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Are CDS Auctions Biased and Inefficient?

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ABSTRACT

We study the design of credit default swaps (CDS) auctions, which determine the payments by CDS sellers to CDS buyers following defaults of bonds. Using a simple model, we find that the current design of CDS auctions leads to biased prices and inefficient allocations. This is because various restrictions imposed in CDS auctions prevent certain investors from participating in the price discovery and allocation process. The imposition of a price cap or floor also gives dealers large influence on the final auction price. We propose an alternative double auction design that delivers more efficient price discovery and allocations.

CREDIT DEFAULT SWAPS (CDS) ARE DEFAULT insurance contracts between buyers of protection ("CDS buyer") and sellers of protection ("CDS seller") that are written against the default of firms or countries. Since the financial crisis, CDS have been among the top financial innovations to receive policy attention and regulatory actions.[1] As of December 2015, global CDS markets

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[1] For example, the Dodd-Frank Act of the United States mandated and later implemented mandatory central clearing of standard over-the-counter derivatives, including CDS. Furthermore, in its Financial Markets Infrastructure Regulation, the European Commission stated that "Derivatives play an important role in the economy but are associated with certain risks. Recent financial crises have highlighted that these risks are not sufficiently mitigated in the over-the-counter (OTC) part of the market, especially in regards to credit default swaps (CDS)." See "Financial stability: New EU rules on central clearing for certain credit derivative contracts," European Commission

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have a notional outstanding of \$12.3 trillion and a gross market value of \$421 billion.[2]

This paper studies the design of CDS auctions, a unique mechanism that determines postdefault recovery value for the purpose of settling CDS. Since recovery value is a fundamental parameter for the pricing, trading, and clearing of CDS contracts, achieving a fair and unbiased auction price is crucial for the proper functioning of CDS markets. For example, using a sample of U.S. corporate bond defaults, Gupta and Sundaram (2012) find that "information generated in CDS auctions is critical for postauction market price formation." In addition to price discovery, an auction protocol also has the key benefit of facilitating cash settlement by producing a transparent price, which overcomes the difficulty of physical settlement when the outstanding amount of CDS exceeds the supply of bonds (see Creditex and Markit (2010)).[3]

The current CDS auction mechanism was initially implemented in 2005, and in 2009 it became the standard method used to settle CDS contracts after default (ISDA (2009)). Over the 2005 to May 2016 period, CDS auctions settled 121 defaults of firms (such as Fannie Mae, Lehman Brothers, and General Motors) and sovereign countries (such as Greece, Argentina, and Ukraine). For many firms, separate CDS auctions are held for senior and subordinate debt.

As we explain in Section I, the current auction mechanism consists of two stages. The first stage involves soliciting market orders, called "physical settlement requests," to buy or sell defaulted bonds. The net market order is called the "open interest." At the same time, dealers quote prices on the bonds used to calculate a price cap or floor. The second stage involves a (one-sided) uniform-price auction subject to the price cap or floor. If the final auction price is p^* per \$1 of face value, the default payment by CDS sellers to CDS buyers is $1 - p^*$.

The primary objective of this paper is to evaluate CDS auctions from a theoretical and market design perspective. We show that the current design of CDS auctions leads to biased prices and inefficient allocations of defaulted bonds. The primary reason is that various restrictions imposed on the auctions prevent certain investors from participating in the price discovery and allocation process. Moreover, the current design leaves ample scope for dealers to strategically manipulate the price to profit from their existing CDS positions. We suggest a double auction design that delivers more efficient prices and allocations.

Our analysis builds on a simple model of divisible auctions that works roughly as follows. There are two dates, $t \in \{0, 1\}$, and a continuum of infinitesimal traders. At $t = 0$, everyone has the same probability distribution over the default of a risky bond. Traders are endowed with i.i.d. private values $\{b_i\}$ for

Press Release, March 1, 2016. In 2012, European regulators banned "naked" sovereign CDS in the European Union.

[2] See the latest semiannual OTC derivatives statistics, Bank for International Settlements, http://www.bis.org/statistics/derstats.htm.

[3] As noted by Creditex and Markit (2010), in the physical settlement of CDS, if the outstanding amount of CDS exceeds the supply of defaulted bonds, the bonds need to be recycled multiple times to settle all CDS contracts, leading to endogenous scarcity of the bonds and artificially high prices.

buying or selling CDS on the bond. Each trader incurs a per-unit cost in trading CDS contracts and a quadratic inventory cost in holding CDS positions. The CDS positions are determined optimally in a double auction, taking into account the actions at $t = 1$. At $t = 1$, with some positive probability, the bond defaults and traders are endowed with high or low valuations $\{v_i\}$ for owning the defaulted bonds. Immediately afterward, a CDS auction is held. Traders select the optimal physical requests in the first stage and the optimal demand schedules in the second stage. Like CDS positions, traders also incur quadratic costs in buying or selling defaulted bonds in the auction. Optimal strategies in the CDS auction as well as predefault CDS trading are solved in a subgame-perfect equilibrium. For simplicity, our model is solved without imposing the price cap or floor in the second stage.

To see why the price is biased and allocations are inefficient, consider the strategy of a trader who has a high value for owning the defaulted bonds at $t = 1$. The trader's CDS position could be positive (CDS buyer), negative (CDS seller), or zero. For simplicity, let us consider a high-value trader with zero CDS position. In practice, this trader could be a specialist in distressed debt who does not trade CDS. This high-value trader wishes to buy defaulted bonds, but the current design of the CDS auction stipulates that only CDS sellers can submit physical requests to buy (i.e., market orders to buy) in the first stage. Thus, demand to buy bonds from this high-value trader is suppressed in the first stage of the auction. If the open interest is to buy, the auction protocol also stipulates that only orders in the opposite direction, that is, sell limit orders, are allowed in the second stage. Thus, demand to buy bonds from this high-value trader is suppressed in the second stage of the auction as well. The same argument applies to a high-value CDS buyer. As a result, if the open interest is to buy, high-value traders with positive or zero CDS positions are excluded from the auction, leading to a downward-biased final auction price.[4]

This result applies symmetrically if the open interest is to sell, but in this case it is the low-value traders with zero or negative CDS positions who are excluded from both stages of the auction. Thus, the final auction price is upward-biased if the open interest is to sell.

Inefficient allocations naturally follow from biased prices. The directions of the inefficiency are summarized in Table I. In particular, high-value traders' allocations are almost always too low (with the only exception being high-value CDS sellers in the high state), and low-value traders' allocations are almost always too high (with the only exception being low-value CDS buyers in the low state).

The restrictions imposed in CDS auctions also have an unintended consequence in CDS markets before default. Because a larger CDS position (in absolute value) relaxes participation constraints in the first stage of CDS auctions,

[4] We can show that other types of traders—including low-value traders (regardless of CDS positions) and high-value CDS sellers—either fully or partially participate in either stage of the auction.

Table I
Price Bias and Inefficient Allocations in CDS Auctions (without Price Cap or Floor)

			Allocations					
			High-Value Traders			Low-Value Traders		
State	Open Interest	Price Bias	CDS Buyer	Zero CDS	CDS Seller	CDS Buyer	Zero CDS	CDS Seller
High	Buy	Downward	Too low	Too low	Mixed	Too high	Too high	Too high
Low	Sell	Upward	Too low	Too low	Too low	Mixed	Too high	Too high

traders with moderate values of trading CDS establish CDS positions that are larger in magnitude than the socially optimal level.

We emphasize that although investors can buy and sell defaulted bonds in the secondary markets, doing so incurs nontrivial transaction costs. For example, Feldhutter, Hotchkiss, and Karakas (2016) find that the round-trip cost of trading defaulted bonds is about 0.9% in the quarter after default. The Amihud illiquidity measure also roughly doubles in the week of default, suggesting higher price impact costs of trading large quantities. More recently, Bao, O'Hara, and Zhou (2016) find that the price impact of trading corporate bonds after a downgrade has increased since the implementation of postcrisis regulation. Dick-Nielsen and Rossi (2016) find similar results in the "intertemporal bid-ask spread" following the index exclusion of some corporate bonds.

By contrast, trading in CDS auctions incurs zero spread and hence is desirable from a social perspective. As long as transaction costs prevent the realization of some gains from trade, the qualitative nature of inefficient allocations would carry through in a model extension with bond trading in the secondary market.

We do not explicitly model price caps or floors. Although they are used in practice, price caps and floors are difficult to set correctly in the first place, and even if they were set correctly, allocations would remain inefficient.[5] In fact, since price caps and floors are determined by dealers' quotes in the first stage, this arrangement leaves ample room for dealers to manipulate the final auction price. This manipulation incentive is similar to that in survey-based financial benchmarks such as the London Interbank Offered Rate (LIBOR). Since LIBOR manipulation is already an established fact,[6] the current CDS auction design raises similar questions.

Our model generates a number of novel predictions regarding quoting behavior, price biases, and postauction trading activity. For example, our model

[5] Details of this result are available upon request.

[6] See, for example, Market Participants Group on Reference Rate Reform (2014) and Official Sector Steering Group (2014) for institutional details and suggested reforms on reference rates such as LIBOR.

predicts that, in the first stage, dealers who are CDS buyers quote relatively low prices, whereas dealers who are CDS sellers quote relatively high prices. Moreover, our model predicts that if the open interest is to buy, low-value CDS traders get too much of an allocation in the auctions and will sell bonds after the auctions, whereas if the open interest is to sell, high-value CDS traders get too little of an allocation in the auctions and will buy bonds after the auctions. These predictions are unique to our model and new to the literature. Tests of these predictions require data on CDS positions by identity or bond transactions by identity. These types of data are available to regulatory agencies.

Our analysis suggests that a double auction design reduces price biases and improves allocation inefficiency. A double auction design is by no means unusual or exotic. Indeed, it is used in open and close auctions on stock exchanges. Under a double auction design, limit orders in the second stage can be submitted in both directions—buy and sell—regardless of the open interest. Two-way orders allow for broader investor participation in the price discovery process. We also argue that the price caps or floors should be dropped (or modified to a wide two-way price band) to weaken dealers' ability to manipulate prices in a particular direction. Importantly, however, the double auction design could still allow physical settlement requests and dealers' quotes to help aggregate information in the first stage.

To the best of our knowledge, two other theoretical models of CDS auctions have been proposed in the literature. The model of Chernov, Gorbenko, and Makarov (2013, CGM) is an extension of the strategic bidding models of Wilson (1979) and Back and Zender (1993). CGM (2013) deliver the important insight that CDS auction participants have incentives to manipulate the second-stage price to profit from their existing CDS positions. Such manipulation is possible and effective because the second stage of the auction is one-sided, that is, only buy orders or only sell orders are allowed. CGM (2013) also model constraints to buy and sell defaulted bonds (short-sales are impossible, and certain investors cannot buy the defaulted bonds at all). Combining these two features, CGM (2013) conclude that the final auction price can be higher or lower than a bond's fundamental value.

More recently, Peivandi (2015) studies CDS auctions using a mechanism design approach that puts an emphasis on participation. He argues that any subset of CDS traders may settle among themselves away from the CDS auction, in a "blocking mechanism," if doing so gives them a better payoff than settling in the CDS auction. In particular, a large CDS trader may use side payments to prevent some of his CDS counterparties from participating in CDS auctions so that the large trader can manipulate the CDS auction price to his advantage. Peivandi (2015) shows that the only way to ensure full participation in CDS auctions is to use a fixed price, which is independent of agents' signals of the defaulted bond's value. That is, in Peivandi's model, full participation and price discovery cannot be achieved together.

Table II compares the economic channels of this paper to those of CGM (2013) and Peivandi (2015). As we can see, CGM (2013) and Peivandi (2015) share the common feature that the second stage of the CDS auction has positive price

Table II
Economic Channels of This Paper Relative to CGM (2013) and Peivandi (2015)

	Price Impact	Constraints to Buy/Sell Bonds	Nonparticipation of a Block	Price Discovery	Allocative Efficiency
CGM (2013)	Yes	Yes	No	No	No
Peivandi (2015)	Yes	No	Yes	No	No
Current paper	No	No	No	Yes	Yes

impact, whereas our model has zero price impact. Under zero price impact, CGM's model would generate underpricing only for open interest to sell (because of funding constraint), and Peivandi's model would not generate block settlement away from the auction (because no one can affect the final auction price). Our model with zero price impact therefore illustrates the economic insights of this paper in the cleanest and most transparent way. For a similar reason, we do not model constraints in buying or selling bonds, nor do we model side settlement. Of course, in practice, all of the economic channels listed in Table II are probably at play. Our results can thus be thought of as complementary to, and not in dispute with, those of CGM (2013) and Peivandi (2015).

