Payment Innovation and Interbank Competition *

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Abstract

This paper presents a theoretical analysis of how innovation in the form of public payment infrastructures, accessible to all banks with equal efficacy, affects competition in the banking sector. We develop a model of two-bank competition in which one bank's deposit offers greater payment convenience than its rival's. In equilibrium, the bank with the convenience advantage captures a larger share of both the deposit market and—via a novel channel involving interest on central bank reserves—the lending market. We show that innovations enabling a uniform level of payment convenience for all consumers help level the playing field between large and small banks. In contrast, paying interest on balances of alternative media of exchange tends to reinforce the advantages of larger banks. Our theoretical findings are supported by recent empirical evidence from payment innovations in Brazil, China, and the United States.

Keywords: payment, bank competition, convenience value, interest on reserves, deposit interest rates, bank lending
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This paper presents a theoretical analysis of how innovation in the form of public payment infrastructures, accessible to all banks with equal efficacy, affects competition in the banking sector. We develop a model of two-bank competition in which one bank's deposit offers greater payment convenience than its rival's. In equilibrium, the bank with the convenience advantage captures a larger share of both the deposit market and—via a novel channel involving interest on central bank reserves—the lending market. We show that innovations enabling a uniform level of payment convenience for all consumers help level the playing field between large and small banks. In contrast, paying interest on balances of alternative media of exchange tends to reinforce the advantages of larger banks. Our theoretical findings are supported by recent empirical evidence from payment innovations in Brazil, China, and the United States.

1 Introduction

Over the last two decades, technological innovations in the payment space have reshaped the competitive landscape of the banking industry. In particular, innovative payment systems in Brazil (Pix), China (Alipay, WeChat Pay, e-CNY), India (UPI), Kenya (M-Pesa), Sweden (Swish), and the United States (PayPal, Venmo, Zelle, Apple Pay) have all increased the convenience, speed, and security of making and receiving payments, in some cases leading to significant improvements in financial inclusion. More recent innovations like stablecoins and tokenized deposits also have the potential to make their own footprint in this large market. According to McKinsey, global payment revenue in 2024 was 2.5 trillion, based on $2.0 quadrillion in value flows and 3.6 trillion transactions worldwide.[1]

The objective of this paper is to theoretically analyze how payment innovations affect interbank competition. We make two main contributions. First, we propose a simple but novel model of interbank competition with ample reserves and identify a deposit-lending linkage via interest on reserves and retained deposits. Second, applying this model framework, we find that payment innovations that provide convenience value to consumers level the playing field between large and small banks, but payment innovations that pay interest on user balances have the opposite effect, magnifying the competitive advantage of large banks.

When banks lend, they create deposits (see McLeay, Radia, and Thomas (2014), Martin, McAndrews, and Skeie (2016), and Jakab and Kumhof (2019)). Historically, banks have been constrained by a reserve requirement that limits their ability to create new deposits. However, since the great financial crisis, the U.S. financial system has been characterized by a large amount of excess central bank reserves, over $3 trillion as of this writing. Moreover, the current reserve requirement is zero. In this "ample reserves" regime, a bank's lending decision is generally not constrained by the amount of reserves it holds. However, reserves still factor into the lending decision, since they are remunerated.

The Federal Reserve implements monetary policy primarily by paying interest on reserves balances (IORB).[2] When the borrower purchases goods and services from a supplier using the newly created deposits, reserves used to settle the deposit transaction flow from the lending bank to the supplier's bank. If the borrower and the supplier use the same bank, reserves are retained by the lending bank, which continues to receive interest on reserves from the central bank. Otherwise, the reserves associated with the newly created deposits leave the lending bank, causing that bank

[1] See https://www.mckinsey.com/industries/financial-services/our-insights/global-payments-report.

[2] In the United States, IORB has been paid since October 2008. Since the financial crisis of 2008-09, interest on excess reserves (IOER) has become the Federal Reserve's main policy tool to adjust interest rates. In March 2020, the Federal Reserve set required reserve amount to zero. In July 2021, the Federal Reserve renamed IOER to interest on reserve balances (IORB), as all reserves were "excess" reserves by that point.

to lose the associated interest on reserves.

This dynamic—deposit recirculation in the economy and possible retention by the same bank—is important because it determines the opportunity cost of lending. Crucially, banks with large numbers of depositors have a lower opportunity cost than banks with small numbers of depositors, since the former has a higher probability of retained deposits. Accounting for differences in the opportunity cost of lending for banks of different sizes enables us to explain qualitative effects of payment innovation on the competitive landscape of the banking industry.

Formally, our model features two banks, one large and one small. The large bank's deposits are assumed to provide a higher payment convenience value than the small bank's. The payment convenience could be a proxy for extensive branch networks, high quality mobile apps, or a broad set of services. The banks exercise market power in setting deposit interest rates and making lending decisions. In equilibrium, the more convenient bank becomes large, in the sense that it captures a higher deposit market share. With a higher deposit market share, the large bank is more likely than it's rival to retain deposits that it creates in the lending process and continues to receive interest on reserves from the central bank. Thus, the large bank's advantage in the deposit market spills over to the lending market, in the sense that certain unprofitable loans for the small bank are profitable for the large bank. In equilibrium, we show that the large bank has a lower deposit interest rate and a higher deposit market share, which are in line with empirical findings from d'Avernas, Eisfeldt, Huang, Stanton, and Wallace (2023). We also show that the large bank has a lower lending standard, compared to the small bank.

Based on this model, we explore the impact of introducing payment innovations on the deposit market, the lending market, and monetary policy pass-through. The payment innovation in our model is characterized by two features: the convenience value it provides and the interest it pays to users. The interest-bearing aspect is relevant in cases where the payment innovation is in the form of new payment media in "closed-loop" systems, such as Alipay, or payment tokens, including CBDCs and stablecoins. Both benefits are identical across users. Moreover, consistent with practical implementations in Brazil, China, India, and the United States, the payment innovation in our model is linked seamlessly to bank deposits. This means that the interest rate paid on the payment token becomes the lower bound on bank deposit interest rates, and the convenience value of the new payment system narrows the convenience gap between deposits at the large bank and the small bank. In other words, in equilibrium, the payment innovation in our model changes the economic value of bank deposits but does not "replace" deposits.

We evaluate the impact of payment innovation by varying these two features one at a time. Increasing the interest rate paid on the payment tokens, while holding its convenience value fixed, raises the deposit interest rates of both banks, bringing their weighted average closer to the interest on reserves. However, by forcing both banks to raise interest rates, this type of payment innovation impedes the small bank's ability to compete with the large bank with a higher deposit interest rate. Thus, a higher interest rate offered by the payment token reduces the market share of the small bank in deposit and lending markets, exacerbating the large bank-small bank gap.

On the other hand, a payment innovation that provides a higher convenience value narrows the convenience gap between the two banks and reduces the market power of the large bank. For example, by linking to an instant payment system, a small community bank partially "catches up" with large global banks in offering payment functionalities. Thus, the small bank gains deposit market share and, because it no longer has to compete solely on interest rate, sets a lower deposit interest rate. Again, by the deposit-lending linkage, the small bank also catches up in the lending market.

Recent empirical evidence supports our theoretical predictions. For example, Sarkisyan (2023) finds that after Pix was introduced in Brazil, small banks gained deposit market share while reducing their deposit interest rates; small banks also increased lending, albeit to a limited extent. Li (2024) finds that after China introduced e-CNY, a CBDC, large banks' deposit market share decreased, deposit interest rate increased, and loan-to-asset ratio decreased.

A key contribution of our analysis to the banking literature is the deposit-lending linkage, arising from the combination of interest on reserves and deposit retention. We modeled the effect of deposits leaving a bank and circulating back to the same bank in an earlier version of this paper (Garratt and Zhu, 2021). Cooperman, Duffie, Luck, Wang, and Yang (2025) also incorporates this feature in a model of bank lending to study how the choice of loan reference rate affects the supply of credit lines.[3] Our model differs in that we characterize bank competition in the deposit market and show that a bank's deposit market share is linked to a bank's lending decision. In Bianchi and Bigio (2022), deposits also circulate across banks, but the reason that deposit flows affect lending in their model is that large deposit withdrawals may force a bank to borrow reserves at a cost. Li, Li, and Sun (2024) emphasize the importance of bank network structure in studying the funding risks associated with reserve flows. Complementary to these studies, lending is not constrained by the level of reserves in our model; instead, it is the combination of interest on reserves and deposit retention that affects banks' lending standards and loan volume.

Models that study bank market power generally do not feature interest on reserves and deposit retention. For instance, Drechsler, Savov, and Schnabl (2017) study how banks' deposit market power affects the transmission of monetary policy to bank lending. Wang, Whited, Wu, and Xiao (2022) quantify how banks' market power in deposit and lending markets affects monetary policy pass-through and bank lending. Unlike us, in these models, the level of deposits affects bank

[3] In their model, corporate credit lines are drawn more heavily when banks' funding markets are stressed. The resulting adverse impact on credit supply ex ante is weaker if credit line drawdowns are expected to be deposited at the same bank.

lending because financing a loan using external funding is costlier than financing it using deposits. d'Avernas, Eisfeldt, Huang, Stanton, and Wallace (2023) emphasize that banks do not compete solely on interest rates but also on business models. For example, large banks offer more "liquidity services," which could explain why large banks offer lower deposit interest rates empirically. We explicitly model heterogeneous convenience values across banks and show their impact on banks' deposit market shares and lending decisions.

This paper also contributes to the burgeoning theoretical literature on payment innovations. For example, Keister and Sanches (2021), Dong and Xiao (2021), Andolfatto (2021), Jiang and Zhu (2021), Agur, Ari, and Dell'Ariccia (2022), and Chiu, Davoodalhosseini, Jiang, and Zhu (2023) study how CBDC interest rate and other design features affect the deposit and lending of commercial banks. Our model differs from this group by focusing on heterogeneous banks and by showing that payment convenience and interest rate have opposite effects on bank competition.

The literature has also pointed out unintended consequences of faster payments on lending. Parlour, Rajan, and Walden (2022) show that more efficient settlements of funds can increase banks' liquidity demand and reduce lending. Li and Li (2024) find a negative empirical association between bank lending and the volatility of payment-driven deposit flows. Ding, Gonzalez, Ma, and Zeng (2025) find that the introduction of Pix increases banks' holdings of liquid assets and risk-taking in lending. Our model is complementary to theirs in that we focus not on liquidity risk but heterogeneity among banks, in particular, small banks benefit from payment innovation not only in payment convenience but also lending.

2 A Model of Bank Competition with Ample Reserves

The economy has a large bank (L) and a small bank (S). The assumption of two banks is, of course, a simplification, but it may not be too far away from the experience of a typical depositor.[4] There are $X = X_L + X_S$ reserves in the banking system, where X_S denotes the reserve holding of the small bank and X_L denotes the reserve holding of the large bank.

For simplicity, the banks start off holding reserves as their only asset, balanced by exactly the same amount of deposits. This simplifying assumption appears restrictive, but as we explain shortly, the main actions in the model are bank lending and associated deposit creation and deposit flows in the banking system via reserves. Thus, the mechanism and results of this paper are robust to alter-

[4] Using survey data, Honka, Hortaçsu, and Vitorino (2017) find that U.S. consumers were, on average, aware of only 6.8 banks and considered 2.5 banks when shopping for a new bank account. More than 80% considered fewer than 3 banks when shopping for a new bank account. Drechsler, Savov, and Schnabl (2017) find that the average HHI of bank branches among U.S. counties is 0.35, with a standard deviation of 0.2; a county with three equally-sized bank branches would have an HHI of 0.33, and a county with two equally-sized bank branches would have an HHI of 0.5.

native specifications of banks' balance sheets, as long as the banks hold sufficiently large amount of reserves.

The total amount of reserves X is exogenously determined by the central bank and is assumed to be large. This premise is consistent with the "ample reserve" regime of the Federal Reserve (see also Martin, McAndrews, and Skeie (2016)). The central bank pays the two commercial banks an exogenously determined interest rate f on their reserve holdings, known as interest on reserve balances (IORB). The large and small banks pay depositors endogenously determined deposit rates r_L and r_S, respectively. Thus, if nothing else happens, bank j's total profit would be $X_j(f - r_j)$.

Interest on reserves is significant for banks. To illustrate, we calculate interest earned on reserve balances for U.S. banks from 2021 to 2024. We follow Afonso, Cipriani, Martinez, and Plosser (2025) and use the call report item "Balances due from Federal Reserve Banks" (RCON/RCFD 0090) to measure the amount of reserves that a bank places at the Fed. Note that only banks with total assets of $300 million or more report this item. Interest on reserves revenue is calculated as the average reserve amount within the year multiplied by the average IORB rate within the year.

The average IORB rate, set by the Federal Reserve, was 0.15%, 1.76%, 5.10%, and 5.21% in 2021, 2022, 2023, and 2024, respectively. We aggregate the interest on reserves revenue and income of all banks. Panel A of Table 1 shows that interest on reserves revenue was a significant source of income for the banking sector during the latest hiking cycle. In 2024, for example, total interest on reserves across 872 banks in the sample was $94 billon, and their total net income was $251 billon. That is, interest on reserves accounted for about 38% of banks' net income in the aggregate in 2024. The corresponding fraction for 2023 was about 40%.

Panel B of Table 1 shows the distribution of IOR revenue for U.S. banks between 2021 and 2024. Averaged across all bank-years (equally weighted), interest on reserves revenue accounts for 15% of banks' net income, with the 10th percentile at 0 and 90th percentile at 42%. The concentration of IOR revenue at the top is salient.

Commercial bank deposits are valuable not just for the interest they pay, but also for the services they provide; we refer to the latter as *convenience value*. The convenience value of deposits in the small bank is normalized to be zero. The convenience value of deposits in the large bank is a random variable $\delta \geq 0$ that has a twice differentiable, cumulative distribution function G. A salient example of convenience value is the availability and quality of banks' mobile apps, as documented by Haendler (2022) and Koont (2023).

There is a unit mass of infinitesimal agents. Each agent potentially plays three roles: entrepreneur (borrower), worker, and depositor. These agents draw i.i.d. convenience values for large bank deposits from the distribution G. We make the following assumption about G throughout the paper:

Assumption 1. *The function G satisfies $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f]$.*

Table 1: Significance of interest on reserves (IOR) for commercial banks

Panel A. Bank IOR revenue and incomes in the aggregate

Year	IORB rate	IOR revenue ($bn)	Net income ($bn)	Total interest income ($bn)	Net interest income ($bn)	IOR revenue /Net income	No. of banks
2021	0.15%	3.62	260.41	511.07	479.67	1.4%	907
2022	1.76%	31.36	244.31	692.53	581.47	12.8%	908
2023	5.10%	93.86	236.87	1071.79	644.53	39.6%	891
2024	5.21%	94.52	251.41	1177.48	643.95	37.6%	872

Panel B. Distribution of IOR revenue

	Count	Mean	S.D.	p10	p25	p50	p75	p90
IOR revenue/Net interest income	3578	0.06	0.11	0.00	0.00	0.02	0.07	0.14
IOR revenue/Total interest income	3578	0.04	0.05	0.00	0.00	0.02	0.04	0.09
IOR revenue/Net income	3578	0.15	0.32	0.00	0.01	0.05	0.18	0.42

Note: This table reports summary statistics of U.S. banks' interest on reserves (IOR) revenue using data in call reports, from 2021 to 2024. In Panel A, we aggregate the IOR revenue and income of all banks. Only banks that report reserves data are included. IOR revenue is calculated as the reserve amount multiplied by the IORB rate. Total interest income (call report item RIAD4107) includes the interest that a bank receives from loans, reserves, bonds, and mortgage backed securities, etc. Net interest income (call report item RIAD4074) is equal to the total interest income minus the interest on deposits and interest expenses on other liabilities. Net income (call report item RIAD4340) considers all income and expense items including noninterest income and expense. In Panel B, we report the distribution of IOR revenue. Each bank-year is an observation.

The condition has the effect of bounding the masses of agents who receive either a very high or a very low convenience value for holding the large bank's deposits, which, in turn, ensures that both banks compete to win depositors. This assumption also implies that $G'(\delta) > 0$ for any $\delta \in [0, f]$.

The model has four periods. At $t = 0$, the two commercial banks set the deposit interest rates r_L and r_S. The central bank sets the interest on reserves f. At the start of the model, a fraction m_L of agents have existing deposits only at the large bank and a fraction $m_S = 1 - m_L$ of agents have existing deposits only at the small bank. The amount of deposits per capita across agents is identical. This means $m_L = X_L/X$ and $m_S = X_S/X$.

