wage (\$1), she deposits it in a bank, chosen endogenously. The worker's value for "convenience" of depositing at the large bank is δ, which

[6] We normalize the size of an individual loan to be \$1, so reserves are in units of the standard loan size. For example, if a loan size is \$1 million and the actual reserve is \$1 trillion, then in our model, X is interpreted as $\$1\, trillion/\$1\, million = 10^6$.

is a nonnegative random variable with the cumulative distribution function G. The convenience value of depositing at the small bank is normalized to be zero.

We will look for an equilibrium in which the market shares, in terms of deposits, of the large bank and the small bank are $\alpha_L > 0.5$ and $\alpha_S = 1 - \alpha_L < 0.5$, both endogenously determined.

The timeline of the model is as follows.

$t = 0$: The banks set the deposit interest rates r_L and r_S. As mentioned above, the CBDC interest rate s is a lower bound on these interest rates, i.e., $r_L \geq s$ and $r_S \geq s$. A fraction m_L of agents have existing deposits at the large bank and a fraction $m_S = 1 - m_L$ of agents have existing deposits at the small bank. The deposit per capita across agents is identical. This means $m_L = X_L/X$ and $m_S = X_S/X$.

$t = 1$: Each agent is endowed with a project and goes to the relationship bank to borrow \$1. Therefore, the relationship bank prices the loan as a monopolist.

$t = 2$: If a loan is granted, a funded entrepreneur pays a randomly matched worker \$1 as wage. The worker chooses the bank to deposit and receives the bank's deposit interest rate. The project succeeds or fails. The banks earn interest on reserves and pay deposit interest rates.

3.2 Bank deposit creation

For the purpose of illustration it is convenient to illustrate the deposit creation process by considering a discrete set-up, in which we characterize the bank's decision to make a single loan. The condition on bank lending that we derive will be applicable to the continuum model in which borrowers (i.e., the entrepreneurs) are infinitesimal.

The tables below show the sequence of changes in the large bank's balance sheet in the loan process. The changes in the small bank's balance sheet in the loan process are entirely analogous.

1. Before lending, the large bank starts with X_L reserves. Its balance sheet looks like:

Asset	Liability
Reserves X_L	Deposits X_L

2. If the large bank makes a loan of \$1, it immediately creates deposit of \$1 in the name of the entrepreneur. The balance sheet of the bank becomes:

Asset	Liability
Reserves X_L Loans 1	Deposits X_L New Deposits 1

3. Eventually, the entrepreneur will spend her money to pay a worker. The large bank anticipates that, in expectation, a fraction α_S of the \$1 new deposit will be transferred to the small bank, leading to a reduction of reserves by the same amount. The fraction α_L remains in the bank because the worker has an account with the same bank. The bank's balance sheet becomes:

Asset	Liability
Reserves $X_L - \alpha_S$	Deposits X_L
Loans 1	New Deposits α_L

If the large bank makes the \$1 loan to entrepreneur i, and charges interest rate R_i, its total expected profit, by counting all items in the balance sheet, will be

$$\underbrace{(X_L - \alpha_S)f}_{\text{Interest on reserves}} + \underbrace{[q_i(1+R_i) - 1]}_{\text{Gross profit on the loan}} - \underbrace{(X_L + \alpha_L)r_L}_{\text{Cost of deposits}}. \tag{1}$$

If the large bank does not make the loan, then its total profit will be

$$X_L(f - r_L). \tag{2}$$

The large bank's marginal profit from making the loan, compared to not making it, is

$$\pi_i = \underbrace{q_i(1+R_i) - (1+f)}_{\text{Net profit on the loan}} + \underbrace{\alpha_L(f - r_L).}_{\text{Profit on deposit}} \tag{3}$$

In the expression of π_i, the net profit on the loan reflects the true opportunity cost of capital.

There are two cases of equilibrium, depending on whether the lower bound s on the deposit interest rates is binding.

3.3 When the CBDC interest rate does not bind deposit interest rates

In this case, $r_L > s$ and $r_S > s$. We can solve the model backward in time.

Deposit market at $t = 2$. Faced with the two deposit interest rates r_L and r_S, an agent with convenience value δ chooses the large bank if and only if

$$r_L + \delta > r_S \Rightarrow \delta > r_S - r_L. \tag{4}$$

Therefore, the eventual market shares of the banks in the newly created deposits are

$$\alpha_L = 1 - G(r_S - r_L), \tag{5}$$
$$\alpha_S = G(r_S - r_L). \tag{6}$$

Loan market at $t = 1$. We have derived in the previous section the marginal profit of a bank in making a loan. While the entrepreneur is infinitesimal here, the same expression (3) still applies.

To derive the lending criterion, the monopolist position of each bank in the lending market implies that a bank can make a take-it-or-leave-it offer to the entrepreneur. The bank's optimal interest rate quote would be $R_i = A - 1$ (or just tiny amount below), and the entrepreneur, who has no alternative source of funds, would accept. The lending bank takes the full surplus.

Hence, the large bank makes the loan if and only if

$$q_i A - (1 + f) + \alpha_L(f - r_L) > 0, \tag{7}$$

or

$$q_i > q_L^* = \frac{1 + f - \alpha_L(f - r_L)}{A}. \tag{8}$$

Exactly the same calculation for the small bank yields the comparable investment threshold

$$q_S^* = \frac{1 + f - \alpha_S(f - r_S)}{A}. \tag{9}$$

Choice of deposit interest rates at $t = 0$. Again, we start with the large bank. The large bank makes profits in two ways. Because the large bank is a monopolist when lending to its customers, its first source of profit is on the loans, $m_L \int_{q_l^*}(qA - 1 - f)dQ(q)$. The second source of the large bank's profit is on the interest rate spread. The existing deposit in the banking sytem is $X = X_L + X_S$. As discussed above, the lending process also creates new deposits. The amount of new deposit created by the large bank is $m_L(1 - Q(q_L^*)$, by the normalization that each loan is of \$1. Likewise, the small bank creates new deposit $m_S(1 - Q(q_S^*)$. When the two banks compete for depositors by setting the deposit interest rates r_L and r_S, we already show above that a fraction $\alpha_L = 1 - G(r_S - r_L)$ of total deposits end up with the large bank, enabling the large bank to collect a spread of $f - r_L$ per unit of deposit held.

Adding up the two components, we can write the large bank's total profit as

$$\begin{aligned}
\Pi_L &= m_L \int_{q_L^*}^{1} (qA - 1 - f)dQ(q) + [X_L + X_S + m_L(1 - Q(q_L^*)) + m_S(1 - Q(q_S^*))]\alpha_L(f - r_L) \\
&= m_L \int_{q_L^*}^{1} [qA - (1 + f) + \alpha_L(f - r_L)]dQ(q) + [X_L + X_S + m_S(1 - Q(q_S^*))]\alpha_L(f - r_L).
\end{aligned} \tag{10}$$

Assuming that Π_L is strictly quasi-concave in r_L (for any given α_L), the sufficient condition for a unique maximum of this function with respect to r_L is

$$\begin{aligned}
\frac{d\Pi_L}{dr_L} &= m_L(1-Q(q_L^*))\frac{d[\alpha_L(f-r_L)]}{dr_L} - m_L\underbrace{[q_L^*A-(1+f)+\alpha_L(f-r_L)]}_{=0}\frac{dq_L^*}{dr_L} \\
&\quad + [X_L+X_S+m_S(1-Q(q_S^*))]\frac{d[\alpha_L(f-r_L)]}{dr_L} - m_S\alpha_L(f-r_L)Q'(q_S^*)\frac{dq_S^*}{dr_L} \\
&= [X_L+X_S+m_L(1-Q(q_L^*))+m_S(1-Q(q_S^*))]\cdot[(f-r_L)G'(r_S-r_L)-1+G(r_S-r_L)] \\
&\quad - m_S\alpha_L(f-r_L)Q'(q_S^*)\frac{(f-r_S)G'(r_S-r_L)}{A}. \qquad (11)
\end{aligned}$$

Likewise, the small bank's total profit is

$$\Pi_S = m_S\int_{q_S^*}^{1}[qA-(1+f)+\alpha_S(f-r_S)]dQ(q) + [X_L+X_S+m_L(1-Q(q_L^*))]\alpha_S(f-r_S). \qquad (12)$$

The first-order condition of the small bank is

$$\begin{aligned}
\frac{d\Pi_S}{dr_S} &= [X_L+X_S+m_L(1-Q(q_L^*))+m_S(1-Q(q_S^*))]\cdot[(f-r_S)G'(r_S-r_L)-G(r_S-r_L)] \\
&\quad - m_L\alpha_S(f-r_S)Q'(q_L^*)\frac{(f-r_L)G'(r_S-r_L)}{A}. \qquad (13)
\end{aligned}$$

For simplicity, let $Q(\cdot)$ be the uniform distribution on $[0,1]$. And further impose a "stationarity" condition that the market shares of deposit α_j are identical to the starting market share m_j. The first-order conditions are simplified to

$$\begin{aligned}
0 = \frac{d\Pi_L}{dr_L} &= [X+\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)]\cdot[(f-r_L)G'(r_S-r_L)-1+G(r_S-r_L)] \\
&\quad - \frac{1}{A}\alpha_S\alpha_L(f-r_L)(f-r_S)G'(r_S-r_L), \qquad (14) \\
0 = \frac{d\Pi_S}{dr_S} &= [X+\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)]\cdot[(f-r_S)G'(r_S-r_L)-G(r_S-r_L)] \\
&\quad - \frac{1}{A}\alpha_L\alpha_S(f-r_L)(f-r_S)G'(r_S-r_L). \qquad (15)
\end{aligned}$$