More specifically, our results lead to a few new insights not in CGM (2013) and Peivandi (2015). First, we identify a new explanation for why CDS auctions generate biased prices: various restrictions prevent the participation of certain types of traders, even without price impact. In CGM (2013), a bond's fundamental value is commonly known after default, so CDS auctions provide no additional price discovery. Second, results on allocation inefficiency are unique to our model because the models of CGM (2013) and Peivandi (2015) have common values. Our allocation efficiency results provide new empirical predictions that can be tested in the data. Third, we show that traders have "excessive CDS positions" before default because a higher CDS position (in absolute value) relaxes participation constraints in the first stage of CDS auctions. Endogenous CDS positions are not modeled by CGM (2013) and Peivandi (2015). Last, we suggest a double auction design, which is distinct from CGM's proposal of an alternative allocation rule (following Kremer and Nyborg (2004)) and state-dependent price cap.

Empirically, CGM (2013) find that, for a sample of 26 CDS auctions on U.S. firms from 2006 to 2011, CDS auction prices tend to be lower than secondary market bond prices before and after auction dates. In CGM's model, this price pattern is generated by strategic bidding and some investors' inability to buy bonds. The V-shaped price pattern is confirmed by Gupta and Sundaram (2012, GS), who also conduct a structural estimation of CDS auctions. GS (2012) further examine Vickery and discriminatory auctions as alternative formats in the second stage, holding the first-stage strategies fixed. In earlier empirical papers with smaller samples, Helwege et al. (2009) find that CDS auction prices and bond prices are close to each other, and Coudert and Gex (2010) provide a detailed discussion on the performance of a few large CDS auctions.

The rest of the paper is organized as follows. Section I summarizes the institutional details concerning CDS auctions. Section II sets up our model. After solving for the unconstrained benchmark equilibrium in Section III, in Section IV we solve for the main equilibrium of the CDS auction model in which constraints are imposed. Manipulation incentives are discussed in Section V, and empirical implications are discussed in Section VI. In Section VII, we propose the double auction design and discuss various practical considerations.

I. Institutional Background on CDS Auctions

This section provides an overview of CDS auctions. Detailed descriptions of the auction mechanism are provided by Creditex and Markit (2010).

A CDS auction comprises two stages. In the first stage, participating dealers[7] submit "physical settlement requests" on behalf of themselves and their clients. These physical settlement requests indicate whether they want to buy or sell the defaulted bonds as well as the quantities of bonds they want to buy or sell. Importantly, only market participants with nonzero CDS positions are allowed to submit physical settlement requests, and these requests must be in the opposite direction of, and not exceeding, their net CDS positions. For example, suppose that bank A has bought CDS protection on \$100 million notional of General Motors bonds. Because bank A will deliver defaulted bonds in physical settlement, the bank can only submit a physical *sell* request with a notional value between \$0 and \$100 million. Similarly, a fund that has sold CDS on \$100 million notional of GM bonds is only allowed to submit a physical *buy* request with a notional value between \$0 and \$100 million.[8] Participants who submit physical requests are obliged to transact at the final price, which is determined in the second stage of the auction and is thus unknown in the first stage. The net of total buy physical requests and total sell physical requests is called the "open interest."

Also in the first stage, but separately from the physical settlement requests, each dealer submits a two-way quote, that is, a bid and an offer. The quotation size (say \$5 million) and the maximum spread (say \$0.02 per \$1 face value of bonds) are predetermined in each auction. Bids and offers that cross each other are eliminated. The average of the best halves of remaining bids and offers becomes the "initial market midpoint" (IMM), which serves as a benchmark for the second stage of the auction. A penalty called the "adjustment amount" is

[7] For example, between 2006 and 2010, participating dealers in CDS auctions include ABN Amro, Bank of America Merrill Lynch, Barclays, Bear Stearns, BNP Paribas, Citigroup, Commerzbank, Credit Suisse, Deutsche Bank, Dresdner, Goldman Sachs, HSBC, ING Bank, JP Morgan Chase, Lehman Brothers, Merrill Lynch, Mitsubishi UFJ, Mizuho, Morgan Stanley, Nomura, Royal Bank of Scotland, Société Générale, and UBS.

[8] To the best of our knowledge, there are no formal external verifications that one's physical settlement request is consistent with one's net CDS position.

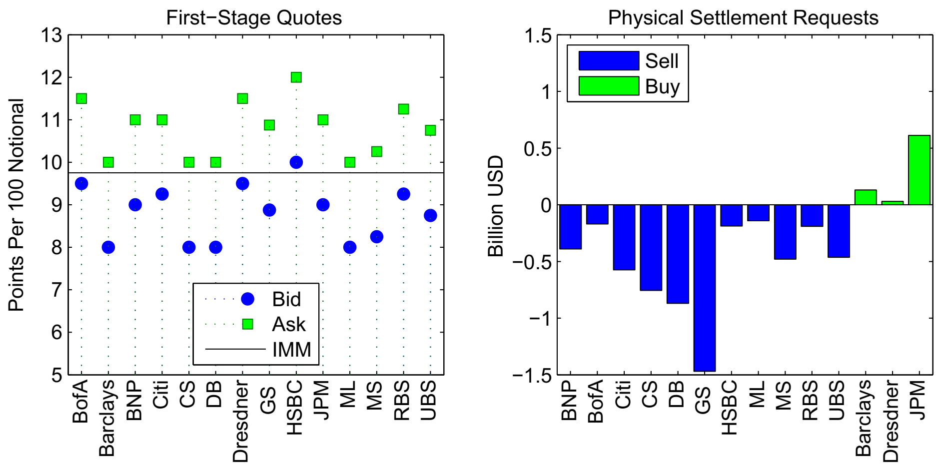

Figure 1. Lehman Brothers CDS auction, first stage. (Color figure can be viewed at wileyonlinelibrary.com)

imposed on dealers whose quotes are off-market (i.e., too far from other dealers' quotes).

The first stage of the auction concludes with the simultaneous publication of (i) the IMM, (ii) the size and direction of the open interest, and (iii) adjustment amounts, if any.

Figure 1 plots the first-stage quotes (left-hand panel) and physical settlement requests (right-hand panel) of the Lehman Brothers auction in October 2008. The bid-ask spread quoted by dealers was fixed at \$2 per \$100 face value, and the IMM was \$9.75. One dealer whose bid and ask were on the same side of the IMM paid an adjustment amount. Of the 14 participating dealers, 11 submitted physical sell requests and three submitted physical buy requests. The open interest to sell was about \$4.92 billion.

In the second stage of the auction, all dealers and market participants—including those without any CDS position—can submit limit orders to match the open interest. Nondealers must submit orders through dealers, and there is no restriction regarding the size of limit orders one can submit. If the first-stage open interest is to sell, then bidders must submit limit orders to buy. If the open interest is to buy, then bidders must submit limit orders to sell. Thus, the second stage is a one-sided market. The final price, p^*, is determined as in a usual uniform-price auction, subject to a price cap or floor. Specifically, for open sell interest, the final price is set at

$$p^* = \min\left(M + \Delta,\, p_b\right), \tag{1}$$

where M is the IMM, Δ is a predetermined "spread" that is usually \$0.01 or \$0.02 per \$1 face value, and p_b is the limit price of the last limit buy order

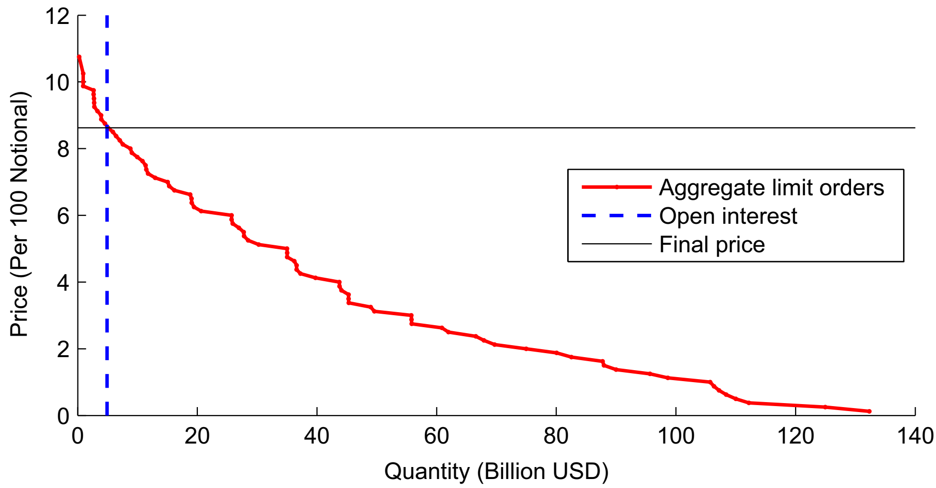

Figure 2. Lehman Brothers CDS auction, second stage. (Color figure can be viewed at wileyonlinelibrary.com)

that is matched. If needed, limit orders with price p^* are rationed pro-rata. Symmetrically, for open buy interest, the final price is set at

$$p^* = \max(M - \Delta, p_s), \tag{2}$$

where p_s is the limit price of the last limit sell order that is matched, with pro-rata allocation at p^* if needed. If the open interest is zero, then the final price is set at the IMM. The announcement of the final price p^* concludes the auction.

After the auction, bond buyers and sellers that are matched in the auction trade the bonds at the price of p^*; this is called "physical settlement." In addition, CDS sellers pay CDS buyers $1 - p^*$ per unit notional of their CDS contract; this is called "cash settlement."

Figure 2 plots the aggregate limit order schedule in the second stage of the Lehman auction. For any given price p, the aggregate limit order at p is the sum of all limit orders to buy at p or above. The sum of all submitted limit orders was over \$130 billion, with limit prices ranging from \$10.75 (the price cap) to \$0.125 per \$100 face value. The final auction price was \$8.625. CDS sellers thus pay CDS buyers \$91.375 per \$100 notional of CDS contract.

II. A Model of CDS Auctions

In this section, we describe the model. Price caps or floors are not formally modeled here for analytical simplicity, but are discussed in Section V.

There is a unit mass of infinitesimal traders on $[0, 1]$. There are two dates, $t = 0, 1$. At $t = 0$, before the potential bond default, each trader $i \in [0, 1]$ is endowed with a benefit $b_i \in [-\overline{b}, \overline{b}]$, $\overline{b} > 0$, for buying a unit of CDS contract. A positive

b_i means trader i is a potential CDS buyer, such as an institution that wishes to hedge its credit exposure. A negative b_i corresponds to a potential CDS seller, such as an institution that wishes to create a synthetic credit exposure through derivatives. We assume that b_i is symmetrically distributed around 0: $\mathbb{P}(b_i \leq z \mid b_i \geq 0) = \mathbb{P}(b_i \geq -z \mid b_i \leq 0) \equiv F(z)$ for every $z \geq 0$. Moreover, trading each unit of CDS incurs a cost of $c > 0$,[9] and holding q units of CDS contracts incurs an inventory cost of $\frac{\gamma}{2}q^2$, with $\gamma > 0$. The linear cost c is a proxy for trading or operational costs, and the quadratic inventory cost is a proxy for convex margin or collateral costs. Each trader i chooses an endogenous CDS position $Q_i \in \mathbb{R}$, where $Q_i > 0$ means buying CDS and $Q_i < 0$ means selling CDS. We assume that $\overline{b} > c$, so some traders will choose $Q_i \neq 0$ in equilibrium. But because $c > 0$, we expect traders with sufficiently small $|b_i|$ to hold zero CDS position.

At $t = 1$, the bond defaults with probability π; if the bond does not default, it pays the face value \$1. Conditional on default, there are two states for the defaulted bonds, high (H) and low (L), with equal ex ante probability. In the high state, a fraction $m > 1/2$ of the traders have value v_H for holding the defaulted bonds, and the remaining fraction $1 - m$ have value v_L, where $v_H > v_L$. In the low state, a fraction m of the traders have value v_L, and the rest have value v_H. Each trader observes his value for the bond immediately after a default, and each trader i's value v_i for a defaulted bond is independent of his value b_i for buying or selling CDS. In practice, a trader's value v_H or v_L may reflect, for example, this trader's expertise in managing the complicated legal process of restructuring or liquidation. Hence, reallocating defaulted bonds to investors who can extract a higher recovery value is socially beneficial.