Following Drechsler, Savov, and Schnabl (2017), we think of cash as consisting of paper currency and zero-interest checking accounts. We assume that the cost of switching between cash and interest-bearing deposit is negligible. By holding cash, especially in the form of zero-interest checking account balances, agents can still access all the payment services provided by the bank.

If the deposit interest rate of a bank were negative, agents would hold cash. Thus, banks face a zero lower bound for deposit interest rates, i.e., $r_S \geq 0, r_L \geq 0$. (To the extent that banks must recover positive fixed costs, the true lower bound on deposit interest rates may be slightly negative, but adding that feature would not change the main insight of the paper.)

At $t = 1$, the agents act as entrepreneurs and workers. Specifically, each agent is endowed with a project, which requires \$1 of investment and pays $A > 1$ with probability q_i and zero with probability of $1 - q_i$, where q_i has the distribution function Q. The gross return A is a commonly known constant and Q is commonly known. The expected payoff per dollar invested is thus $q_i A$. Each agent can only borrow from the bank where she keeps her deposit (the "relationship" bank), and as a result, each bank prices the loan as a monopolist.[5] If the loan is granted by the bank, the entrepreneur pays \$1 to a randomly selected agent from the same population. The selected agent plays the role as a worker and completes the project. (The same infinitesimal agent could be an entrepreneur for her own project and a worker for someone else.) The main point of introducing workers beyond entrepreneurs is to model deposit flows in the economy. The entrepreneur-worker pair could also be modeled as a firm-worker pair, a firm-supplier pair, or a household-firm pair, as long as money lent to the first agent by a bank flows to the second agent.

At $t = 2$, agents play the role of depositors. Workers who receive wages choose where to deposit the wage. The depositor chooses either the large bank or the small bank, after considering her own convenience value for the large bank's deposits and the deposit interest rates of both banks.

At $t = 3$, the projects succeed or fail. The banks earn interest on reserves and pay depositors according to their deposit holdings and the deposit interest rates.

2.1 Bank lending and deposit creation

Without loss of generality, we model a bank's lending and deposit creation process for a single loan. The condition on bank lending that we derive applies to the continuum model in which borrowers (i.e., entrepreneurs) are infinitesimal.

The tables below show the sequence of changes in the large bank's balance sheet in the loan process. The changes in the small bank's balance sheet in the loan process are entirely analogous.

1. Before lending, the large bank starts with X_L reserves. Its balance sheet looks like:

Asset	Liability
Reserves X_L	Deposits X_L

[5] While this assumption is restrictive, empirical evidence shows that payment data are informative about credit quality. Thus, the bank that hold's a consumer's deposits has an information advantage in the lending market, even if it is not literally a monopolist. See Parlour, Rajan, and Zhu (2022) and discussions of related literature.

2. If the large bank makes a loan of \$1, it immediately creates a new deposit of \$1 in the name of the entrepreneur. The balance sheet of the bank becomes:

Asset	Liability
Reserves X_L	Deposits X_L
Loans 1	New Deposits 1

3. Eventually, the entrepreneur will spend her money to pay a worker, and the worker deposits the money in the bank she prefers. The large bank anticipates that, in expectation, an endogenous fraction α_S of the \$1 new deposit will eventually be transferred to the small bank, leading to a reduction of the large bank's reserves by the same amount. An endogenous fraction $\alpha_L \equiv 1 - \alpha_S$ of the new deposit remains in the large bank. The large bank's expected balance sheet becomes:

Asset	Liability
Reserves $X_L - \alpha_S$	Deposits X_L
Loans 1	New Deposits α_L

It is worth highlighting that, in our model, banks create deposits as a result of lending. Literally reflecting how banks operate in practice, this process has also become more important after the Federal Reserve implemented its "ample reserve" system in the wake of the 2008-09 financial crisis. In the "ample reserve" regime, the Fed guides interbank interest rates by paying interest on reserves f, rather than by adjusting the quantity of reserves. Because reserves are abundant, over \$3 trillion as of this writing,[6] they are large compared to new loans made by banks. Thus, if newly created deposits leave a bank, the corresponding outflow of reserves is small compared to the bank's outstanding amount of reserves. In other words, bank lending in the ample reserve regime is no longer constrained by the amount of reserves. Indeed, in 2020, the Federal Reserve removed the reserve requirements altogether, cementing the separation of lending and reserves. This reality is reflected in our model, as the amount of reserves X is assumed to be large compared to the size of newly created loans, normalized to \$1. See McLeay, Radia, and Thomas (2014), Martin, McAndrews, and Skeie (2016), and Jakab and Kumhof (2019) for discussions of this lending-creates-deposit process. In practice, lending is also constrained by capital requirement, which we do not model here.

We now turn to the banks' lending decisions. If the large bank makes the \$1 loan to entrepreneur i, and charges the interest rate R_i, its total expected profit, by counting all items in the balance sheet, will be

$$\underbrace{(X_L - \alpha_S)f}_{\text{Interest on reserves}} + \underbrace{[q_i(1+R_i) - 1]}_{\text{Gross profit on the loan}} - \underbrace{(X_L + \alpha_L)r_L}_{\text{Cost of deposits}}. \tag{1}$$

[6]See https://fred.stlouisfed.org/series/TOTRESNS.

If the large bank does not make the loan, then its total profit will be

$$X_L(f - r_L). \tag{2}$$

Thus, the large bank's marginal profit from making the loan, compared to not making it, is

$$\pi_i = \underbrace{q_i(1 + R_i) - (1 + f)}_{\text{Net profit on the loan}} + \underbrace{\alpha_L(f - r_L)}_{\text{Profit on deposit}}. \tag{3}$$

In the expression of π_i, the net profit made from the loan reflects the true opportunity cost of capital f. The large bank makes an additional profit equal to $\alpha_L(f - r_L)$. This is because each $1 lent out stays with the large bank with probability α_L and earns the IORB-deposit spread of $f - r_L$. The corresponding term for the small bank's marginal profit of lending is $\alpha_S(f - r_S)$. In the equilibrium we characterize, it will be the case that $\alpha_L(f - r_L) > \alpha_S(f - r_S)$, i.e., the large bank's convenience value of deposits translates into an advantage in the lending market.

How relevant is the profit on the retained deposit in the lending process? Our model has only two banks for simplicity, but this idea is generalizable to a market with an arbitrary number of banks. For example, during 2010-2020, based on Call Reports data, the four largest banks captured 35% of the U.S. deposit market. Their deposit market shares were large and stable during this period, with Bank of America capturing 11% market share, JPMorgan Chase 10%, Wells Fargo 10%, and Citi 4%. If we treat the U.S. as a national market for deposits and loans, these numbers suggest that deposits created by three of the four largest banks in their lending process would stay with the same bank with probability around 10%. For a deposit spread of 3% ($f - r$), this translates into a 30 bps increase in loan profit per $1 lent.

This simple back-of-the-envelope calculation could underestimate the importance of the deposit-lending linkage via interest on reserves and deposit retention. First, the local banking markets are more concentrated. Second, for loans of certain types or large sizes, there may only be a small number of banks that have the expertise or balance sheet to compete. Third, banks have strong incentives to "bank for the supply chain" to maximize the benefit of this deposit-lending linkage. For instance, the bank that holds the account of a business would wish to offer deposit accounts to its employees and suppliers. In the last situation, the redistribution of deposits in the banking system would be directed, not random.

Cooperman, Duffie, Luck, Wang, and Yang (2025) find that when COVID struck, for every dollar withdrawn from banks, an average of 94 cents was placed into low-interest-rate corporate deposit accounts at the same set of banks. If we interpret the set of banks in their sample as analogous to our "large bank," their study implies that $\alpha_L \approx 0.94$.

2.2 Equilibrium

We solve the model backward in time.

Deposit market at $t = 2$. A depositor with a large-bank convenience value of δ faces two choices:

	Large bank Deposit	Small bank Deposit
Convenience value	δ	0
Interest rate	r_L	r_S

Obviously, the small bank attracts no depositors if $r_S \leq r_L$ since $\delta \geq 0$ and has a continuous distribution. So $r_S > r_L$ in equilibrium. This implies that a depositor with convenience value δ chooses the large bank if and only if

$$\delta > r_S - r_L. \tag{4}$$

Therefore, the eventual market shares of the banks in the newly created deposits are

$$\alpha_L = 1 - G(r_S - r_L), \tag{5}$$

$$\alpha_S = G(r_S - r_L). \tag{6}$$

Loan market at $t = 1$. Previously, we derived the marginal profit of a bank from making a loan, and the profit of the large bank is given by Equation (3).

The monopolist position of each bank in the lending market implies that a bank can make a take-it-or-leave-it offer to the entrepreneur. The bank's optimal interest rate quote would be $R_i = A - 1$ (or just a tiny amount below), and the entrepreneur, who has no alternative source of funds, would accept. The bank takes the full surplus from the loan.

Hence, plugging $R_i = A - 1$ into Equation (3), the large bank makes the loan if and only if

$$q_i A - (1 + f) + \alpha_L(f - r_L) > 0, \tag{7}$$

or

$$q_i > q_L^* = \frac{1 + f - \alpha_L(f - r_L)}{A}. \tag{8}$$

Analogous calculation for the small bank yields the comparable investment threshold

$$q_S^* = \frac{1 + f - \alpha_S(f - r_S)}{A}. \tag{9}$$

Choice of deposit rates at $t = 0$. Again, we start with the large bank, which makes profits in two ways. First, its total profit from lending is $m_L \int_{q_L^*} (qA - 1 - f) dQ(q)$. Second, the large bank also makes profit from the deposit interest rate spread $f - r_L$. The existing deposit in the banking system is $X = X_L + X_S$. As discussed above, the lending process also creates new deposits. The amount of new deposit created by the large bank is $m_L(1 - Q(q_L^*))$; recall that each loan is normalized to be \$1. Likewise, the small bank creates new deposit $m_S(1 - Q(q_S^*))$. When the two banks compete for depositors by setting deposit interest rates r_L and r_S, we already show above that a fraction $\alpha_L = 1 - G(r_S - r_L)$ of total deposits end up with the large bank, and the large bank collects a spread of $f - r_L$ per unit of deposit held. Here, total deposits include existing deposit X in the banking system and newly created deposits $m_L(1 - Q(q_L^*)) + m_S(1 - Q(q_S^*))$.

Adding up the two sources of profit, we can write the large bank's total profit as

$$
\begin{aligned}
\Pi_L &= m_L \int_{q_L^*}^{1} (qA - 1 - f) dQ(q) + [X_L + X_S + m_L(1 - Q(q_L^*)) + m_S(1 - Q(q_S^*))]\alpha_L(f - r_L) \\
&= m_L \int_{q_L^*}^{1} [qA - (1 + f) + \alpha_L(f - r_L)] dQ(q) + [X_L + X_S + m_S(1 - Q(q_S^*))]\alpha_L(f - r_L).
\end{aligned}
\tag{10}
$$

Likewise, the small bank's total profit is

$$
\Pi_S = m_S \int_{q_S^*}^{1} [qA - (1 + f) + \alpha_S(f - r_S)] dQ(q) + [X_L + X_S + m_L(1 - Q(q_L^*))]\alpha_S(f - r_S).
\tag{11}
$$

As discussed before, banks face a zero lower bound for deposit rates, i.e., $r_L \geq 0$, and $r_S \geq 0$. There are two cases. The first is that $r_L > 0$, so that the zero lower bound does not constrain the banks' deposit rates. We call the first case the *unconstrained equilibrium.* The second case is that $r_L = 0$, i.e., the zero lower bound binds the large bank's deposit rate. We call the second case the *constrained equilibrium.*

Unconstrained equilibrium. Assuming that Π_L is strictly quasi-concave in r_L, the sufficient condition for a unique maximum of the function Π_L with respect to r_L is

$$
\begin{aligned}
\frac{d\Pi_L}{dr_L} &= m_L(1-Q(q_L^*))\frac{d[\alpha_L(f-r_L)]}{dr_L} - m_L \underbrace{[q_L^* A - (1+f) + \alpha_L(f-r_L)]}_{=0} \frac{dq_L^*}{dr_L} \\
&\quad + [X_L + X_S + m_S(1-Q(q_S^*))]\frac{d[\alpha_L(f-r_L)]}{dr_L} - m_S\alpha_L(f-r_L)Q'(q_S^*)\frac{dq_S^*}{dr_L} \\
&= [X_L + X_S + m_L(1-Q(q_L^*)) + m_S(1-Q(q_S^*))] \cdot [(f-r_L)G'(r_S-r_L) - 1 + G(r_S-r_L)] \\
&\quad - m_S\alpha_L(f-r_L)Q'(q_S^*)\frac{(f-r_S)G'(r_S-r_L)}{A} = 0. \qquad (12)
\end{aligned}
$$

Likewise, the first-order condition of the small bank is

$$
\begin{aligned}
\frac{d\Pi_S}{dr_S} &= [X_L + X_S + m_L(1-Q(q_L^*)) + m_S(1-Q(q_S^*))] \cdot [(f-r_S)G'(r_S-r_L) - G(r_S-r_L)] \\
&\quad - m_L\alpha_S(f-r_S)Q'(q_L^*)\frac{(f-r_L)G'(r_S-r_L)}{A} = 0. \qquad (13)
\end{aligned}
$$

For simplicity, let $Q(\cdot)$ be the uniform distribution on $[0,1]$. And we further impose a stationarity condition that the market shares of deposits $\{\alpha_j\}$ are identical to the starting market shares $\{m_j\}$, that is, $m_j = \alpha_j$.

The first-order conditions simplify to

$$
\begin{aligned}
0 = \frac{d\Pi_L}{dr_L} &= [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f-r_L)G'(r_S-r_L) - 1 + G(r_S-r_L)] \\
&\quad - \frac{1}{A}\alpha_S\alpha_L(f-r_L)(f-r_S)G'(r_S-r_L), \qquad (14) \\
0 = \frac{d\Pi_S}{dr_S} &= [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f-r_S)G'(r_S-r_L) - G(r_S-r_L)] \\
&\quad - \frac{1}{A}\alpha_L\alpha_S(f-r_L)(f-r_S)G'(r_S-r_L). \qquad (15)
\end{aligned}
$$

From the above conditions we derive

$$
(r_S - r_L)G'(r_S-r_L) + 2G(r_S-r_L) = 1. \qquad (16)
$$

Constrained equilibrium. In the constrained equilibrium, the zero lower bound on deposit interest rates becomes binding for the large bank.

The small bank's profit function and first-order condition are as before:

$$0 = \frac{d\Pi_S}{dr_S} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S) - G(r_S)] - \frac{1}{A}\alpha_L\alpha_S f(f - r_S)G'(r_S). \tag{17}$$

By contrast, the large bank's first-order condition takes an inequality because the conjectured optimal solution is at the left corner:

$$0 > \frac{d\Pi_L}{dr_L}\Big|_{r_L=0} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [fG'(r_S) - 1 + G(r_S)] - \frac{1}{A}\alpha_S\alpha_L f(f - r_S)G'(r_S). \tag{18}$$

That is, the large bank wishes to further reduce its deposit interest rate r_L but is constrained by the lower bound of zero.

We show that if the IORB rate f is low, deposit and lending markets are characterized by the constrained equilibrium, and if f is high, they enter the unconstrained equilibrium. Let f^* denote the threshold value at which the economy transits from the constrained equilibrium to the unconstrained equilibrium. We solve for f^*, from the following FOCs.

$$\begin{aligned}
0 = \frac{d\Pi_S}{dr_S} &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f^* - r_S)G'(r_S) - G(r_S)] \\
&- \frac{1}{A}\alpha_L\alpha_S f^*(f^* - r_S)G'(r_S). \\
0 = \frac{d\Pi_L}{dr_L}\Big|_{r_L=0} &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [f^*G'(r_S) - 1 + G(r_S)] \\
&- \frac{1}{A}\alpha_S\alpha_L f^*(f^* - r_S)G'(r_S).
\end{aligned} \tag{19}$$

We require that the level of reserves X be sufficiently large. Given a functional form of G and parameters f and A, we can specify a lower bound for X under which the two cases of equilibrium and comparative statics hold. We also require the IORB rate not to exceed one, corresponding to an annual IORB rate below 100%. The parameter conditions are summarized in the following assumption, which we apply throughout the rest of the paper.