Proposition 1. *Suppose that the profit function Π_j is quasi-concave in r_j, $j \in \{L,S\}$. Let r_L and r_S solve equations (14)–(15). If $r_L > s$ and $r_S > s$, then it is an unconstrained equilibrium that the banks set r_L and r_S as their deposit interest rates. In this equilibrium:*

1. *The large bank sets a lower deposit interest rate ($r_L < r_S$) and has a larger market share ($\alpha_L > \alpha_S$) than the small bank.*

2. *The large bank uses a looser lending standard than the small bank does ($q_L^* < q_S^*$).*

Further intuition of the equilibrium may be gained by considering the uniform distribution of G. Suppose that $G(\delta) = \delta/\Delta$, where $\delta \in [0, \Delta]$ for a sufficiently large Δ. Then $G'(\cdot) = 1/\Delta$. The two first-order conditions reduce to

$$\frac{f - r_L}{\Delta} = 1 - \frac{r_S - r_L}{\Delta} + B, \tag{16}$$

$$\frac{f - r_S}{\Delta} = \frac{r_S - r_L}{\Delta} + B, \tag{17}$$

where

$$B \equiv \frac{\frac{1}{A\Delta}\alpha_L \alpha_S (f - r_L)(f - r_S)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \tag{18}$$

As the total reserve X becomes large, B becomes close to zero. So the equilibrium deposit interest rates of the two banks become approximately $r_L \approx f - \frac{2}{3}\Delta$ and $r_S \approx f - \frac{1}{3}\Delta$.

3.4 When the CBDC interest rate binds the banks' deposit interest rates

The second case of the equilibrium is if the CBDC interest rate s becomes binding for at least one of the two banks. It is easy to show s cannot bind for both banks unless $s = f$.[7] Thus, we look for a constrained equilibrium in which $r_S > s$ and $r_f = s$.

The small bank's profit function and first-order condition are as before:

$$\begin{aligned} \frac{d\Pi_S}{dr_S} &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S - s) - G(r_S - s)] \\ &\quad - \frac{1}{A}\alpha_L \alpha_S (f - s)(f - r_S)G'(r_S - s)) = 0. \end{aligned} \tag{19}$$

By contrast, the large bank's first order condition takes an inequality because the conjectured optimal solution is at the left corner:

$$\begin{aligned} 0 > \left.\frac{d\Pi_L}{dr_L}\right|_{r_L \downarrow s} &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - s)G'(r_S - s) - 1 + G(r_S - s)] \\ &\quad - \frac{1}{A}\alpha_S \alpha_L (f - s)(f - r_S)G'(r_S - s). \end{aligned} \tag{20}$$

Proposition 2. *Suppose that the profit function Π_j is quasi-concave in r_j, $j \in \{L, S\}$. Let r_S solve equation* (19). *If, at r_S, equation* (20) *also holds, then it is a constrained equilibrium that the large bank sets s and the small bank sets r_S as their deposit interest rates. In this equilibrium:*

1. *The large bank sets a lower deposit interest rate ($s < r_S$) but has a larger market share ($\alpha_L > \alpha_S$) than the small bank.*

2. *The large bank uses a looser lending standard than the small bank does ($q_L^* < q_S^*$).*

[7] If $r_L = r_S = s < f$, then the small bank has market share $G(r_S - r_L) = 0$. It can profitably deviate by raising the deposit interest rate by a tiny amount and capturing some of the interest rate spread $f - r_S$.

4 Impact of IOR on Market Outcomes

Before proceeding to examine the impact of CBDC policy, we first examine how well the model captures monetary policy transmission in the current environment with large excess reserves. In particular, setting the CBDC interest rate to 0, we compute the impact of a change in the IOR rate on the deposit market. Key to this analysis is the recognition that as the IOR rate increases from 0, we transition from a constrained equilibrium to an unconstrained one.

4.1 Monetary policy transmission in the deposit market

As discussed in the introduction, deposit interest rates tend to be non-responsive to changes in IOR rates at low levels, but are responsive at higher levels. These stylized facts are captured in our comparative static results.

Proposition 3. *For a sufficiently large X, the comparative statics with respect to f in the deposit market are given in the following table.*

	Constrained		*Unconstrained*	
	Large	*Small*	*Large*	*Small*
Deposit interest rates r_L and r_S	*Flat*	$\uparrow$	$\uparrow$	$\uparrow$
Deposit market shares α_L and α_S	$\downarrow$	$\uparrow$	*Flat*	*Flat*
Weighted average deposit interest rate	$\uparrow$		$\uparrow$	

Figure 3 shows the deposit interest rates and market share of the two banks as functions of the interest on reserves f for a uniform distribution $G(\delta) = \delta/0.035$. We observe that the deposit interest rate offered by the large bank remains at zero until the IOR rises to a high level (left panel), which in this case is approximately 2.33%. In addition, the market share of the large bank is larger for IOR closer to zero (right panel).

These two properties combined produce predicted average deposit interest rates that are lower and less responsive to changes in the IOR rate f when it is near zero. When f is near zero, the share of deposits at the large bank is near 1 and the large bank offers a deposit interest rate equal to 0. As f increases, the share of deposits held by the small bank increases and the small bank offers a positive rate, so the the average deposit interest rate increases slowly. Eventually, as f increases further, we move to the interior solution described in Proposition 1, where deposit shares are constant (with respect to f) and deposit interest rates rise proportionately with f.

We illustrate the predictive performance of our model using U.S. data on deposit interest rates from 1986 to 2021. The opportunity cost of funds for banks is determined in part by either the IOR rate or the federal funds rate, whichever is larger. In the period before the 2008-09 crisis the relevant rate was the federal funds rate. Figure 4 shows actual and predicted U.S. deposit interest rates from 1986Q1 to 2008Q2 relative to the federal fund rate. The choice of the upper bound on the convenience value for deposits at the large bank

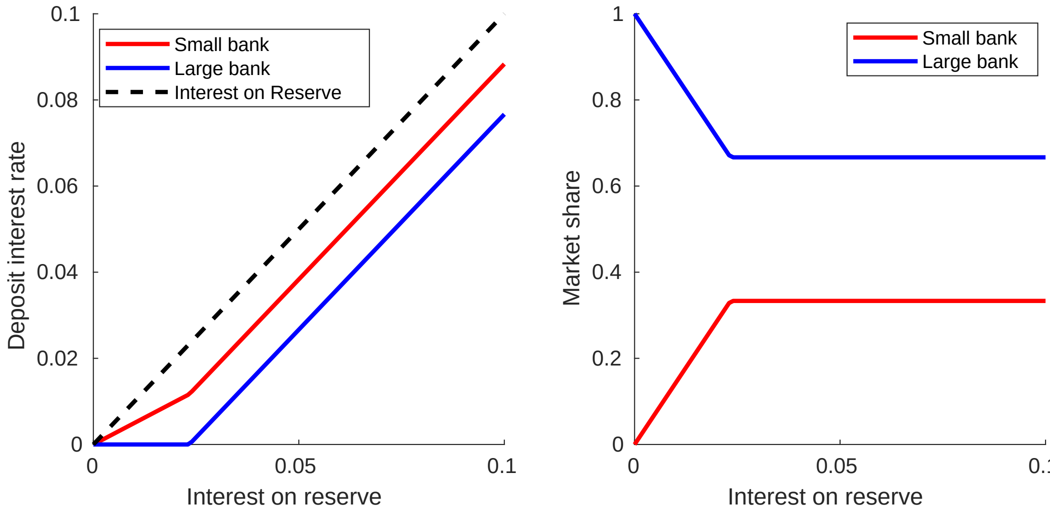

Figure 3: Equilibrium deposit market. Model parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, s = 0$.

is somewhat arbitrary, but 3.5% seems reasonable as a maximum value and the resulting predicted data series seems to fit the actual data reasonably well. The main deviation of the predicted series from the actual one is that our predicted rates respond too quickly (and hence to much) to changes in the federal funds rate. This is expected since we do not build in any "stickiness" into the deposit interest rate that would be necessary to match the actual data more closely. Given our assumed distribution for the convenience parameter for large bank deposits, our predicted equilibrium involves a binding zero lower bound on the rate at the large bank when the reference rate is less than about 2.33%. This is true in the data during the pre-crisis period from 2001Q3 to 2004Q3. During this period we predict deposit interest rates near zero, however actual deposit interest rates remained close to, and even slightly above, the federal funds rate.

In the period after the crisis the relevant rate for computing the opportunity cost of loaned funds was (generally) the IOR rate. Figure 5 shows actual and predicted U.S. deposit interest rates from May 2009 to February 2021 relative to the IOR rate. This period is characterized by a long stretch of near zero rates in the Federal Funds market and an IOR rate of 25 basis points. Again consistent with the market-lag story, deposit interest rates fell slowly during this period toward zero until the Fed began to raise rates in December 2015. The Fed raised the IOR rate multiple time reaching a peak of 2.40% from December 2018 to April 2019, but deposit interest rate reacted very slowly. Again, our model reacts too quickly to the changes in the IOR rate, but still our predicted rate stays quite low relative to the increases in the IOR rate. This is because at the low IOR rates that existed up until December 2018, the zero lower bound is binding in our model and hence average market rate are determined largely by large bank deposit interest rates which are at zero.

