

Does a Central Clearing Counterparty Reduce Counterparty Risk?

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We show whether central clearing of a particular class of derivatives lowers counterparty risk. For plausible cases, adding a central clearing counterparty (CCP) for a class of derivatives such as credit default swaps reduces netting efficiency, leading to an increase in average exposure to counterparty default. Further, clearing different classes of derivatives in separate CCPs always increases counterparty exposures relative to clearing the combined set of derivatives in a single CCP. We provide theory as well as illustrative numerical examples of these results that are calibrated to notional derivatives position data for major banks. (*JEL* G01, G14, G18, G28)

1. Introduction

A key element of the new regulatory approach to financial stability is the central clearing of derivatives. A central clearing counterparty (CCP) stands between over-the-counter (OTC) derivatives counterparties, insulating them from each other's default. Effective clearing mitigates systemic risk by lowering the likelihood that defaults propagate from counterparty to counterparty. Clearing could also reduce the degree to which the solvency problems of a market participant are suddenly compounded by a flight of its OTC derivative counterparties, such as when the solvency of Bear Stearns and Lehman Brothers was in question.¹ Finally, central clearing reduces the risk of disruptions to financial

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¹ Many of Bear Stearns's counterparties asked other dealers for novations, by which those dealers would effectively stand between Bear Stearns and its counterparties, absorbing the risk of a failure by Bear Stearns. See "Fear, Rumors Touched Off Fatal Run on Bear Stearns," by Kate Kelly, *WSJ.com*, May 28, 2008. Kelly reported, "Hedge funds flooded Credit Suisse Group's brokerage unit with requests to take over trades opposite Bear Stearns. In a blast e-mail sent out that afternoon, Credit Suisse stock and bond traders were told that all such novation requests involving Bear Stearns and any other 'exceptions' to normal business required the approval

markets through fire sales of derivatives positions or of collateral held against derivatives positions.

The Dodd-Frank Act of the United States Congress, passed in July 2010, stipulates that all sufficiently standard derivatives traded by major market participants must be cleared in regulated CCPs. The [European Commission \(2010\)](#) has taken similar steps.

Our objective is to model whether the central clearing of a particular class of derivatives increases or reduces counterparty exposures. For plausible cases, adding a new CCP dedicated to a class of derivatives such as credit default swaps (CDS) reduces netting efficiency, increases collateral demands, and leads to higher average exposure to counterparty default. We further show that counterparty credit risk in the OTC derivatives market is exacerbated by a multiplicity of CCPs. Using recent data on the OTC derivatives positions of U.S. banks, we provide illustrative numerical examples of the adverse counterparty-risk impact of splitting clearing across CCPs. We also prove that counterparty risk is always reduced by merging the clearing activities of multiple CCPs into a single CCP. For CDS alone, approved CCPs include two based in the United States and several based in Europe. A number of additional CDS clearing houses have been proposed for the United States, Europe, and Asia.² Attempts to obtain the effective benefits of concentrating clearing into fewer CCPs through cross-CCP “interoperability” agreements appear to be stalled by both technical and strategic impediments.

While the central clearing of derivatives can in principle offer substantial reductions in counterparty risk, we provide a foundation for concerns that these benefits may be lost through a fragmentation of clearing services.

Our results are based on a simple model, but clarify an important trade-off between two types of netting opportunities: bilateral netting between pairs of counterparties across different underlying assets, versus multilateral netting among many clearing participants across a single class of derivatives, such as credit default swaps (CDS). The introduction of a CCP for a particular class such as standard credit derivatives is effective only if the opportunity for multilateral netting in that class dominates the resulting loss in bilateral netting opportunities across all uncleared derivatives, such as uncleared CDS and

of credit-risk managers.” In “Bringing Down Bear Stearns,” *Vanity Fair*, August 2008, Bryan Burroughs writes: “That same day Bear executives noticed a worrisome development whose potential significance they would not appreciate for weeks. It involved an avalanche of what are called ‘novation’ requests. When a firm wants to rid itself of a contract that carries credit risk with another firm, in this case Bear Stearns, it can either sell the contract back to Bear or, in a novation request, to a third firm for a fee. By Tuesday afternoon, three big Wall Street companies—Goldman Sachs, Credit Suisse, and Deutsche Bank—were experiencing a torrent of novation requests for Bear instruments.”

² U.S.-approved CCPs for CDS are those of the ICE Trust and the CME Group. Proposed U.S. CDS CCPs include those of Euronext Liffe and Eurex (part of the Deutsche Börse). Current and proposed European CCPs include those of ICE Clear Europe, NYSE-LIFFE/BClear, and LCH.Clearnet, Eurex, and LCH.Clearnet SA (a French subsidiary of LCH.Clearnet, dedicated to Eurozone CDS clearing). Those proposed for Asia include initiatives of Japan Securities Clearing Corporation (JSCC) and Tokyo Financial Exchange (TFX).

uncleared OTC derivatives of equities, interest rates, commodities, and foreign exchange, among others.

The intuition of our results is simple. Suppose that Dealer *A* is exposed to Dealer *B* by \$100 million on CDS, while at the same time Dealer *B* is exposed to Dealer *A* by \$150 million on interest-rate swaps. The bilateral exposure is the net, \$50 million. The introduction of central clearing dedicated to CDS eliminates the bilateral netting benefits and increases the exposure between these two dealers, before collateral, from \$50 million to \$150 million. In addition to any collateral posted by Dealer *A* to the CCP for CDS, Dealer *A* would need to post a significant amount of additional collateral to Dealer *B*. Collateral is a scarce resource, especially in a credit crisis.

The introduction of a CCP for CDS can nevertheless be effective when there are extensive opportunities for multilateral netting. For example, if Dealer *A* is exposed by \$100 million to Dealer *B* through a CDS, while Dealer *B* is exposed to Dealer *C* for \$100 million on the same CDS, and Dealer *C* is simultaneously exposed to Dealer *A* for the same amount on the same CDS, then a CCP eliminates this unnecessary circle of exposures. The introduction of a CCP therefore involves an important tradeoff between bilateral netting without the CCP and multilateral netting through the CCP.

Naturally, our results show that introducing a CCP for a particular set of derivatives reduces average counterparty exposures if and only if the number of clearing participants is sufficiently large relative to the exposure on derivatives that continue to be bilaterally netted. For example, our model suggests that clearing CDS through a dedicated CCP improves netting efficiency for twelve similarly sized dealers if and only if the fraction of a typical dealer's total expected exposure attributable to cleared CDS is at least 66% of the total expected exposure of remaining bilaterally netted classes of derivatives. It is far from obvious that clearable exposures in the CDS market are nearly this large.