Assumption 2. *The total reserve X satisfies $X + 1 - \frac{1+f}{A} > \max\{3f^2G'(\delta) + \frac{1}{G'(\delta)}, f + \frac{1}{8G'(\delta)}, \frac{3}{AG'(\delta)}, \frac{1}{A}, \frac{f}{2A}\}$ for any $\delta \in [0, f]$. The IORB rate f satisfies $f \in (0, 1]$.*

A sufficiently large total reserve amount X ensures that the deposit spread and deposit market share play an important role in both banks' total profits. This assumption does not merely accentuate a bank's profit from offering deposit, since in our model, due to the retained deposit channel, deposit spread and deposit market share also affect loan profit. The technical details behind these conditions are provided in the appendix, but as an overview, the condition that $X + 1 - \frac{1+f}{A}$ is larger than $3f^2 G'(\delta) + \frac{1}{G'(\delta)}$ and $f + \frac{1}{8G'(\delta)}$ guarantees well-behaved second order conditions of both banks' optimization problems; and the condition that $X + 1 - \frac{1+f}{A}$ is greater than $\frac{3}{AG'(\delta)}$, $\frac{1}{A}$ and $\frac{f}{2A}$ is necessary for establishing a cutoff IORB rate f^* that separates the constrained and unconstrained cases of the equilibrium and for establishing the comparative statics results.[7]

We summarize the existence and properties of the unconstrained equilibrium in the following proposition.

Proposition 1. *Under Assumption 1 and Assumption 2, the profit function Π_j is quasi-concave in r_j, $j \in \{L, S\}$, and there exists a cutoff IORB rate f^*, such that if $f > f^*$, there exist $r_L > 0$ and $r_S > 0$ that solve Equations (14)–(15). In this unconstrained equilibrium:*

1. *The large bank sets a lower deposit rate and has a larger market share than the small bank. That is, $0 < r_L < r_S < f$ and $\alpha_L > \alpha_S$.*
2. *The large bank uses a looser lending standard than the small bank ($q_L^* < q_S^*$).*
3. *The total loan volume $V = \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$ is decreasing in f.*
4. *If $X \to \infty$, the deposit interest rates of the large bank and the small bank, r_L and r_S, both move one-for-one with f.*

Proofs are in the appendix. The following example helps illustrate the intuition. Suppose that $G(\delta) = \delta/\Delta$, where $\delta \in [0, \Delta]$ for a sufficiently large Δ. Then $G'(\cdot) = 1/\Delta$ and the two first-order conditions reduce to

$$\frac{f - r_L}{\Delta} = 1 - \frac{r_S - r_L}{\Delta} + B, \tag{20}$$

$$\frac{f - r_S}{\Delta} = \frac{r_S - r_L}{\Delta} + B, \tag{21}$$

where

$$B \equiv \frac{\frac{1}{A\Delta}\alpha_L \alpha_S (f - r_L)(f - r_S)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \tag{22}$$

As the total reserve X becomes large, B becomes close to zero. So the equilibrium deposit interest rates of the two banks become approximately $r_L \approx f - \frac{2}{3}\Delta$ and $r_S \approx f - \frac{1}{3}\Delta$. And the

[7] Note that if G represents the uniform distribution or the exponential distribution, Assumption 2 is relatively easy to satisfy. If $G(x) = x/\Delta$ for $x \in [0, \Delta]$, the assumption is equivalent to $X + 1 - \frac{1+f}{A} > \max\{3f^2/\Delta + \Delta, f + \frac{1}{8}\Delta, \frac{3\Delta}{A}, \frac{1}{A}, \frac{f}{2A}\}$. If $G(x) = 1 - e^{-x/\mu}$ for $x \in [0, \infty)$, the assumption is satisfied if $X + 1 - \frac{1+f}{A} > \max\{3f^2/\mu + \mu e^{f/\mu}, f + \frac{1}{8}\mu e^{f/\mu}, \frac{3}{A}\mu e^{f/\mu}, \frac{1}{A}, \frac{f}{2A}\}$. Numerical results show that, for the uniform distribution or the exponential distribution of convenience values, the magnitude of X has a minimal effect on the equilibrium outcome as long as the IOR rate f is reasonably small.

cut-off value of IORB rate is approximately $f^* \approx \frac{2}{3}\Delta$. As f increases, both banks raise deposit interest rates by the same amount, implying steady deposit market shares. However, both banks increase their lending standards (minimal success probability) q_L^* and q_S^*, so lending by each bank declines, and so does the aggregate lending.

In the unconstrained equilibrium with abundant reserves (i.e., sufficiently large X), deposit interest rates of both the large and the small banks move one-for-one with the IORB rate f, implying effective pass-through of monetary policy. The levels of deposit interest rate remain below f, however. Thus, in our model with abundant reserves, the spread between IORB and the deposit interest rate is the primary source of profits for the banks. In this case, the two banks compete primarily in the deposit market, and in equilibrium this competition leads to market shares being invariant to changes in f if f is sufficiently high.

We summarize the existence and the properties of the constrained equilibrium in the following proposition.

Proposition 2. *Under Assumption 1 and Assumption 2, the profit function Π_j is quasi-concave in r_j, $j \in \{L, S\}$, with the same cutoff f^* as in Proposition 1. If $f < f^*$, there exists an r_S that solves Equation (17) and satisfies Equation (18). In this constrained equilibrium:*

1. *The large bank sets a lower deposit interest rate and has a larger market share than the small bank. That is, $0 = r_L < r_S < f$ and $\alpha_L > \alpha_S$.*
2. *The large bank uses a looser lending standard than the small bank ($q_L^* < q_S^*$).*
3. *The total loan volume $V = \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$ is decreasing in f.*
4. *The deposit interest rate of the small bank, r_S, is increasing in X.*
5. *As $X \to \infty$, the deposit interest rate of the small bank, r_S, moves less than one-for-one with f.*

Proofs are in the appendix. In the constrained equilibrium, the large bank's deposit interest rate is zero, the lower bound; the small bank's deposit rate reacts to f but less than one-for-one. Consequently, the pass-through of f to the average deposit rate is much weaker in the constrained equilibrium than in the unconstrained equilibrium.

The impact of f on aggregate lending volume is subtler because f affects both the opportunity cost of lending and the market shares. Using $\alpha_L = 1 - \alpha_S$, we can decompose the impact of f on loan volume V:

$$\frac{dV}{df} = \frac{d\alpha_L}{df}(1-q_L^*) + \frac{d\alpha_S}{df}(1-q_S^*) - \alpha_L\frac{dq_L^*}{df} - \alpha_S\frac{dq_S^*}{df} = \underbrace{\frac{d\alpha_S}{df}(q_L^* - q_S^*)}_{\text{Composition}} \underbrace{-\alpha_L\frac{dq_L^*}{df} - \alpha_S\frac{dq_S^*}{df}}_{\text{Cost of capital}}. \quad (23)$$

The cost of capital channel is straightforward. An increase in f causes both banks to raise their lending standards, q_L^* and q_S^*, which reduces lending for both banks. In addition, an increase in f causes the small bank's deposit interest rate r_S to increase, so the small bank gains deposit market share. Because the large bank's lending standard is lower, marginal borrowers that could obtain a loan from the large bank cannot obtain it from the small bank. The composition effect also reduces lending. In sum, the total loan volume is decreasing in f.

The comparative statics of in both cases of equilibrium with respect to f are further illustrated in Figure 1. Again, the small bank sets a higher deposit interest rate and has a lower deposit market share than the large bank. Moreover, because of the deposit-lending linkage via IORB and deposit retention, the small bank also has a higher lending standard and a lower loan volume than the large bank. Under a sufficiently high IORB, the market shares of the two banks become stable but do not converge.

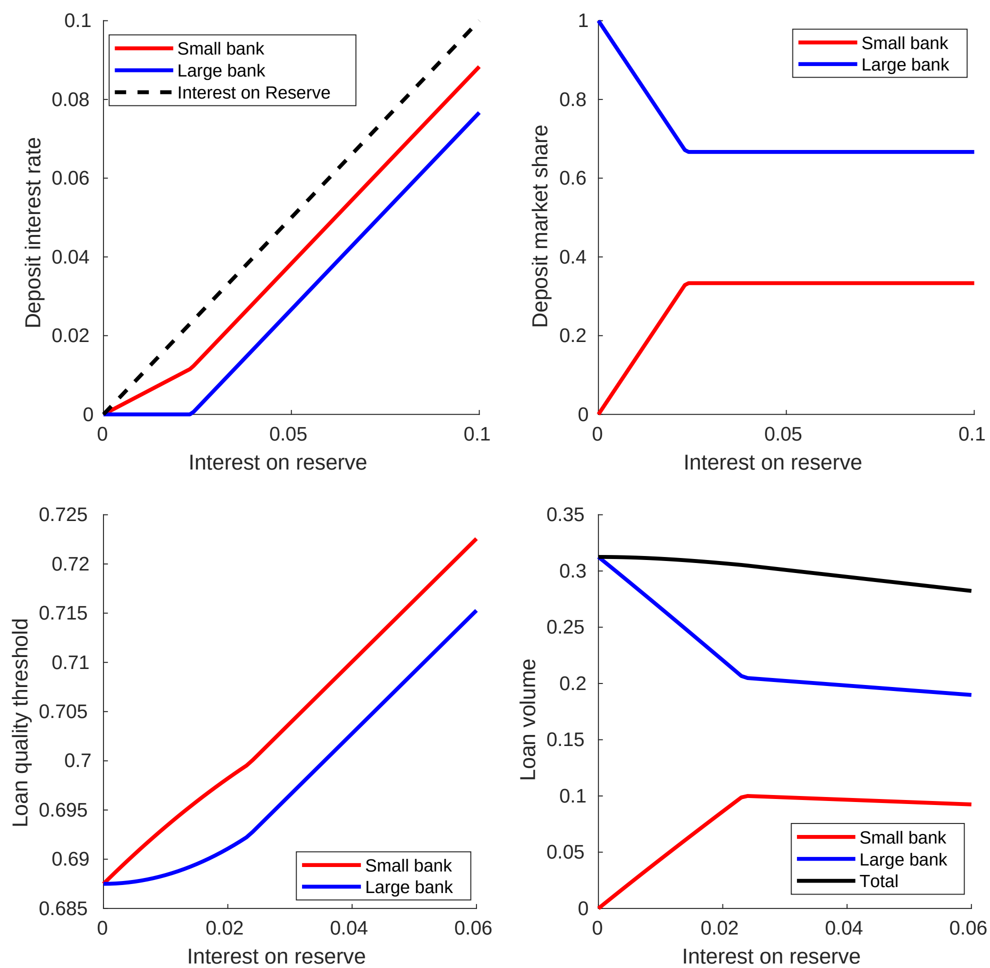

Figure 1: Comparative statics with respect to IORB f. Parameters: $G(\delta) = \delta/0.035$, $A = 1.5$, $X = 10$

Our theoretical predictions are consistent with empirical evidence from d'Avernas, Eisfeldt, Huang, Stanton, and Wallace (2023). They find that over the last two decades, small banks in the U.S. consistently set higher deposit interest rates than large banks do. Moreover, during the 2018-2020 hiking cycle, small banks raised their deposit interest rates from zero, albeit less than one-for-one with IORB, while large banks barely moved their deposit interest rates from the zero lower bound.

3 Introducing Payment Innovation

We now introduce payment innovation to the bank competition model, motivated by real-world implementations of payment innovations in Brazil, China, India, and the United States, among other countries. Specifically, we consider a payment innovation that is operationally linked to deposits in commercial banks. For instance, Pix in Brazil and UPI in India connect commercial banks and enable bank users to send and receive payment instantaneously. In China, Alipay and WeChat Pay are digital wallets that offer instantaneous payments among users, and users can move money between their digital wallets and bank accounts easily. The e-CNY, China's central bank digital currency, offers similar functionality and access model via commercial banks. PayPal, Apple Pay, and Venmo are similar examples in the United States, as they are all linked to bank accounts or credit card accounts.[8]

The payment innovation in our model provides a uniform per-dollar convenience value $v \geq 0$ to all agents. This convenience value v could be interpreted as the speed, security, or anonymity of payment services or as the value of auxiliary services linked to the payment innovation.[9]

When payment innovation is implemented as an alternative medium of exchange in a closed-loop system (e.g., Alipay), or a payment token (e.g., CBDCs or stablecoins), we also allow the possibility that it accrues interest $s \geq 0$. For instance, Alipay in China pays interest on digital wallet balances through its affiliate Yu'ebao (which is essentially a money market product that is linked with Alipay wallet). In the United States, while the GENIUS Act prohibits stablecoin issuers from paying interests directly to stablecoin holders, the banking industry raised concerns that unless the prohibition is interpreted broadly, it could be circumvented, whereby stablecoin holders receive interests or rewards for depositing their stablecoins at digital asset intermediaries (e.g., trading platforms), and these intermediaries receive payments from the stablecoin issuers.[10]

[8] Piazzesi and Schneider (2020) study a standalone CBDC (separate from banks) and how it affects lending by unbundling deposits and credit lines.

[9] It is also possible to model heterogeneous convenience value derived from the payment innovation. If v varies across agents, we can write $v = \bar{v} + \tilde{v}$, where $\bar{v}$ is the average convenience value of the payment innovation, and $\tilde{v}$ represents the individual deviation from the average. Our model results would all hold if we let G in the model describe the distribution of $\delta - \tilde{v}$ instead of δ.

[10] https://www.aba.com/about-us/press-room/press-releases/stablecoin-state-associations-joint-

Because a depositor can seamlessly link her bank deposit account to the payment innovation, she can obtain a convenience value in payments that is equal to the maximum of the two options. Thus, a depositor at the large bank receives the convenience value $\max(\delta, v)$ and a depositor at the small bank receives the convenience value $\max(0, v) = v$. The convenience value v offered by the payment innovation thus narrows the gap between payment convenience levels that depositors receive from the two banks.

We solve the model backward in time, just like the baseline model.

Deposit market at $t = 2$. A depositor can pick either the large bank or the small bank. For interest-bearing payment tokens, the depositor can also choose to keep her cash as bank deposit or interest-bearing payment token. Therefore, a depositor with a large-bank convenience value of δ faces four choices:

	Large bank		Small bank	
	Deposit	Payment token	Deposit	Payment token
Convenience value	$\max(\delta, v)$	$\max(\delta, v)$	v	v
Interest rate	r_L	s	r_S	s

Two results are apparent. First, the interest rate s on the payment token is a lower bound on banks' deposit interest rates, i.e., $r_L \geq s$ and $r_S \geq s$. Otherwise, depositors would convert bank deposits to the payment token, and the banks would lose their interest rate spread $f - r_L$ and $f - r_S$. In equilibrium, depositors use the payment innovation as a payment pipe but do not keep any excess balance there. Second, as in the baseline model, the small bank attracts no depositors if $r_S < r_L$. So $r_S \geq r_L$ in equilibrium. For technical simplicity, whenever a depositor is indifferent between two choices, their preference is the small bank, the large bank, and the payment token, in this order.[11]

We will characterize an equilibrium in which $r_S > r_L$. This implies that a depositor with convenience value δ chooses the large bank if and only if

$$\delta > v \text{ and } r_L + \delta > r_S + v \Rightarrow \delta > r_S - r_L + v. \tag{24}$$

Therefore, the eventual market shares of the banks in the newly created deposits are

$$\alpha_L = 1 - G(r_S - r_L + v), \tag{25}$$

$$\alpha_S = G(r_S - r_L + v). \tag{26}$$

comment-letter.

[11]The tie-breaking rule between banks and the payment token is without loss of generality because a bank can always offer a deposit rate that is just above s to make depositors strictly prefer bank deposits. The rule is needed to preserve continuity in depositors' choices as parameters change.