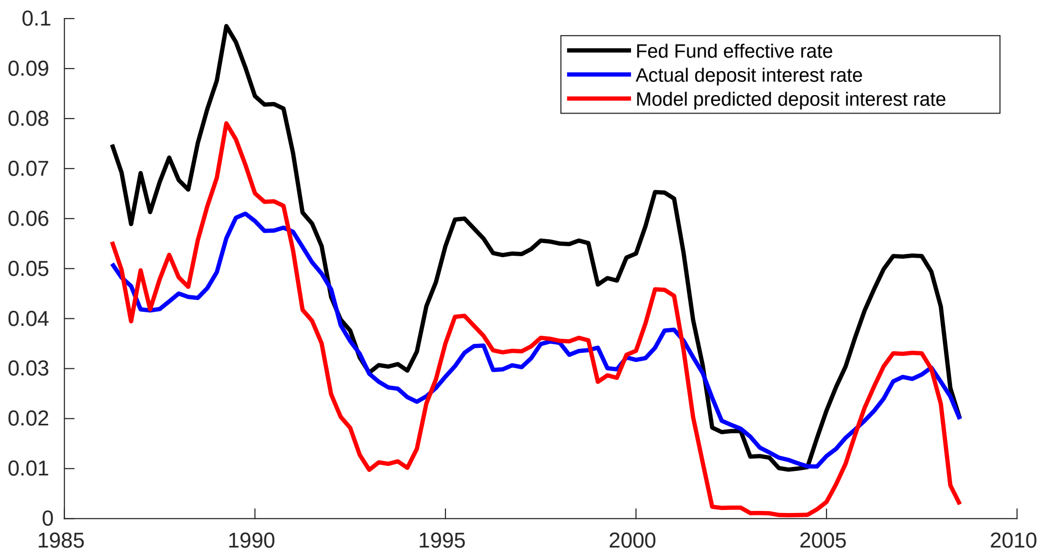

Figure 4: Actual and predicted U.S. deposit interest rates from 1986Q1 to 2008Q2. Domestic deposit interest rates are quarterly, calculated from call reports, as total interest expense on domestic deposit divided by total domestic deposit, multiplied by 4. During this period, interest on reserves is taken to be the actual Federal Funds rate. The model-implied interest rate is the weighted average of the large bank's and the small bank's deposit interest rates, weighted by their market shares. Model parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, s = 0$.

4.2 Monetary policy and bank lending

The impact of increasing IOR rate on the deposit market carries over to the lending market. The most interesting case is the constrained case, which applies for lower IOR rates. In this region, an increase in the IOR rate raises the opportunity cost of lending for both banks, which leads to higher quality thresholds. At the same time it raises the deposit interest rate at the small bank, but not the large bank, resulting in an increase in deposit market share for the small bank. Both impacts on lending are negative for the large bank, but for the small bank the impacts work in opposite directions: a higher IOR rate results in a higher opportunity cost of lending and hence a higher quality threshold, but greater market share results in more opportunities to lend. Hence, for the small bank, lending can increase or decrease as the IOR rate rises. The overall effect on loan volume is unambiguously negative, however, since, as the IOR rate rises, depositors move from the large bank to the small bank where the quality threshold is higher. In the unconstrained case, which applies for higher IOR rates, there is no change in market shares as the IOR rate rises. However, both banks adopt higher quality thresholds and so both banks lend less, and total lending declines.

These features of the model are summarized in the following proposition.

Proposition 4. *For a sufficiently large X, the comparative statics with respect to f in the*

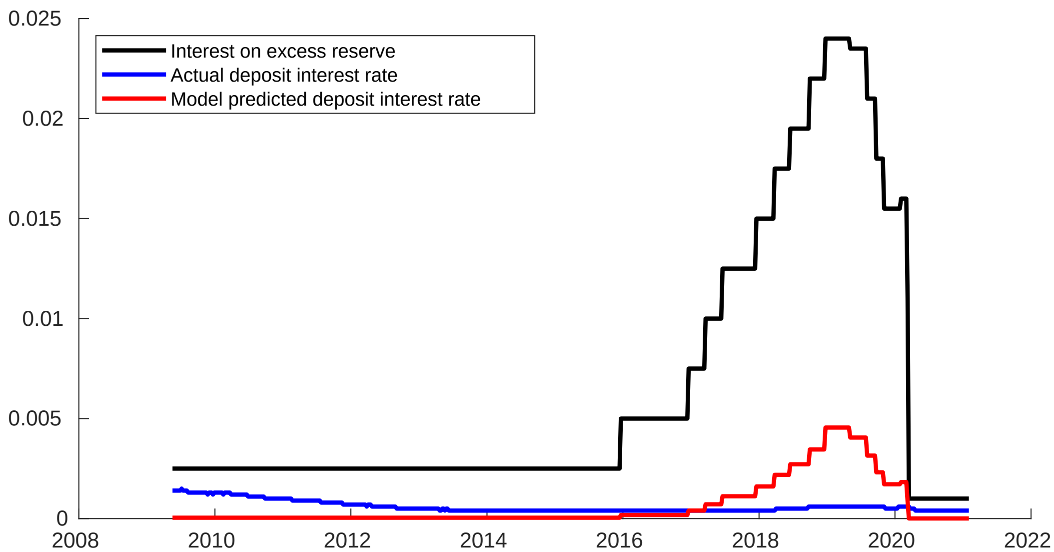

Figure 5: Actual and predicted U.S. deposit interest rates from May 18, 2009 to February 1, 2021. Weekly deposit interest rates for amount less than \$100,000 are obtained from FDIC through FRED. During this period, interest on reserves is taken to be the interest on excess reserves (IOER). The model-implied interest rate is the weighted average of the large bank's and the small bank's deposit interest rates, weighted by their market shares. Model parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, s = 0$.

lending market are given in the following table.

	Constrained		*Unconstrained*	
	Large	*Small*	*Large*	*Small*
Loan quality thresholds q_L^* *and* q_S^*	$\uparrow$	*unclear*	$\uparrow$	$\uparrow$
Loan volume $\alpha_L(1-q_L^*)$ *and* $\alpha_S(1-q_S^*)$	$\downarrow$	*unclear*	$\downarrow$	$\downarrow$
Total loan volume, i.e., total deposit created	$\downarrow$		$\downarrow$	

Figure 6 shows the lending standard and loan volume of the two banks as functions of the IOR rate f for a uniform distribution $G(\delta) = \delta/0.035$. In this example, loan quality thresholds increase in f (left panel). But the small bank's loan volume increases in f in the constrained equilibrium, which means that positive impact of an increasing market share for the small bank dominates the negative impact of a higher opportunity cost of funds. Once the equilibrium transitions into the unconstrained region with f higher than approximately 2.33%, the market shares become invariant to f, and the small bank's loan volume declines in f.

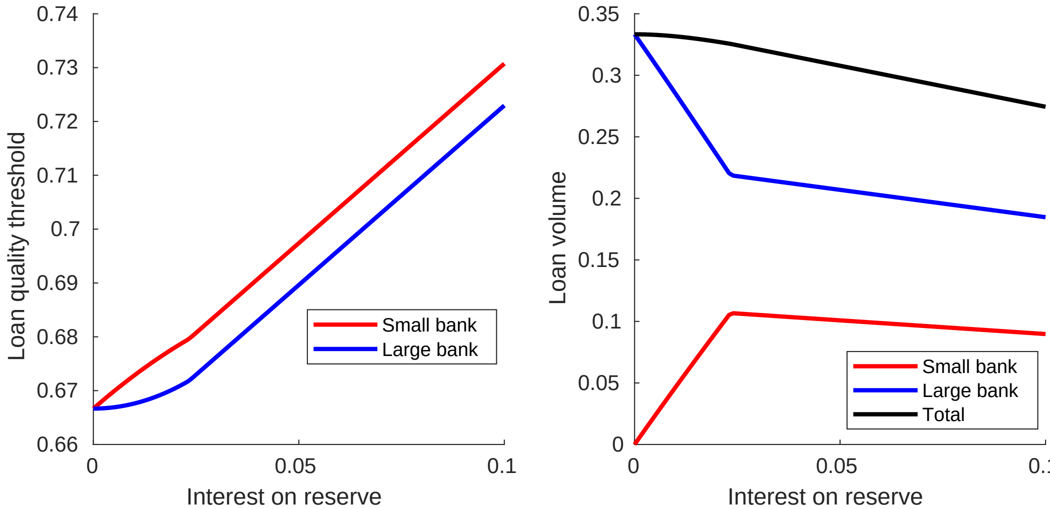

Figure 6: Equilibrium lending market. Model parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, s = 0$.

5 CBDC Interest Rate Policy

When the federal reserve introduced the overnight reverse repo program (ON RRP) as a temporary facility to support its IOR policy, it began by testing the facility by varying the rate between 1 basis point and 10 basis points, while holding the IOR rate fixed at 25 basis points. Here we examine how market outcomes change as s varies from a rate of 0 to f, while holding f fixed.