After default and the private values $\{v_i\}$ are realized, the following two-stage auction is held:

(1) In stage 1, each trader $i \in [0, 1]$ submits a physical settlement request r_i that satisfies $r_i \cdot Q_i \leq 0$ and $|r_i| \leq |Q_i|$.
Let

$$R \equiv \int_i r_i \, di \tag{3}$$

be the open interest in the first stage of the auction.

(2) (a) If $R < 0$, then in stage 2 each trader $i \in [0, 1]$ submits a demand schedule $x_i : [0, 1] \to [0, \infty)$ to buy bonds.
(b) If $R > 0$, then in stage 2 each trader $i \in [0, 1]$ submits a supply schedule $x_i : [0, 1] \to (-\infty, 0]$ to sell bonds.

(3) The final auction price p^* is defined by

$$\int_i (r_i + x_i(p^*)) \, di = 0. \tag{4}$$

[9] The qualitative nature of the equilibrium would be the same if one assumed a fixed transaction cost, or the sum of fixed and proportional costs.

(4) The bond allocation to trader i is $r_i + x_i(p^*)$.

We assume that buying or selling a net position q of defaulted bonds in the CDS auction incurs the inventory cost of $\frac{\lambda}{2}q^2$, for some $\lambda > 0$, with the same motivation of convex collateral or margin cost.

Summarizing, the time-0 expected utility of trader i is

$$U_i = \left(b_i - p^{CDS}\right)Q_i - c|Q_i| - \frac{\gamma}{2}Q_i^2$$
$$+\pi \cdot \mathbb{E}\left[(1-p^*)Q_i + \left(v_i - p^*\right)\left(r_i + x_i(p^*)\right) - \frac{\lambda}{2}\left(r_i + x_i(p^*)\right)^2\right]. \quad (5)$$

The first line of U_i summarizes the time-0 profit of trading Q_i units of CDS, including the per-unit cost and quadratic inventory cost. The second line of U_i summarizes the profit of trader i in the CDS auction: $(1-p^*)Q_i$ is the payout from CDS settlement, $(v_i - p^*)(r_i + x_i(p^*))$ is the profit of trading the defaulted bonds, and $\frac{\lambda}{2}(r_i + x_i(p^*))^2$ is the inventory cost of creating the bond position $r_i + x_i$ in CDS auctions. This type of linear-quadratic utility is also used by Vives (2011), Rostek and Weretka (2012), and Du and Zhu (2016), among others. The constants γ and λ that dictate the inventory costs are required to be positive, but can be arbitrarily small.

We make two final remarks on the model setup. First, we do not explicitly model initial bond positions because they are not an essential element for our main result. One could add initial bond positions to the model by assuming that trader i is endowed with initial bond position z_i, such that $\int_i z_i\, di = Z$, the total bond supply. Exogenous initial bond positions like these could come from institutional constraints. For instance, if a corporate bond fund receives money inflows or outflows, then it has to buy or sell corporate bonds. The quadratic cost $\frac{\lambda}{2}(r_i + x_i(p^*))^2$ should be interpreted as the cost of creating a bond position in CDS auctions that is above and beyond the predefault bond holdings. With initial bond positions, the utility function U_i becomes

$$U_i' = \left(b_i - p^{CDS}\right)Q_i - c|Q_i| - \frac{\gamma}{2}Q_i^2 + (1-\pi)z_i \cdot 1$$
$$+\pi \cdot \mathbb{E}\left[v_i z_i + (1-p^*)Q_i + \left(v_i - p^*\right)\left(r_i + x_i(p^*)\right) - \frac{\lambda}{2}\left(r_i + x_i(p^*)\right)^2\right]. \quad (6)$$

Clearly, the extra term involving z_i is $(1 - \pi + \pi\mathbb{E}[v_i])z_i$ at $t = 0$ and is either $v_i z_i$ (given default) or z_i (given no default) at $t = 1$. Hence, the part of the utility function U_i' involving z_i is a constant to dealer i on each date and does not affect his optimal strategies. Without loss of generality, we can work with U_i in equation (5).

Second, we do not explicitly model the trading of bonds after default but before the CDS auction. This time window is fairly short in reality (about three weeks in the case of Lehman Brothers). Moreover, as discussed in the introduction, trading defaulted bonds in the secondary market is costly, which prevents some beneficial trades from taking place before the auction. The qualitative nature of our results is therefore likely to carry through if bonds are also traded after default but before the CDS auction.

III. The Competitive Equilibrium Benchmark

As a benchmark, we first consider the competitive equilibrium. In our setting with infinitesimal traders, the competitive equilibrium at $t = 1$ can be implemented by a double auction in which each trader submits an unconstrained demand schedule, taking the bond price as given. The first-order condition of U_i with respect to bonds gives the demand for bonds:

$$x_i^c = \frac{v_i - p}{\lambda}. \tag{7}$$

Given the market-clearing condition $\int_i x_i^c \, di = 0$, the competitive equilibrium prices in the high state and low state are, respectively,

$$p_H^c = mv_H + (1-m)v_L, \tag{8}$$

$$p_L^c = (1-m)v_H + mv_L. \tag{9}$$

The allocations in the competitive equilibrium are efficient, so trader i's efficient allocation of defaulted bonds is $(v_i - p_H^c)/\lambda$ in the high state and $(v_i - p_L^c)/\lambda$ in the low state.

Next, we turn to the competitive equilibrium at $t = 0$ in the CDS market before a default. Trader i's competitive (i.e., efficient) allocation of defaulted bonds is clearly independent of his CDS position Q_i. This implies that the last two terms in the expectation $\mathbb{E}[(v_i - p^*)(r_i + x_i(p^*)) - \frac{\lambda}{2}(r_i + x_i(p^*))^2]$ are not functions of Q_i. Moreover, $\mathbb{E}[1 - p^*]$ is equal to the ex ante mean, $1 - (v_H + v_L)/2$. Thus, the first-order condition of U_i with respect to Q_i gives the following demand for CDS:

$$Q_i = \begin{cases} \frac{1}{\gamma} \max\left(b_i - p^{CDS} - c + \pi\left(1 - \frac{v_H + v_L}{2}\right), 0\right) & \text{if } b_i \geq 0, \\ \frac{1}{\gamma} \min\left(b_i - p^{CDS} + c + \pi\left(1 - \frac{v_H + v_L}{2}\right), 0\right) & \text{if } b_i \leq 0. \end{cases} \tag{10}$$

Since the distribution of b_i is symmetric around zero, the market-clearing condition $\int_i Q_i \, di = 0$ for CDS implies

$$p^{CDS} = \pi\left(1 - \frac{v_H + v_L}{2}\right). \tag{11}$$

Thus, in the competitive equilibrium trader i chooses

$$Q_i^c = \begin{cases} \frac{1}{\gamma} \max(b_i - c, 0) & \text{if } b_i \geq 0, \\ \frac{1}{\gamma} \min(b_i + c, 0) & \text{if } b_i \leq 0. \end{cases} \tag{12}$$

In summary, the strategy (12) at $t = 0$ and the strategy (7) at $t = 1$ constitute the unique equilibrium in the competitive benchmark.

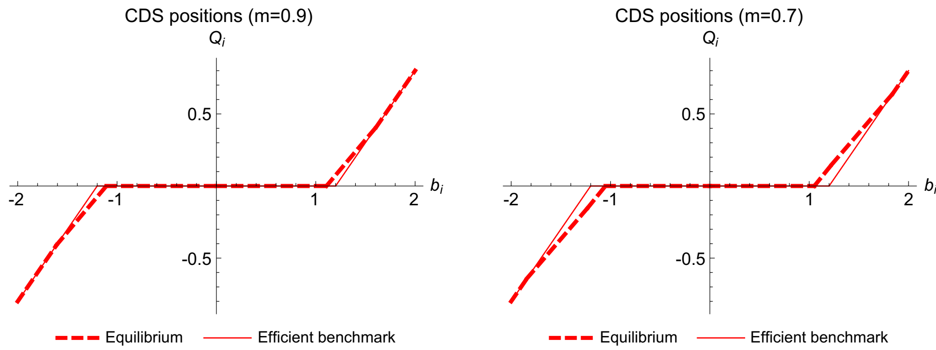

Figure 3. CDS positions from Proposition 4 (left) and Proposition 5 (right). Parameters: $v_H = 1, v_L = 0, \lambda = 1, \gamma = 1, \pi = 0.5, c = 1.2$, and b is uniformly distributed on $[-2, 2]$. (Color figure can be viewed at wileyonlinelibrary.com)

IV. Equilibrium of CDS Auctions and Predefault CDS Trading

This section provides an equilibrium analysis of CDS auctions and predefault CDS trading.

A. Intuition behind the Equilibrium in a Numerical Example

Before formally stating the equilibrium strategies, we first provide intuition behind the equilibrium outcomes based on a numerical example and a few plots. The formal statements of equilibrium strategies are in Section IV.B and Section IV.C. Since this discussion of intuition is presented before the formal results, we focus on the most salient features of the equilibrium in this subsection and leave details to the next two subsections. This sequence is chosen to prioritize the general intuition of the equilibrium over (certain complicated-looking) mathematical formulas. Readers who prefer to see the formal results first may jump to Section IV.B and Section IV.C and then return to this subsection.

Throughout this subsection, we compare the equilibrium behavior and the hypothetical efficient benchmark shown in Section III. We use the following parameterization for the numerical examples: $v_H = 1$, $v_L = 0$, $\lambda = 1$, $\gamma = 1$, $\pi = 0.5$, $c = 1.2$, and b is uniformly distributed on $[-2, 2]$.

Predefault CDS trading. Figure 3 shows the time-0 equilibrium choice of CDS position, Q_i, as a function of the benefit of CDS trading, b_i. The thin solid line is the efficient benchmark of Section III, and the thick dashed line is the equilibrium CDS positions.

The main observation from this figure is that only traders with sufficiently positive or sufficiently negative b_i trade CDS. The rest hold zero CDS positions. Because of the CDS auction rule that $|r_i| \leq |Q_i|$, these zero-CDS-position traders cannot submit a physical settlement request in the first stage of CDS auctions. We thus anticipate, and will see shortly, that these traders suffer from inefficient allocations of defaulted bonds.

A secondary observation from Figure 3 is that, for a certain range of b_i, traders' CDS positions tend to be "too large" relative to the efficient benchmark (i.e., more positive Q_i if $b_i > 0$ and more negative Q_i if $b_i < 0$). Again, this is because of the CDS auction rule $|r_i| \leq |Q_i|$. As we explain in more detail below, by trading CDS positions that are too large, a trader gets some flexibility in the CDS auctions. This incentive of trading too-large CDS does not apply to traders with b_i close to zero because of the trading cost c, nor does it apply to any trader with a large $|b_i|$, since such a trader already picks a large $|Q_i|$ that does not constrain him in the CDS auction.

CDS auction strategies. Now we turn to the strategies in the CDS auction. The intuition here is best understood by asking which types of traders are constrained in the two stages?

Figure 4 plots the first-stage physical requests and second-stage allocations in equilibrium, both as functions of b_i. Here, we have "plugged in" the equilibrium mapping from b_i to Q_i, as shown in Figure 3. The three left-hand subplots correspond to the parameter case with a sufficiently large m, shown formally in Proposition 1. The three right-hand subplots correspond to the parameter case with a sufficiently small m, shown formally in Proposition 2.

The top two subplots of Figure 4 show the first-stage physical request r_i as a function of b_i. As we show in Section IV.C, traders with moderate b_i choose $Q_i = 0$ because of the cost of trading CDS. Since $|r_i| \leq |Q_i|$, those traders are forced to pick $r_i = 0$, that is, they are prevented from participating in the first stage. Moreover, a trader with a sufficiently positive b_i will choose a positive Q_i and can only submit a sell physical request (because r_i and Q_i must have opposite signs). If, however, this trader turns out to have a high value for holding the bonds and wishes to buy, the constraint $r_i Q_i \leq 0$ prevents him from sending a physical request to buy. As a result, a high-value trader with sufficiently positive b_i chooses $r_i = 0$. Similarly, a low-value trader with sufficiently negative b_i chooses $r_i = 0$. The only traders who send a nonzero physical requests are those who have (i) a sufficiently positive b_i and a realized low value v_L or (ii) a sufficiently negative b_i and a realized high value v_H.

The next four subplots of Figure 4 show the second-stage allocations, depending on whether m is high or low and on whether the open interest is to buy or sell. Let us first focus on the high state, which leads to an open interest to buy ($R > 0$). In this case, the two middle subplots show that all high-value traders are prevented from participating in the second stage. This is because only sell orders are allowed for $R > 0$ but high-value traders only wish to buy. By contrast, all low-value traders participate in the second stage. Consequently, in the second stage the ratio of high-value traders to low-value traders is too low, leading to a downward-biased auction price.