The loan market at $t = 1$ and the choice of deposit rates at $t = 0$ are fully analogous to the bank competition model described above. As discussed before, the payment token puts a lower bound on commercial banks' deposit interest rates, i.e., $r_L \geq s$, and $r_S \geq s$. There are two cases. The first is that $r_L > s$, so that the payment token interest rate does not constrain either bank's deposit interest rate. We call the first case the *unconstrained equilibrium.* The second case is that $r_L = s$, i.e., the payment token interest rate binds the large bank's deposit interest rate. We call the second case the *constrained equilibrium.*

Unconstrained equilibrium. The first-order conditions of the large and the small banks are:

$$0 = \frac{d\Pi_L}{dr_L} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_L)G'(r_S - r_L + v) - 1 + G(r_S - r_L + v)] - \frac{1}{A}\alpha_S\alpha_L(f - r_L)(f - r_S)G'(r_S - r_L + v), \tag{27}$$

$$0 = \frac{d\Pi_S}{dr_S} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S - r_L + v) - G(r_S - r_L + v)] - \frac{1}{A}\alpha_L\alpha_S(f - r_L)(f - r_S)G'(r_S - r_L + v). \tag{28}$$

From the above conditions, we derive

$$(r_S - r_L)G'(r_S - r_L + v) + 2G(r_S - r_L + v) = 1. \tag{29}$$

Constrained equilibrium. In this equilibrium, the payment token interest rate s becomes binding for the large bank. Recall the tie-breaking rule that at $r_L = s$, marginal depositors would choose to deposit in the large bank rather than the payment token. The small bank's profit function and first-order condition are as before:

$$0 = \frac{d\Pi_S}{dr_S} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - r_S)G'(r_S - s + v) - G(r_S - s + v)] - \frac{1}{A}\alpha_L\alpha_S(f - s)(f - r_S)G'(r_S - s + v). \tag{30}$$

The large bank's first order condition takes an inequality because the conjectured optimal solution is at the left corner:

$$0 > \frac{d\Pi_L}{dr_L}\Big|_{r_L = s} = [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] \cdot [(f - s)G'(r_S - s + v) - 1 + G(r_S - s + v)] - \frac{1}{A}\alpha_S\alpha_L(f - s)(f - r_S)G'(r_S - s + v). \tag{31}$$

We can again solve for the cut-off value of IORB, f^*, that separates the two cases of equilibria, from the following FOCs:

$$
\begin{aligned}
0 &= [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f^* - r_S)G'(r_S - s + v) - G(r_S - s + v)] \\
&\quad - \frac{1}{A}\alpha_L\alpha_S(f^* - s)(f^* - r_S)G'(r_S - s + v), \\
0 &= [X + \alpha_L(1-q_L^*) + \alpha_S(1-q_S^*)] \cdot [(f^* - s)G'(r_S - s + v) - 1 + G(r_S - s + v)] \\
&\quad - \frac{1}{A}\alpha_S\alpha_L(f^* - s)(f^* - r_S)G'(r_S - s + v). \qquad (32)
\end{aligned}
$$

Since the payment token interest rate s changes the lower bound of deposit rates, and its convenience value v changes the convenience gap between large and small banks, we need to slightly modify Assumption 1 and Assumption 2 to accommodate these changes. The modifications are summarized in the following assumption.

Assumption 3. *The function G satisfies $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f-s+v]$. The total reserve X satisfies $X + 1 - \frac{1+f}{A} > \max\{3f^2G'(\delta) + \frac{1}{G'(\delta)}, f + \frac{1}{8G'(\delta)}, \frac{3}{AG'(\delta)}, \frac{1}{A}, \frac{f}{2A}\}$ for any $\delta \in [0, f - s + v]$. The convenience value v satisfies $v < \bar{v}$ for some $\bar{v}$ such that $G(v) < 0.5$ and $(f - s - \frac{1}{4}f^2)G'(v) - G(v) > 0$. The IORB rate f satisfies $f \in (0, 1]$.*

The following proposition summarizes the properties of the unconstrained and constrained equilibria.

Proposition 3. *Under Assumption 3, the profit function Π_j is quasi-concave in r_j, $j \in \{L, S\}$. There exists a cutoff f^*, such that if $f > f^*$, an unconstrained equilibrium exists in which the large bank sets a lower deposit rate ($s < r_L < r_S < f$) and has a larger deposit market share ($\alpha_L > \alpha_S$) than the small bank. If $f < f^*$, a constrained equilibrium exists, in which $s = r_L < r_S < f$ and $\alpha_L > \alpha_S$. In both cases, the large bank uses a looser lending standard than the small bank ($q_L^* < q_S^*$).*

Proofs are in the appendix. The condition $v < \bar{v}$ for some $\bar{v}$ ensures that the payment token does not increase the market share of the small bank so much that it fully eliminates the convenience value advantage of the large bank's deposits. Consequently, the small bank still needs to compete by offering a higher deposit interest rate than the large bank does.

Further intuition of the equilibrium may be gained by considering the same example as in the baseline model. Suppose that $G(\delta) = \delta/\Delta$, where $\delta \in [0, \Delta]$ for a sufficiently large Δ. Then $G'(\cdot) = 1/\Delta$. The two first-order conditions in the unconstrained equilibrium reduce to

$$\frac{f - r_L}{\Delta} = 1 - \frac{r_S - r_L + v}{\Delta} + B, \qquad (33)$$

$$\frac{f - r_S}{\Delta} = \frac{r_S - r_L + v}{\Delta} + B, \qquad (34)$$

where

$$B \equiv \frac{\frac{1}{A\Delta}\alpha_L \alpha_S (f - r_L)(f - r_S)}{X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)} > 0. \tag{35}$$

As the total reserves X becomes large, B becomes close to zero, and the equilibrium deposit interest rates of the two banks become approximately $r_L \approx f - \frac{2}{3}\Delta + \frac{1}{3}v$ and $r_S \approx f - \frac{1}{3}\Delta - \frac{1}{3}v = r_L + \frac{1}{3}\Delta - \frac{2}{3}v$. Thus, an increase in the convenience value of payment innovation reduces the spread between the two bank's deposit interest rates. In the constrained equilibrium, the deposit interest rates of the two banks are approximately $r_L = s$, $r_S \approx \frac{f+s-v}{2}$, showing that the small bank's deposit interest rate moves less than one-for-one with IORB f. The cut-off value of interest on reserve is approximately $f^* \approx s + \frac{2}{3}\Delta - \frac{1}{3}v$, which is increasing in s and decreasing in v.

4 Impact of Payment Innovation on Interbank Competition

The payment innovation in our model is defined by two features: the interest rate s and the convenience value v. We will consider each feature in turn.

4.1 Impact of interest rate s

Here we examine how market outcomes change as the payment token interest rate s varies from 0 to the interest on reserve rate f, while holding f fixed.[12] We focus on the case where, given a fixed value of v, f is sufficiently low such that the constrained equilibrium applies. In the unconstrained equilibrium, market outcomes are invariant to the payment token interest rate s.

Before we provide a formal statement of the comparative statics, it is useful to illustrate them in an example. The top row of Figure 2 plots the behavior in the deposit markets as the payment token interest rate rises from 0 to $f = 2\%$. The charts are computed numerically using a uniform distribution for G and a zero payment token convenience value ($v = 0$). As we see in the top left plot, a rising s increases the deposit interest rates of both banks as well as the weighted average deposit interest rates. The top right plot shows the corresponding changes in deposit market shares α_L and α_S. Since the large bank's deposit interest rate rises faster than the small bank's, the large bank gains market share from the small bank. Intuitively, the small bank competes with the large bank primarily by offering a higher deposit interest rate. As s increases, the maximum spread $f - s$ shrinks, limiting the small bank's ability to compete with a higher deposit interest rate. Once

[12] When the Federal Reserve introduced the overnight reverse repo program (ONRRP) as a temporary facility to support its interest on reserve policy, it tested the facility by varying the ONRRP rate between 1 basis point and 10 basis points, while holding the interest on reserve fixed at 25 basis points.

the deposit interest rates are equal at f, the large bank captures the entire deposit market since its deposits carry a higher convenience value.

The bottom row of Figure 2 illustrates the impact of a rising payment token interest rate s on the lending market. A rising s increases banks' opportunity cost of lending. As both α_S and $f - r_S$ decrease in s, so does the expected profit from the deposit interest rate spread $\alpha_S(f - r_S)$. Thus, the small bank's loan quality threshold, $q_S^* = \frac{1+f-\alpha_S(f-r_S)}{A}$, increases in s, and its loan volume, $\alpha_S(1 - q_S^*)$, decreases in s. In this example, the large bank's loan quality threshold, $q_L^* = \frac{1+f-\alpha_L(f-r_L)}{A}$, increases in s, and its loan volume $\alpha_L(1-q_L^*)$, also increases due to its larger market share. In this example, the total loan volume declines in s.

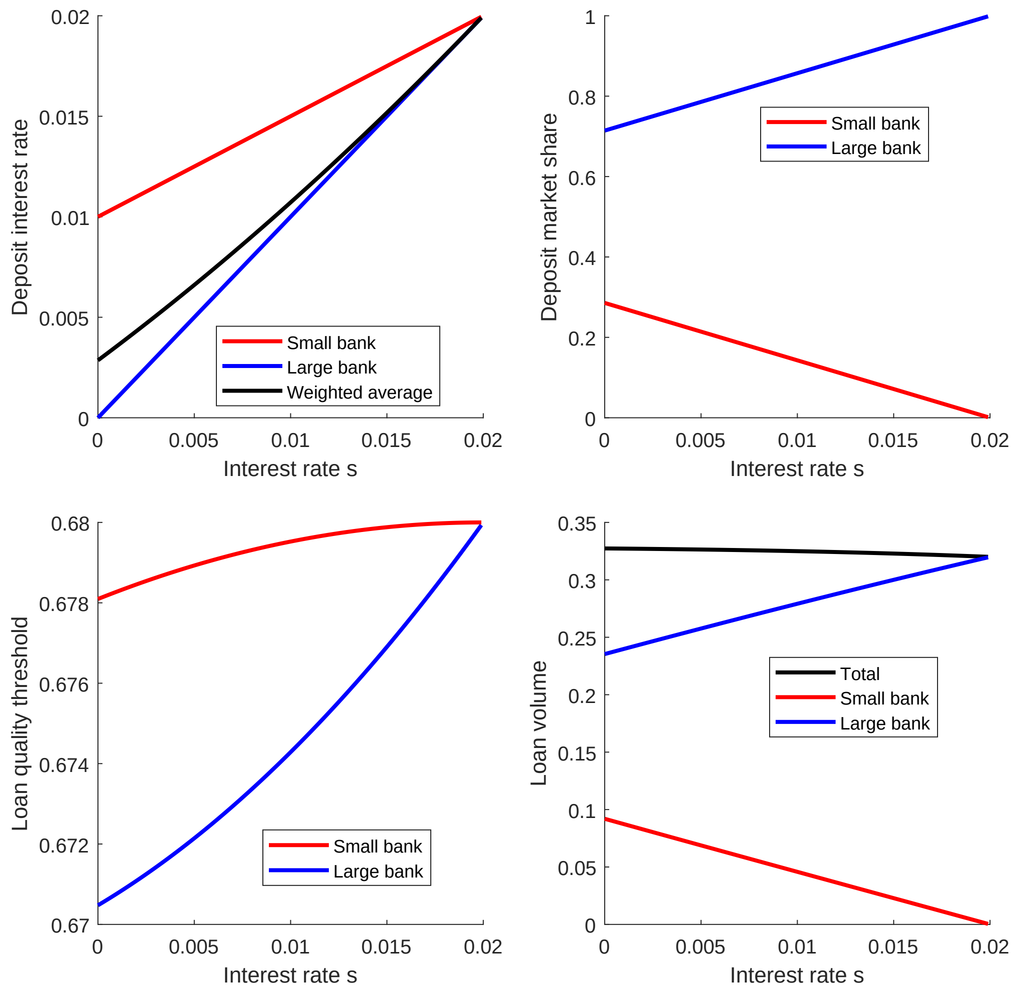

Figure 2: Impact of payment token interest rate on deposit and lending markets. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02, v = 0$.

The following proposition characterizes the impact of payment token interest rate s on the deposit and lending markets.

Proposition 4. *In the equilibrium described in Proposition 3, as $X \to \infty$, increasing the interest rate s of the payment token in a constrained equilibrium has the following impact on the deposit and lending markets:*

As s increases	*Constrained equilibrium*	
	Large	*Small*
Deposit interest rates r_L and r_S	$\uparrow$	$\uparrow$
Deposit market shares α_L and α_S	$\uparrow$	$\downarrow$
Weighted average deposit interest rate	$\uparrow$	
Loan quality thresholds q_L^* and q_S^*	$\uparrow$	$\uparrow$
Loan volume $\alpha_L(1-q_L^*)$ and $\alpha_S(1-q_S^*)$	$\uparrow$ *or* $\downarrow$	$\downarrow$
Total loan volume, i.e., total deposit created	$\downarrow$ *if* $G'' \leq 0$	

Moreover, a higher s also increases f^*, the cutoff value of IORB that separates the two cases of equilibria.

Proofs are in the appendix.[13] Most of the qualitative aspects of comparative statics illustrated in Figure 2 apply generally, and are analytically proven in Proposition 4. The exceptions are that the large bank's loan volume can increase or decrease in s; moreover, if $G'' > 0$, the total loan volume may increase or decrease in s.[14]

A larger s increases the cutoff value f^*. Recall that the large bank's deposit interest rate is fixed at $r_L = s$ for $f \leq f^*$. Thus, if s is larger, a higher interest on reserve is required for the large bank's deposit interest rate to start responding to monetary policy.

In sum, while interest-bearing payment innovations raise the overall level of deposit interest rates, they also reinforce the advantages of the large bank in the deposit market and possibly the loan market.

4.2 Impact of convenience value v

A convenient payment innovation reduces the large bank's convenience advantage and hence levels the playing field even without paying any interest. Before stating the formal proposition, we illustrate the impact of a convenient payment innovation in Figure 3, where s is set to zero and v varies.

[13]While the proposition is analytically proven for $X \to \infty$, numerical results suggest that, if G is uniform or exponential, the comparative statics in this proposition still hold for small X as long as the IOR rate f is reasonably small.

[14]An example illustrating the ambiguity in large bank's loan volumes is seen by setting $G(\delta) = \delta/0.035, A = 1.05, X = 10, f = 0.02, v = 0$. Under these parameters, in the constrained equilibrium, the large bank's loan volume first increases and then decreases in s.

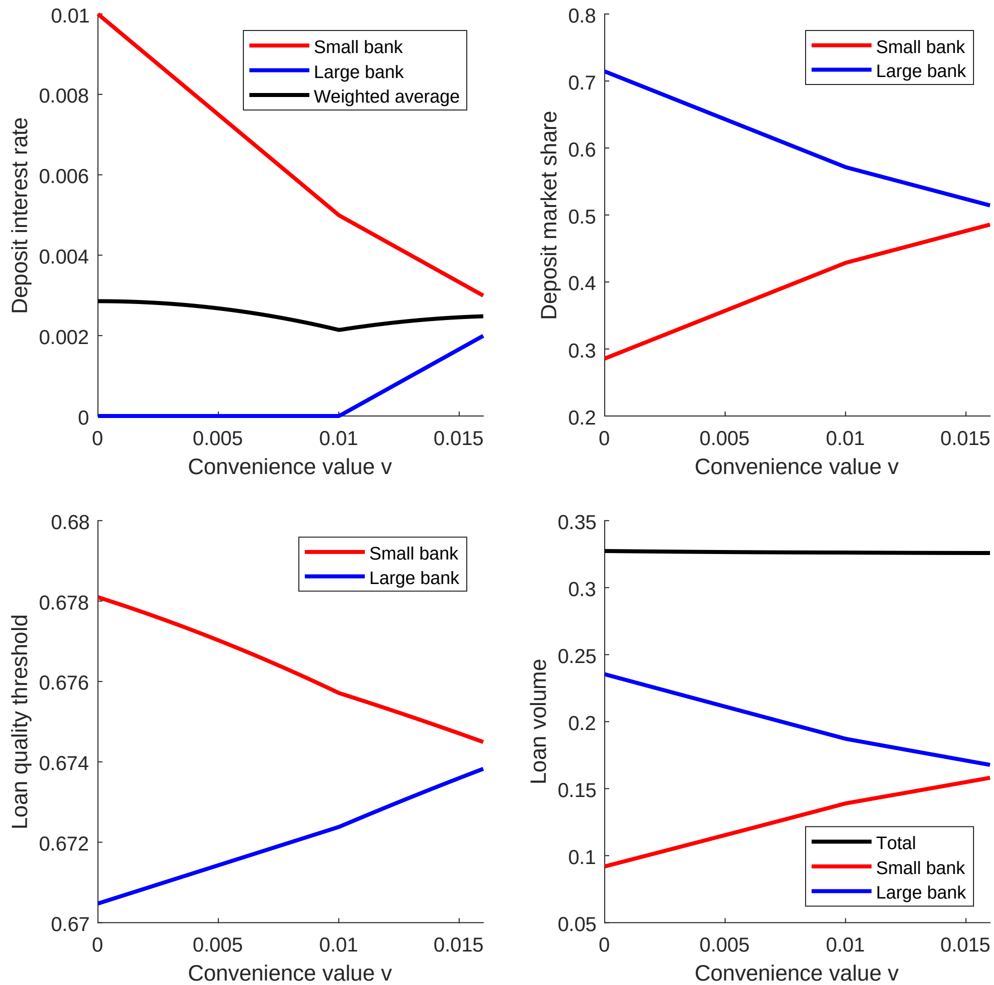

Figure 3: Impact of convenience value on deposit and lending markets. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.02, s = 0$. The equilibrium transitions from constrained to unconstrained at $v = 0.01$.