Our results show that a single central clearing counterparty that clears both credit derivatives and interest-rate swaps is likely to offer significant reductions in expected counterparty exposures, even for a relatively small number of clearing participants. For example, in a simple illustrative calculation based on data provided by U.S. banks, we show that once 75% of interest-rate swaps are cleared, the incremental reduction in before-collateral average expected counterparty exposures obtained by clearing 75% of credit derivatives in a separate CCP is negligible, because of the loss of bilateral netting opportunities. In the same setting, however, clearing these credit derivatives in the same CCP used for interest-rate swaps reduces average expected exposures by about 7%, despite the loss of bilateral netting opportunities. Relative to the case of fully bilateral netting (no clearing), substantial benefits can be obtained by the joint clearing of the four major classes of derivatives monitored by the Office of the Comptroller of the Currency. Our rough estimates suggest that the joint clearing of 75% of interest-rate swaps and credit derivatives, along with 40% of

other derivatives classes, results in a 37% reduction in pre-collateral expected counterparty exposures, relative to a market without CCPs.

2. Netting Efficiency in an OTC Market

We consider N market participants whose over-the-counter derivative exposures to each other are of concern. These entities may have the opportunity to novate some OTC derivative positions to a central clearing counterparty (CCP). For example, if entities i and j have a CDS position by which i buys protection from j , then both i and j can novate to a CCP, who is then the seller of protection to i and the buyer of protection from j . Novation to a CCP is sometimes called “clearing,” although the term “clearing” is often used in other contexts.³

We allow for K classes of derivatives. These classes could be defined by the underlying asset classes, such as credit, interest rates, foreign exchange, commodities, and equities. One can also construct derivatives classes by grouping more than one underlying asset type, or by separating the derivatives in a given asset into the subset of derivatives that are sufficiently standard to be cleared and the complementary set of derivatives that are too customized or thinly traded to justify clearing.

We take the perspective of a regulator or industry coordinator that is considering the design of the market clearing architecture from the viewpoint of exposures of market participants to each other on a typical day in the future, well after the architecture has been chosen and market participants have determined their approaches to clearing.

For entities i and j , let X_{ij}^k be the amount that j will owe i in all positions in some derivatives class k , before considering the benefits of netting across asset classes, collateral, and default recovery.⁴ We call $\max(X_{ij}^k, 0)$ the *exposure* of i to j in derivative class k . This exposure is the amount entity i risks losing upon the default of entity j . Similarly, the exposure of entity j to i is $\max(-X_{ij}^k, 0)$ because, by definition,

$$X_{ij}^k = -X_{ji}^k. \quad (1)$$

Before setting up a CCP, X_{ij}^k is uncertain in part because the derivatives positions between i and j that will exist on this typical future day, possibly years in the future, are yet to be determined. The volatilities and correlations of prices on that future date are also uncertain at the point of market design. The exposure also reflects uncertainty regarding the degree of netting that is achieved in the master swap agreements, due in part to uncertain changes in market prices before this future date. In addition, the uncertainty in X_{ij}^k includes the risks

³ See Bliss and Steigerwald (2006), Pirrong (2009), and Stulz (2010) for discussions of CCPs in the context of an over-the-counter derivatives market.

⁴ Bliss and Kaufman (2006) provide an analysis of the legal implications of settlement of OTC exposures at default.

associated with marks to market that will occur between the times at which additional collateral can be requested and received, respectively.

We emphasize that the uncertainty regarding X_{ij}^k that matters from the viewpoint of market design some months or years before the exposure date *cannot* be based on conditioning information that becomes available just before this future date. In particular, a market designer cannot know in advance the notional sizes of derivatives positions that will be in place at a future time. Position uncertainty, both size and direction, is to be incorporated into the probability distribution of the future exposures. This distinction is also important for other CCP design problems, such as the sizing of the CCP's capital, initial margin requirements, and default guarantee fund, which are chosen well in advance, with the objective of being sufficient for default management across a wide variety of portfolios that could be submitted for clearing and across a wide variety of market price volatilities and correlations.

For now, we suppose that all (X_{ij}^k) are of the same variance and are independent across asset classes and pairs of entities, excluding the obvious case represented by (1). We later relax all of these assumptions. For simplicity, we assume symmetry in the distributions of exposures across all pairs of entities. This implies in particular that $E(X_{ij}^k) = 0$. We will also relax the symmetry assumption. With N entities and K asset classes, there are $K \times N \times (N - 1)/2$ exposure distributions to be specified. Symmetry allows a dramatic reduction in the dimension of the problem.

A reasonable measure of the netting efficiency offered by a market structure is the total expected counterparty exposures of a typical entity, say entity i , to the default of the other $(N - 1)$ entities, before collateral is considered. Under bilateral netting, the exposure of i to any one of its counterparties, say j , is netted across all K derivative classes, but exposures to different counterparties cannot be netted. Before introducing a CCP, therefore, the total netting efficiency is

$$\phi_{N,K} = \sum_{j \neq i} E \left[\max \left(\sum_{k=1}^K X_{ij}^k, 0 \right) \right], \quad (2)$$

where we have used symmetry by fixing attention on a particular entity i . Assuming normality of X_{ij}^k , and symmetry across all $N - 1$ counterparties, we have

$$\phi_{N,K} = (N - 1) \sigma \sqrt{\frac{K}{2\pi}}, \quad (3)$$

where σ is the standard deviation of X_{ij}^k .

For given collateralization standards, the risk of loss caused by a counterparty default is typically increasing in average expected exposure. (Under normality and symmetry, essentially any reasonable risk measure is increasing in expected exposure.) Risk of loss from counterparty default is a first-order consideration for systemic risk analysis.

Going beyond counterparty default risk, as expected exposures go up, the expected amount of collateral that must be supplied goes up. Collateral use is expensive. In an OTC market without a CCP, whatever collateral is supplied by one counterparty is received by another, so the net use of collateral is always zero. The need to supply collateral is nevertheless onerous, for several reasons. First, some individual counterparties on a given day will supply more collateral to others than others supplied to them. The net drain on the assets that could be supplied as collateral is costly, because of the lost opportunity to use that collateral for secured borrowing, as a cash management buffer, or for securities lending as a rent-earning business. Second, there is a question of the timing of collateral settlement. One must often supply collateral to a particular counterparty on a given day before collateral is received from another counterparty. If this were not the case, for instance, there would be no specials in treasury repo markets. This sort of frictional demand for collateral, analogous to the demand for money that arises from a limited velocity of circulation of money, is considered by [Duffie, Gârleanu, and Pedersen \(2002\)](#). So long as the average cost of supplying collateral to others is larger, on average, than the average benefit of receiving collateral from others, a market with poorer netting efficiency is also a market with higher net cost of collateral use. For a simple illustration, if the amount of collateral to be supplied is on average some multiple U of exposure, and if the average benefit b per unit of collateral value received is less than the average cost c per unit of collateral value supplied, then the average net expected cost to an entity of collateral usage arising from counterparty exposure is $(c - b)U\phi_{N,K}$, where $\phi_{N,K}$ is the average total expected exposure measure defined above. Under market stress, collateral demand from derivative counterparties could exacerbate the liquidity problem of an already weakened dealer bank, as explained by [Duffie \(2010\)](#).