We focus on the case of low f where the constrained equilibrium applies. This case is most relevant to the current economic environment in the United States. (In the unconstrained equilibrium, market outcomes are invariant to the CBDC interest rate s by definition.)

Proposition 5. *Suppose that $G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f]$. Then, in the constrained equilibrium, for a sufficiently large X, an increasing CBDC interest rate has the following impact on the deposit and lending markets:*

	As s increases	
	Large	*Small*
Deposit interest rates r_L and r_S	$\uparrow$	$\uparrow$
Deposit market shares α_L and α_S	$\uparrow$	$\downarrow$
Weighted average deposit interest rate	$\uparrow$	
Loan quality thresholds q_L^* and q_S^*	*unclear*	$\uparrow$
Loan volume $\alpha_L(1 - Q(q_L^*))$ and $\alpha_S(1 - Q(q_S^*))$	*unclear*	$\downarrow$
Total loan volume, i.e., total deposit created	*unclear*	

Moreover, if $G''(\delta) \leq 0$ for all δ, then the total lending volume decreases in s.

5.1 Impact of the CBDC interest rate s on the deposit market

Figure 7 plots behavior in the deposit markets as the CBDC interest rate rises from 0 to $f = 2\%$. The charts are computed numerically using a uniform distribution for G. As we see in the left panel of Figure 7, eventually deposit rates reach their maximum profitable level for each bank at the fixed IOR rate f. The right panel of Figure 7 shows the corresponding changes in market share, which are easily computed from (5) and (6). Since the large bank's deposit rate rises faster than the small bank's, the market shares diverge. Once the deposit rates are equal at f the large bank obtains the entire market of depositors.

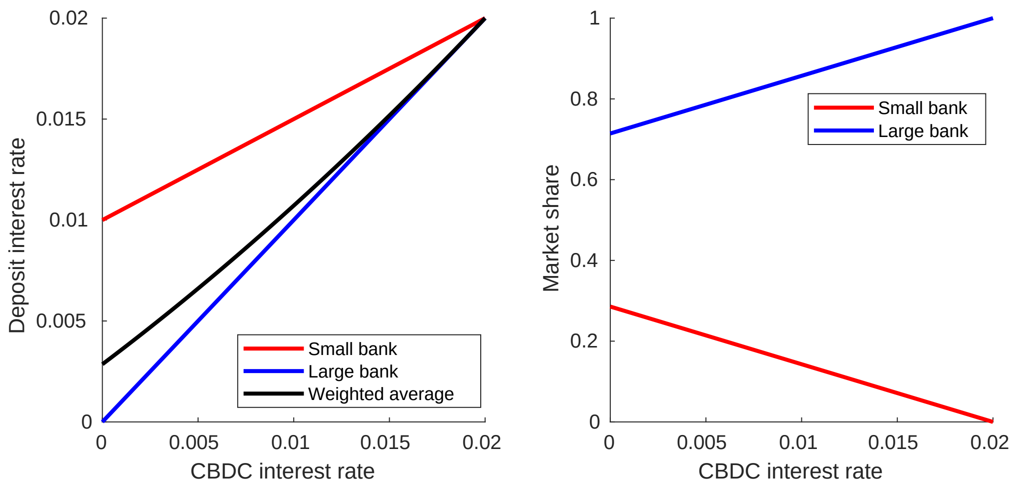

Figure 7: Impact of CBDC interest rate on deposit market. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02$.

5.2 Impact of the CBDC interest rate s on the lending market

The lending thresholds of the large and small bank given in and depend on the how deposit interest rates r_L and r_S and market shares α_L and α_S change when we vary s.

Different from the IOR rate f, the CBDC interest rate s is not the opportunity cost of funds; f is. Yet, the CBDC interest rate changes the incentives to make loans via the expected profit on interest rate spread, $\alpha_j(f - r_j)$. Because, as established above, both α_S and $f - r_S$ decrease in s, so does $\alpha_S(f - r_S)$. Thus, the small bank's loan quality threshold and total loan volume decrease in the CBDC interest rate. The large bank's loan market outcome is generally ambiguous in s, although in this example, the large bank's quality threshold rises in s and its loan volume declines in s. The total loan volume also declines in s in this example.

Figure 8 illustrates the impact of CBDC interest rate s on the lending market with a uniform G. In this example, both banks' lending standards go up in s. A higher CBDC interest rate reduces the small bank's lending but increases the large bank's, and the net result is lower amount of lending.

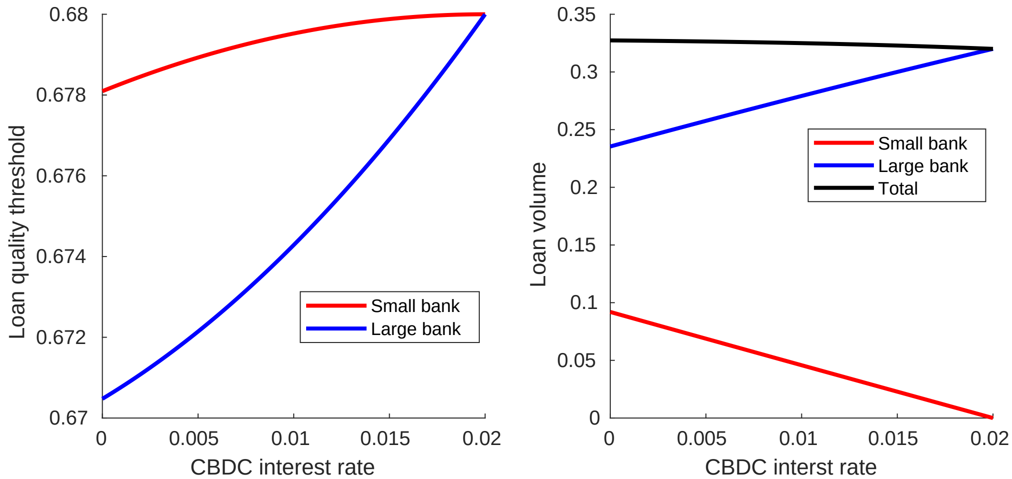

Figure 8: Impact of CBDC interest rate on lending market. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02$.

6 A Convenient CBDC

So far, we have considered a scenario where the CBDC inherits the convenience value of the commercial banks that "host" the CBDC accounts. A main effect of introducing such an interest-bearing CBDC is that it tends to reduce the market share of the small bank. In this section, we consider an independent CBDC that has its own convenience value.

6.1 CBDC with its own convenience value

Recall that the small bank's deposits have a convenience value of zero and the large bank's deposits have a convenience value that varies across agents with distribution G. We assume that the CBDC's own convenience value is v_{CBDC}, and it is the same across all agents. The CBDC accounts are still offered by commercial banks, but if a depositor uses a CBDC account hosted by bank j, the convenience value received by the depositor is $\max(v_{CBDC}, v_j)$, where v means convenience value. Each agent has four choices now:

	Large bank		Small bank	
	Deposit	CBDC	Deposit	CBDC
Convenience value	$\max(\delta, v_{CBDC})$	$\max(\delta, v_{CBDC})$	v_{CBDC}	v_{CBDC}
Interest rate	r_L	s	r_S	s

For agents with large-bank preference $\delta \leq v_{CBDC}$, CBDC convenience dominates:

For $\delta \le v_{CBDC}$	Large bank		Small bank	
	Deposit	CBDC	Deposit	CBDC
Convenience value	v_{CBDC}	v_{CBDC}	v_{CBDC}	v_{CBDC}
Interest rate	r_L	s	r_S	s

For agents with large-bank preference $\delta > v_{CBDC}$, large bank still has more convenience:

For $\delta > v_{CBDC}$	Large bank		Small bank	
	Deposit	CBDC	Deposit	CBDC
Convenience value	δ	δ	v_{CBDC}	v_{CBDC}
Interest rate	r_L	s	r_S	s

Obviously, the small bank's deposit account attracts no depositors if $r_S < r_L$. So $r_S \ge r_L$. The CBDC interest rate s remains a lower bound for r_L and r_S, i.e., $r_S \ge r_L \ge s$. For this reason, the CBDC is not used in equilibrium, so we still only compare the commercial bank deposits. We look for an equilibrium with $r_S > r_L$. Given the tables above, agents with $\delta > r_S - r_L + v_{CBDC}$ use the large bank and others use small bank, i.e., $\alpha_S = G(r_S - r_L + v_{CBDC})$. Write v_c as a shorthand for v_{CBDC}, where c represents "central bank" and "convenience."