The top row shows the outcomes in the deposit market. As v rises, the inconvenience disadvantage of the small bank shrinks. As long as the large bank's deposit interest rate remains at the floor of zero, the small bank can afford to lower its deposit interest rate and still capture a growing market share. Once v becomes large enough, the large bank responds by raising its deposit interest rate; however, the small bank can still afford to continue lowering its deposit interest rate for the same reason that the convenience gap between the two banks continues to shrink. Throughout this process, the large bank loses market share to the small bank, and the pace of deposit reallocation is slower after the large bank lifts its deposit interest rate above zero. The overall impact of increasing the convenience value is the convergence of the deposit interest rates and deposit market shares for the two banks.[15]

[15] A modest convenience value may be enough to fully level the playing field. Under the uniform distribution

The convenience value has a nuanced impact on the weighted average deposit interest rates. In a constrained equilibrium, a higher v results in a lower weighted average deposit interest rate if the large bank's deposit rate is at the lower bound. That is, a convenient payment innovation slows down the transmission of monetary policy. Once the economy transitions to an unconstrained equilibrium with a sufficiently high v, however, a higher convenience value increases the average deposit interest rate, speeding the transmission of monetary policy.

The bottom row of Figure 3 shows the outcomes in the lending market. Because the two deposit interest rates and the deposit market shares converge as a result of a rising convenience value v, it is unsurprising that the loan quality thresholds and loan volume of the two banks are also converging. In this example, the total loan volume is almost invariant to v (mildly decreasing), and the most salient effect is the reallocation of loans from the large bank to the small one.

The next proposition summarizes the comparative statics with respect to v. As in the previous subsection, most of the comparative statics as illustrated for a uniform G apply generally.

Proposition 5. *In the equilibrium described in Proposition 3, as $X \to \infty$, increasing the convenience value v has the following impact on the deposit and lending markets:*

As v increases	*Constrained*		*Unconstrained*	
	Large	*Small*	*Large*	*Small*
Deposit interest rates r_L and r_S	*Flat(=s)*	$\downarrow$	$\uparrow$	$\downarrow$
Deposit market shares α_L and α_S	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Weighted average deposit interest rate	$\downarrow$ *if* $G'' \leq 0$		$\uparrow$ *if* $G'' \geq 0$	
Loan quality thresholds q_L^* and q_S^*	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
Loan volume $\alpha_L(1-q_L^*)$ and $\alpha_S(1-q_S^*)$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
Total loan volume, i.e., total deposit created	$\uparrow$ *or* $\downarrow$		$\downarrow$ *if* $G'' \geq 0$	

Moreover, if $G''(\delta) \geq 0$, a higher v reduces f^*, the cutoff value of IORB that separates the two cases of equilibrium.

Proofs are in the appendix.[16] The overarching pattern of these comparative statics is that a higher convenience value levels the playing field by increasing the small bank's deposit market share and loan volume.

The impact of v on the weighted average interest rate, $\alpha_S r_S + \alpha_L r_L$, is ambiguous. An increase in v shifts market share to the small bank and reduces the small bank's deposit interest rate, but the

of large-bank preference $\delta \sim U[0, \Delta]$, once v rises to the point $v = \Delta/2$, depositors with $\delta > \Delta/2$ strictly prefer the large bank, and depositors with $\delta < \Delta/2$ strictly prefer the small bank. That is, the deposit market shares become equal; and so do the deposit interest rates, loan quality thresholds, and loan volume.

[16]Again, while the proposition is analytically proven for $X \to \infty$, numerical results suggest that, if G is uniform or exponential, the comparative statics in this proposition still hold for small X as long as the IOR rate f is reasonably small.

small bank has a higher deposit interest rate to start with, so the overall effect on market-wide deposit interest rate can go in either direction. In the constrained equilibrium, a concave G means that relatively more depositors have a weak (but still positive) preference for the large bank's deposits, so a higher v quickly eliminates the large bank's advantage. As a result, the small bank can afford to reduce its deposit interest rate quickly, leading to a lower weighted average $\alpha_S r_S + \alpha_L r_L$. In the unconstrained equilibrium, r_L increases in v. A convex G means that relatively more depositors have a strong preference for the large bank's deposits, so the large bank raises r_L aggressively in response to a higher v, compared to the reduction in r_S, leading to a higher weighted-average deposit interest rate $\alpha_S r_S + \alpha_L r_L$.

The total loan volume unambiguously decreases in v in the unconstrained equilibrium if G is weakly convex. Intuitively, as explained above, the weighted average deposit rate increases in v if G is weakly convex, which discourages lending by compressing the interest spread earned on the retained deposit. If G is strictly concave, however, the change in total loan volume resulting from an increase in v can be in either direction because the weighted average deposit interest rate can go either way.

The convenience value of payment innovation has an ambiguous impact on total loan volume in the constrained case, as illustrated by the numerical example in Figure 4 under a different set of parameters, including $s = 1.25\%$ and $f = 3\%$. These parameters lead to a constrained equilibrium, with $r_L = s$. In this example, the total loan volume (left axis) is first decreasing in v and then increasing in v. A higher v shifts deposit from the large bank to the small bank; this reallocation by itself reduces overall lending because the small bank has a higher lending standard. However, a higher v also leads to a lower deposit interest rate at the small bank, thus a lower opportunity cost of lending. A closer inspection of the graph suggests that the magnitude of the reallocation of deposits and lending from the large bank to the small bank dominates the magnitude of changes in the total loan volume.

Finally, a higher v decreases the cutoff value f^* if G is weakly convex. This condition implies that relatively more depositors have a strong preference for the large bank's deposits. A higher v reduces the large bank's convenience advantage for this group, so the large bank starts to compete more aggressively on the deposit interest rate under a lower IORB. This comparative static, together with the comparative static of weighted average deposit interest rate, points to a possible channel that a higher v could speed up the pass-through of monetary policy from IORB to deposit interest rates.

The theoretical predictions in Proposition 5 are supported by empirical evidence on payment innovation. For example, Sarkisyan (2023) studies the impact of the introduction of Pix in Brazil, an instant payment system that maps to a payment innovation with a positive convenience value v and zero interest rate s in our model. After the introduction of Pix, small banks gained more deposits

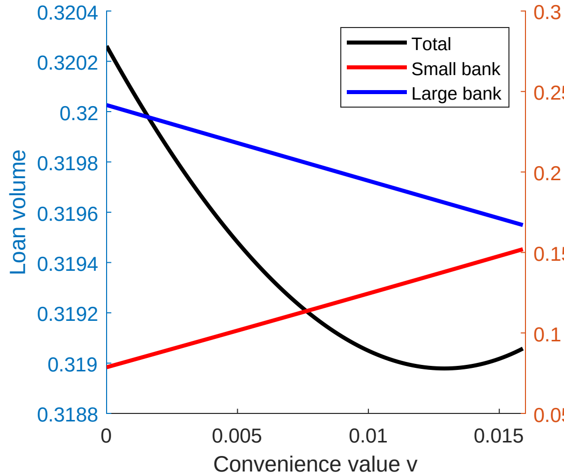

Figure 4: Convenience value and loan volume. Total loan volume on the left axis. Loan volume of the two banks is on the right axis. Parameters: $G(\delta) = \delta/0.035, A = 1.5, X = 10, f = 0.03, s = 0.0125$. All values of v in the depicted range correspond to a constrained equilibrium.

and reduced their deposit interest rates, compared to large banks. He also finds that small banks increased lending compared to large banks after Pix introduction, although the effect is limited. In his sample, "large" banks are those with more than 50 million depositors.

Evidence from China's e-CNY, a CBDC, provides additional support. As the e-CNY does not pay interest, it is also a payment innovation with a positive v and zero s. Li (2024) finds that after China introduced the e-CNY, large banks' deposit market share decreased, deposit interest rates increased, and loan-to-asset ratios decreased, relative to small banks—all in line with the predictions from Proposition 5. In her sample, "large" banks are those with assets above the median.

The United States is still in an early stage of implementing instant payment systems between banks, but other technological improvements also provide insight. Jiang, Yu, and Zhang (2022) find that after the introduction of 3G mobile network in the U.S., less branch-reliant banks set higher deposit interest rates, compared to more branch-reliant banks. They measure branch-reliance as the number of bank branches per unit of deposit, and in their sample, the largest U.S. G-SIBs are less branch reliant. While a faster mobile network is a technological innovation whose benefits are not limited to payment, their empirical finding is consistent with our theoretical predictions in the deposit market.

5 Concluding Remarks

This paper proposes a simple model of interbank competition and analyzes how payment innovation changes the competitive landscape. The theory provides a parsimonious framework for linking and interpreting a number of important and yet seemingly disconnected facts about the banking industry:

Fact 1. In the current regime of ample reserves, bank lending is not mechanically constrained by the level of reserves.

Fact 2. Banks create deposits through lending, and newly created deposits used for payment may return to the same bank and continue to earn interest on reserves.

Fact 3. Large banks adopt digital banking faster and provide a broader set of services. Compared to small banks, large banks' deposit interest rates are lower and less responsive to changes in monetary policy.

Fact 4. The introduction of payment innovations in the banking industry narrows the gap between large and small banks. Compared to large banks, small banks gain deposit market shares, reduce deposit interest rates, and increase lending.

The theory also makes two novel predictions that could be tested in the data:

Prediction 1. All else equal, banks that have larger deposit market shares have lower lending standards.

Prediction 2. Interest-bearing payment innovations will make large banks larger.

Our theoretical analysis could be extended in several directions. One possible direction is to allow more than two banks. This extension would not change the qualitative nature of our result, but it could be more flexibly applied to the data, from a local market with 2-3 banks to a national market with many.

Another direction is to model direct interbank competition in lending. Because a large bank understands that "money lent out" (i.e., newly created deposit) is more likely to return to the bank, the large bank would continue to have a lower lending standard. The difference is that the large bank would set a more competitive loan interest rate and give the borrower more surplus, compared to the baseline model. Because the large bank would win all loans that meet its standard but not the small bank's, this extension would predict an even more pronounced dominance by the large bank in lending.

A third possibility is to add short-term investment vehicles such as money market mutual funds and Treasury Bills. These investments pay higher interest rates than bank deposits but cannot process payments or make loans. Upon closer inspection, however, this extension is not a large departure from our current model. Why? When an investor buys shares in a money market mutual fund, the investor's deposit flows from the investor's bank to the fund's bank and eventually to the asset seller's bank. Because deposit flows are settled in central bank reserves, and entities having reserve accounts constitute a closed interbank payment system, "lost deposits" always find their way back to some bank. The exception is when the ultimate seller of the Treasury Bill is a bank; see Blickle, Li, Lu, and Ma (2025) for discussions.

Fourth, what if the payment innovation is not offered through existing banks but through a different entity? In Kenya, for example, the mobile money M-Pesa can be created and redeemed with licensed agents, and M-Pesa is transferred via the mobile phone network. In the United States, stablecoins are transferred on blockchains. Because M-Pesa agents and stablecoin issuers need to deposit the "money" in the banking system, the same logic as before brings us back to the interbank competition model. An important caveat here is that the stablecoin issuer may, one day, become a narrow bank that can also access central bank reserves.[17] The narrow bank does not lend, but its payment token (stablecoin) can provide convenience. In our model, the small bank that offers zero payment convenience would not be able to compete with the stablecoin bank, and deposits and reserves will drain from the small bank to the stablecoin bank and the large bank.

Finally, and perhaps most importantly, our model could be a useful building block for policy analysis of payment innovation. For instance, if a payment innovation provides both convenience and interest, what is the impact on bank competition and monetary policy pass-through? And is there any combination of these two features that provide a Pareto improvements to consumers, large banks, and small banks? The last question is particularly relevant for the political economy of innovation and competition.

[17] In December 2025, the Office of the Comptroller of the Currency (OCC) granted conditional approval for the national trust bank Charter applications from Circle, Ripple, BitGo, Fidelity Digital Assets, and Paxos. See https://www.occ.treas.gov/news-issuances/news-releases/2025/nr-occ-2025-125.html. Access to the Federal Reserve requires separate applications.

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Appendix A: Proofs

Proof of Proposition 1

Let f^*, $r_S = a$ be the solution of Equation (19). Write $B(r_S, r_L; f) = \frac{\frac{1}{A}\alpha_L\alpha_S(f-r_L)(f-r_S)G'(r_S-r_L)}{X+\alpha_L(1-q_L^*)+\alpha_S(1-q_S^*)}$. After plugging in the expression of q_L^* and q_S^* from Equation (8) and (9), we have $B(r_S, r_L; f) = \frac{\frac{1}{A}\alpha_L\alpha_S(f-r_L)(f-r_S)G'(r_S-r_L)}{X+1-(1+f)/A+[\alpha_L^2(f-r_L)+\alpha_S^2(f-r_S)]/A}$. Then Equation (19) can be written as

$$(f^* - a)G'(a) - G(a) = B(a, 0; f^*), \tag{36}$$

$$f^*G'(a) - 1 + G(a) = B(a, 0; f^*). \tag{37}$$

We show that the solution exists. First, we show that when X is larger than a threshold, for any given a, there exists a solution f^* to Equation (37). This equation can be written as $Mf^{*2} - Nf^* + L = 0$, with M, N and L defined as below.

$$M = \frac{1}{A}(1 + \alpha_L\alpha_S - \alpha_L^2 - \alpha_S^2), \tag{38}$$

$$N = X + 1 - \frac{1}{A} + \frac{a}{A}(\alpha_L\alpha_S - \alpha_S^2) + \frac{1}{A}\frac{1 - G(a)}{G'(a)}(1 - \alpha_L^2 - \alpha_S^2), \tag{39}$$

$$L = \frac{1 - G(a)}{G'(a)}(X + 1 - \frac{1}{A} - \frac{1}{A}a\alpha_S^2). \tag{40}$$

$G'(a)$ can be in the denominator, since Assumption 1 implies that $G'(\delta) > 0$ for any $\delta \in [0, f]$. Since $\alpha_L + \alpha_S = 1$ and $\alpha_L > \alpha_S$ in equilibrium, we have $0 \le M < \frac{3}{4A}$, and $N \ge X + 1 - \frac{1}{A}$. Consider the case that $M > 0$, in which we have a quadratic equation of f^*. Under the assumption that $X+1-\frac{1}{A} > \frac{3}{AG'(\delta)}$ for any $\delta \in [0, f]$, we have $N^2 - 4ML > 0$ and $N > 0$. Based on these, we know that there exists at least one positive solution f^* to the equation $Mf^{*2} - Nf^* + L = 0$. Under the case that $M = 0$, we have $\alpha_S = 0$. Obviously, under the same assumption that $X + 1 - \frac{1}{A} > \frac{3}{AG'(\delta)}$, we have $N > 0, L > 0$ and a positive solution f^* to the equation exists. Second, since Equation (37) has a solution f^*, we plug the expression $f^* = \frac{1-G(a)+B(a,0;f^*)}{G'(a)}$ into Equation (36), which becomes $1 - aG'(a) - 2G(a) = 0$. The function $g(a) = 1 - aG'(a) - 2G(a)$ is continuous. Also, we have $g(0) > 0$ since $G(0) = 0$ and $G'(0)$ is bounded (because G is twice differentiable). We also have $g(a) < 0$ when a is sufficiently large since $G(\infty) = 1$ and $G'(a) > 0$. Thus a positive solution to $g(a) = 0$ exits. Consequently, the solution $f^* > 0$, $r_S = a > 0$ to Equation (19) exits. We also have $a < f^*$ because of the first equation, as on the right hand side $B(a, 0; f^*) > 0$.