Although average expected exposure, after netting and before collateral, is a reasonable measure of a market's netting efficiency and is closely related to systemic risk, this measure misses some important aspects of systemic risk. Most importantly, this measure does not consider the joint determination of defaults across entities. In particular, as opposed to the joint solvency analysis of [Eisenberg and Noe \(2001\)](#), our netting efficiency measure does not consider the implications of jointly determined defaults in a network of entities. For example, the likelihood that entity i cannot cover its payments to j plays a causal role in determining the likelihood that entity j cannot cover its payments to entity m , and so on. Adding a CCP could in principle increase or decrease the potential for jointly determined defaults, depending on the capitalization of the CCP and of the clearing entities, and on the collateralization standards of bilateral netting and central clearing. In addition to the capital that it holds, a CCP is typically backed by member guarantees. (See the appendix of [Duffie, Li, and Lubke 2010](#).) An analysis of the implications of a CCP for the joint solvency of its members is beyond the scope of our research.

In addition to the benefits of a CCP from the viewpoint of netting and of insulating counterparties from each other’s default, a well-run central clearing counterparty can also offer improved and more harmonized trade and collateral settlement procedures than those that apply to uncleared derivatives, as suggested by the [Bank for International Settlements \(2007\)](#).

The assumption of normality clearly does not apply well to the changes in market value of many types of individual derivatives positions, such as individual CDS contracts, which have heavily skewed and fat-tailed market values due to jump-to-default risk. Bearing in mind that the modeled distribution of the exposures includes uncertainty regarding position sizes and directions, aggregating within the class of standard CDS could result in a net exposure of one entity to another that is substantially less skewed and less fat-tailed, given the diversification across underlying names and the effect of aggregating across long and short positions. For example, two dealers running large and active matched-book CDS intermediation businesses could have almost no skew in the distributions of their exposures to each other. That said, one cannot claim enough diversification from position diversification to justify normality based on a central-limit-theorem argument.

3. Netting Efficiency with a CCP

We consider the implications of a CCP for one class of derivatives, say class K . Taking the previously described setting, suppose that all positions in class K are novated to the same CCP. The expected exposure of entity i to this CCP is then

$$\gamma_N = E \left[\max \left(\sum_{j \neq i} X_{ij}^K, 0 \right) \right] = \sqrt{\frac{N-1}{2\pi}} \sigma. \tag{4}$$

In practice, the exposure of a clearing participant to a CCP has two components. The first part is the direct exposure to the failure of the CCP, as to any other counterparty. We have explicitly modeled this source of exposure. The second part of the exposure to the CCP is indirect, in the form of new contributions by the entity to the CCP guarantee fund that are payable in the event that one or more other members of the CCP fail. The latter exposure depends in part on the CCP rules for collateral, guarantee funds, and default management.⁵ We have not modeled these indirect exposures. Our measure of netting efficiency is thus likely to be somewhat biased in favor of clearing.

The expected exposure of entity i to the other $N-1$ entities for the remaining $K-1$ classes of derivatives is $\phi_{N,K-1}$. Thus, with a CCP for one class of derivatives, the average expected exposure is

$$\phi_{N,K-1} + \gamma_N. \tag{5}$$

⁵ See Appendices A and B of [Duffie, Li, and Lubke \(2010\)](#).

Introducing a CCP for this single class of derivatives therefore improves netting efficiency if and only if $\gamma_N + \phi_{N,K-1} < \phi_{N,K}$, which applies if and only if

$$K < \frac{N^2}{4(N-1)}. \quad (6)$$

Because a CCP normally collects initial margin from its members, but does not provide initial margins to them, a CCP usually does not post as much collateral to its counterparties as it receives from them. Thus, the comparison (6) overstates the benefits of a CCP from the viewpoint of collateral efficiency.

Based on (6), if there are $K = 2$ symmetric classes of uncleared derivatives, then central clearing of one of the classes improves netting efficiency if and only if there are at least seven entities clearing. If there are four symmetric classes of derivatives, then central clearing of one of the classes improves efficiency if and only if there are at least fifteen entities clearing. A CCP is always preferred, in terms of netting efficiency, if it handles all classes of derivatives (which is, in effect, the case of $K = 1$).

In Appendix A, we allow for correlations across derivatives classes, and show that the benefit of introducing central clearing increases if there is positive cross-class exposure correlation. We also point out that counterparties have an incentive to create exposures with each other that are negatively correlated across asset classes, in order to hedge their counterparty risks.

It could be argued that the exposure of an entity to a CCP is likely to be of less concern than its exposure to another entity, because a CCP is likely to be well regulated, bearing in mind the systemic risk posed by the potential failure of a CCP. We do not model this “benefit” of a CCP; our average expected exposure measure weights all counterparty exposures equally. Arguing the other way, the centrality of a CCP implies that its failure risk could be more toxic than that of other market participants.⁶ Likewise, we do not consider this effect.⁷

Our measure of netting efficiency is based on the total of the expected exposures of an entity to its counterparties. This measure does not consider concentration risk. Even putting aside the systemic risk of a CCP caused by its centrality, a CCP tends to represent a concentration of exposure to its counterparties. In our simple setting, this is true whenever the number of entities clearing one of the classes of derivatives is greater than the number of derivatives classes; that is, $N > K$. Specifically, the expected exposure of an entity

⁶ Examples of clearing-house failures include those of Caisse de Liquidation, Paris (1974), the Kuala Lumpur Commodity Clearing House (1983), and the Hong Kong Futures Guarantee Corporation (1987). See Hills et al. (1999). In 1995, the Singapore International Monetary Exchange borrowed \$300 million to cover potential shortfalls associated with the futures positions of Barings Bank. See Edward Gargan, “Singapore Defends Itself Over Barings,” *New York Times*, March 2, 1995.

⁷ For a more comprehensive review of policy issues regarding OTC derivatives market infrastructure, see Duffie, Li, and Lubke (2010) and European Central Bank (2009).

to its CCP, as a multiple of that entity’s expected exposure to each of its other counterparties, is $\sqrt{(N - 1)/(K - 1)}$. For instance, if there are $N = 10$ entities and $K = 5$ classes of equally risky derivatives, then after novation of positions in one class to a CCP, the expected exposure of an entity to the CCP is 50% more than its exposure to any other counterparty.⁸

3.1 Derivatives classes with different degrees of risk

We now generalize by considering the netting efficiency allowed by the central clearing of a class of derivatives that could have particularly large exposures relative to other classes of derivatives. That is, we now allow the expected exposure $E[\max(X_{ij}^k, 0)]$ of class k to be different from that of another class. Our other assumptions are maintained. A class could include derivatives with more than one underlying asset type. For example, we could group together all CDS and all interest-rate swaps into a single class for clearing purposes.