6.2 Equilibrium and comparative statics

As before, the unconstrained equilibrium features $r_S > s$ and $r_L > s$. The first-order conditions are

$$
\begin{aligned}
0 = \frac{d\Pi_L}{dr_L} &= [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f - r_L)G'(r_S - r_L + v_c) - 1 + G(r_S - r_L + v_c)] \\
&\quad - \frac{1}{A}\alpha_S\alpha_L(f - r_L)(f - r_S)G'(r_S - r_L + v_c), \qquad (21) \\
0 = \frac{d\Pi_S}{dr_S} &= [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f - r_S)G'(r_S - r_L + v_c) - G(r_S - r_L + v_c)] \\
&\quad - \frac{1}{A}\alpha_L\alpha_S(f - r_L)(f - r_S)G'(r_S - r_L + v_c). \qquad (22)
\end{aligned}
$$

From the above conditions we derive

$$
(r_S - r_L)G'(r_S - r_L + v_c) + 2G(r_S - r_L + v_c) = 1 \qquad (23)
$$

In the constrained equilibrium, the small bank's optimal solution is interior and the large bank's is corner. The small bank and the large bank's first-order conditions are, respectively,

$$
\begin{aligned}
0 &= [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f - r_S)G'(r_S - s + v_c) - G(r_S - s + v_c)] \\
&\quad - \frac{1}{A}\alpha_L\alpha_S(f - s)(f - r_S)G'(r_S - s + v_c), \qquad (24) \\
0 &> [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f - s)G'(r_S - s + v_c) - 1 + G(r_S - s + v_c)] \\
&\quad - \frac{1}{A}\alpha_L\alpha_S(f - s)(f - r_S)G'(r_S - s + v_c). \qquad (25)
\end{aligned}
$$

The comparative statics with respect to v_c is given by the next proposition.

Proposition 6. *Suppose that G satisfies $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v_c]$. For sufficiently large X, the impacts of increasing v_c are given in the following table:*

As v_c increases	*Constrained*		*Unconstrained*	
	Large	*Small*	*Large*	*Small*
Deposit interest rates r_L and r_S	*Flat(=s)*	$\downarrow$	$\uparrow$	$\downarrow$
Deposit market shares α_L and α_S	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Weighted average deposit interest rate	*unclear*		$\uparrow$ *if* $0 \le G''(\delta) < \frac{G'(\delta)}{f}$	
Loan quality thresholds q_L^* and q_S^*	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Loan volume $\alpha_L(1-q_L^*)$ and $\alpha_S(1-q_S^*)$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Total loan volume, i.e., total deposit created	*unclear*		*unclear*	

6.3 Impact of CBDC convenience value v on deposit market.

A convenient CBDC reduces the large bank's convenience advantage and hence has an impact even when its interest rate is zero. We illustrate the impact of a convenient CBDC by considering this polar case in Figure 9. As v rises, the inconvenience disadvantage of the small bank shrinks. As long as the large bank's deposit interest rate remains at the floor rate, the small bank can afford to lower its interest rate and still capture a growing market share. Once v get large enough, the large bank responds by raising its interest rate; however, the small bank can still afford to continue lowering its deposit interest rate for the same reason that the convenience gap between the two banks continues to shrink. Throughout this process the large bank loses market share and the small bank gains market share, albeit at a slower rate once the large bank is no longer constrained. The overall impact of increasing the CBDC convenience value is a "convergence" of the two banks in terms of deposit interest rates and market shares. Note that we restrict parameters such that in equilibrium, $r_S > r_L$.

The interesting implication of this analysis is that in a constrained equilibrium, a convenient CBDC weakens the transmission of monetary policy to the deposit market through IOR. For any given f a higher value of v results in a lower deposit interest rate when the large bank's deposit interest rate is at the lower bound. Once the economy transitions to an unconstrained equilibrium, which occurs for a sufficiently large v, a higher CBDC convenience value increases the average deposit interest rate, speeding up the transition of monetary policy. Hence, in the unconstrained equilibrium, both monetary policy transmission and the small bank's market share in the deposit market increase in v.

6.4 Impact of CBDC convenience value v on loan market.

As discussed above, a salient feature of a convenience CBDC is that it shrinks the difference between the large bank and the small bank, hence "leveling the playing field." As v increases, the two deposit interest rates and the deposit market shares get closer to each other. It is, therefore, unsurprising that the loan quality thresholds and loan volume of the two banks

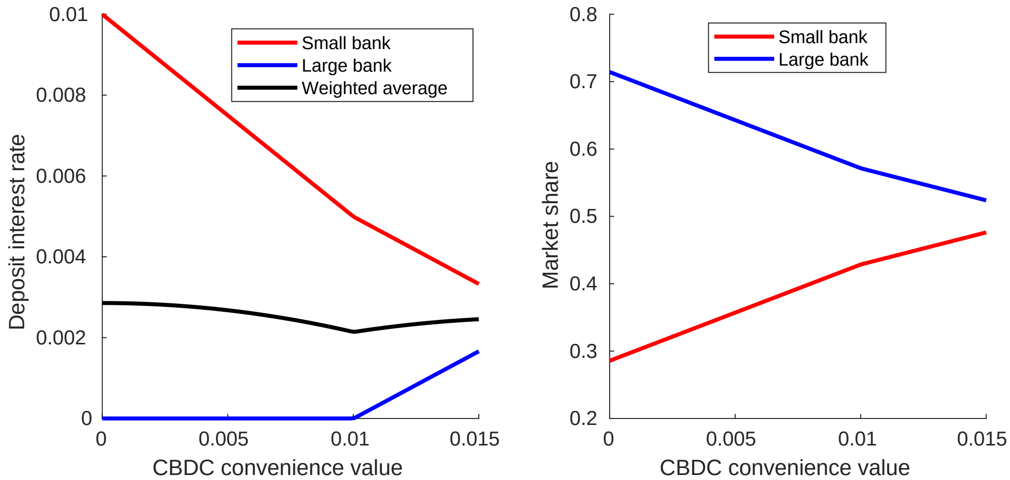

Figure 9: Impact of CBDC convenience value on deposit market. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02, s = 0$.

are also getting closer to each other as v rises. See Figure 10. In this example, the total lending volume is slightly decreasing in v but the magnitude is small that it looks most flat.

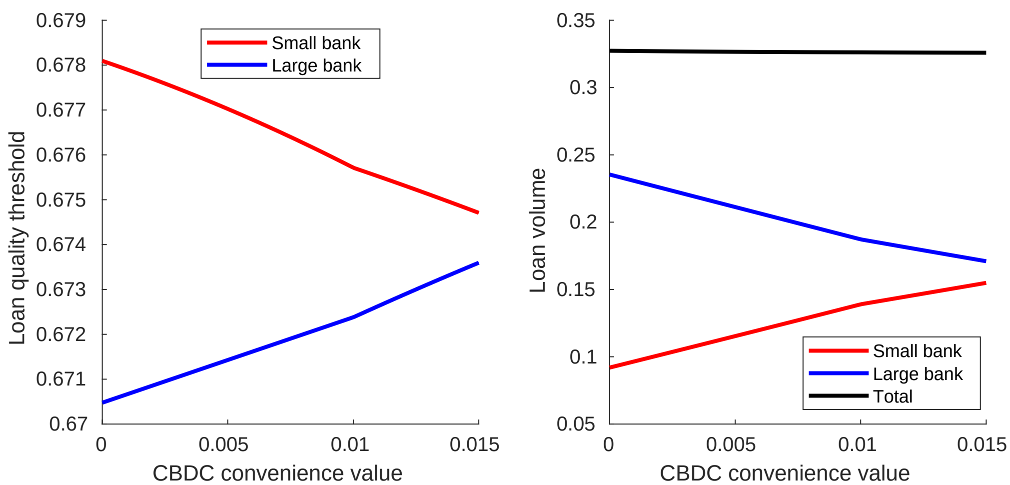

Figure 10: Impact of CBDC convenience value on lending market. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02, s = 0$.

7 Concluding Remarks

Our objective in this study is to understand how an interest-bearing CBDC might impact deposit and lending markets. Our institutional setup is most suited for the current environment in the United States, where there are large quantities of excess reserves and an administered rate, the interest on reserves, that determines banks' opportunity cost of lending. We construct a model that describes how banks set deposit interest rates and decide on the project quality threshold for lending. In this regard we believe it is crucial to consider multiple, heterogeneous banks. Heterogeneous banks are required to capture differences in depositor preferences for banks of different sizes, where size is proxy for an array of preferred services and economies of scale provided by larger banks. It turns out that this modelling feature is key to understanding the impact of the zero lower bound, since differences in preferences for deposits at banks of various sizes lead to an array of equilibrium scenarios that differ in terms of whether or not the zero-lower bound is binding and how responsive deposit interest rates are to changes in the IOR rate.

The first test of our model is its ability to capture a key feature of deposit interest rates: the lack of responsiveness to changes in the IOR rate at low levels. This rigidity arises in our model because banks have different opportunity costs of credit provision due to the differing likelihoods that loan reserves will return as new deposits. This introduces a channel through which changes in the opportunity cost of credit provision — federal funds rate or IOR, whichever is higher — leads to changes not only in deposit interest rates at the small and large banks, but also in the share of depositors at each bank.

Our initial approach to modelling a CBDC is to assume it would be offered via the commercial banks and inherit the convenience of the host institution. In this setting, an interest-bearing CBDC leads to a lower market share for small banks.Under some conditions, all deposit interest rates increase in response to the introduction of an interest-bearing CBDC, in which case the trade-off for loan market is the usual quality vs quantity one: A higher CBDC interest rate will increase the quality of loan portfolios, but reduce quantity.

A CBDC could be designed so that CBDC accounts provided their own convenience value. This case is interesting because it fits the proposal outlined in the Banking for All Act. Here we identified an interesting trade-off. A CBDC can be designed that levels the paying field for competition between banks, but in doing so can weaken the transmission of monetary policy through the IOR rate. Surprisingly, a convenience CBDC can strengthen the transmission of monetary policy if the CBDC convenience value is sufficiently high.