When $f > f^*$, we show that there exist r_L and r_S that satisfy the first order conditions in

Equation (14)–(15). The first order conditions are equivalent to

$$(f - r_S)G'(r_S - r_L) - G(r_S - r_L) = B(r_S, r_L; f), \tag{41}$$

$$(f - r_L)G'(r_S - r_L) - 1 + G(r_S - r_L) = B(r_S, r_L; f). \tag{42}$$

Taking the difference between the two equations, we have $(r_S - r_L)G'(r_S - r_L) = 1 - 2G(r_S - r_L)$. We again denote $g(x) = 1 - xG'(x) - 2G(x)$, and find the solution to $g(x) = 0$. Since we just showed that the solution $r_S = a$ to Equation (19) makes $g(a) = 0$, we know that $x = a$ is a solution. In fact, this is the unique solution on $x \in [0, f]$, since $g'(x) = -xG''(x) - 3G'(x) < 0$ on $x \in [0, f]$ under Assumption 1. Thus, we have $r_S - r_L = a$. Now we only need to solve the first equation while letting $r_S - r_L = a$. That is to find r_S such that $(f - r_S)G'(a) - G(a) - B(r_S, r_S - a; f) = 0$. Let $n(x; f) = (f - x)G'(a) - G(a) - B(x, x - a; f)$. Since $n(x; f)$ is continuous in x, we only need to prove that $n(a; f) > 0$ and $n(f; f) < 0$ to show that there exists a solution $x \in (a, f)$ to $n(x; f) = 0$. Write the denominator of $B(r_S, r_L; f)$ as W, then we have

$$\begin{aligned}\frac{\partial B(r_S, r_L; f)}{\partial f} &= \frac{(2f - r_L - r_S)W + \frac{1-\alpha_L^2-\alpha_S^2}{A}(f - r_L)(f - r_S)}{AW^2}\alpha_L \alpha_S G'(r_S - r_L) \\ &< (\frac{f}{2AW} + \frac{f^2}{4A^2W^2})G'(r_S - r_L).\end{aligned} \tag{43}$$

Under the assumption that $X + 1 - \frac{1+f}{A} > 1/A$, we have $W > 1/A$ and $\frac{\partial B(r_S, r_L; f)}{\partial f} < G'(r_S - r_L)$ for any given $f \in (0, 1]$. Thus, we have $\partial n(x; f)/\partial f = G'(a) - \partial B(x, x - a; f)/\partial f > 0$ for any given $x \geq a$ and any given $f \in (0, 1]$. Because of this, note that $n(a; f)$ is positive: since $f > f^*$, we have $n(a; f) = (f - a)G'(a) - G(a) - B(a, 0; f) > n(a; f^*) = (f^* - a)G'(a) - G(a) - B(a, 0; f^*) = 0$. Also, $n(f; f) = -G(a) < 0$, so there must be a solution $x \in (a, f)$ to $n(x; f) = 0$. Thus, there exist $r_S \in (a, f)$ and $r_L = r_S - a$ that satisfy the first order conditions in Equation (14)–(15).

The following properties can be established in this unconstrained equilibrium. First, because $r_S \in (a, f)$, $r_L = r_S - a$, and $0 < a < f^* < f$, we have $0 < r_L < r_S < f$. We can also show $\alpha_L > \alpha_S$. Since the difference between the FOCs suggests in equilibrium we have $(r_S - r_L)G'(r_S - r_L) = 1 - 2G(r_S - r_L)$, we must have $\alpha_S = G(r_S - r_L) < 0.5$ and consequently $\alpha_L = 1 - \alpha_S > \alpha_S$, otherwise the left hand side of this equation is positive yet the right hand side is negative. Second, we show that the total loan volume $V = \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$ is decreasing in f. Note that r_S and r_L are increasing in f. To show this, first note that $\partial B(x, x - a; f)/\partial x = \frac{1}{A}\alpha_L \alpha_S G'(a)[(2x - 2f - a)W + (\alpha_L^2 + \alpha_S^2)(f - x + a)(f - x)/A]/W^2 > \frac{-f}{2A}G'(a)/W$ for any given $x \geq a$. Under the assumption that $X + 1 - \frac{1+f}{A} > \frac{f}{2A}$, we have $\partial B(x, x - a; f)/\partial x > -G'(a)$. As a result, we have $\partial n(x; f)/\partial x = -G'(a) - \partial B(x, x - a; f)/\partial x < 0$ for any given $x \geq a$. We also knew that $\partial n(x; f)/\partial f > 0$. Let $x(f)$ be the solution to $n(x; f) = 0$. We have $dx(f)/f =$

$-\frac{\partial n}{\partial f}/\frac{\partial n}{\partial x} > 0$. Thus, r_S and $r_L = r_S - a$ are both increasing in f. After plugging in the expression of q_L^* and q_S^* from Equation (8) and (9), the total loan volume can be written as $V = 1 - \frac{1+f}{A} + [\alpha_L^2(f - r_L) + \alpha_S^2(f - r_S)]/A$. We have $dV/df = -1/A + (\alpha_L^2 + \alpha_S^2 - dr_L/df - dr_S/df)/A < 0$. Third, when $X \to \infty$, we have $B(r_S, r_L; f) \to 0$. Since $r_S - r_L$ does not change with f, r_L and r_S need to move one-for-one with f to make the Equation (41) and (42) hold.

Finally, we verify that the second order conditions hold under Assumption 1. We first show that the derivative $\Gamma_L = d\Pi_L/dr_L$ is strictly decreasing in r_L when $0 < r_L < r_S$, given any $r_S \in [0, f]$. We rewrite the Γ_L and Γ_S from Equation (14) and (15) as

$$\begin{aligned}
\Gamma_L = \frac{d\Pi_L}{dr_L} &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) - \frac{1}{A}\alpha_S\alpha_L(f - r_S)] \cdot (f - r_L)G'(r_S - r_L) \\
&\quad - [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)]\alpha_L, \qquad (44) \\
\Gamma_S = \frac{d\Pi_S}{dr_S} &= [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) - \frac{1}{A}\alpha_L\alpha_S(f - r_L)] \cdot (f - r_S)G'(r_S - r_L) \\
&\quad - [X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)]\alpha_S. \qquad (45)
\end{aligned}$$

Write $T_1 = X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) - \frac{1}{A}\alpha_S\alpha_L(f - r_S)$, and $T_2 = X + \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$. Then we can write $\Gamma_L = T_1(f - r_L)G'(r_S - r_L) - T_2\alpha_L$. Plugging the expression of q_L^* and q_S^* from Equation (8) and (9) into T_1 and T_2, we have

$$T_1 = X + 1 - \frac{1+f}{A} + \frac{1}{A}[\alpha_L^2(f - r_L) + \alpha_S^2(f - r_S) - \alpha_S\alpha_L(f - r_S)], \qquad (46)$$

$$T_2 = X + 1 - \frac{1+f}{A} + \frac{1}{A}[\alpha_L^2(f - r_L) + \alpha_S^2(f - r_S)]. \qquad (47)$$

Under the equilibrium condition $0 < r_L < r_S < f$, we have $\alpha_L > 1/2 > \alpha_S$ and $\alpha_L^2(f - r_L) > \alpha_S\alpha_L(f - r_S)$. Based on these, under the assumption that $X + 1 - \frac{1+f}{A} > 0$, we have $T_1 > 0$ and $T_2 > 0$. To show that Γ_L is strictly decreasing in r_L, we take derivative of Γ_L with respect to r_L while fixing r_S, and show that $d\Gamma_L/dr_L < 0$. We have

$$\frac{d\Gamma_L}{dr_L} = \frac{dT_1}{dr_L}(f - r_L)G'(r_S - r_L) + T_1\frac{d(f - r_L)G'(r_S - r_L)}{dr_L} - \frac{dT_2}{dr_L}\alpha_L - T_2\frac{d\alpha_L}{dr_L}. \qquad (48)$$

First, we have $T_1\frac{d(f-r_L)G'(r_S-r_L)}{dr_L} < 0$, as $T_1 > 0$ and $\frac{d(f-r_L)G'(r_S-r_L)}{dr_L} = -G'(r_S - r_L) - (f - r_L)G''(r_S - r_L) < 0$ under Assumption 1. Second, we have $dT_1/dr_L = 2\alpha_L(f - r_L)G'(r_S - r_L) + (\alpha_L - 3\alpha_S)(f - r_S)G'(r_S - r_L) - \alpha_L^2 < 3fG'(r_S - r_L) - 1/4$, and $dT_2/dr_L = 2\alpha_L(f - r_L)G'(r_S - r_L) - 2\alpha_S(f - r_S)G'(r_S - r_L) - \alpha_L^2 \geq -1$, and $T_2\frac{d\alpha_L}{dr_L} > (X + 1 - \frac{1+f}{A})G'(r_S - r_L)$. These three inequalities suggest that

$$\frac{dT_1}{dr_L}(f-r_L)G'(r_S-r_L)-\frac{dT_2}{dr_L}\alpha_L-T_2\frac{d\alpha_L}{dr_L}<[3fG'(r_S-r_L)-1/4](f-r_L)G'(r_S-r_L)$$
$$+\alpha_L-(X+1-\frac{1+f}{A})G'(r_S-r_L). \quad (49)$$

Under the assumption that $3f^2G'(\delta)+1/G'(\delta)\leq X+1-\frac{1+f}{A}$, we have $\frac{dT_1}{dr_2}(f-r_L)G'(r_S-r_L)-\frac{dT_2}{dr_L}\alpha_L-T_2\frac{d\alpha_L}{dr_L}<0$, thus $d\Gamma_L/dr_L<0$.

Similarly, we show that the derivative $\Gamma_S=d\Pi_S/dr_S$ is strictly decreasing in r_S when $0<r_L<r_S$, given any $r_L\in[0,f]$. Write

$$T_3=X+1-\frac{1+f}{A}+\frac{1}{A}[\alpha_L^2(f-r_L)+\alpha_S^2(f-r_S)-\alpha_S\alpha_L(f-r_L)], \quad (50)$$
$$T_4=X+1-\frac{1+f}{A}+\frac{1}{A}[\alpha_L^2(f-r_L)+\alpha_S^2(f-r_S)]. \quad (51)$$

then we can write $\Gamma_S=T_3(f-r_S)G'(r_S-r_L)-T_4\alpha_S$. Under the equilibrium condition $0<r_L<r_S<f$, we have $\alpha_L>1/2>\alpha_S$ and $\alpha_L^2(f-r_L)>\alpha_S\alpha_L(f-r_L)$. Based on these, under the assumption that $X+1-\frac{1+f}{A}>0$, we have $T_3>0$ and $T_4>0$. To show that Γ_S is strictly decreasing in r_S, we take derivative of Γ_S with respect to r_S while fixing r_L, and show that $d\Gamma_S/dr_S<0$. We have

$$\frac{d\Gamma_S}{dr_S}=\frac{dT_3}{dr_S}(f-r_S)G'(r_S-r_L)+T_3\frac{d(f-r_S)G'(r_S-r_L)}{dr_S}-\frac{dT_4}{dr_S}\alpha_S-T_4\frac{d\alpha_S}{dr_S}. \quad (52)$$

First, we have $T_3\frac{d(f-r_S)G'(r_S-r_L)}{dr_S}<0$, as $T_3>0$ and $\frac{d(f-r_S)G'(r_S-r_L)}{dr_S}=-G'(r_S-r_L)+(f-r_S)G''(r_S-r_L)<0$ under Assumption 1. Second, we have $dT_3/dr_S=2\alpha_S(f-r_S)G'(r_S-r_L)+(\alpha_S-3\alpha_L)(f-r_L)G'(r_S-r_L)-\alpha_S^2<(3\alpha_S-3\alpha_L)(f-r_L)G'(r_S-r_L)-\alpha_S^2<0$, $dT_4/dr_S=2\alpha_S(f-r_S)G'(r_S-r_L)-2\alpha_L(f-r_L)G'(r_S-r_L)-\alpha_S^2\geq-2fG'(r_S-r_L)-\alpha_S^2$, and $T_4\frac{d\alpha_S}{dr_S}>(X+1-\frac{1+f}{A})G'(r_S-r_L)$. These three inequalities suggest that

$$\frac{dT_3}{dr_S}(f-r_S)G'(r_S-r_L)-\frac{dT_4}{dr_S}\alpha_S-T_4\frac{d\alpha_S}{dr_S}<0+[2fG'(r_S-r_L)+\alpha_S^2]\alpha_S \quad (53)$$
$$-(X+1-\frac{1+f}{A})G'(r_S-r_L). \quad (54)$$

Under the assumption that $f+\frac{1}{8G'(\delta)}\leq X+1-\frac{1+f}{A}$, we have $\frac{dT_3}{dr_S}(f-r_S)G'(r_S-r_L)-\frac{dT_4}{dr_S}\alpha_S-T_4\frac{d\alpha_S}{dr_S}<0$, thus $d\Gamma_S/dr_S<0$.

Proof of Proposition 2

Let $f^*, r_S = a$ be the solution of Equation (19). When $f < f^*$, we show that there exists $r_S > 0$ that satisfies the first order conditions in Equation (17)–(18). These conditions are equivalent to

$$(f - r_S)G'(r_S) - G(r_S) = B(r_S, 0; f), \tag{55}$$

$$fG'(r_S) - 1 + G(r_S) < B(r_S, 0; f). \tag{56}$$

We first show that given f there exists a solution $r_S \in (0, f)$ to the first equation. Then we verify that this solution satisfies the inequality in the second equation. Let $u(x; f) = (f - x)G'(x) - G(x) - B(x, 0; f)$. Then the first equation is equivalent to $u(x; f) = 0$. Note that $B(r_S, r_L; f) = \frac{\frac{1}{A}\alpha_L\alpha_S(f-r_L)(f-r_S)G'(r_S-r_L)}{X+1-(1+f)/A+[\alpha_L^2(f-r_L)+\alpha_S^2(f-r_S)]/A} < \frac{\frac{1}{4A}f^2G'(r_S-r_L)}{X+1-(1+f)/A}$. Under the assumption that $X+1-(1+f)/A > \frac{f}{4A}$, we have $B(r_S, r_L; f) < fG'(r_s - r_L)$. Note that $u(x; f)$ is continuous in x, and $u(0; f) = fG'(0) - G(0) - B(0, 0; f) = fG'(0) - B(0, 0; f) > 0$. Also, according to the proof of Proposition 1, under the assumption that $X + 1 - \frac{1+f}{A} > 1/A$, we have $\frac{\partial u(x;f)}{\partial f}|_{x=a} = G'(a) - \frac{\partial B(a,0;f)}{\partial f} > 0$ for any given $f \in (0, 1]$. Thus, when $f < f^*$, we have $u(a; f) = (f - a)G'(a) - G(a) - B(a, 0; f) < u(a; f^*) = (f^* - a)G'(a) - G(a) - B(a, 0; f^*) = 0$. As a result, we know that there is a solution $x \in (0, a)$ to $u(x; f) = 0$. This solution is r_S.

Now we verify that the solution $r_S \in (0, a)$ satisfies the inequality in Equation (56). Utilizing $B(r_S, 0; f) = (f - r_S)G'(r_S) - G(r_S)$ (Equation (55)) , the inequality becomes $1 - r_SG'(r_S) - 2G(r_S) > 0$. According to the proof of Proposition 1, we know that $g(x) = 1 - xG'(x) - 2G(x)$ is strictly decreasing in x under Assumption 1 and $g(a) = 0$. Since $r_S < a$, we know that $g(r_S) = 1 - r_SG'(r_S) - 2G(r_S) > g(a) = 0$, hence the inequality holds.