Suppose that derivatives in class K are under consideration for clearing. The ratio of an entity’s expected exposure with a given counterparty in this asset class to the total expected exposure with the same counterparty in all other classes combined is

$$R = \frac{E \left[\max \left(X_{ij}^K, 0 \right) \right]}{E \left[\max \left(\sum_{k < K} X_{ij}^k, 0 \right) \right]} \tag{7}$$

For example, if all classes have equal expected exposures, then $R = 1/\sqrt{K - 1}$, using the fact that expected exposures are proportional to standard deviations. If class- K exposures are twice as big (in terms of expected exposure) as each of the other $K - 1$ classes, then $R = 2\sqrt{1/(K - 1)}$. A calculation analogous to that shown previously for the symmetric case leads to the following result.

Proposition 1. The introduction of a CCP for a particular class of derivatives leads to a reduction in average expected counterparty exposures if and only if

$$R > \frac{2\sqrt{N - 1}}{N - 2}, \tag{8}$$

where R is the ratio of the pre-CCP expected entity-to-entity exposures of the class in question to the expected entity-to-entity exposures of all other classes combined.

For example, we can take the case of $N = 12$ entities, approximately the number of entities that partnered with ICE Trust in its CCP for clearing credit

⁸ When comparing instead to the expected exposure to a counterparty that existed *before* novation to a CCP, this concentration ratio is $\sqrt{(N - 1)/K}$, which is 1.34 in our example. This represents a 34% increase in concentration due to “clearing,” under our simple assumptions. For $N = 20$ entities and $K = 5$ classes of derivatives, the corresponding increase in concentration is 94%.

default swaps.⁹ Under our assumptions, with $N = 12$, clearing the derivatives in a particular class through a CCP improves netting efficiency if and only if the fraction R of an entity's expected exposure attributable to this class is at least 66% of the total expected exposure of all remaining bilaterally netted classes derivatives. With $N = 26$, the cutoff level drops to $R = 41.7\%$. Although the CDS market poses a large amount of exposure risk, with a total notional market size of roughly \$25 trillion, it would be difficult to make the case that it represents as much as 41.7% of dealers' expected exposures in all other "uncleared" derivatives classes combined.

The Bank for International Settlements (BIS) provides data on OTC derivatives exposures of dealers in several major asset classes. The latest available data, for June 2010, are shown in Table 1. Although these data show merely gross current credit exposures, and therefore do not incorporate the add-on exposure uncertainty associated with uncertain future position sizes and risky marks to market, they do give a rough indication of the relative amount of exposure in each of the major underlying asset classes, before netting across classes and before collateral. The effect of bilateral netting across classes reduced the total gross exposures shown in Table 1 from \$24.7 trillion to \$3.6 trillion.¹⁰

In light of Proposition 1, it would be hard to base a case for the netting benefits of a central clearing counterparty dedicated to credit default swaps on the magnitudes of OTC derivatives credit exposures shown in Table 1. Credit derivatives account for less than 7% of the total gross exposures. If one assumes that total counterparty expected exposures of a given dealer are proportional, class by class, to the gross credit exposures shown in Table 1,

Table 1
Gross credit exposures in OTC derivative markets as of June 2010

Asset class	Exposure (\$ billions)
CDS	1,666
Commodity	457
Equity Linked	706
Interest Rate	17,533
Foreign Exchange	2,524
Unallocated	1,788
Total	24,673
Total after netting	3,578

Source: Bank for International Settlements.

⁹ As of 2010, the participants of the ICE Trust were Bank of America, Barclays Capital, BNP Paribas, Citibank, Credit Suisse, Deutsche Bank, Goldman Sachs, HSBC, JPMorgan Chase, Merrill Lynch, Morgan Stanley, Nomura, Royal Bank of Scotland, and UBS. See https://www.theice.com/publicdocs/ice-trust/ICE_Trust_Participant_List.pdf.

¹⁰ Because of the manner in which these data are collected, the net exposures do not include the effects of credit default swaps held by non-U.S. dealers.

and that X_{ij}^k are independent across k , the implied ratio R of expected exposures on credit derivatives to expected exposures on the total of other classes is less than¹¹ 10%. This would in turn imply, from Proposition 1, that a central clearing counterparty dedicated to CDS reduces average expected counterparty exposures if there are more than 460 entities clearing together. After adding to gross exposures the add-on effect of highly volatile CDS marks to market (relative to other asset classes), the threshold number of entities necessary to justify a central clearing counterparty dedicated to CDS is likely to be lower.

Exposures on credit derivatives among dealers have been reduced significantly since June 2008 due to CDS compression trades.¹² According to DTCC DerivServ data, dealer CDS positions continued to shrink throughout 2008–2010. The total size of the CDS market in terms of notional positions in June 2010 was less than half of mid-2008 levels.¹³

The data in Table 1 suggest that there is a much stronger case for the joint clearing of CDS and interest-rate swaps, which together accounted for about 80% of the total gross exposures. Indeed, interest-rate swaps on their own represent large enough exposures to justify a dedicated central clearing counterparty, and a significant fraction of interest-rate swaps are already cleared through CCPs.¹⁴

Ironically, our model suggests that it is easier to justify the netting benefits of a central clearing counterparty dedicated to a particular class of derivatives after a different CCP has already been set up for a different class of derivatives. In this sense, “one mistake justifies another.” For example, the threshold size of the CDS market that justifies the netting benefits of a CDS-dedicated CCP is lowered once a significant fraction of interest-rate swaps are cleared.

One could argue that CDS exposure is rather special, because of jump-to-default risk and because default risk tends to be correlated with systemic risk.

¹¹ To calculate the implied ratio R , denote by Z_k the total gross exposure on derivatives of class k , for $k = 1, 2, \dots, K$. Assume that the total expected counterparty exposure on class k is a fixed fraction α of Z_k , and that these expected counterparty exposures are independent across k . Without loss of generality, let class K be centrally cleared while all remaining classes are bilaterally netted. Then the implied ratio of total expected counterparty exposure on class K to that on classes 1 to $K - 1$ combined is

$$R = \frac{\alpha Z_K}{\sqrt{\sum_{k=1}^{K-1} (\alpha Z_k)^2}} = \frac{Z_K}{\sqrt{\sum_{k=1}^{K-1} Z_k^2}}$$

¹² According to a press release by Markit of July 2, 2008, a compression trade “involves terminating existing trades and replacing them with a far fewer number of new ‘replacement trades’ which have the same risk profile and cash flows as the initial portfolio, but with less capital exposure. The initiative, available to both the U.S. and European CDS markets, will be managed jointly by Creditex and Markit and has the support of 13 major CDS market participants.” See “Markit and Creditex Announce Launch of Innovative Trade Compression Platform to Reduce Operational Risk in CDS Market,” July 2, 2008, at www.markit.com.