An interesting aspect of our results is that the provision of CBDC impacts equilibrium outcomes even though the currency is not held in equilibrium. This is also true in Chiu et al. (2019) and Garratt and Lee (2020), where the *option* to use CBDC changes the equilibrium outcome even it is not exercised. An exception is Keister and Sanches (2020), where the CBDC has specific liquidity benefits that leads to its use. The idea that a central bank introduces a program to influence market rates by increasing the bargaining power of lenders is not new. Early descriptions of the overnight reverse repurchase agreement facility (ON RRP) that the Federal Reserve Bank of New York began testing in September 2013 indicated that "the option to invest in ON RRPs also would provide bargaining power to investors in their negotiations with borrowers in money markets, so even if actual ON RRP take-up is not very large, such a facility would help provide a floor on short-term interest rates..." (Frost et al. 2015, p.7).

The results of our paper could be extended in multiple directions. One possible extension is to add short-term investment vehicles such as money market mutual funds and repurchase agreements (repos) that typically pay higher interest rates than bank deposits but cannot be easily used for processing payments. If the CBDC pays a sufficiently high interest rate, it is possible that money would flow out of these short-term investment vehicles into the CBDC, i.e., investors would earn returns from the Fed rather than short-term Treasury Bills. This additional channel is unlikely to affect lending because money market investors do not make loans anyway. Another possibility is to consider heterogeneous CBDC interest rates paid to banks of different sizes, which adds yet another degree of freedom in the central bank's toolkit, in particular if the central bank wishes to fine-tune the competitive positions of large and small banks. These extensions are left for future research.

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Appendix: Proofs

Proof of Proposition 1

Taking the difference of the two FOCs, we have $(r_S - r_L)G'(r_S - r_L) = 1 - 2G(r_S - r_L)$. If $r_S \leq r_L$, because the lower bound of G's domain is zero, the left-hand side is nonpositive but the right-hand size is 1, contradiction. So $r_S > r_L$, and both sides are positive in equilibrium.

Let

$$B \equiv \frac{\frac{1}{A}\alpha_L\alpha_S(f - r_L)(f - r_S)G'(r_S - r_L)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \tag{26}$$

The two FOCs are separately written as

$$(f - r_L)G'(r_S - r_L) = \alpha_L + B, \tag{27}$$

$$(f - r_S)G'(r_S - r_L) = \alpha_S + B. \tag{28}$$

So both r_L and r_S are below f. Take the ratio:

$$\frac{f - r_L}{f - r_S} = \frac{\alpha_L + B}{\alpha_S + B} > 1 > \frac{\alpha_S}{\alpha_L}. \tag{29}$$

Hence, $(f - r_L)\alpha_L > (f - r_S)\alpha_S$, and $q_L^* < q_S^*$.

Proof of Proposition 2

The proof of this proposition is similar to that of Proposition 1 and omitted.

Proof of Proposition 3

First consider the unconstrained case. We have shown before that α_j is invariant to f. To calculate dr_j/df, let $\Gamma_j = d\Pi_j/dr_j$. By the second-order condition, Γ_j is decreasing in r_j. Take derivative of Γ_j with respect to f at the equilibrium values, we have

$$0 = \frac{\partial\Gamma_L}{\partial f} + \frac{\partial\Gamma_L}{\partial r_L}\frac{dr_L}{df}, \tag{30}$$

where, writing total loan volume as $V = \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$, we have

$$\begin{aligned}
\frac{\partial\Gamma_L}{\partial f} &= \left(-\alpha_L\frac{1-\alpha_L}{A} - \alpha_S\frac{1-\alpha_S}{A}\right)[(f - r_L)G'(r_S - r_L) - 1 + G(r_S - r_L)] \\
&\quad + (X + V)G'(r_S - r_L) - \frac{1}{A}\alpha_S\alpha_L G'(r_S - r_L)(2f - r_S - r_L) \\
&= -\frac{2}{A}\alpha_L\alpha_S\frac{\frac{1}{A}\alpha_S\alpha_L(f - r_L)(f - r_S)G'(r_S - r_L)}{X + V} \\
&\quad + (X + V)G'(r_S - r_L) - \frac{1}{A}\alpha_S\alpha_L G'(r_S - r_L)(2f - r_S - r_L).
\end{aligned} \tag{31}$$

By construction, $1/A$, α_L, and α_S are all less than one. So

$$\frac{\partial \Gamma_L}{\partial f} > G'(r_S - r_L)\left[-\frac{2(f-r_L)(f-r_S)}{X+V} + (X+V) - (2f - r_S - r_L)\right]. \tag{32}$$

For a sufficiently large X, the above expression is positive, so $dr_L/df > 0$. Likewise, $dr_S/df > 0$. The weighted average interest rate $\alpha_S r_S + \alpha_L r_L$ increases in f because both r_S and r_L increase in f and the market shares are invariant to f in the unconstrained equilibrium.

To calculate how r_S is affected by f in the constrained case, take the total derivative of (19) with respect to f:

$$0 = \frac{\partial \Gamma_S}{\partial f} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{df}. \tag{33}$$

The second-order condition requires $\partial \Gamma_S / \partial r_S < 0$. We have

$$\begin{aligned}
\frac{\partial \Gamma_S}{\partial f} &= \left(-\alpha_L \frac{1-\alpha_L}{A} - \alpha_S \frac{1-\alpha_S}{A}\right)[(f-r_S)G'(r_S - s) - G(r_S - s)] + (X+V)G'(r_S - s) \\
&\quad - \frac{1}{A}\alpha_L \alpha_S (2f - s - r_S) G'(r_S - s) \\
&= -\frac{2}{A}\alpha_L \alpha_S \frac{\frac{1}{A}\alpha_S \alpha_L (f-r_S)(f-s)G'(r_S-s)}{X+V} + (X+V)G'(r_S - s) \\
&\quad - \frac{1}{A}\alpha_L \alpha_S (2f - s - r_S) G'(r_S - s)
\end{aligned} \tag{34}$$

By the same argument as before, for a sufficiently large $X > 0$, we have $\partial \Gamma_S / \partial f > 0$, and hence $dr_S/df > 0$. The weighted average interest rate $\alpha_S r_S + \alpha_L s$ increases in f because $\frac{d}{df}(\alpha_S r_S + \alpha_L s) = \alpha_S \frac{dr_S}{df} + \frac{d\alpha_S}{df} r_S + \frac{d\alpha_L}{df} s = \alpha_S \frac{dr_S}{df} + (r_S - s)\frac{d\alpha_S}{df} > 0$, as both terms are positive.

Proof of Proposition 4

In the unconstrained equilibrium, because r_j increases in f, $\frac{dq_j^*}{df} = \frac{1-\alpha_j + \alpha_j \frac{dr_j}{df}}{A} > 0$. Loan volume issued by bank j is $\alpha_j(1-q_j^*)$. Because q_j^* increases in f and α_j is invariant to f, each bank's loan volume decreases in f, and so does the total loan volume.

In the constrained equilibrium, $q_L^* = \frac{1}{A}[1+f-\alpha_L(f-s)]$. So $\frac{dq_L^*}{df} = \frac{1}{A}[1-\alpha_L - \frac{d\alpha_L}{df}(f-s)] > 0$ because $f - s \geq 0$ and $\frac{d\alpha_L}{df} < 0$. Since α_L decreases in f and q_L^* increases in f, large bank's loan volume $\alpha_L(1-q_L^*)$ decreases in f.

For $q_S^* = \frac{1}{A}[1 + f - \alpha_S(f - r_S)]$, we have $\frac{dq_S^*}{df} = \frac{1}{A}[1 - \frac{d\alpha_S}{df}(f - r_S) - \alpha_S(1 - \frac{dr_S}{df})] = \frac{1}{A}[1 - G'(r_S - s)\frac{dr_S}{df}(f - r_S) - G(r_S - s)(1 - \frac{dr_S}{df})] = \frac{1}{A}[1 - G(r_S - s) + \frac{dr_S}{df}(G(r_S - s) - G'(r_S - s)(f - r_S))] = \frac{1}{A}[1 - G(r_S - s) - \frac{dr_S}{df}\frac{\alpha_L \alpha_S (f-s)(f-r_S)G'(r_S-s)/A}{X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)}]$, where the last equality uses the first-order condition of the small bank. If $G'(\cdot)$ is bounded, then one can choose a large enough X to make q_S^* increase in f as well; otherwise it is ambiguous. For the same reason, the small bank's loan volume $\alpha_S^*(1-q_S^*)$ may increase or decrease in f because α_S^* increases in f and $1 - q_S^*$ either decreases in f or is ambiguous.

Total loan volume is $\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*) = 1-\alpha_L\frac{1+f-\alpha_L(f-s)}{A}-\alpha_S\frac{1+f-\alpha_S(f-r_S)}{A} = 1-\frac{1+f}{A}+\frac{\alpha_L^2(f-s)+\alpha_S^2(f-r_S)}{A}$. Its derivative with respect to f is $-\frac{1}{A}+\frac{1}{A}[2\alpha_L\frac{d\alpha_L}{df}(f-s)+\alpha_L^2+2\alpha_S\frac{d\alpha_S}{df}(f-r_S)+\alpha_S^2(1-\frac{dr_S}{df})] = -\frac{1-\alpha_L^2-\alpha_S^2}{A}+\frac{1}{A}[-2\alpha_L\frac{d\alpha_S}{df}(f-s)+2\alpha_S\frac{d\alpha_S}{df}(f-r_S)-\alpha_S^2\frac{dr_S}{df}]$. Because $d\alpha_S/df>0$, $\alpha_L>\alpha_S$, and $f-s>f-r_S$, the term $-2\alpha_L\frac{d\alpha_S}{df}(f-s)+2\alpha_S\frac{d\alpha_S}{df}(f-r_S)$ is negative. The other two terms are also negative. Hence, total loan volume declines in f.