The following properties can be established in this constrained equilibrium. First, we have just shown $0 = r_L < r_S < a < f$. We can also show $\alpha_L > \alpha_S$. If we take the difference between Equation (55) and (56), we have $\alpha_L - \alpha_S - r_SG'(r_S) > 0$. It follows that $\alpha_L > \alpha_S$. Second, we show that the total loan volume $V = \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$ is decreasing in f. Note that r_S and α_S are both increasing in f. This is because $r_S(f)$ is the solution to $u(x; f) = 0$, and as shown in the proof of Proposition 1, $\partial u(x; f)/\partial f = G'(x) - \partial B(x, 0; f)/\partial f > 0$ under the assumption that $X + 1 - \frac{1+f}{A} > 1/A$. Also, if we write the denominator of $B(r_S, 0; f)$ as W, and write the numerator of $B(r_S, 0; f)$ as S, we have

$$\begin{aligned}\frac{\partial B(r_S, 0; f)}{\partial r_S} &= \frac{(f - r_S)[(1 - 2\alpha_S)G'(r_S)^2 + \alpha_L\alpha_SG''(r_S)] - \alpha_L\alpha_SG'(r_S)}{AW/f} \\ &+ \frac{[2\alpha_LfG'(r_S) + \alpha_S^2 - 2\alpha_S(f - r_S)G'(r_S)]S}{AW^2} \\ &> \frac{f\alpha_L\alpha_S[(f - r_S)G''(r_S) - G'(r_S)]}{AW}.\end{aligned} \tag{57}$$

Under Assumption 1, we have $\partial B(r_S, 0; f)/\partial r_S > \frac{-2f\alpha_L\alpha_S G'(r_S)}{AW}$. Under the assumption that $X + 1 - \frac{1+f}{A} > \frac{f}{2A}$, we have $\partial B(r_S, 0; f)/\partial r_S > -G'(r_S)$, which leads to $\partial u(r_S; f)/\partial r_S = (f - r_S)G''(r_S) - 2G'(r_S) - \partial B(r_S, 0; f)/\partial r_S < 0$ under Assumption 1. As a result $dr_S(f)/df = -\frac{\partial u(r_S;f)}{\partial f}/\frac{\partial u(r_S;f)}{\partial r_S} > 0$, Thus, $\alpha_S = G(r_S)$ is also increasing in f. After plugging in the expression of q_L^* and q_S^* from Equation (8) and (9), the total loan volume can be written as $V = 1 - \frac{1+f}{A} + [\alpha_L^2 f + \alpha_S^2(f - r_S)]/A$. We have $dV/df = -1/A + (\frac{d\alpha_L^2 f}{df} + \frac{d\alpha_S^2(f-r_S)}{df})/A$. Note that $\frac{d\alpha_S^2(f-r_S)}{df} = \alpha_S^2\frac{d(f-r_S)}{df} + \frac{d\alpha_S^2}{df}(f - r_S) < \alpha_S^2 + \frac{d\alpha_S^2}{df}f$. And $\frac{d\alpha_L^2 f}{df} = \alpha_L^2 + \frac{d\alpha_L^2}{df}f$. Thus, we have $\frac{d\alpha_L^2 f}{df} + \frac{d\alpha_S^2(f-r_S)}{df} < \alpha_L^2 + \alpha_S^2 + 2f(\alpha_S - \alpha_L)\frac{d\alpha_S}{df} < \alpha_L^2 + \alpha_S^2 \leq 1$. That leads to $dV/df < 0$.

Third, we show that r_S is increasing in X. Note that

$$\frac{\partial u(r_S; f)}{\partial X} = -\frac{\partial B(r_S, 0; f)}{\partial X} = \frac{\frac{1}{A}\alpha_L\alpha_S(f - r_L)(f - r_S)G'(r_S - r_L)}{[X + 1 - (1 + f)/A + [\alpha_L^2(f - r_L) + \alpha_S^2(f - r_S)]/A]^2} > 0. \tag{58}$$

We have just shown that $\partial u(r_S; f)/\partial r_S < 0$. Thus we have $dr_S/dX = -\frac{\partial u(r_S;f)}{\partial X}/\frac{\partial u(r_S;f)}{\partial r_S} > 0$.

Fourth, regarding the sensitivity of dr_S/df, when $X \to \infty$, we have $B(x, 0; f) \to 0$, and the expression of $u(x; f)$ simplifies to $u(x; f) = (f - x)G'(x) - G(x)$. In this case, we have $\partial u(x; f)/\partial f = G'(x) > 0$, and $\partial u(x; f)/\partial x = (f-x)G''(x)-2G'(x) < 0$ for any $x \in [0, f]$ under Assumption 1. Let $x(f)$ be the solution to $u(x; f) = 0$. We have $dx(f)/df = -\frac{\partial u(x;f)}{\partial f}/\frac{\partial u(x;f)}{\partial x} \in (1/3, 1)$ under Assumption 1. Thus the deposit rate of the small bank r_S moves less than one-for-one with f in the constrained equilibrium.

Completely analogous to the proof of Proposition 1, we can verify that the second order condition for the small bank holds.

Proof of Proposition 3

The proof is analogous to the proofs of Proposition 1 and Proposition 2. One additional assumption for this proposition is that $v < \bar{v}$, where $\bar{v}$ is a cutoff value of v such that $G(v) < 0.5$ and $(f - s - f^2/4)G'(v) - G(v) > 0$. To see that such a cutoff exists, note that both conditions are satisfied when $v = 0$. Hence, by continuity, they hold for some strictly positive v. Note that when G is uniform, $(f - s - f^2/4)G'(v) - G(v) > 0$ is equivalent to $v < f - s - f^2/4$, and when G is exponential, this condition is approximately equivalent to $v < f - s - f^2/4$.

Denote $\tilde{B}(r_S, r_L; f, v) = \frac{\frac{1}{A}\alpha_L\alpha_S(f-r_L)(f-r_S)G'(r_S-r_L+v)}{X+1-(1+f)/A+[\alpha_L^2(f-r_L)+\alpha_S^2(f-r_S)]/A}$. Let f^*, $r_S = a$ be the solution to Equation (32). Then we have

$$(f^* - a)G'(a - s + v) - G(a - s + v) = \tilde{B}(a, s; f^*, v), \tag{59}$$

$$(f^* - s)G'(a - s + v) - 1 + G(a - s + v) = \tilde{B}(a, s; f^*, v). \tag{60}$$

We do a transformation of the variables: denote $\phi^* = f^* - s$ and $m = a - s$. Then the equations are equivalent to

$$(\phi^* - m)G'(m+v) - G(m+v) = \tilde{B}(m, 0; \phi^*, v), \tag{61}$$

$$\phi^* G'(m+v) - 1 + G(m+v) = \tilde{B}(m, 0; \phi^*, v). \tag{62}$$

We show that the solution (ϕ^*, m) exists. As in the proof of Proposition 1, under the assumption that $X + 1 - \frac{1}{A} > \frac{3}{AG'(\delta)}$ for any $\delta \in [0, f - s + v]$, for any given m, there exists at least one positive solution ϕ^* to Equation (62). Second, since Equation (62) has a solution, we plug $\phi^* = \frac{1-G(m+v)+\tilde{B}(m,0;\phi^*,v)}{G'(m+v)}$ into Equation (61), which becomes $1 - mG'(m+v) - 2G(m+v) = 0$. The function $g(m) = 1 - mG'(m+v) - 2G(m+v)$ is continuous. Under the assumptions that $G(v) < 0.5$ and G is twice differentiable (hence $G'(m+v)$ is bounded), we have $g(0) > 0$. Also, $g(m) < 0$ when m is sufficiently large, since $G(\infty) = 1$. Thus, a solution $m > 0$ to $g(m) = 0$ exists. It follows that the solution $f^* > 0$, $a = s + m > s$ to Equation (32) exists. Note that $a < f^*$ because of the first equation.

When $f > f^*$, we show that there exist r_L and r_S that satisfy the first order conditions in Equation (27)–(28). The first order conditions are

$$(f - r_S)G'(r_S - r_L + v) - G(r_S - r_L + v) = \tilde{B}(r_S, r_L; f, v), \tag{63}$$

$$(f - r_L)G'(r_S - r_L + v) - 1 + G(r_S - r_L + v) = \tilde{B}(r_S, r_L; f, v). \tag{64}$$

Taking difference between the two equations, we have $(r_S - r_L)G'(r_S - r_L + v) = 1 - 2G(r_S - r_L + v)$. Again, denote $g(m) = 1 - mG'(m+v) - 2G(m+v) = 0$. Recall that $m = a - s$ (a is defined above) is a solution to $g(m) = 0$. And because $s < a < f^* < f$, we know that $m = a - s \in [0, f - s]$. In fact, $g(m) = 0$ has a unique solution on $m \in [0, f - s]$ since $g'(m) = -mG''(m+v) - 3G'(m+v) < 0$ under Assumption 3.

Thus, $r_S - r_L = a - s$. Now we solve the first equation, while imposing $r_S - r_L = a - s$. The equation becomes $\tilde{n}(r_S) = (f - r_S)G'(a - s + v) - G(a - s + v) - \tilde{B}(r_S, r_S - a + s; f, v) = 0$. Under the assumption that $X + 1 - \frac{1+f}{A} > 1/A$, analogous to the proof of Proposition 1, we can show that $\tilde{n}(r_S) = 0$ has a solution $r_S \in (a, f)$.

When $f < f^*$, we show that there exists $r_S \in (s, a)$ that satisfies the first order conditions in Equation (30)–(31). The first order conditions are:

$$(f - r_S)G'(r_S - s + v) - G(r_S - s + v) = \tilde{B}(r_S, s; f, v), \tag{65}$$

$$(f - s)G'(r_S - s + v) - 1 + G(r_S - s + v) < \tilde{B}(r_S, s; f, v). \tag{66}$$

Let $\tilde{u}(x; f, v) = (f - x)G'(x - s + v) - G(x - s + v) - \tilde{B}(x, s; f, v)$. We first show that given

f there exists a solution $x \in (s, a)$ to $\tilde{u}(x; f, v) = 0$, which is the first equation. Second, we show that this solution satisfies the inequality stated by the second equation.

To show that $\tilde{u}(x; f, v) = 0$ has a solution $x \in (s, a)$, first note that under the assumption that $X + 1 - (1+f)/A > 1/A$, we have $\tilde{B}(r_S, r_L; f, v) < \frac{1}{4} f^2 G'(r_s - r_L + v)$. $\tilde{u}(x; f, v)$ is continuous in x. Also, $\tilde{u}(s; f, v) = (f - s)G'(v) - G(v) - \tilde{B}(s, s; f, v) > (f - s - f^2/4)G'(v) - G(v) > 0$. Moreover, $\tilde{u}(a; f, v) < 0$, because $\tilde{u}(a; f^*, v) = 0$, $f < f^*$ and that $\tilde{u}(x; f, v)$ is increasing in f. The last fact is because, as shown in the proof of Proposition 1, under the assumption that $X + 1 - \frac{1+f}{A} > 1/A$ and $f \in (0, 1]$, we have $\partial B(r_S, r_L; f, v)/\partial f < G'(r_S - r_L + v)$, and as a result $\partial u(x; f, v)/\partial f = G'(x - s + v) - \partial B(x, s; f, v)/\partial f > 0$. In sum, $\tilde{u}(x; f, v) = 0$ has a solution $x \in (s, a)$.

Now we show that the solution $r_S = x$ satisfies the inequality stated by the second equation. This is equivalent to showing $(r_S - s)G'(r_S - s + v) - 1 + 2G(r_S - s + v) < 0$. Equations (63) and (64) suggest that $(a - s)G'(a - s + v) - 1 + 2G(a - s + v) = 0$. Note that under Assumption 3 the function $g(m) = 1 - mG'(m + v) - 2G(m + v)$ is strictly decreasing in m. Since $r_S < a$, we have $g(r_S) > g(a) = 0$. As a result, $(r_S - s)G'(r_S - s + v) - 1 + 2G(r_S - s + v) = -g(r_S) < 0$. Thus, the inequality is satisfied.

The steps to verify the second order conditions, and to prove the properties of deposit market shares and loan qualities under Assumption 3 are analogous to the proofs of Proposition 1 and Proposition 2, hence we omitted them.

Proof of Proposition 4

A. Deposit market

Since the large bank is constrained by the lower bound, its deposit rate r_L rises in step with the payment token interest rate s. To show that r_S is increasing in s, let $\Gamma_S = d\Pi_S/dr_S$, and start with the expression

$$0 = \frac{\partial \Gamma_S}{\partial s} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{ds}. \tag{67}$$

We have verified the second order condition $\partial \Gamma_S/\partial r_S < 0$. Thus, a sufficient condition for $dr_S/ds > 0$ is $\partial \Gamma_S/\partial s > 0$. Writing total loan volume as $V = \alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$, we have

$$\begin{aligned}\frac{\partial \Gamma_S}{\partial s} &= [(f - r_S)G'(r_S - s + v) - G(r_S - s + v)]\frac{\partial V}{\partial s} \\ &\quad + (X + V)[-(f - r_S)G''(r_S - s + v) + G'(r_S - s + v)] \\ &\qquad + \frac{1}{A}(f - r_S)\frac{\partial}{\partial s}[\alpha_L \alpha_S (f - s)G'(r_S - s + v)].\end{aligned} \tag{68}$$

The first and third terms are bounded, by G being twice-differentiable. Under the assumption that

$X \to \infty$, the second term dominates. Under Assumption 1, we have $-(f - r_S)G''(r_S - s + v) + G'(r_S - s + v) > 0$, so a sufficiently large X implies that $\partial\Gamma_S/\partial s > 0$. Thus, we have $dr_S/ds > 0$.

Next, we show that α_L is increasing in s and α_S is decreasing in s. We first prove that $r_S - s$ decreases in s. We have

$$\begin{aligned}\frac{\partial\Gamma_S}{\partial r_S} &= X\frac{\partial}{\partial r_S}[(f - r_S)G'(r_S - s + v) - G(r_S - s + v)] \\ &+ \frac{\partial}{\partial r_S}\{V \cdot [(f - r_S)G'(r_S - s + v) - G(r_S - s + v)]\} \\ &- \frac{\partial}{\partial r_S}\left[\frac{1}{A}\alpha_L\alpha_S(f - s)(f - r_S)G'(r_S - s + v)\right]. \end{aligned} \tag{69}$$

Under the assumption that $X \to \infty$, the first term dominates the other terms. The first term equals $X[(f - r_S)G''(r_S - s + v) - 2G'(r_S - s + v)]$. We have,

$$\frac{dr_S}{ds} = -\frac{\partial\Gamma_S/\partial s}{\partial\Gamma_S/\partial r_S} \to \frac{(f - r_S)G''(r_S - s + v) - G'(r_S - s + v)}{(f - r_S)G''(r_S - s + v) - 2G'(r_S - s + v)}, \tag{70}$$

as $X \to \infty$. We have $d(r_S - s)/ds = \frac{G'(r_S - s + v)}{(f - r_S)G''(r_S - s + v) - 2G'(r_S - s + v)}$, where the denominator is negative under Assumption 3 and the numerator is positive. Hence, $d(r_S - s)/ds < 0$. This implies that $\alpha_S = G(r_S - s + v)$ is decreasing in s, and α_L is increasing in s.

We now show that the weighted average interest rate $\alpha_S r_S + \alpha_L s$ is increasing in s. The derivative of $\alpha_S r_S + \alpha_L s$ with respect to s is

$$\frac{d(\alpha_S r_S + \alpha_L s)}{ds} = \frac{d\alpha_S}{ds}r_S + \alpha_S\frac{dr_S}{ds} + \frac{d\alpha_L}{ds}s + \alpha_L = [(r_S - s)G'(r_S - s + v) + \alpha_S]\frac{d(r_S - s)}{ds} + 1. \tag{71}$$

By the calculation above, $d(r_S - s)/ds \to \frac{G'(r_S - s)}{(f - r_S)G''(r_S - s + v) - 2G'(r_S - s + v)} > -\frac{f}{f + r_S}$ as $X \to \infty$ and under Assumption 3. Thus we have

$$\frac{d(\alpha_S r_S + \alpha_L s)}{ds} > 1 - \frac{f}{f + r_S}[(r_S - s)G'(r_S - s + v) + \alpha_S] \geq 1 - \frac{f}{f + r_S} > 0, \tag{72}$$

where the second last inequality follows from the large bank's first-order condition as $X \to \infty$: $(f - s)G'(r_S - s + v) + G(r_S - s + v) \leq 1$.

B. Loan market

Now we turn to loan market outcomes. Since α_S decreases in s and r_S increases in s, we know that $\alpha_S(f - r_S)$ is decreasing in s and q_S^* is increasing in s. The small bank's loan volume, $\alpha_S(1 - q_S^*)$, is then decreasing in s.

For the large bank's loan quality q_L^*, we have

$$\frac{dq_L^*}{ds} = -\frac{1}{A}\Big[G'(r_S - s + v)(1 - \frac{dr_S}{ds})(f - s) - 1 + G(r_S - s + v)\Big]. \tag{73}$$

For the first term in the brackets, we know that $G'(r_S - s + v)(1 - \frac{dr_S}{ds})(f - s) < (f - s)G'(r_S - s + v)$, since $dr_S/ds > 0$. Also, from the large bank's optimality condition, as $X \to \infty$, we have $(f - s)G'(r_S - s + v) - 1 + G(r_S - s + v) \leq 0$. That means $dq_L^*/ds > 0$. However, the impact of s on the large bank's loan volume $\alpha_L(1 - q_L^*)$ is ambiguous.