¹³ See <http://www.dtcc.com/products/derivserv/data/index.php>.

¹⁴ According to a February 3, 2009, press release on its website, LCH.Clearnet stated that it clears about 50 percent of the OTC global interest-rate-swap market in a CCP for interest-rate swaps. However, Duffie, Li, and Lubke (2010) provide a lower estimate of 35% for dealer-to-dealer clearing based on a survey of dealers by the Federal Reserve Bank of New York. U.S.-based CCPs for interest-rate swaps include CME Cleared Swaps and IDGC. Ledrut and Upper (2007) provide details on the central clearing of interest-rate swaps.

Given the typical practice of daily re-collateralization, the revaluation of CDS positions caused by any defaults on a given day would need to be extremely large in order to build a strong case for separate CDS clearing on the implications of jump-to-default risk. Our results show that jump-to-default risk is better reduced through bilateral netting or joint clearing with interest-rate swaps, unless the jump-to-default risk is large relative to that of all other OTC derivatives exposures. A large fraction, about one-third, of the gross credit exposures shown in Table 1 are multi-name CDS products, mainly in the form of index contracts such as CDX and iTraxx, which represent equal-weighted CDS positions in over one hundred corporate borrowers. These products have relatively small jump-to-default risk compared with single-name CDS.

The benefit of multilateral netting among a large set of entities is reduced by a concentration of exposures among a small subset of the entities. For example, among U.S. banks, the latest data available through the Office of the Comptroller of the Currency as of this writing show that the five largest derivative dealers—JPMorgan Chase, Bank of America, Goldman Sachs, Morgan Stanley, and Citigroup—account for about 95% of total notional credit derivatives positions held by all U.S. banks. The effective number of U.S. CDS market participants for purposes of our analysis may not be much more than five. The proposal for derivatives clearing becomes relatively more attractive if a single CCP handles clearing for all standard CDS positions of large global dealers, including those in Europe and the United States, and much more attractive if credit derivatives are cleared together with interest-rate swaps in the same central clearing counterparty.

3.2 An example of exposure reduction

We now provide a simple illustrative example of exposure reduction under various clearing scenarios for the six largest U.S. derivatives dealers. Table 2 shows the notional amounts of OTC derivatives contracts reported to the Office of the Comptroller of the Currency.¹⁵ Because we do not have similar data for non-U.S. banks, we assume there are six other derivatives dealers with the same total notional amounts of derivatives by class, giving a total of $N = 12$ major dealers globally.

Let S_i^k be the aggregate (notional) size of the positions of dealer i in derivatives class k . We suppose that the standard deviation of exposures due to class- k derivatives is a scaling m_k of the associated notional position S_i^k . Here, m_k incorporates both the effect of market value on a typical future date (which is uncertain from the current perspective), as well as the effect of volatility of

¹⁵ The Office of the Comptroller of the Currency does not provide notional amounts by the underlying asset classes, such as interest-rate, credit, equity, and so on. However, we note that almost all swap contracts are interest-rate swaps, and that almost all credit derivatives are credit default swaps. For that reason we use the notional amounts of swaps contracts as proxies for those of interest-rate swaps, and the notional amounts of credit derivatives as proxies for those of CDS.

Table 2
Notional sizes of six largest U.S. derivatives dealers

	Forwards	Swaps	Options	Credit	Total
Bank 1	8177	51203	10059	6376	75815
Bank 2	8984	49478	5918	5590	69970
Bank 3	1651	31521	6980	5762	45914
Bank 4	5718	24367	4064	5482	39631
Bank 5	5536	16375	6384	2764	31059
Bank 6	1198	2192	477	268	4135
Total	31264	175136	33882	26242	266524

This table shows the notional sizes of six largest U.S. derivatives dealer banks, published by the Office of the Comptroller of the Currency, as of 2009 Q3. The identities of the banks are omitted.

changes in market value between that day and the time by which additional collateral could partially be collected before the counterparty fails. We also assume that the exposure of dealer i to dealer j on class k is proportional to S_j^k . Thus, the standard deviation of the pre-collateral pre-clearing exposure of dealer i to dealer j on derivatives class k is

$$m_k S_i^k \frac{S_j^k}{\sum_{h \neq i} S_h^k}. \tag{9}$$

We let α^k be the fraction of notional positions in derivatives of class k that are centrally cleared. Keeping our normality and independence assumptions, we have

$$X_{ij}^k \sim N \left(0, \left(m_k S_i^k \frac{S_j^k}{\sum_{h \neq i} S_h^k} \right)^2 \right). \tag{10}$$

The expected exposure of dealer i to a CCP dedicated to class- k derivatives is thus

$$E \left[\max \left(\sum_{j \neq i} \alpha^k X_{ij}^k, 0 \right) \right] = \frac{1}{\sqrt{2\pi}} \alpha^k m_k S_i^k \left[\frac{\sum_{j \neq i} (S_j^k)^2}{\left(\sum_{j \neq i} S_j^k \right)^2} \right]^{\frac{1}{2}}. \tag{11}$$

The expected exposure of dealer i to dealer j on all uncleared positions is

$$E \left[\max \left(\sum_{k=1}^K (1 - \alpha^k) X_{ij}^k, 0 \right) \right] = \frac{1}{\sqrt{2\pi}} \left[\sum_{k=1}^K \left((1 - \alpha^k) m_k S_i^k \frac{S_j^k}{\sum_{j \neq i} S_j^k} \right)^2 \right]^{\frac{1}{2}}. \tag{12}$$

Table 3 shows the dealers' pre-collateral expected exposures under various clearing approaches. These exposures are shown as multiples of total exposures

Table 3
Expected counterparty exposures under various clearing approaches

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Fractions cleared on CCP(s)								
Forwards	0	0	0	0	0	0	0	0.4	0.4
Swaps	0	0	0	0	0.75	0.75	0.75	0.75	0.75
Options	0	0	0	0	0	0	0	0.4	0.4
Credit	0	1	1	0.75	0	0.75	0.75	0.75	0.75
Number of CCP	–	Same	Mult.	Same	Same	Mult.	Same	Mult.	Same
	Total exposures as fractions of exposures without CCPs								
Bank 1	1	1.05	1.09	1.03	0.88	0.89	0.83	0.79	0.63
Bank 2	1	1.05	1.09	1.03	0.84	0.85	0.79	0.76	0.62
Bank 3	1	1.05	1.10	1.02	0.88	0.85	0.78	0.76	0.61
Bank 4	1	1.04	1.10	1.01	0.94	0.91	0.83	0.80	0.63
Bank 5	1	1.05	1.09	1.03	1.00	1.02	0.97	0.86	0.69
Bank 6	1	1.04	1.06	1.03	1.00	1.02	0.99	0.83	0.70
Total (ratio)	1	1.05	1.09	1.03	0.90	0.90	0.83	0.79	0.63