The opposite case, the low state, displays symmetric behavior. The two bottom subplots show that all low-value traders are prevented from participating in the second stage because the open interest is to sell ($R < 0$). As a result, the final auction price is too high.

Biased auction price. Figure 5 plots the final auction price as a function of m. The high-state equilibrium price is denoted p^*_B since the open interest is to buy,

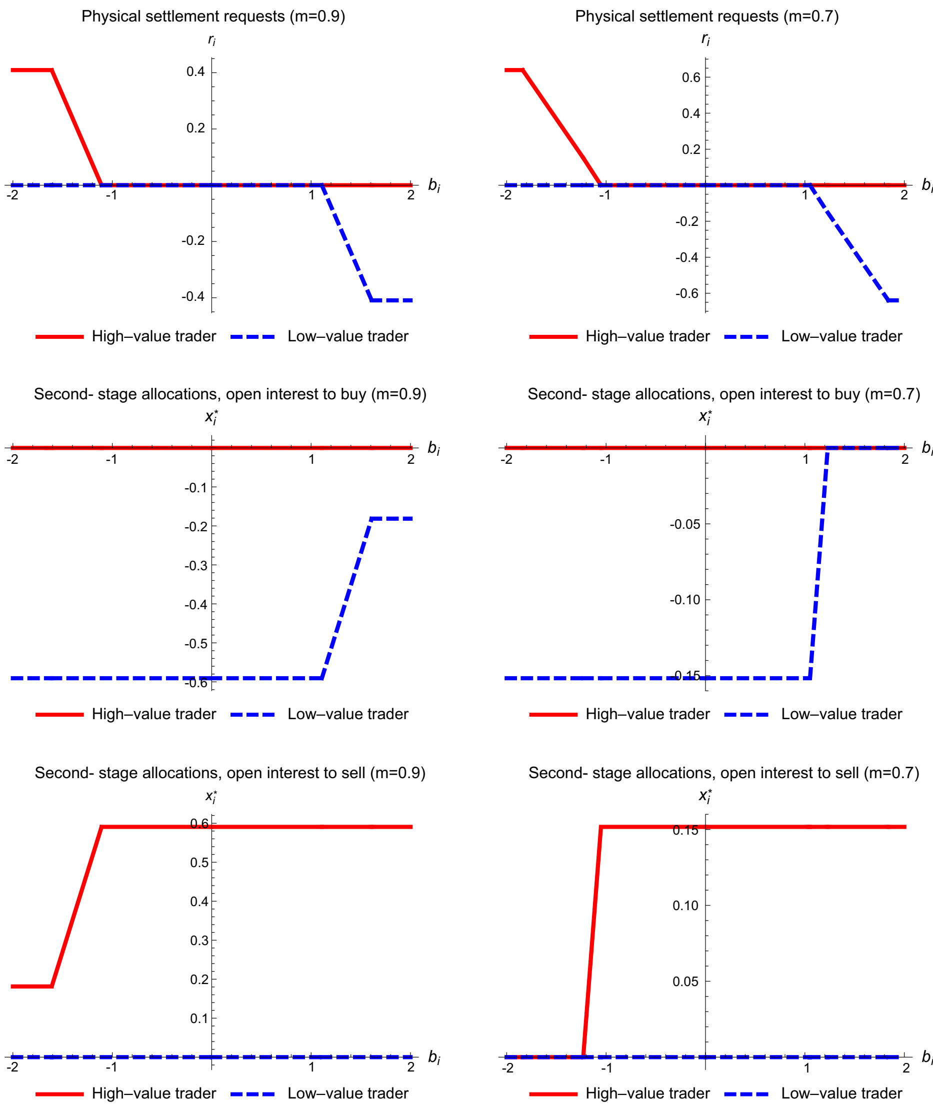

Figure 4. First-stage physical requests and second-stage allocations, given the CDS positions from Figure 3. Parameters: $v_H = 1$, $v_L = 0$, $\lambda = 1$, $\gamma = 1$, $\pi = 0.5$, $c = 1.2$, and b is uniformly distributed on $[-2, 2]$. Left panels are from Proposition 1 ($\alpha = 1$), and right panels are from Proposition 2. (Color figure can be viewed at wileyonlinelibrary.com)

and the low-state equilibrium price is denoted p_S^* since the open interest is to sell. We observe that $p_B^* < p_H^c$ and $p_S^* > p_L^c$ for all $m \in (0.5, 1)$. Again, because the high state leads to an open interest to buy and only low-value traders are willing to sell (at the equilibrium price) in the second stage, the final auction price is downward-biased relative to the efficient benchmark. Likewise, the low state produces an upward-biased price.

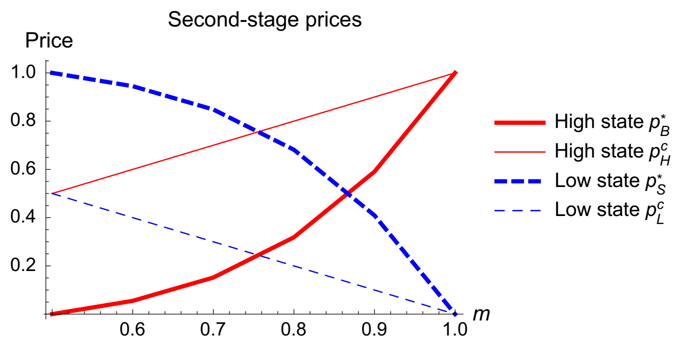

Figure 5. Final auction prices (parameters as in Figure 4). (Color figure can be viewed at wileyonlinelibrary.com)

We also observe from Figure 5 that the price bias is the most severe for m close to 1/2. This is because m close to 1/2 generates a very small open interest in magnitude. Indeed, as $m \to 1/2$, $R \to 0$. In the high state, if R is small, then the substantial participation of low-value traders in the second stage dominates the small open interest to buy. As a result, the auction price is heavily downward-biased. Similarly, in the low state, there is a very small open sell interest, and high-value traders dominate price discovery in the second stage, resulting in a heavily upward-biased price.

Inefficient allocations. It is therefore not a surprise that the equilibrium allocations of bonds are inefficient. Figure 6 depicts the final allocations. In the high state, allocations to low-value traders are uniformly too high. High-value traders buy either too much (if Q_i is sufficiently negative) or too little (if Q_i is positive or mildly negative). In the low state, allocations to high-value traders are uniformly too low. Low-value traders sell either too much (if Q_i is sufficiently positive) or too little (if Q_i is negative or mildly positive).

B. Equilibrium of the Two-Stage CDS Auction

Having illustrated the intuition of the results through a numerical example, we now provide the formal equilibrium results for the two-stage CDS auction (all proofs are provided in the Appendix A). In this subsection, we take the preauction CDS positions as given. In particular, we suppose that the predefault CDS positions $\{Q_i\}$ are symmetrically distributed around 0: $\mathbb{P}(Q_i \le z \mid Q_i \ge 0) = \mathbb{P}(Q_i \ge -z \mid Q_i \le 0) \equiv G(z)$ for every $z \ge 0$. The distribution of CDS positions is endogenized in Section IV.C.

Given the first-stage open interest, the second-stage strategy is straightforward. Since traders are infinitesimal, each trader takes the price p^* in the second stage as given and wants to get as close to his optimal bond allocation, $\frac{v_i - p^*}{\lambda}$, as possible.

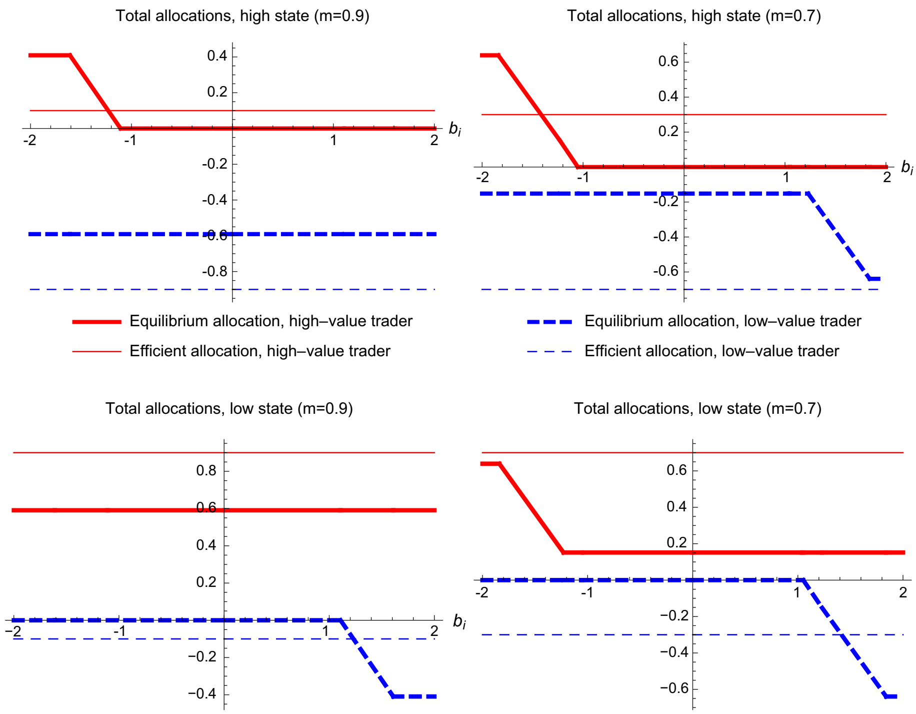

Figure 6. Total allocations in two stages (parameters as in Figure 4). (Color figure can be viewed at wileyonlinelibrary.com)

LEMMA 1: *Let R be the open interest. In any equilibrium, in the second stage each trader i submits the demand / supply schedule*

$$x_i(p) = \begin{cases} \max\left(-r_i + \frac{v_i - p}{\lambda}, 0\right), & \text{if } R < 0, \\ \min\left(-r_i + \frac{v_i - p}{\lambda}, 0\right), & \text{if } R > 0. \end{cases} \tag{13}$$

The following propositions show the equilibrium strategy in both stages of the CDS auction. Proposition 1 corresponds to the case $p_B^* \geq p_S^*$ and the left-hand subplots of Figures 3, 4, and 6. Proposition 2 corresponds to the case $p_B^* < p_S^*$ and the right-hand subplots of Figures 3, 4, and 6.

PROPOSITION 1: *Suppose*

$$-(1-m)\frac{v_H - v_L}{2\lambda} + \frac{m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - v_L}{2\lambda}\right) dG(Q_i) \geq 0. \tag{14}$$

Let p_B^* and p_S^* be the unique solution to

$$(1-m)\frac{v_L - p_B^*}{\lambda} + \frac{m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - p_B^*}{\lambda}\right) dG(Q_i) = 0, \tag{15}$$

$$p_S^* = v_L + v_H - p_B^*. \tag{16}$$

We have the following equilibrium in the two-stage auction:

- *In the first stage, every trader i submits*

$$r_i = \begin{cases} \min\left(-Q_i, \dfrac{v_H - \alpha p_B^* - (1-\alpha)p_S^*}{\lambda}\right) & \textit{if } v_i = v_H, Q_i < 0 \\ \max\left(-Q_i, \dfrac{v_L - \alpha p_S^* - (1-\alpha)p_B^*}{\lambda}\right) & \textit{if } v_i = v_L, Q_i > 0 \\ 0 & \textit{otherwise} \end{cases} \tag{17}$$

  for any $\alpha \in [0, 1]$.
- *In the second stage, every trader i submits the demand schedule in equation (13). In the high state, the open interest is to buy, and the final price is p_B^*; in the low state, the open interest is to sell, and the final price is p_S^*. We have $p_B^* \geq p_S^*$.*
- *The equilibrium outcome is unique, in the sense that the total allocations of bonds $r_i + x_i(p^*)$ and the prices p_B^* and p_S^* are the same for any choice of α in the first stage.*

PROPOSITION 2: *Suppose*

$$-(1-m)\frac{v_H - v_L}{2\lambda} + \frac{m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - v_L}{2\lambda}\right) dG(Q_i) < 0. \tag{18}$$

Let p_B^* and p_S^* be the unique solution to

$$\frac{1-m}{2}\frac{v_L - p_B^*}{\lambda} + \frac{2m-1}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - mp_B^* - (1-m)p_S^*}{\lambda}\right) dG(Q_i) + \frac{1-m}{2}\int_{Q_i \geq 0} \min\left(\frac{v_L - p_B^*}{\lambda} + Q_i, 0\right) dG(Q_i) = 0, \tag{19}$$

$$p_S^* = v_L + v_H - p_B^*. \tag{20}$$

We have the following equilibrium in the two-stage auction:

- *In the first stage, every trader i submits*

$$r_i = \begin{cases} \min\left(-Q_i, \dfrac{v_H - mp_B^* - (1-m)p_S^*}{\lambda}\right) & \textit{if } v_i = v_H, Q_i < 0 \\ \max\left(-Q_i, \dfrac{v_L - mp_S^* - (1-m)p_B^*}{\lambda}\right) & \textit{if } v_i = v_L, Q_i > 0 \\ 0 & \textit{otherwise.} \end{cases} \tag{21}$$

- *In the second stage, every trader i submits the demand schedule in equation (13). In the high state, the open interest is to buy, and the final price is p_B^*; in the low state, the open interest is to sell, and the final price is p_S^*. We have $p_B^* < p_S^*$.*

PROPOSITION 3: *In both Propositions 1 and 2, the equilibrium price p_S^* in the low state is higher than the competitive price p_L^c, and the equilibrium price p_B^* in the high state is lower than the competitive price p_H^c.*

Moreover, the equilibrium allocation of bonds is inefficient in both Propositions 1 and 2, in the following manner:

State	*Open Interest*	*Allocations*					
		High-Value Traders			*Low-Value Traders*		
		CDS Buyer	*Zero CDS*	*CDS Seller*	*CDS Buyer*	*Zero CDS*	*CDS Seller*
High	*Buy*	*Too low*	*Too low*	*Mixed*	*Too high*	*Too high*	*Too high*
Low	*Sell*	*Too low*	*Too low*	*Too low*	*Mixed*	*Too high*	*Too high*

The general shapes of the equilibrium behaviors in Propositions 1, 2, and 3 are described in the previous subsection, so we now look at the equations in more detail.

Starting from condition (14) of Proposition 1, this condition guarantees that $p_B^* \geq p_S^*$. To see why, consider the allocation of bonds at the hypothetical price $(v_H + v_L)/2$ and in the high state. The first term on the left-hand side of condition (14) is the total allocation for the low-value traders. Those traders are not constrained in the high state, so their allocation is $(1-m)\frac{v_L-(v_H+v_L)/2}{\lambda} = -(1-m)\frac{v_H-v_L}{2\lambda}$. The second term on the left-hand side of condition (14) is the total allocation for the high-value CDS sellers, who have submitted buy physical requests but do not buy in the second stage because the open interest is to buy. High-value CDS buyers are prevented from trading altogether. Thus, condition (14) says that at a price of $(v_H + v_L)/2$, there is excess demand for bonds, so $p_B^* \geq (v_H + v_L)/2$. But since the equilibrium is symmetric, $p_B^* \geq (v_H + v_L)/2$ if and only if $p_S^* \leq (v_H + v_L)/2$, or $p_B^* \geq p_S^*$. By a similar argument, if the state is low, condition (14) implies an excess supply of bonds at the hypothetical price $(v_H + v_L)/2$ and hence $p_S^* \leq (v_H + v_L)/2$, which also leads to $p_B^* \geq p_S^*$.

The equation defining p_B^*, (15), simply follows from market-clearing at the second stage. In the high state, the two terms on the left-hand side of equation (15) represent the allocations of low-value traders and high-value CDS sellers, respectively, at the price p_B^*. The price p_S^* follows from symmetry.

Likewise, condition (18) in Proposition 2 guarantees that $p_B^* < p_S^*$. The equation that defines p_B^*, (19), is similar to equation (15) but involves three terms. By comparing the top-right and top-left subplots of Figure 6, we see that the extra term comes from more "subtypes" of low-value traders. Again, p_S^* follows by symmetry.

Now let us move to first-stage strategies. There is a subtle difference between the two equilibrium cases. In Proposition 1, with $p_B^* \geq p_S^*$, traders are indifferent between a range of physical requests. For example, a high-value trader with a highly negative Q_i (that does not constrain any r_i) is indifferent between $r_i = \frac{v_H - \alpha p_B^* - (1-\alpha)p_S^*}{\lambda}$ for every $\alpha \in [0, 1]$. This is because he can sell back $x_i = -r_i + \frac{v_H - p_B^*}{\lambda} < 0$ units if $R > 0$ and buy back $x_i = -r_i + \frac{v_H - p_S^*}{\lambda} > 0$ units if $R < 0$, where the two inequalities follow from $p_B^* \geq p_S^*$, so his total allocation of bonds is the same.

In Proposition 2, with $p_B^* < p_S^*$, a high-value trader does not wish to trade more in stage 2 given a physical request $r_i = \frac{v_H - \alpha p_B^* - (1-\alpha)p_S^*}{\lambda}$, for any $\alpha \in [0, 1]$. To see this, note that $r_i = \frac{v_H - \alpha p_B^* - (1-\alpha)p_S^*}{\lambda} \in [\frac{v_H - p_S^*}{\lambda}, \frac{v_H - p_B^*}{\lambda}]$. If $R > 0$, the price is p_B^* and the high-value trader can only sell; but his first-stage trade r_i is already smaller than his desired trade $\frac{v_H - p_B^*}{\lambda}$. Symmetrically, if $R < 0$, the price is p_S^* and the high-value trader can only buy; but at p_S^* his first-stage trade r_i is already larger than his desired trade $\frac{v_H - p_S^*}{\lambda}$. Thus, in both cases, the high-value trader does not trade in the second stage. The uniquely optimal r_i is obtained by taking $\alpha = m$ because by Bayes's rule the high-value trader believes that the high state occurs with probability m.

C. Predefault CDS Trades

We now endogenize the distribution of CDS positions $\{Q_i\}$, or the function G in the previous section. At $t = 0$, trader i chooses Q_i based on the benefit b_i and cost c of trading CDS, while also taking into account the equilibrium outcomes in the CDS auction if a default occurs. As before, there are two equilibrium cases.

PROPOSITION 4: *Suppose*

$$-(1-m)\frac{v_H - v_L}{2\lambda} + \frac{m}{2}\int_0^{\overline{b}} \min\left(\frac{(b_i + \pi m(v_H - v_L)/4 - c)^+}{\gamma + \pi m\lambda/2}, \frac{v_H - v_L}{2\lambda}\right) dF(b_i) \geq 0. \tag{22}$$

Let p_B^* and p_S^* be the unique solution to

$$(1-m)\frac{v_L - p_B^*}{\lambda} + \frac{m}{2}\int_0^{\overline{b}} \min\left(\frac{(b_i + \pi m(v_H - p_B^*)/2 - c)^+}{\gamma + \pi m\lambda/2}, \frac{v_H - p_B^*}{\lambda}\right) dF(b_i) = 0, \tag{23}$$

$$p_S^* = v_L + v_H - p_B^*. \tag{24}$$

We have the following equilibrium:

- *Before the auction, we have for* $b_i \geq 0$

$$Q_i(b_i) = \begin{cases} \dfrac{(b_i + \pi m(p_S^* - v_L)/2 - c)^+}{\gamma + \pi m\lambda/2} > \dfrac{b_i - c}{\gamma} & \text{if } b_i \leq c + \dfrac{\gamma(p_S^* - v_L)}{\lambda}, \\ \dfrac{b_i - c}{\gamma} & \text{if } b_i > c + \dfrac{\gamma(p_S^* - v_L)}{\lambda}, \end{cases} \tag{25}$$

$Q_i(b_i) = -Q_i(-b_i)$ *if* $b_i \leq 0$*, and*

$$p^{CDS} = \pi\left(1 - \frac{v_H + v_L}{2}\right). \tag{26}$$

- *Equilibrium strategies in the CDS auction are given by Proposition 1, where* p_B^* *is the final auction price in the high state,* p_S^* *is the final auction price in the low state, and* $p_B^* \geq p_S^*$.

PROPOSITION 5: *Suppose*

$$-(1-m)\frac{v_H - v_L}{2\lambda} + \frac{m}{2}\int_0^{\bar{b}} \min\left(\frac{(b_i + \pi m(v_H - v_L)/4 - c)^+}{\gamma + \pi m\lambda/2}, \frac{v_H - v_L}{2\lambda}\right) dF(b_i) < 0. \tag{27}$$

Let p_B^* *and* p_S^* *be uniquely defined by*

$$\begin{aligned} &\frac{1-m}{2}\frac{v_L - p_B^*}{\lambda} + \frac{2m-1}{2} \\ &\cdot \int_0^{\bar{b}} \min\left(\frac{(b_i + \pi m(v_H - p_B^*)/2 - c)^+}{\gamma + \pi m\lambda/2},\right. \\ &\left.\frac{(b_i + \pi(v_H - mp_B^* - (1-m)p_S^*)/2 - c)^+}{\gamma + \pi\lambda/2}, \frac{v_H - mp_B^* - (1-m)p_S^*}{\lambda}\right) dF(b_i) \\ &+ \frac{1-m}{2}\int_0^{\bar{b}} \min\left(\frac{v_L - p_B^*}{\lambda} + \frac{(b_i + \pi m(v_H - p_B^*)/2 - c)^+}{\gamma + \pi m\lambda/2}, 0\right) dF(b_i) = 0, \end{aligned} \tag{28}$$

$$p_S^* = v_L + v_H - p_B^*. \tag{29}$$

We have the following equilibrium:

- *Before the auction, we have for* $b_i \geq 0$

$$Q_i(b_i) = \begin{cases} \frac{(b_i + (p_S^* - v_L)\pi m/2 - c)^+}{\gamma + \pi m\lambda/2} > \frac{b_i - c}{\gamma} & \text{if } \frac{(b_i + (p_S^* - v_L)\pi m/2 - c)^+}{\gamma + \pi m\lambda/2} \leq \frac{p_B^* - v_L}{\lambda} \\ \frac{b_i + (mp_S^* + (1-m)p_B^* - v_L)\pi/2 - c}{\gamma + \pi\lambda/2} > \frac{b_i - c}{\gamma} & \text{if } \frac{p_B^* - v_L}{\lambda} < \frac{b_i + (mp_S^* + (1-m)p_B^* - v_L)\pi/2 - c}{\gamma + \pi\lambda/2} \\ & \qquad \leq \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda} \\ \frac{b_i - c}{\gamma} & \text{if } \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda} < \frac{b_i - c}{\gamma}, \end{cases} \tag{30}$$

$Q_i(b_i) = -Q_i(-b_i)$ *if* $b_i \leq 0$, *and*

$$p^{CDS} = \pi\left(1 - \frac{v_H + v_L}{2}\right). \tag{31}$$

- *Equilibrium strategies in the CDS auction are given by Proposition 2, where* p_B^* *is the final auction price in the high state,* p_S^* *is the final auction price in the low state, and* $p_B^* < p_S^*$.

While Propositions 4 and 5 contain quite a few equations, most of them are analogous to those in Propositions 1 and 2. For example, condition (22) follows from substituting equation (25) into condition (14), and condition (27) follows from substituting equation (30) into condition (18). The equations defining p_B^* in the two cases are obtained similarly.

Therefore, the only equations left to discuss are the equilibrium choices of Q_i. Although the equilibrium price of CDS in Propositions 4 and 5 is the same as that in a competitive equilibrium (see Section III), the equilibrium CDS positions are generally larger in magnitude than their competitive counterparts.

Let us first examine Proposition 4 and, without loss of generality, consider a potential CDS buyer with $b_i \geq 0$. Traders anticipate equilibrium behavior from Proposition 1 in the CDS auction after a default. Suppose that the state of the default is low and trader i also has a low value for the bond. Then from Proposition 1, trader i has a total bond allocation of $\max(-Q_i, (v_L - p_S^*)/\lambda)$ in the auction. Thus, if $Q_i > (p_S^* - v_L)/\lambda$, his CDS position Q_i does not affect his bond allocation. In this case, Q_i is determined entirely by the cost c and benefit b_i of trading CDS before default, exactly as in the competitive equilibrium. This corresponds to the second case in equation (25). If $Q_i < (p_S^* - v_L)/\lambda$, then increasing Q_i moves trader i's bond allocation closer to the optimal quantity of $(v_L - p_S^*)/\lambda$ in the low state, and such incentive results in a larger Q_i compared with a competitive equilibrium. This corresponds to the first case in equation (25).