We now show that the total loan volume is $\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)$ is decreasing in s if $G''(\delta) \leq 0$.As $X \to \infty$, the two banks' first-order conditions imply that

$$\begin{aligned}(f - s)G'(r_S - s + v) - \underbrace{(1 - G(r_S - s + v))}_{\alpha_L} &\leq 0,\\ (f - r_S)G'(r_S - s + v) - \underbrace{G(r_S - s + v)}_{\alpha_S} &= 0.\end{aligned} \tag{74}$$

Multiplying the first equation by α_L and and the second equation by α_S, we get

$$\begin{aligned}\alpha_L(f - s)G'(r_S - s + v) - \alpha_L^2 &\leq 0,\\ \alpha_S(f - r_S)G'(r_S - s + v) - \alpha_S^2 &= 0.\end{aligned} \tag{75}$$

Next, take the derivative of the total loan volume with respect to s, and plug in the two conditions we just derived, to get

$$\begin{aligned}\frac{d}{ds}[\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*)] &= \frac{1}{A}[2\alpha_S(f - r_S) - 2\alpha_L(f - s)]G'(r_S - s + v)\left(\frac{dr_S}{ds} - 1\right)\\ &\quad - \frac{1}{A}\alpha_L^2 - \frac{1}{A}\alpha_S^2\frac{dr_S}{ds} \qquad (76)\\ &\leq \frac{1}{A}(2\alpha_L^2 - 2\alpha_S^2)\left(1 - \frac{dr_S}{ds}\right) - \frac{1}{A}\alpha_L^2 - \frac{1}{A}\alpha_S^2\frac{dr_S}{ds}\\ &= \frac{1}{A}\left[\left(1 - 2\frac{dr_S}{ds}\right)\alpha_L^2 + \left(\frac{dr_S}{ds} - 2\right)\alpha_S^2\right]. \qquad (77)\end{aligned}$$

Because $\frac{dr_S}{ds} < 1$, we have $(\frac{dr_S}{ds} - 2)\alpha_S^2 < 0$. If $G''(\delta) \leq 0$ and $X \to \infty$, we know from the expression of $\frac{dr_S}{ds}$ above that $\frac{dr_S}{ds} \geq \frac{1}{2}$. That means $(1 - 2\frac{dr_S}{ds})\alpha_L^2 \leq 0$. Thus, the total loan volume is decreasing in s.

C. Impact of s on monetary policy pass-through

We now show that the cut-off value f^* that separates the constrained equilibrium and the unconstrained equilibrium is increasing in s. Given v and s, we solve for f^*. Let f^*, $r_S = a$

be the solution to Equation (59) and (60). Taking the difference of the two equations, we have $(a-s)G'(a-s+v)-1+2G(a-s+v)=0$. Note that $a-s=m(v)$ is a function solely of v. When $X\to\infty$, Equation (59) suggests that

$$f^* = s + \frac{G(m(v)+v)}{G'(m(v)+v)} + m(v). \tag{78}$$

We see that f^* increases one-for-one with s when $X\to\infty$.

Proof of Proposition 5

First we consider the unconstrained equilibrium and then the constrained one.

1. The unconstrained equilibrium

A. Deposit market

We know that $s<r_L<r_S<f$, and $\alpha_S<\frac{1}{2}<\alpha_L$. Let $\Gamma_S=d\Pi_S/dr_S$, $\Gamma_L=d\Pi_L/dr_L$. To calculate how r_L and r_S are affected by v, we take derivative of Γ_L and Γ_S at the equilibrium values and obtain

$$0 = \frac{\partial \Gamma_L}{\partial v} + \frac{\partial \Gamma_L}{\partial r_L}\frac{dr_L}{dv} + \frac{\partial \Gamma_L}{\partial r_S}\frac{dr_S}{dv}, \tag{79}$$
$$0 = \frac{\partial \Gamma_S}{\partial v} + \frac{\partial \Gamma_S}{\partial r_L}\frac{dr_L}{dv} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{dv}. \tag{80}$$

We solve for $\frac{dr_L}{dv}$ and $\frac{dr_S}{dv}$ from above equations. Denote $A_v=\frac{\partial\Gamma_L}{\partial v}$, $A_L=\frac{\partial\Gamma_L}{\partial r_L}$, $A_S=\frac{\partial\Gamma_L}{\partial r_S}$, $B_v=\frac{\partial\Gamma_S}{\partial v}$, $B_L=\frac{\partial\Gamma_S}{\partial r_L}$, and $B_S=\frac{\partial\Gamma_S}{\partial r_S}$. Then we have,

$$\frac{dr_L}{dv} = \frac{A_S B_v - B_S A_v}{A_L B_S - B_L A_S}, \tag{81}$$
$$\frac{dr_S}{dv} = \frac{B_L A_v - A_L B_v}{A_L B_S - B_L A_S}. \tag{82}$$

When $X\to\infty$, we have $A_v\approx A_S\approx X[(f-r_L)G''(r_S-r_L+v)+G'(r_S-r_L+v)]$, $B_v\approx -B_L\approx X[(f-r_S)G''(r_S-r_L+v)-G'(r_S-r_L+v)]$, $A_L\approx X[-(f-r_L)G''(r_S-r_L+v)-2G'(r_S-r_L+v)]$, and $B_S\approx X[(f-r_S)G''(r_S-r_L+v)-2G'(r_S-r_L+v)]$. Hence, $\frac{dr_L}{dv}\to\frac{(f-r_L)G''(r_S-r_L+v)+G'(r_S-r_L+v)}{(r_S-r_L)G''(r_S-r_L+v)+2G'(r_S-r_L+v)}$, and $\frac{dr_S}{dv}\to\frac{(f-r_S)G''(r_S-r_L+v)-G'(r_S-r_L+v)}{(r_S-r_L)G''(r_S-r_L+v)+2G'(r_S-r_L+v)}$. Under Assumption 3, we have $(r_S-r_L)G''(r_S-r_L+v)+2G'(r_S-r_L+v)>0$, $(f-r_L)G''(r_S-r_L+v)+G'(r_S-r_L+v)>0$, and $(f-r_S)G''(r_S-r_L+v)-G'(r_S-r_L+v)<0$. Thus, we have $\frac{dr_L}{dv}>0$ and $\frac{dr_S}{dv}<0$. In sum, r_L is increasing in v and r_S is decreasing in v.

For deposit market share $\alpha_S=G(r_S-r_L+v)$, we take the difference of the two FOCs, and have

$$(r_S - r_L)G'(r_S - r_L + v) + 2G(r_S - r_L + v) = 1. \tag{83}$$

Write $y = r_S - r_L + v$, and take derivative of the above equation with respect to v, then we have

$$[3G'(y) + (r_S - r_L)G''(y)]\frac{dy}{dv} - G'(y) = 0. \tag{84}$$

Under Assumption 3, we have $3G'(y) + (r_S - r_L)G''(y) > 0$, hence $\frac{dy}{dv} > 0$. Thus, α_S is increasing in v, and α_L is decreasing in v.

The weighted average deposit rate is $\alpha_S r_S + \alpha_L r_L = \alpha_S(r_S - r_L) + r_L$. Its derivative with respect to v is

$$\begin{aligned}\frac{d(\alpha_S r_S + \alpha_L r_L)}{dv} &= \frac{d\alpha_S}{dv}(r_S - r_L) + \alpha_S\frac{d(r_S - r_L)}{dv} + \frac{dr_L}{dv} \\ &> \frac{d\alpha_S}{dv}(r_S - r_L) + \frac{1}{2}\frac{d(r_S - r_L)}{dv} + \frac{dr_L}{dv} = \underbrace{\frac{d\alpha_S}{dv}}_{>0}(r_S - r_L) + \frac{1}{2}\frac{d(r_L + r_S)}{dv},\end{aligned} \tag{85}$$

where the inequality follows from $\alpha_S < \frac{1}{2}$ and that $r_S - r_L$ is decreasing in v. As $X \to \infty$, we have

$$\frac{d(r_S + r_L)}{dv} \to \frac{(2f - r_L - r_S)G''(r_S - r_L + v)}{(r_S - r_L)G''(r_S - r_L + v) + 2G'(r_S - r_L + v)}. \tag{86}$$

The denominator is positive under Assumption 3. As $r_L + r_S < 2f$, when $G''(\delta) \geq 0$, we have $\frac{d(r_L + r_S)}{dv} \geq 0$, and it follows that $\alpha_S r_S + \alpha_L r_L$ increases in v.

B. Loan market

For loan quality thresholds, since α_L is decreasing in v and r_L is increasing in v, q_L^* is increasing in v. Since α_S is increasing in v and r_S is decreasing in v, q_S^* decreasing in v.

For loan volumes, $\alpha_L(1 - q_L^*)$ is decreasing in v, since α_L is decreasing and q_L^* is increasing. Similarly, $\alpha_S(1 - q_S^*)$ is increasing in v.

Total loan volume equals $\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) = 1 - \frac{1+f}{A} + \frac{\alpha_L^2(f - r_L) + \alpha_S^2(f - r_S)}{A}$. Its derivative with respect to v is

$$\frac{1}{A}\left\{[2\alpha_S(f - r_S) - 2\alpha_L(f - r_L)]\frac{d\alpha_S}{dv} - \alpha_L^2\frac{dr_L}{dv} - \alpha_S^2\frac{dr_S}{dv}\right\}, \tag{87}$$

where $2\alpha_S(f - r_S) - 2\alpha_L(f - r_L) < 0$, $\frac{d\alpha_S}{dv} > 0$, $\frac{dr_L}{dv} > 0$, and $\frac{dr_S}{dv} < 0$. We know that $-\alpha_L^2\frac{dr_L}{dv} - \alpha_S^2\frac{dr_S}{dv} < -\alpha_S^2\frac{d(r_L + r_S)}{dv}$. If $G''(\delta) \geq 0$, we know from above that $\frac{d(r_L + r_S)}{dv} \geq 0$, so $-\alpha_L^2\frac{dr_L}{dv} - \alpha_S^2\frac{dr_S}{dv} \leq$

0, and Equation (87) is negative. If $G''(\delta) < 0$, however, the sign of the equation is ambiguous.

2. The constrained equilibrium

A. Deposit market

To calculate how r_S is affected by v, we take the derivative of Γ_S at the equilibrium values and obtain

$$0 = \frac{\partial \Gamma_S}{\partial v} + \frac{\partial \Gamma_S}{\partial r_S}\frac{dr_S}{dv}. \tag{88}$$

When $X \to \infty$, the term $X[(f - r_S)G''(r_S - s + v) - G'(r_S - s + v)]$ dominates $\partial\Gamma_S/\partial v$. Under Assumption 3 we have $-G'(\delta)/f < G''(\delta) < G'(\delta)/f$ for any $\delta \in [0, f - s + v]$, thus we have $\partial\Gamma_S/\partial v < 0$. The second-order condition implies that $\partial\Gamma_S/\partial r_S < 0$. Hence $dr_S/dv < 0$, i.e., r_S is decreasing in v.

Now we examine how deposit market share $\alpha_S = G(r_S - s + v)$ is affected by v. When $X \to \infty$, we have

$$\frac{dr_S}{dv} = -\frac{\partial \Gamma_S}{\partial v}\Big/\frac{\partial \Gamma_S}{\partial r_S} \to -\frac{(f - r_S)G''(r_S - s + v) - G'(r_S - s + v)}{(f - r_S)G''(r_S - s + v) - 2G'(r_S - s + v)}. \tag{89}$$

Hence,

$$\frac{d(r_S - s + v)}{dv} = \frac{dr_S}{dv} + 1 = \frac{-G'(r_S - s + v)}{(f - r_S)G''(r_S - s + v) - 2G'(r_s - s + v)}, \tag{90}$$

where the numerator and the denominator are both negative. So $\frac{d(r_S - s + v)}{dv} > 0$, i.e., α_S is increasing in v and α_L is decreasing in v.

We show that if $G''(\delta) \le 0$ for any $\delta \in [0, f - s + v]$, then the weighted average deposit rate is weakly decreasing in v. We take the derivative of the weighted average deposit rate with respect to v:

$$\begin{aligned}\frac{d}{dv}(\alpha_L s + \alpha_S r_S) &= \frac{d\alpha_S}{dv}(r_S - s) + \alpha_S \frac{dr_S}{dv} \\ &= \frac{(f - s)dr_S/dv + r_S - s}{f - r_S}\alpha_S,\end{aligned} \tag{91}$$

where the second equality uses $d\alpha_S/dv \to \frac{\alpha_S}{(f - r_S)}(dr_S/dv + 1)$, implied by the small bank's FOC when $X \to \infty$. We have $\frac{d}{dv}(\alpha_L s + \alpha_S r_S) \le 0$ if and only if $(f - s)dr_S/dv + r_S - s \le 0$. Write $y = r_S - r_L + v$. Plugging in $\frac{dr_S}{dv} = -\frac{(f - r_S)G''(y) - G'(y)}{(f - r_S)G''(y) - 2G'(y)}$, we have $\frac{d}{dv}(\alpha_L s + \alpha_S r_S) \le 0$ if and only if $G''(r_S - s + v) \le \frac{f + s - 2r_S}{(f - r_S)^2}G'(r_S - s + v)$. For this to hold when $G''(\delta) \le 0$, we only need to show that $f + s - 2r_S \geq 0$. Since r_S is decreasing in v, we only need to show that, given s, $f + s - 2r_S \geq 0$ when $v = 0$. Let $l(x) = (f - x)G'(x - s) - G(x - s)$, then $l(r_S) = 0$, and $\frac{dl(x)}{dx} = (f - x)G''(x - s) - 2G'(x - s) < 0$ under Assumption 3. To show that $f + s - 2r_S \geq 0$, we only need to show $l(\frac{1}{2}(f + s)) \leq 0$, equivalent to $\frac{f-s}{2}G'(\frac{f-s}{2}) - G(\frac{f-s}{2}) \leq 0$. This is true because if we let $m(x) = xG'(x) - G(x)$, then $m(0) = 0$ and $\frac{dm(x)}{dx} = xG''(x) \leq 0$ under the assumption that $G''(\delta) \leq 0$. As a result, we have $m(\frac{f-s}{2}) = \frac{f-s}{2}G'(\frac{f-s}{2}) - G(\frac{f-s}{2}) \leq 0$. Based on this proof, the weighted average deposit rate is strictly decreasing in v, whenever $v > 0$.

B. Loan market

For loan quality thresholds and individual banks' loan volumes, the same proofs for the unconstrained equilibrium apply and are omitted.

Total loan volume equals $\alpha_L(1 - q_L^*) + \alpha_S(1 - q_S^*) = 1 - \dfrac{1 + f}{A} + \dfrac{\alpha_L^2(f - s) + \alpha_S^2(f - r_S)}{A}$. Its derivative with respect to v is

$$\frac{1}{A}\left\{[2\alpha_S(f - r_S) - 2\alpha_L(f - s)]\frac{d\alpha_S}{dv} - \alpha_S^2\frac{dr_S}{dv}\right\}. \tag{92}$$

Its sign is ambiguous because while the first term in the brackets is negative, the second term is positive.

C. Impact of v on monetary policy pass-through

We now show that the cut-off value f^* that separates the constrained equilibrium and the unconstrained equilibrium is decreasing in v. Let f^*, $r_S = a$ be the solution to Equation (59) and (60). Taking the difference of the two equations, and writing $a - s = m(v)$, we have $\Phi(m(v), v) = m(v)G'(m(v) + v) - 1 + 2G(m(v) + v) = 0$. We take derivative of Φ with respect to v to solve for $\frac{dm(v)}{dv}$:

$$0 = \frac{\partial\Phi}{\partial v} + \frac{\partial\Phi}{\partial m(v)}\frac{dm(v)}{dv}. \tag{93}$$

We have $\frac{dm(v)}{dv} = -\frac{\partial\Phi}{\partial v}/\frac{\partial\Phi}{\partial m(v)} = -\frac{m(v)G''(m(v)+v)+2G'(m(v)+v)}{m(v)G''(m(v)+v)+3G'(m(v)+v)}$. When $X \to \infty$, Equation (59) suggests that $f^* \to s + \frac{G(m(v)+v)}{G'(m(v)+v)} + m(v)$. Denoting $y = m(v) + v = a - s + v$, as $X \to \infty$, we have

$$\begin{aligned}\frac{df^*}{dv} &\to \frac{d}{dv}\left\{\frac{G(y)}{G'(y)}\right\} + \frac{dm(v)}{dv} \\ &= \frac{G'(y)^2[-m(v)G''(y) - G'(y)] - G''(y)G'(y)G(y)}{G'(y)^2[m(v)G''(y) + 3G'(y)]}.\end{aligned} \tag{94}$$

As $m(v) = a - s > 0$, if G satisfies that $G''(\delta) \geq 0$ for any $\delta \in [0, f - s + v]$, then the numerator is negative and the denominator is positive. In this case we have $\frac{df^*}{dv} < 0$.