This table shows the expected counterparty derivatives exposures of dealers under various clearing approaches, as multiples of total exposures when all classes are bilaterally netted. “Mult.” refers to the case of multiple CCPs, each clearing one class of derivatives. “Same” refers to the case of a single CCP clearing all derivatives classes considered. The estimates are based on $N = 12$ dealers, the six dealers of Table 2 and six others with the same exposures class by class. The standard-deviation scaling m_k for non-interest-rate swap derivatives is assumed to be three times that for interest-rate derivatives.

for the case in which all derivatives are bilaterally netted. For this example, we use a standard-deviation scaling m_k for non-interest-rate-swap derivatives that is three times that for interest-rate derivatives.¹⁶

Relative to the no-clearing base case, the introduction of a CCP that clears 100% of credit derivatives actually increases market wide expected exposures by about 5% in this setting (Column 2), as suggested by our theory. If a CDS-dedicated CCP clears 75% of CDS, then expected exposures are about 3% higher than for the case of fully bilateral netting (Column 4).

If we divide CDS positions into two classes, say “European” and “U.S.,” of equal total notional sizes, then clearing the U.S. and European CDS separately increases expected exposures by 9% relative to bilateral netting (Column 3).

Estimated expected exposures are reduced by about 10% relative to bilateral netting if 75% of interest-rate swaps are centrally cleared (Column 5). Morgan Stanley (2009) forecasts the clearing of about 75% of dealer-to-dealer interest-rate swaps and CDS in the next two to three years. With this 75% level

¹⁶ From BIS data as of June 2009, the market value of interest-rate swaps is roughly 3.5% of the notional amounts. The market value of all the other derivatives classes combined is about 5.9% of the notional amounts. These numbers suggest a ratio of roughly 1.67 to 1 for the current valuations of non-interest-rate swaps to interest-rate swaps, per unit notional. We scale up from 1.67 to 3 in order to allow for the volatility of changes in market value between the time of valuation and the time by which additional collateral could be received before a potential default.

of interest-rate-swap clearing, adding a CDS-only CCP has a negligible effect on expected exposures (Column 6). If, however, 75% of CDS and interest-rate swaps are cleared by the same CCP, then expected exposures are reduced by 17% compared with bilateral netting (Column 7).

Clearing a moderately large fraction of all classes of derivatives in the same CCP reduces average estimated exposures by 37% (Column 9). This high degree of netting efficiency is not achieved if the same amounts are cleared centrally but separately (Column 8).

Figure 1 illustrates total expected exposures under various clearing approaches. We fix the fraction of interest-rate swaps that are centrally cleared to be 35%, the estimate of clearing obtained in a late 2009 dealer survey conducted by the Federal Reserve Bank of New York.¹⁷ When CDS and interest-rate swaps are cleared together in the same CCP, the reduction in exposure is positive and convex in the cleared fraction of CDS. Total expected exposures are strictly higher if CDS are cleared separately from interest-rate swaps.

Our numerical results highlight the tradeoff between bilateral and multilateral netting. Our results indicate that counterparty exposures can be reduced significantly if the same CCP jointly clears multiple classes of derivatives. Joint clearing also reduces margin requirements. Singh (2009) estimates that if two-thirds of all OTC derivatives are cleared through CCPs using current CCP approaches, then roughly \$400 billion in additional clearing margin and guarantee funds will be needed.

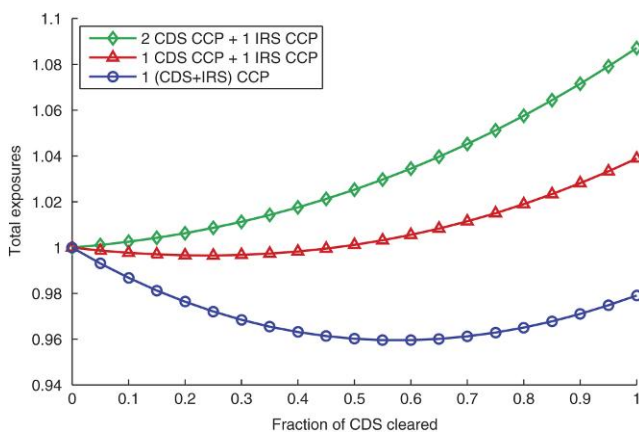


Figure 1
Total expected exposures under various clearing approaches

This figure shows the total expected exposures under various clearing approaches as a function of the cleared fraction of CDS. In all cases, we assume that 35% of interest-rate swaps (IRS) are cleared. In the case of two CDS-dedicated CCPs, we assume that the total notional sizes of CDS cleared on the two CDS CCPs are equal.

¹⁷ See Duffie, Li, and Lubke (2010).

4. Netting Efficiency with Multiple CCPs

In this section, going beyond the illustrative estimates of Section 3.3, we prove a general result on the loss of netting efficiency caused by dedicating different CCPs to each of several classes of derivatives, as opposed to the joint clearing of various classes of derivatives in a single CCP.

We drop our normality and symmetry assumptions, and allow for an arbitrary joint distribution of X_{ij}^k . We suppose that C of the K classes of derivatives are centrally cleared, while the remaining $K - C$ classes are bilaterally netted. Without loss of generality, classes $1, 2, \dots, C$ are cleared through CCPs. The expected exposure of entity i on uncleared derivatives is

$$\phi_{i,N,K-C} = E \left[\max \left(\sum_{k=C+1}^K \sum_{j \neq i} X_{ij}^k, 0 \right) \right]. \quad (13)$$

With a single CCP that clears all of the first C classes, the total expected exposure of entity i is

$$U_i = E \left[\max \left(\sum_{k=1}^C \sum_{j \neq i} X_{ij}^k, 0 \right) \right] + \phi_{i,N,K-C}. \quad (14)$$

With the separate clearing of the first C classes of derivatives, the expected exposure of entity i to the C different CCPs is instead

$$\begin{aligned} \hat{U}_i &= E \left[\sum_{k=1}^C \max \left(\sum_{j \neq i} X_{ij}^k, 0 \right) \right] + \phi_{i,N,K-C} \\ &\geq E \left[\max \left(\sum_{k=1}^C \sum_{j \neq i} X_{ij}^k, 0 \right) \right] + \phi_{i,N,K-C} \\ &= U_i, \end{aligned} \quad (15)$$

using the convexity of $\max(\cdot)$ and Jensen's inequality. That is, each entity has higher expected counterparty exposure with multiple CCPs than with a single CCP. This result formalizes the intuition of the example given in Section 3.3.