Proof of Proposition 5

Since the large bank is constrained by the lower bound, its deposit rate raises in step with the CBDC interest rate s. Meanwhile, the small bank adjusts its equilibrium deposit interest rate at a slower pace, continuing to balance its ability to maintain depositors while its profit margin shrinks. To see how r_S is affected by s, start with the expression

$$0 = \frac{\partial\Gamma_S}{\partial s}+\frac{\partial\Gamma_S}{\partial r_S}\frac{dr_S}{ds}. \tag{35}$$

Because $\partial\Gamma_S/\partial r_S<0$, a sufficient condition for $dr_S/ds>0$ is $\partial\Gamma_S/\partial s>0$. We have

$$\begin{aligned}\frac{\partial\Gamma_S}{\partial s} = &\left(-\alpha_L\frac{\alpha_L}{A}\right)[(f-r_S)G'(r_S-s)-G(r_S-s)]+(X+V)[-(f-r_S)G''(r_S-s)+G'(r_S-s)]\\ &+\frac{1}{A}(f-r_S)\frac{\partial}{\partial s}[\alpha_L\alpha_S(f-s)G'(r_S-s)].\end{aligned} \tag{36}$$

On any closed region of f and s, the first and third term are bounded, by G being differentiable. So if X is sufficiently large, the second term dominates. Under the assumption that $G''(\delta)<G'(\delta)/f$ for $\delta\in[0,f]$, we have $-(f-r_S)G''(r_S-s)+G'(r_S-s)>0$, so a sufficiently large X would imply that $\partial\Gamma_S/\partial s>0$, and so is dr_S/ds.

Next, we show that r_S-s decreases in s. We have

$$\begin{aligned}\frac{\partial\Gamma_s}{\partial r_S} = &X\frac{\partial}{\partial r_S}[(f-r_S)G'(r_S-s)-G(r_S-s)]\\ &+\frac{\partial\Gamma_s}{\partial r_S}\{[\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)]\cdot[(f-r_S)G'(r_S-s)-G(r_S-s)]\}\\ &-\frac{\partial\Gamma_s}{\partial r_S}\left[\frac{1}{A}\alpha_L\alpha_S(f-s)(f-r_S)G'(r_S-s)\right].\end{aligned} \tag{37}$$

The second and the third term are bounded on any closed region of r_S. The first term equals $X[(f-r_S)G''(r_S-s)-2G(r_S-s)]$. Hence,

$$\frac{dr_S}{ds} = -\frac{\partial\Gamma_s/\partial s}{\partial\Gamma_s/\partial r_S}\to\frac{(f-r_S)G''(r_S-s)-G'(r_S-s)}{(f-r_S)G''(r_S-s)-2G'(r_S-s)}, \tag{38}$$

as X becomes sufficiently large. The condition $G''(\delta)<G'(\delta)/f$ implies that both the numerator and the denominator are negative, and that the denominator is larger in magnitude. Hence, $dr_S/ds<1$ and $d(r_S-s)/ds<0$ for sufficiently large X. This implies that $\alpha_S=G(r_S-s)$ is decreasing in s.

The weighted average interest rate is $\alpha_S r_S + \alpha_L s$. Its derivative with respect to s is

$$\frac{d(\alpha_S r_S + \alpha_L s)}{ds} = \frac{d\alpha_S}{ds} r_S + \alpha_S \frac{dr_S}{ds} + \frac{d\alpha_L}{ds} s + \alpha_L = [(r_S - s)G'(r_S - s) + \alpha_S]\left(\frac{dr_S}{ds} - 1\right) + 1. \quad (39)$$

By the calculation earlier, as X becomes large, $\frac{dr_S}{ds} - 1 \to \frac{G'(r_S-s)}{(f-r_S)G''(r_S-s)-2G'(r_S-s)} > -\frac{f}{f+r_S}$, where the inequality follows from $G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f]$. So, as X becomes large,

$$\frac{d(\alpha_S r_S + \alpha_L s)}{ds} > 1 - \frac{f}{f + r_S}[(r_S - s)G'(r_S - s) + \alpha_S] \geq 1 - \frac{f}{f + r_S} > 0, \quad (40)$$

where the second last inequality follows from the large bank's FOC that, $\lim_{X\to\infty}(f - s)G'(r_S - s) + G(r_S - s) \leq 1$.

Now we turn to loan market outcomes. Since α_S decreases in s and r_S increases in it, $\alpha_S(f - r_S)$ is decreasing in s and q_S^* is increasing in s. The small bank's loan volume, $\alpha_S(1 - q_S^*)$, is also decreasing in s.

But the impact of s on q_L^* is ambiguous because α_L increases in s but $f - s$ decreases in s. Also ambiguous is the impact of s on the large bank's loan volume $\alpha_L(1 - q_L^*)$.

The total loan volume is $\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$. Its derivative with respect to s is

$$\frac{1}{A}[2\alpha_S(f - r_S) - 2\alpha_L(f - s)]G'(r_S - s)\left(\frac{dr_S}{ds} - 1\right) - \frac{1}{A}\alpha_L^2 - \frac{1}{A}\alpha_S^2\frac{dr_S}{ds}. \quad (41)$$

While the first term is positive, the last two terms are negative. It is, however, possible to show that this derivative is negative if $G''(\delta) \leq 0$ and X is sufficiently large. As X becomes large, the two first-order conditions imply that

$$\begin{aligned} \lim_{X\to\infty}(f - s)G'(r_S - s) - \underbrace{(1 - G(r_S - s))}_{\alpha_L} &\leq 0, \\ \lim_{X\to\infty}(f - r_S)G'(r_S - s) - \underbrace{G(r_S - s)}_{\alpha_S} &= 0. \end{aligned} \quad (42)$$

Multiplying α_L to the first equation and α_S to the second equation, we have

$$\begin{aligned} \lim_{X\to\infty}\alpha_L(f - s)G'(r_S - s) - \alpha_L^2 &\leq 0, \\ \lim_{X\to\infty}\alpha_S(f - r_S)G'(r_S - s) - \alpha_S^2 &= 0. \end{aligned} \quad (43)$$

Plugging this in Equation (41), we have, as X becomes large,

$$\begin{aligned} \lim_{X\to\infty}\frac{d}{ds}(\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)) &\leq \frac{1}{A}(2\alpha_L^2 - 2\alpha_S^2)\left(1 - \frac{dr_S}{ds}\right) - \frac{1}{A}\alpha_L^2 - \frac{1}{A}\alpha_S^2\frac{dr_S}{ds} \\ &= \frac{1}{A}\left[\left(1 - 2\frac{dr_S}{ds}\right)\alpha_L^2 + \left(\frac{dr_S}{ds} - 2\right)\alpha_S^2\right], \end{aligned} \quad (44)$$

where the substitution of $\alpha_L(f_s)G'(r_S - s)$ uses the fact that $\frac{dr_S}{ds} - 1 < 0$. Because $\frac{dr_S}{ds} < 1$, $(\frac{dr_S}{ds} - 2)\alpha_S^2 < 0$. If $G'' \leq 0$ and X is sufficiently large, we know from the expression of $\frac{dr_S}{ds}$ above that $\frac{dr_S}{ds} \geq \frac{1}{2}$. That means $(1 - 2\frac{dr_S}{ds})\alpha_L^2 \leq 0$ as well. So the total loan is decreasing in s in the limit. Because the limit is strictly negative, it is also negative for finite but large enough X.

Proof of Proposition 6

We proceed in two steps, first the unconstrained equilibrium and then the constrained one.

The unconstrained equilibrium

The equilibrium is characterized by two FOCs:

$$\begin{aligned}
0 = &[X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_L)G'(r_S - r_L + v_c) - 1 + G(r_S - r_L + v_c)] \\
&- \frac{1}{A}\alpha_S\alpha_L(f - r_L)(f - r_S)G'(r_S - r_L + v_c), \qquad (45) \\
0 = &[X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S - r_L + v_c) - G(r_S - r_L + v_c)] \\
&- \frac{1}{A}\alpha_L\alpha_S(f - r_L)(f - r_S)G'(r_S - r_L + v_c). \qquad (46)
\end{aligned}$$

Call the two FOCs Γ_L and Γ_S, respectively. When X is sufficiently large, we know that $f > r_L, f > r_S$, since otherwise the two FOCs must be negative. As in the case when CBDC does not carry its own convenience value, we know $r_L < r_S < f$, and $\alpha_S < \alpha_L$.