In Proposition 5, traders anticipate equilibrium behavior from Proposition 2 in the CDS auction after default. If $0 \leq Q_i \leq \frac{p_B^* - v_L}{\lambda}$ for a low-value CDS buyer, then in Proposition 2 Q_i only affects his total bond position in the low state as before; this corresponds to the first case in equation (30). If $\frac{p_B^* - v_L}{\lambda} < Q_i < \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}$, then in Proposition 2, $-Q_i$ is the total bond allocation in both high and low states for a low-value CDS buyer, which causes additional distortion in the preauction incentive for Q_i as summarized in the second case in equation (30). Finally, if $Q_i \geq \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}$, the bond allocation in the auction is unaffected by Q_i; this is the third case in equation (30).

In sum, as we discussed in the previous subsection, traders with moderate b_i enter predefault CDS positions that are too large (in magnitude) relative to the first-best. This is because a larger CDS position relaxes a trader's constraint in CDS auctions.

D. A Short Discussion of Multiple Equilibria

The equilibrium characterized in this section can generate both directions of open interest, $R > 0$ and $R < 0$, depending on the underlying state being high or low. This equilibrium can therefore match the empirical observation that some CDS auctions have open interest to sell and some open interest to buy.

This equilibrium is not the only one, however. In the Internet Appendix of this paper, we characterize two one-sided equilibria and show that they also do not yield the competitive price or competitive allocation.[10] In one equilibrium, the open interest is always to buy regardless of the state; in the other, the open interest is always to sell regardless of the state. Intuitively, these equilibria may arise due to coordination. For example, if low-value traders only participate in the second stage (using limit orders) and high-value traders participate in both stages, the open interest is always to buy. And conditional on always having a buy open interest, it is self-fulfilling that (i) high-value traders always submit buy market orders in the first stage, anticipating the low-state price (their second-stage limit orders are executed in the high state, selling back some of their first-stage orders), and (ii) low-value traders always submit limit sell orders in the second stage (there is no benefit for them to use the first stage). A symmetric self-fulfilling logic applies to the equilibrium that always has an open interest to sell. Clearly, both one-sided equilibria are counterfactual.

The equilibrium of this section and the two one-sided equilibria in the Internet Appendix are the only equilibria under pure strategies (see the Internet Appendix for more discussions). Since only the equilibrium characterized in this section can generate both directions of open interest, it is the one that we use throughout the paper.

V. A Brief Discussion of Price Caps/Floors and Dealers' Incentive to Manipulate the First-Stage Quotes

In Section IV, we show that the two-stage CDS auctions without a price cap or floor lead to biased prices and inefficient allocations. In this section, we briefly discuss the effect of imposing a price cap or floor that is formed by dealers' quotes in the first stage of the auction. Since the price cap or floor could be binding in the second stage, dealers may have incentives to manipulate the first-stage quotes to push the second-stage price in their favor.

Let the price cap be $\overline{p}$, which applies if $R < 0$, and let the price floor be $\underline{p}$, which applies if $R > 0$. Given the auction rule, if $R < 0$, the final auction price is $\min(\overline{p}, p_S^*)$, where p_S^* is the hypothetical market-clearing price without the price cap; if $R > 0$, the final auction price is $\max(\underline{p}, p_B^*)$, where p_B^* is the hypothetical market-clearing price without the price floor. Since the price cap or floor is determined by a relatively small group of dealers, it leads to manipulation incentives. For example, suppose that dealers are predominantly CDS buyers. Because they benefit from a low final auction price p^*, dealers have

[10] The Internet Appendix may be found in the online version of this article.

incentives to quote prices that are too low in the first stage, leading to a $\overline{p}$ and $\underline{p}$ that are also too low. All else equal, a strictly lower $\overline{p}$ or $\underline{p}$ means, respectively, a weakly lower $\min(\overline{p}, p_S^*)$ or $\max(\underline{p}, p_B^*)$ and hence a lower expected final auction price. Conversely, if dealers are predominantly CDS buyers, they tend to quote prices that are too high in the first stage, leading to an inflated price cap/floor and a higher expected final auction price. These theoretical results are straightforward and omitted.

Manipulation incentives like these are more than theoretical possibilities. A close analogy relates to LIBOR, an interest rate benchmark underlying trillions of dollars of derivatives contracts such as interest rate swaps. Every day, LIBOR is fixed at a truncated mean of quoted interest rates from LIBOR panel banks, just like the CDS auction midpoint price is fixed at a truncated mean of dealers' quotes. The difference is that LIBOR is the price at which the derivatives contracts settle on, whereas the CDS auction midpoint (adjusted by the cap amount) provides a one-way bound on the final auction price. But similar to the CDS auction setting, the LIBOR panel banks' profits and losses from their own derivatives books depend on LIBOR, so the LIBOR fixing method induces strong manipulation incentives. Unsurprisingly, manipulations of LIBOR have happened, and the banks involved in the scandals have paid billions of dollars in fines.[11] Although this analogy is by no means evidence of manipulation in CDS auctions, it serves as a reminder that certain incentives created by market designs are too strong to resist.

VI. Empirical Implications

This section discusses empirical predictions of our model. The following two propositions follow from the formal analysis of Section IV and the discussion of Section V.

PREDICTION 1: *All else equal,*

(1) *If the open interest is to buy, low-value traders get too large an allocation in the auctions and will sell bonds after the auctions. This effect is stronger for CDS sellers and traders with zero CDS positions.*
(2) *If the open interest is to sell, high-value traders get too little an allocation in the auctions and will buy bonds after the auctions. This effect is stronger for CDS buyers and traders with zero CDS positions.*

PREDICTION 2: *All else equal,*

(1) *In the first stage, dealers who are net CDS buyers quote lower prices than dealers who are net CDS sellers.*
(2) *The final auction price is lower if dealers' preauction CDS positions are more positive (or less negative).*

[11] For a comprehensive review, see https://en.wikipedia.org/wiki/Libor_scandal.

The publicly available CDS auction data report the first-stage market orders and second-stage limit orders by dealer name. These data do not distinguish dealers' own orders and customers' orders that are channeled through dealers.

To test Prediction 1, one needs proxies for the high-value traders and the low-value traders. For dealers, valuations may be inferred from their trading behaviors before CDS auctions. A higher-value dealer would be one that buy bonds at higher prices, controlling for other covariates. Alternatively, to the extent that dealers wish to keep a small absolute inventory level, a higher-value dealer would be one that has abnormally low (or negative) bond inventory. These proxies will require proprietary data that report dealers' identities in the secondary corporate bond markets, such as the unmasked TRACE data that are managed by the Financial Industry Regulatory Authority and available to regulators.

To test Prediction 2, one would need to combine publicly available CDS auction data with dealers' CDS positions. CDS positions exist in the Depository Trust and Clearing Corporation's Trade Information Warehouse and other regulatory agencies (at least for relatively recent CDS auctions). Note that Prediction 2 does not make a direct statement on the *level* of quotes or final prices in CDS auctions because, as we discussed in the introduction, a few other economic channels also potentially affect them. It would be ideal to test Prediction 2 controlling for those channels.

VII. Discussion: Current CDS Auction Design versus Double Auction

We have shown that the current design of CDS auctions leads to biased prices and inefficient allocations of defaulted bonds. Various restrictions imposed on both stages of CDS auctions prevent certain investors from fully participating in the price discovery process. Moreover, since dealers' first-stage quotes may bind the final auction price from above or below, this design also leaves ample room for potential manipulation. In our model, a double auction achieves the first best (see Section III). It is worth noting that these results are obtained in a model with infinitesimal traders who have zero impact on the price.

In markets with imperfect competition, a double auction is not fully efficient. This is because the strategic avoidance of price impact makes trading too slow relative to the first-best. This result is shown in the static models of Vives (2011), Rostek and Weretka (2012), and Ausubel et al. (2014), as well as the dynamic models of Vayanos (1999) and Du and Zhu (2016), among others. Appendix B reproduces the argument in a static setting, showing that the allocation inefficiency in double auctions is of order $O(1/n)$, where n is the number of auction participants.

Since the double auction protocol is not fully efficient, it is not obvious that it is better than the current CDS auction design. In particular, one may suspect that the current format of CDS auctions is designed to address some practical concerns not covered in our model. In the remainder of this section, we discuss potential motivations for the current CDS auction design, and discuss whether a double auction design can achieve the same objective equally well or better. Although this section involves no formal modeling, we draw upon the widespread implementation of double auctions in stock exchanges as a natural comparison to CDS auctions. Our general conclusion is that, even after taking into account various practical considerations, a double auction design—enriched, if necessary—is still more efficient than the current CDS auction design.

A. A Double Auction Design

Let us start by proposing a double auction design of CDS auctions:

- Step DA-1(a) In the first stage, physical settlement requests may still be submitted, and they are filled at the final auction price.
- Step DA-1(b) In the first stage, simultaneous to Step DA-1(a), dealers may still make quotes on the defaulted bonds, but their quotes no longer bind the second-stage price.
- Step DA-2 In the second stage, traders can submit *both* buy and sell limit orders, regardless of the open interest from the first stage. There is no price cap or floor. The final auction price matches supply and demand.

This double auction design is nothing exotic or unusual. It is similar to the standard mechanism used in open auctions and close auctions in stock exchanges such as NYSE and NASDAQ, with minor differences in details.

In principle, the only essential step in a double auction design is Step DA-2, as in Section III. By including Steps DA-1(a) and DA-1(b), the above implementation preserves the first-stage physical requests and quotes in the current CDS auction design. Note that, given Step DA-2, Step DA-1(a) is in fact unnecessary. A trader can move his market order in Step DA-1(a) to Step DA-2, with no effect in the equilibrium outcome. In this sense, one may also add arbitrary restrictions on Step DA-1(a), such as requiring that a trader's physical settlement request be opposite in direction to his CDS position. Moreover, the dealers' quotes in Step DA-1(b) could be useful for reducing information asymmetry. In models with asymmetric information and price impact, a lower degree of information asymmetry typically improves allocation efficiency (see Du and Zhu (2016) and references therein).

Before elaborating the difference between the one-sided CDS auction design and the double auction design, we emphasize that both mechanisms overcome the concerns that give rise to CDS auctions in the first place. Creditex and Markit (2010) give two main reasons why an auction protocol was introduced. First, an auction protocol produces a unique price at which investors can choose to settle in cash. Second, if CDS outstanding is greater than the volume of bonds outstanding, in bilateral physical settlements "the bonds would have to be 'recycled' a number of times through the market to settle all the CDS trades" (Creditex and Markit (2010)). This recycling of bonds creates an endogenous supply shortage and may artificially raise the price of defaulted bonds. By concentrating all buyers and sellers to one point in time, an auction protocol effectively eliminates the supply shortage problem.

However, while the above reasons justify an auction protocol to settle CDS, they do not justify a one-sided auction design. We now compare the one-sided CDS auction and the double auction.

B. Why Impose Restrictions in CDS Auctions?

Compared with the double auction design above, the current CDS auction design introduces two interconnected restrictions:

(1) In the first stage, dealers' quotes form a one-sided price constraint (cap or floor) on the final auction price.
(2) In the second stage, only limit orders that are on the opposite side of the open interest are permitted.

Below, we discuss a few practical considerations that may motivate these two restrictions.

Price manipulation in the second stage. According to Creditex and Markit (2010), the price cap and floor are introduced to "avoid a large limit order being submitted off-market to try and manipulate the results, particularly in the case of a small open interest." For instance, a large CDS seller benefits from a higher final auction price, and a large CDS buyer benefits from a lower final price. They thus have corresponding incentives to manipulate the price.

It is far from clear that a one-way price constraint is the best way to mitigate manipulation incentives. First, finding the correct price cap or floor is not easy. If the price cap is set too high, CDS sellers can still manipulate the price upward; but if the price cap is set too low, the price cap itself becomes an inefficient constraint.[12] A similar problem applies to the price floor. Second, using a price cap or floor gives dealers the ability to manipulate the final auction price, as we explain in Section V.

A more natural way to mitigate price manipulation is to allow limit orders in both directions in the second stage of the double auction (Step DA-2 above). Because CDS contracts have zero net supply, the increased profit to all CDS sellers of pushing the price up by ϵ is equal to the increased profit to all CDS buyers of pushing the price down by ϵ. That is, manipulation incentives apply to both sides and they should offset each other, at least in part. In Appendix B, we solve an extended double auction model with imperfect competition and show that, although each trader's demand schedule reflects his CDS position, these CDS positions have zero net effect on the final auction price because they add up to zero.