Proposition 2. For an arbitrary joint distribution of (X_{ij}^k) , each entity's total expected counterparty exposure with $C > 1$ CCPs clearing derivative classes separately is greater than or equal to its total expected exposures with a single CCP clearing all C classes jointly.

Similarly, any increase in joint clearing—that is, any reduction in the number C of CCPs obtained by combining different classes of derivatives clearing

into the same CCP—reduces expected exposures. These comparisons are strict under non-degeneracy assumptions on the joint distribution of (X_{ij}^k) .

In Appendix A, we examine the case of separate CCPs for two groups of market participants. We show that whenever introducing a unique CCP for all market participants strictly reduces counterparty exposures, it is always more efficient to have one CCP than separate CCPs for each group of market participants.

5. Conclusion

We show that the separate central clearing of one class of derivatives, such as credit default swaps, could reduce netting efficiency, leading to higher expected counterparty exposures and collateral demands. When multiple derivatives classes are cleared, it is always more efficient to clear them on the same CCP rather than on different CCPs. An obvious policy recommendation is a move toward the joint clearing of standard interest-rate swaps and credit default swaps in the same clearing house.

The interoperability of CCPs, by which at least some of the benefits of joint clearing can be obtained through agreements among CCPs and their participants, can in principle achieve significant reductions in counterparty risk, although obtaining effective interoperability agreements currently presents a number of legal and financial engineering challenges, in addition to business-incentive hurdles. For related discussions of interoperability, see EuroCCP (2010), Kalogeropoulos, Russo, and Schönenberger (2007), and Mägerle and Nellen (2011).

Appendix

A. Cross-class exposure correlation

We now allow for the possibility that derivatives exposures are correlated across asset classes. For simplicity, we suppose that the correlation ρ between X_{ij}^k and X_{ij}^m does not depend on i, j , or the particular pair (k, m) of asset classes. (We continue to assume joint normality, symmetry, and equal variances.)

For entity-to-entity exposures, it would be reasonable to assume that ρ is small in magnitude, bearing in mind that this correlation depends in part on whether the exposure between i and j in one particular derivatives contract is likely to be of the same sign as that of its exposure in another. For pairs of dealers with large matched-book operations, one might anticipate that ρ is close to zero.

The average total expected exposure without a CCP is

$$\phi_{N,K} = \frac{1}{\sqrt{2\pi}} \sigma (N-1) \sqrt{K(1+(K-1)\rho)}. \tag{16}$$

With a CCP for class- K positions only, the average total expected exposure is

$$\gamma_N + \phi_{N,K-1} = \frac{1}{\sqrt{2\pi}} \sigma \left(\sqrt{N-1} + (N-1) \sqrt{(K-1)(1+(K-2)\rho)} \right). \tag{17}$$

The reduction in average expected exposure due to the introduction of a CCP for one class of derivatives is therefore

$$\theta(N, K) = \phi_{N,K} - (\gamma_N + \phi_{N,K-1}). \tag{18}$$

Proposition 3. The introduction of a CCP for one class of derivatives reduces the average total expected exposure of an entity if and only if

$$\theta(N, K) > 0 \Leftrightarrow \beta_K > \frac{1}{\sqrt{N-1}}, \tag{19}$$

where

$$\beta_K = \frac{1 + 2\rho(K-1)}{\sqrt{K(1+(K-1)\rho)} + \sqrt{(K-1)(1+(K-2)\rho)}}. \tag{20}$$

This result follows from the fact that

$$\theta(N, K) = \frac{1}{\sqrt{2\pi}}\sigma(N-1) \left(\sqrt{K(1+(K-1)\rho)} - \sqrt{(K-1)(1+(K-2)\rho)} - \frac{1}{\sqrt{N-1}} \right). \tag{21}$$

Rearranging terms, we have the result.

Figure 2 shows the mean reduction in average total expected exposure for various combinations of N , K , and ρ . (The reduction is scaled to the case of $\sigma = 1$.) Increasing the correlation between positions increases the relative netting benefits of a CCP, because between-entity

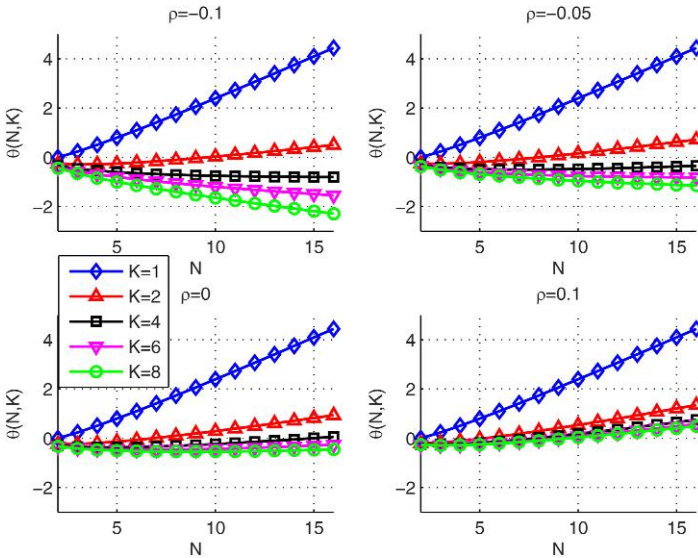


Figure 2
Reductions in average expected exposures with a single CCP

This figure shows the reductions (θ) in average expected exposures associated with clearing one class of derivatives with a single central clearing counterparty, based on N entities, K classes of derivatives, and a cross-class exposure correlation of ρ . The reductions are scaled for the case of $\sigma = 1$.

netting is not as beneficial if cross-class exposures are positively correlated.¹⁸ Indeed, one can show that

$$\frac{\partial \theta}{\partial \rho} = \frac{1}{\sqrt{2\pi}} \sigma (N-1) \frac{1}{2} \left[\frac{\sqrt{K}(K-1)}{\sqrt{1+(K-1)\rho}} - \frac{\sqrt{K-1}(K-2)}{\sqrt{1+(K-2)\rho}} \right] > 0. \tag{22}$$

Because dealers may have a tendency, especially when their counterparties are distressed, of entering derivatives trades that offset exposures arising in other classes of derivatives, we believe that extra emphasis should be placed on the case of negative ρ .

We calculate, treating N as though a real number, that

$$\frac{\partial \theta(N, K)}{\partial N} = \frac{1}{\sqrt{2\pi}} \sigma \left(\beta_K - \frac{1}{2\sqrt{N-1}} \right) \tag{23}$$

$$\frac{\partial^2 \theta(N, K)}{\partial N^2} = \frac{\sigma}{\sqrt{2\pi}} \frac{1}{4(N-1)^{3/2}} > 0. \tag{24}$$

The convexity of $\theta(N, K)$ with respect to N is evident from Figure 2.