To calculate how r_L and r_S are affected by v_c, we take derivative of Γ_L and Γ_S at the equilibrium values and obtain

$$\begin{aligned}
0 &= \frac{\partial \Gamma_L}{\partial v_c} + \frac{\partial \Gamma_L}{\partial r_L}\frac{dr_L}{dv_c}, \qquad (47) \\
0 &= \frac{\partial \Gamma_S}{\partial v_c} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{dv_c}. \qquad (48)
\end{aligned}$$

The second-order condition implies that $\partial\Gamma_j/\partial r_j < 0$. When X is sufficiently large, the term $X[(f - r_L)G''(r_S - r_L + v_c) + G'(r_S - r_L + v_c)]$ dominates $\frac{\partial \Gamma_L}{\partial v_c}$. And the term $X[(f - r_S)G''(r_S - r_L + v_c) - G'(r_S - r_L + v_c)]$ dominates $\frac{\partial \Gamma_S}{\partial v_c}$. When $-G'(y)/f < G''(y) < G'(y)/f$, $(f - r_L)G''(r_S - r_L + v_c) + G'(r_S - r_L + v_c)$ is positive, so $\frac{dr_L}{dv_c} > 0$. Also, $(f - r_S)G''(r_S - r_L + v_c) - G'(r_S - r_L + v_c)$ is negative, so $\frac{dr_S}{dv_c} < 0$. So r_L is increasing in v_c, and r_S is decreasing in v_c.

For deposit market share $\alpha_S = G(r_S - r_L + v_c)$, we take the difference of the two FOCs, and have

$$(r_S - r_L)G'(r_S - r_L + v_c) + 2G(r_S - r_L + v_c) = 1. \qquad (49)$$

Write $y = r_S - r_L + v_c$, and take derivative of the above equation with respect to v_c, then we have

$$[3G'(y) + (r_S - r_L)G''(y)]\frac{dy}{dv_c} - G'(y) = 0 \tag{50}$$

When $-G'(y)/f < G''(y) < G'(y)/f$, we know that $3G'(y) + (r_S - r_L)G''(y) > 0$, hence $\frac{dy}{dv_c} > 0$. So α_S is increasing in v_c, and α_L is decreasing in v_c.

The weighted average deposit interest rate is $\alpha_S r_S + \alpha_L r_L = \alpha_S(r_S - r_L) + r_L$. Its derivative with respect to v_c is

$$\begin{aligned}\frac{d(\alpha_S r_S + \alpha_L r_L)}{dv_c} &= \frac{d\alpha_S}{dv_c}(r_S - r_L) + \alpha_S\frac{d(r_S - r_L)}{dv_c} + \frac{dr_L}{dv_c} \qquad (51)\\ &> \frac{d\alpha_S}{dv_c}(r_S - r_L) + \frac{1}{2}\frac{d(r_S - r_L)}{dv_c} + \frac{dr_L}{dv_c} = \underbrace{\frac{d\alpha_S}{dv_c}}_{>0}(r_S - r_L) + \frac{1}{2}\frac{d(r_L + r_S)}{dv_c},\end{aligned}$$

where the inequality follows from $\alpha_S < \frac{1}{2}$ and $r_S - r_L$ decreasing in v_c. As X becomes large,

$$\begin{aligned}\frac{d(r_S + r_L)}{dv_c} &\to \frac{(f - r_L)G''(y) + G'(y)}{(f - r_L)G''(y) + 2G'(y)} - \frac{(f - r_S)G''(y) - G'(y)}{(f - r_S)G''(y) - 2G'(y)}\\ &= \frac{(r_L + r_S - 2f)G''(y)G'(y)}{[(f - r_L)G''(y) + 2G'(y)][(f - r_S)G''(y) - 2G'(y)]}. \qquad (52)\end{aligned}$$

The denominator is negative as $G''(y) < G'(y)/f$, and $r_L + r_S - 2f < 0$. So if $G''(y) \geq 0$, we have $\frac{d(r_L+r_S)}{dv_c} \geq 0$, and hence $\alpha_S r_S + \alpha_L r_L$ increases in v_c.

For loan quality thresholds, since α_L is decreasing in v_c and r_L is increasing in v_c, q_L^* is increasing in v_c. Since α_S is increasing in v_c and r_S is decreasing in v_c, q_S^* decreasing in v_c.

For loan volumes, $\alpha_L(1-q_L^*)$ is decreasing in v_c, since α_L is decreasing and q_L^* is increasing. Similarly, $\alpha_S(1 - q_S^*)$ is increasing in v_c.

Total loan volume equals $\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) = 1 - \frac{1+f}{A} + \frac{\alpha_L^2(f-r_L)+\alpha_S^2(f-r_S)}{A}$. Its derivative with respect to v_c is

$$\frac{1}{A}\left\{[2\alpha_S(f - r_S) - 2\alpha_L(f - r_L)]\frac{d\alpha_S}{dv_c} - \alpha_L^2\frac{dr_L}{dv_c} - \alpha_S^2\frac{dr_S}{dv_c}\right\}. \tag{53}$$

We know that $2\alpha_S(f - r_S) - 2\alpha_L(f - r_L) < 0$, $\frac{d\alpha_S}{dv_c} > 0$, $\frac{dr_L}{dv_c} > 0$, and $\frac{dr_S}{dv_c} < 0$, so the equation is ambiguous. We know $-\alpha_L^2\frac{dr_L}{dv_c} - \alpha_S^2\frac{dr_S}{dv_c} < -\alpha_S^2\frac{d(r_L+r_S)}{dv_c}$. When X becomes large, $\frac{d(r_L+r_S)}{dv_c} \to \frac{(f-r_L)G''(y)+G'(y)}{(f-r_S)G''(y)+2G'(y)} - \frac{(f-r_S)G''(y)-G'(y)}{(f-r_S)G''(y)-2G'(y)} = \frac{(r_L+r_S-2f)G''(y)G'(y)}{[(f-r_S)G''(y)+2G'(y)][(f-r_S)G''(y)-2G'(y)]}$. We know the denominator is negative, $(r_L + r_S - 2f) < 0$, so if $G''(y) \geq 0$, we have $-\alpha_L^2\frac{dr_L}{dv_c} - \alpha_S^2\frac{dr_S}{dv_c} \leq 0$, and so Equation (53) is negative. If $G''(y) < 0$, however, the sign of the equation is ambiguous.

The constrained equilibrium

The constrained equilibrium is characterized by two FOCs:

$$
\begin{aligned}
0 >& [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f-s)G'(r_S - s + v_c) - 1 + G(r_S - s + v_c)] \\
&- \frac{1}{A}\alpha_S\alpha_L(f-s)(f-r_S)G'(r_S - s + v_c), \qquad (54)\\
0 =& [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f-r_S)G'(r_S - s + v_c) - G(r_S - s + v_c)] \\
&- \frac{1}{A}\alpha_L\alpha_S(f-s)(f-r_S)G'(r_S - s + v_c). \qquad (55)
\end{aligned}
$$

To calculate how r_S is affected by v_c, we take derivative of Γ_S at the equilibrium values and obtain

$$0 = \frac{\partial \Gamma_S}{\partial v_c} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{dv_c}. \qquad (56)$$

When X is sufficiently large, the term $X[(f-s)G''(r_S - s + v_c) - G'(r_S - s + v_c)]$ dominates $\frac{\partial \Gamma_S}{\partial v_c}$. Since $G''(\delta) < G'(\delta)/f$, we know that $\frac{\partial \Gamma_S}{\partial v_c} < 0$. Hence $\frac{dr_S}{dv_c} < 0$, i.e., r_S is decreasing in v_c.

For deposit market share $\alpha_S = G(r_S - s + v_c)$, when X becomes sufficiently large, we have

$$\frac{dr_S}{dv_c} = -\frac{\partial \Gamma_S}{\partial v_c} / \frac{\partial \Gamma_S}{\partial r_S} \to -\frac{(f-r_S)G''(r_S - s + v_c) - G'(r_S - s + v_c)}{(f-r_S)G''(r_S - s + v_c) - 2G'(r_S - s + v_c)}. \qquad (57)$$

We know that

$$\frac{d(r_S - s + v_c)}{dv_c} = \frac{dr_S}{dv_c} + 1 = \frac{-G'(r_S - s + v_c)}{(f-r_S)G''(r_S - s + v_c) - 2G'(r_s - s + v_c)}, \qquad (58)$$

where the numerator and the denominator are both negative. So $\frac{d(r_S - s + v_c)}{dv_c} > 0$, and α_S is increasing in v_c.

For loan quality thresholds, since α_L is decreasing in v_c and r_L is increasing in v_c, q_L^* is increasing in v_c. Since α_S is increasing in v_c and r_S is decreasing in v_c, q_S^* decreasing in v_c.

For loan volumes, $\alpha_L(1-q_L^*)$ is decreasing in v_c, since α_L is decreasing and q_L^* is increasing. Similarly, $\alpha_S(1-q_S^*)$ is increasing in v_c.

Total loan volume equals $\alpha_L(1-q_L^*) + \alpha_S(1-q_S^*) = 1 - \frac{1+f}{A} + \frac{\alpha_L^2(f-s) + \alpha_S^2(f-r_S)}{A}$. Its derivative with respect to v_c is

$$\frac{1}{A}\left\{[2\alpha_S(f-r_S) - 2\alpha_L(f-s)]\frac{d\alpha_S}{dv_c} - \alpha_S^2\frac{dr_S}{dv_c}\right\}. \qquad (59)$$

Its sign is ambiguous for a similar same reason as the unconstrained case.