Supply or demand shock. In practice, a price constraint could be useful to guard against an unexpected supply or demand shock. For example, if only

[12] In our simple model with two states, the open interest reveals the efficient price. But in reality, traders' values for the bonds are likely more complicated, and finding the efficient price cap or floor is likely much more difficult.

a very small fraction of investors show up in CDS auctions (for rational or behavioral reasons), the resulting price may overshoot either way. In this case, a two-way price constraint seems more suitable than a one-way price constraint because, again, the latter gives dealers too much discretion in moving the price in a certain direction. With a two-way price constraint, the midpoint could be calculated from dealers' quotes or secondary market prices (where data are available), and the "spread" should be wide enough to reflect the volatility of the defaulted bonds and thereby discourage incentives to manipulate the quotes.

The U.S. equity markets offer some useful comparisons on such price constraints. For instance, in the NASDAQ closing auction, the lowest permissable price is the NASDAQ best bid minus the greater of $0.50 and 10% of the NASDAQ midpoint price, and the highest permissable price is the NASDAQ best ask plus the greater of $0.50 and 10% of the NASDAQ midpoint price.[13] Therefore, the price range in a NASDAQ close auction is at least 20% of the NASDAQ midpoint. In NYSE Arca close auctions, the price range is from 10% below to 10% above the last transaction price if the stock price is above $10, and from 25% below to 25% above the last transaction price if the stock price is below $10.[14] During continuous trading hours, the two-way price range of individual stocks is governed by the limit up–limit down mechanism, which sets three levels of price bands: 5%, 10%, and 20% of the average price of the stock over the immediately preceding five minutes. If these price bands are breached for more than 15 seconds, trading is halted for five minutes.[15] All these price ranges are designed to prevent extreme volatility in equity markets that may not be driven by fundamentals.

Given the illiquidity of defaulted bonds or loans, it seems reasonable to set a two-way price range that is comparable to equities or wider. In the current CDS auctions, the "cap amount" of $0.01 or $0.02 per $1 face value seems quite low.

Participation. In informal conversations, a couple of market participants have suggested that an advantage of the two restrictions—one-way price constraint and one-sided limit orders in the second stage—is to create additional uncertainty about price and allocation in the second stage of the auction, as such uncertainty should encourage CDS buyers and sellers to participate in the first stage of CDS auctions. For example, a CDS buyer who also holds defaulted bonds may find it risky to wait until the second stage to sell the bonds because he will not be allowed to submit sell orders if the open interest is to sell. This risk may prompt the CDS buyer to submit a physical settlement request in

[13] See https://www.nasdaqtrader.com/content/ProductsServices/Trading/Crosses/openclose_faqs.pdf

[14] See https://www.nyse.com/publicdocs/nyse/markets/nyse-arca/NYSE_Arca_Auctions_Brochure.pdf.

[15] The limit up–limit down mechanism for individual stocks should be distinguished from the marketwide circuit breaker, which is triggered if the S&P 500 index has a single-day decline of more than 7%, 13%, or 20% relative to the prior day's closing price. Breaching the 7% and 13% thresholds leads to a marketwide trading halt for 15 minutes, whereas breaching the 20% threshold ends marketwide trading for the remainder of the day. For more details, see https://www.sec.gov/investor/alerts/circuitbreakersbulletin.htm.

the first stage. If all CDS traders participate in the first stage and submit full physical settlement requests (i.e., $r_i = -Q_i$), then the net open interest would be zero and CDS auctions would achieve the same outcome as cash settlement.

Although we do not formally model the cost of participating in CDS auctions, our model can be viewed as one with zero participation cost. We show that the restrictions imposed in CDS auctions reduce participation by certain types of CDS traders, rather than encourage participation (see Section IV). This leads to biased prices and inefficient allocations. Moreover, the open interest is rarely zero in the data, suggesting that the restrictions in CDS auctions cannot attract full participation in the first stage of the auction. By contrast, the double auction design allows every CDS trader to participate if the cost of doing so is zero. By continuity, we infer that, as long as the participation cost is not too high, the double auction design is still better than the current design in terms of encouraging broad participation.

Summary. In summary, we have difficulty justifying the one-sided CDS auction design, even after considering various frictions and practical concerns. To be clear, while we argue that the double auction design is an improvement over the current CDS auction design, we are by no means suggesting that the double auction design would be the optimal mechanism in practice. Identifying the optimal mechanism in dynamic markets with frictions is a difficult problem that is beyond the scope of this paper.

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Appendix A

A. *Proof of Proposition 1*

We conjecture that $p_S^* \leq p_B^*$ and that in the high state $R > 0$, while in the low state $R < 0$.[16] We derive the equilibrium based on this conjecture, and then verify this conjecture under some parameter conditions.

We first show that the first-stage strategy in equation (17) is optimal, which has four cases.

Case 1. If the trader has a high value for the bond and is a CDS seller, then he wants to buy bonds but is constrained to buy at most $-Q_i$ units in the first stage. Given this constraint, it is optimal for him to submit $r_i = \min(\frac{v_H - \alpha p_B^* - (1-\alpha)p_S^*}{\lambda}, -Q_i)$ for any $\alpha \in [0, 1]$. Indeed, if the open interest is to buy, then he sells back some of his r_i, that is, $x_i = -\min(\frac{v_H - \alpha p_B^* - (1-\alpha)p_S^*}{\lambda}, -Q_i) + \frac{v_H - p_B^*}{\lambda} \leq 0$, which results in a total

[16] In equilibrium we cannot have $R<0$ in the high state and $R>0$ in the low state. Because high-value traders want to buy, if $R>0$ in the low state, then we must have $R>0$ in the high state as well (since there are more high-value traders in the high state). However, we do have a one-sided equilibrium in which $R>0$ in both high and low states and a one-sided equilibrium in which $R<0$ in both high and low states. See the Internet Appendix for those equilibria.

allocation of $\min((v_H - p_B^*)/\lambda, -Q_i)$ that is as close as possible to his optimal allocation $(v_H - p_B^*)/\lambda$. If the open interest is to sell, then he buys $x_i = -\min(\frac{v_H - \alpha p_B^* - (1-\alpha) p_S^*}{\lambda}, -Q_i) + \frac{v_H - p_B^*}{\lambda} \geq 0$ additional units in the second stage, exactly achieving his optimal allocation $(v_H - p_S^*)/\lambda$.

Case 2. The case for a low-value CDS buyer is analogous to Case 1.

Case 3. If the trader has a high value for the bond and has a positive or zero CDS position, then he cannot submit buy orders in the first stage. Clearly, his optimal request is to set $r_i = 0$ in the first stage.

Case 4. The case for a low-value trader with a zero or negative CDS position is analogous to Case 3.

Aggregating the allocations across the two stages, we have the following market-clearing condition given an open interest to buy (i.e., in the high state):

$$(1-m)\frac{v_L - p_B^*}{\lambda} + \frac{m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - p_B^*}{\lambda}\right) dG(Q_i) = 0. \tag{A1}$$

In equation (A1), all low-value traders sell in the second stage (regardless of their physical requests in the first stage) and achieve their optimal allocation of $(v_L - p_B^*)/\lambda$ (the first term). High-value traders with positive or zero CDS positions do not trade in either stage: they only want to buy, are constrained to sell in the first stage because of their CDS positions, and are constrained to sell in the second stage because of the open interest to buy. In the second term of equation (A1), high-value CDS sellers buy in the first stage; they sell in the second stage and achieve their optimal allocation of $(v_H - p_B^*)/\lambda$ if $-Q_i \geq (v_H - p_B^*)/\lambda$ (if $Q_i = 0$, the integrand is just zero).

Analogously, the market-clearing condition given an open interest to sell (i.e., in the low state) is

$$(1-m)\frac{v_H - p_S^*}{\lambda} + \frac{m}{2}\int_{Q_i \geq 0} \max\left(-Q_i, \frac{v_L - p_S^*}{\lambda}\right) dG(Q_i) = 0. \tag{A2}$$

The left-hand side of equation (A1) is clearly decreasing in p_B^*, is positive when $p_B^* = v_L$, and is negative when $p_B^* = v_H$. Thus, there exists a unique $p_B^* \in (v_L, v_H)$ that satisfies equation (A1). Likewise, the left-hand side of equation (A2) is clearly decreasing in p_S^*, is positive when $p_S^* = v_L$, and is negative when $p_S^* = v_H$. Thus, there exists a unique $p_S^* \in (v_L, v_H)$ that satisfies equation (A2). Moreover, the solution (p_B^*, p_S^*) clearly satisfies $v_H - p_S^* = p_B^* - v_L$. Thus, $p_B^* \geq p_S^*$ if and only if $p_B^* \geq (v_H + v_L)/2$, that is, the left-hand side of equation (A1) is nonnegative when $p_B^* = (v_H + v_L)/2$; this is simply condition (14).

Finally, given the identity $v_H - p_S^* = p_B^* - v_L$, the first-stage strategy in equation (17) implies that $R > 0$ in the high state and $R < 0$ in the low state, as we have conjectured.

This completes the derivation and verification of the equilibrium in which $p_B^* \geq p_S^*$.

B. Proof of Proposition 2

We conjecture that $p_B^* < p_S^*$ and that in the high state $R > 0$, while in the low state $R < 0$. We derive the equilibrium based on this conjecture, and then verify this conjecture under some parameter conditions.

We first show that the first-stage strategy in equation (21) is optimal, which has four cases.

- Case 1. If trader i has a high value for the bond and is a CDS seller, then he wants to buy bonds but is constrained to buy at most $-Q_i$ units in the first stage. For any $r_i \in [(v_H - p_S^*)/\lambda, (v_H - p_B^*)/\lambda]$, by equation (13) trader i buys zero following a sell open interest because $-r_i + (v_H - p_S^*)/\lambda \le 0$, and sells zero following a buy open interest because $-r_i + (v_H - p_B^*)/\lambda \ge 0$, where we have used the conjecture that $p_B^* < p_S^*$. That is, trader i does not trade in the second stage. By Bayes's rule, trader i puts the probability m on the high state ($R > 0$). Hence, his unconstrained optimal r_i is $(v_H - mp_B^* - (1-m)p_S^*)/\lambda$, and his constrained optimal r_i is $\min((v_H - mp_B^* - (1-m)p_S^*)/\lambda, -Q_i)$.
- Case 2. The case for a low-value CDS buyer is analogous to Case 1.
- Case 3. If the trader has a high value for the bond and has a positive or zero CDS position, then he cannot submit buy orders in the first stage. Clearly, his optimal request is to set $r_i = 0$ in the first stage.
- Case 4. The case for a low-value trader with a zero or negative CDS position is analogous to Case 3.

Aggregating the allocations across the two stages, we have the following market-clearing condition given a buy open interest (i.e., in the high state):

$$\frac{1-m}{2}\frac{v_L - p_B^*}{\lambda} + \frac{1-m}{2}\int_{Q_i \ge 0} \max\left(-Q_i, \frac{v_L - mp_S^* - (1-m)p_B^*}{\lambda}\right) dG(Q_i)$$
$$+ \frac{1-m}{2}\int_{Q_i \ge 0} \min\left(\frac{v_L - p_B^*}{\lambda} + Q_i, 0\right) dG(Q_i)$$
$$+ \frac{m}{2}\int_{Q_i \ge 0} \min\left(Q_i, \frac{v_H - mp_B^* - (1-m)p_S^*}{\lambda}\right) dG(Q_i) = 0. \quad \text{(A3)}$$

In equation (A3), low-value traders with negative or zero CDS positions trade only in the second stage and achieve their optimal allocation of $(v_L - p_B^*)/\lambda$; see the first term and the $Q_i = 0$ part of the third term of equation (A3). The second term of equation (A3) is the total physical request from low-value CDS buyers (if $Q_i = 0$, the integrand is just zero). In the third term of equation (A3), low-value CDS buyers with physical request $r_i = -Q_i \ge (v_L - p_B^*)/\lambda$ submit sell order in the second stage and end up trading $(v_L - p_B^*)/\lambda - (-Q_i) \le 0$. Low-value CDS buyers do not trade in the second stage because they have sold too much in the first stage. The fourth term of equation (A3) is the total physical request from high-value CDS sellers (if $Q_i = 0$, the integrand is just zero); they do not trade in the second stage because they want to buy but the open interest is also to buy. High-value CDS buyers do not trade in either stage.