B. Separate CCPs by entity groups

In this appendix, we consider the cost of having two CCPs, each dedicated to a particular group of entities, for the same class of derivatives. This separation of CCPs is different from that in Section 4. We return to our original assumption of independence of exposures across classes of exposures. We assume that the entities are partitioned into two groups for separate clearing, Group A with M entities and Group B with $N - M$ entities. We allow for the possibility that entities within a group have higher exposures with each other than they do with entities in the other group. Specifically, if entities i and j are in different groups, while i and n are in the same group, we let

$$q = \frac{E[\max(X_{ij}^k, 0)]}{E[\max(X_{in}^k, 0)]} \tag{25}$$

be the ratio of cross-group expected exposures to within-group expected exposures. We will always assume, naturally, that $q \leq 1$. Our assumptions are otherwise as before.

With the introduction of CCPs for class- K derivatives, one for each group, we suppose that all entities continue to bilaterally net exposures on the remaining $K - 1$ classes, that they clear class- K derivatives within their own group, and that they continue to bilaterally net exposures on class- K derivatives with those counterparties that are not in their own group. The total expected exposure of an entity in Group A , for instance, is therefore

$$\phi_{M, K-1} + q\phi_{N-M+1, K} + \gamma_M = \frac{1}{\sqrt{2\pi}} \sigma \left((M-1)\sqrt{K-1} + q(N-M)\sqrt{K} + \sqrt{M-1} \right). \tag{26}$$

For $M = N/2$, with N even, the average total expected entity exposure (in both groups) is

$$\begin{aligned} & \frac{1}{2} (\phi_{M, K-1} + q\phi_{N-M+1, K} + \gamma_M + \phi_{N-M, K-1} + q\phi_{M+1, K} + \gamma_{N-M}) \\ &= \frac{1}{\sqrt{2\pi}} \sigma \left[\left(\frac{N}{2} - 1 \right) \sqrt{K-1} + \frac{qN}{2} \sqrt{K} + \sqrt{\frac{N}{2} - 1} \right]. \end{aligned} \tag{27}$$

¹⁸ For a fixed number N of entities, as the number K of derivatives classes gets large, β_K converges to $\sqrt{\rho}$, for $\rho > 0$. Thus, in this sense of increasingly many classes of derivatives, or more generally as the expected exposure in the class to be centrally cleared becomes small relative to that in other classes of derivatives, a CCP is asymptotically efficient if and only if $\rho > 1/(N-1)$.

Similarly, with only one CCP, the average total expected entity exposure is

$$\frac{1}{\sqrt{2\pi}} \sigma \left[\left(\frac{N(1+q)}{2} - 1 \right) \sqrt{K-1} + \sqrt{\frac{N(1+q^2)}{2} - 1} \right]. \tag{28}$$

We let $\theta(N, K, M)$ be the reduction in expected exposures associated with two CCPs, over using one CCP for the same class of derivatives for all entities. For the case of $M = N/2$, we calculate that

$$\theta(N, K, N/2) = \frac{1}{\sqrt{2\pi}} \sigma \left[-\frac{qN}{2(\sqrt{K} + \sqrt{K-1})} - \sqrt{\frac{N}{2} - 1} + \sqrt{\frac{N(1+q^2)}{2} - 1} \right]. \tag{29}$$

For $M = N/2$, having two CCPs is more efficient than having one CCP if and only if

$$\theta(N, K, N/2) > 0 \Leftrightarrow \sqrt{K} + \sqrt{K-1} > \frac{1}{q} \left(\sqrt{\frac{N}{2} - 1} + \sqrt{\frac{N(1+q^2)}{2} - 1} \right). \tag{30}$$

Without any CCP, the expected exposure is

$$\frac{1}{\sqrt{2\pi}} \sigma \left(\frac{N(1+q)}{2} - 1 \right) \sqrt{K}. \tag{31}$$

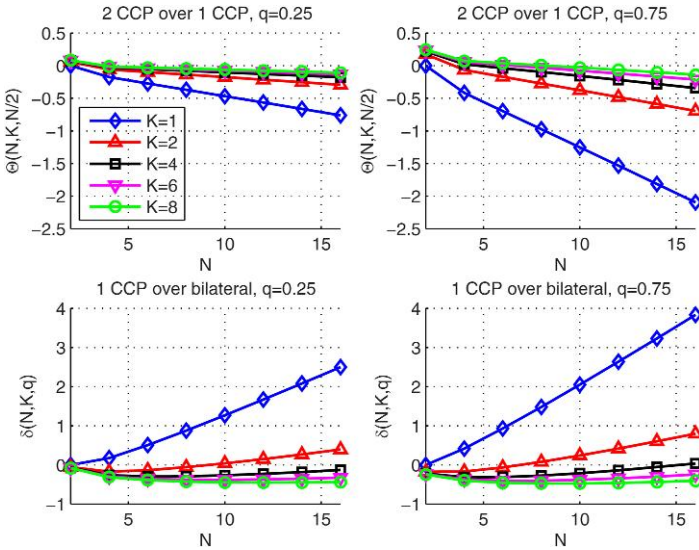


Figure 3
Reductions in average total expected exposure allowed by having two CCPs
 The top panel shows the reductions (θ) in average total expected exposures allowed by having two CCPs, one for each group of entities, relative to having one CCP for all entities. The bottom panel shows the reductions (δ) in average total expected exposures allowed by having one CCP relative to none (fully bilateral netting of exposures). The reductions are normalized by taking $\sigma = 1$.

Provided $M = N/2$, a unique CCP for all class- K derivatives reduces average expected exposure, relative to no CCP, by

$$\delta(N, K, q) = \frac{1}{\sqrt{2\pi}}\sigma \left[\left(\frac{N(1+q)}{2} - 1 \right) (\sqrt{K} - \sqrt{K-1}) - \sqrt{\frac{N(1+q^2)}{2} - 1} \right]. \quad (32)$$

Having a single CCP for all entities improves efficiency, relative to having none, if and only if

$$\delta(N, K, q) > 0 \Leftrightarrow \sqrt{K} + \sqrt{K-1} < \frac{\frac{N(1+q)}{2} - 1}{\sqrt{\frac{N(1+q^2)}{2} - 1}}. \quad (33)$$

Comparing Equation (30) and Equation (33), for equal-sized groups of entities, one can show that whenever introducing a unique CCP for all entities strictly improves efficiency, it is always more efficient to have one CCP than to have separate CCPs for each group of entities. This implication can also be observed in Figure 3.

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