Analogously, the market-clearing condition given a sell open interest (i.e., in the low state) is

$$\frac{1-m}{2}\frac{v_H - p_S^*}{\lambda} + \frac{1-m}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - mp_B^* - (1-m)p_S^*}{\lambda}\right) dG(Q_i)$$
$$+ \frac{1-m}{2}\int_{Q_i \geq 0} \max\left(\frac{v_H - p_S^*}{\lambda} - Q_i, 0\right) dG(Q_i)$$
$$+ \frac{m}{2}\int_{Q_i \geq 0} \max\left(-Q_i, \frac{v_L - mp_S^* - (1-m)p_B^*}{\lambda}\right) dG(Q_i) = 0. \quad \text{(A4)}$$

To solve equations (A3) and (A4), we define the following functions. For any fixed value of $p_S^* - v_L$, let $h_1(p_S^* - v_L)$ be the value of $v_H - p_B^*$ such that equation (A3) holds. (This is well defined because the left-hand side of equation (A3) is decreasing in p_B^*.) Likewise, for any fixed value of $v_H - p_B^*$, let $h_2(v_H - p_B^*)$ be the value of $p_S^* - v_L$ such that equation (A4) holds. By inspecting equations (A3) and (A4) we see that $h_1 = h_2$; moreover, h_1 and h_2 are strictly increasing functions. This implies that any solution (p_B^*, p_S^*) to equations (A3) and (A4) satisfies the symmetry condition

$$v_H - p_B^* = p_S^* - v_L. \quad \text{(A5)}$$

If not (suppose $v_H - p_B^* > p_S^* - v_L$), then we have $p_S^* - v_L = h_2(v_H - p_B^*) > h_1(p_S^* - v_L) = v_H - p_B^*$, that is, a contradiction.

Under condition (A5), equations (A3) and (A4) simplify to

$$\frac{1-m}{2}\frac{v_L - p_B^*}{\lambda} + \frac{2m-1}{2}\int_{Q_i \geq 0} \min\left(Q_i, \frac{v_H - mp_B^* - (1-m)p_S^*}{\lambda}\right) dG(Q_i)$$
$$+ \frac{1-m}{2}\int_{Q_i \geq 0} \min\left(\frac{v_L - p_B^*}{\lambda} + Q_i, 0\right) dG(Q_i) = 0. \quad \text{(A6)}$$

Under condition (A5), $mp_B^* + (1-m)p_S^*$ is increasing in p_B^* since $m > 1/2$. Thus, under condition (A5), the left-hand side of equation (A6) is decreasing in p_B^*, is positive when $p_B^* = v_L$, and is negative when $p_B^* = v_H$, so equation (A6) admits a unique solution $p_B^* \in (v_L, v_H)$. Moreover, this solution satisfies $p_B^* < (v_H + v_L)/2$ (i.e., $p_B^* < p_S^*$) if and only if the left-hand side of equation (A6) is negative when $p_B^* = (v_H + v_L)/2$, which is equivalent to condition (18) by simple algebra.

Finally, given equation (A5), the first-stage strategy in equation (21) implies that $R > 0$ in the high state and $R < 0$ in the low state, as we have conjectured.

This completes the derivation and verification of the equilibrium under which $p_B^* < p_S^*$.

C. Proof of Proposition 3

In Proposition 1, the final price p_B^* in the high state is defined by equation (A1), which excludes some high-value traders; that is, for every value of p_B^*, the left-hand side of equation (A1) is strictly less than

$$(1-m)\frac{v_L - p_B^*}{\lambda} + m\frac{v_H - p_B^*}{\lambda}. \tag{A7}$$

The above expression is equal to zero when $p_B^* = p_H^c$. Therefore, the solution p_B^* that satisfies equation (A1) must be less than p_H^c. Since $p_S^* = v_H + v_L - p_B^*$ and $p_L^c = v_H + v_L - p_H^c$, we have $p_S^* > p_L^c$ for Proposition 1.

In Proposition 2, we have $p_B^* < (v_H + v_L)/2 < p_S^*$. Since $p_H^c > (v_H + v_L)/2 > p_L^c$ by definition, we also have $p_B^* < p_H^c$ and $p_S^* > p_L^c$.

The inefficiency of allocations follows directly from the biases in prices. We focus on the high state, since the proof for the low state is symmetric and hence omitted.

(1) Low-value traders: In Proposition 1, they get a bond allocation of $\frac{v_L - p_B^*}{\lambda} > \frac{v_L - p_H^c}{\lambda}$. In Proposition 2, they get a bond allocation of at least $\frac{v_L - mp_S^* - (1-m)p_B^*}{\lambda}$, which is larger than the efficient allocation of $\frac{v_L - p_H^c}{\lambda}$ because $mp_S^* + (1-m)p_B^* = mp_S^* + (1-m)(v_L + v_H - p_S^*) < mv_H + (1-m)v_L = p_H^c$.

(2) High-value CDS buyers and zero-CDS-position traders: In both Propositions 1 and 2, they are prevented from participating in the auction and receive zero allocation, which is clearly lower than their efficient allocation of $\frac{v_H - p_H^c}{\lambda}$.

(3) High-value CDS sellers: In Proposition 1, their allocation is $\min(-Q_i, \frac{v_H - p_B^*}{\lambda})$, which can be lower than or higher than their efficient allocation of $\frac{v_H - p_H^c}{\lambda}$. Specifically, if $|Q_i|$ is sufficiently small, $\min(-Q_i, \frac{v_H - p_B^*}{\lambda}) = -Q_i < \frac{v_H - p_H^c}{\lambda}$, whereas if $|Q_i|$ is sufficiently large, $\min(-Q_i, \frac{v_H - p_B^*}{\lambda}) = \frac{v_H - p_B^*}{\lambda} > \frac{v_H - p_H^c}{\lambda}$. In Proposition 2, their allocation is $\min(-Q_i, \frac{v_H - mp_B^* - (1-m)p_S^*}{\lambda})$, which can be lower than or higher than their efficient allocation of $\frac{v_H - p_H^c}{\lambda}$, following similar logic.

D. Proof of Proposition 4

Suppose in the CDS auction we have the equilibrium in Proposition 1 (where $p_B^* \geq p_S^*$).

Without loss of generality, let us focus on an investor with a benefit $b_i \geq 0$ (wants to buy CDS). Investor i chooses $Q_i \geq 0$ and obtains the following quantity of bonds from the two-stage auction:

Investor i with $Q_i \geq 0$	$r_i + x_i(p^*)$
High state, $v_i = v_H$	0
High state, $v_i = v_L$	$(v_L - p_B^*)/\lambda$
Low state, $v_i = v_H$	$(v_H - p_S^*)/\lambda$
Low state, $v_i = v_L$	$\max((v_L - p_S^*)/\lambda, -Q_i)$

We can see that the only case in which Q_i plays a role in the auction is the low state with $v_i = v_L$.

Investor i solves the following problem at stage 0 (ignoring terms that are independent of Q_i):

$$\max_{Q_i \geq 0}(b_i - p^{CDS} - c)Q_i - \frac{\gamma}{2}(Q_i)^2 + \pi Q_i \left(1 - \frac{v_H + v_L}{2}\right)$$
$$+ \frac{\pi m}{2}\left((v_L - p_S^*)\max\left(\frac{v_L - p_S^*}{\lambda}, -Q_i\right) - \frac{\lambda}{2}\max\left(\frac{v_L - p_S^*}{\lambda}, -Q_i\right)^2\right), \quad \text{(A8)}$$

where by symmetry $p_B^* + p_S^* = (v_H + v_L)/2$.

Define

$$b_i' \equiv b_i - p^{CDS} - c + \pi\left(1 - \frac{v_H + v_L}{2}\right). \quad \text{(A9)}$$

There are two cases, depending on the value of $\max(\frac{v_L - p_S^*}{\lambda}, -Q_i)$.

First, conjecture $Q_i \leq \frac{p_S^* - v_L}{\lambda}$. The problem (A8) reduces to

$$\max_{0 \leq Q_i \leq (p_S^* - v_L)/\lambda} \left(b_i' - \frac{\pi m}{2}(v_L - p_S^*)\right) Q_i - \left(\frac{\gamma}{2} + \frac{\pi m \lambda}{4}\right)(Q_i)^2, \quad \text{(A10)}$$

whose solution is

$$Q_i = \frac{\left(b_i' + (p_S^* - v_L)\,\pi m/2\right)^+}{\gamma + \pi m \lambda/2} \leq \frac{p_S^* - v_L}{\lambda}, \quad \text{(A11)}$$

where the last inequality is equivalent to

$$\frac{b_i'}{\gamma} \leq \frac{p_S^* - v_L}{\lambda}. \quad \text{(A12)}$$

Second, conjecture $Q_i > \frac{p_S^* - v_L}{\lambda}$. The problem (A8) reduces to

$$\max_{Q_i > (p_S^* - v_L)/\lambda} b_i' Q_i - \frac{\gamma}{2}(Q_i)^2, \quad \text{(A13)}$$

whose solution is

$$Q_i = \frac{b_i'}{\gamma} > \frac{p_S^* - v_L}{\lambda}, \quad \text{(A14)}$$

where the last inequality is just the complement of the condition for the first case.

Therefore, the solution to problem (A8) is

$$Q_i = \begin{cases} \dfrac{(b_i' + (p_S^* - v_L)\,\pi m/2)^+}{\gamma + \pi m\lambda/2} & \text{if } b_i' \le \dfrac{\gamma\,(p_S^* - v_L)}{\lambda} \\ \dfrac{b_i'}{\gamma} & \text{if } \dfrac{\gamma\,(p_S^* - v_L)}{\lambda} < b_i'. \end{cases} \tag{A15}$$

When $b_i < 0$, the optimal Q_i is computed symmetrically. To clear all CDS positions $\{Q_i\}$, we must have

$$p^{CDS} = \pi\left(1 - \frac{v_H + v_L}{2}\right). \tag{A16}$$

Given the strategy in equation (A15) and the counterpart when $b_i < 0$, the market-clearing condition for the auction price p_B^* in the high state is equation (23). Clearly, the left-hand side of equation (23) is strictly decreasing in p_B^*, is positive when $p_B^* = v_L$, and is negative when $p_B^* = v_H$. Thus, it has a unique solution in p_B^*. Moreover, this solution p_B^* is larger than $p_S^* = v_H + v_L - p_B^*$ if and only if the left-hand side of equation (23) is positive when $p_B^* = (v_H + v_L)/2$, that is, condition (22).

E.1 Proof of Proposition 5

Suppose in the CDS auction we have the equilibrium in Proposition 2 (where $p_B^* < p_S^*$).

Without loss of generality let us focus on a trader with a benefit $b_i \ge 0$ (wants to buy CDS). Trader i chooses $Q_i \ge 0$ and obtains the following quantity of bonds from the two-stage auction:

Investor i with $Q_i \ge 0$	$r_i + x_i(p^*)$	
High state, $v_i = v_H$	0	
High state, $v_i = v_L$	$(v_L - p_B^*)/\lambda$	if $(v_L - p_B^*)/\lambda \le -Q_i$
	$\max\left((v_L - mp_S^* - (1-m)p_B^*)/\lambda, -Q_i\right)$	if $(v_L - p_B^*)/\lambda > -Q_i$
Low state, $v_i = v_H$	$(v_H - p_S^*)/\lambda$	
Low state, $v_i = v_L$	$\max\left((v_L - mp_S^* - (1-m)p_B^*)/\lambda, -Q_i\right)$	

We can see that of the four cases, two involve Q_i, namely, those in which $v_i = v_L$.

Investor i solves the following problem at stage 0 (ignoring terms that are independent of Q_i):

$$\max_{Q_i \geq 0}(b_i - p^{CDS} - c)Q_i - \frac{\gamma}{2}(Q_i)^2 + \pi Q_i\left(1 - \frac{v_H + v_L}{2}\right)$$
$$+\frac{\pi(1-m)}{2}\left((v_L - p_B^*)\max\left(\min\left(-Q_i, \frac{v_L - p_B^*}{\lambda}\right), \frac{v_L - mp_S^* - (1-m)p_B^*}{\lambda}\right)\right.$$
$$\left.-\frac{\lambda}{2}\max\left(\min\left(-Q_i, \frac{v_L - p_B^*}{\lambda}\right), \frac{v_L - mp_S^* - (1-m)p_B^*}{\lambda}\right)^2\right)$$
$$+\frac{\pi m}{2}\left((v_L - p_S^*)\max\left(-Q_i, \frac{v_L - mp_S^* - (1-m)p_B^*}{\lambda}\right)\right.$$
$$\left.-\frac{\lambda}{2}\max\left(-Q_i, \frac{v_L - mp_S^* - (1-m)p_B^*}{\lambda}\right)^2\right), \tag{A17}$$

where by symmetry $p_B^* + p_S^* = (v_H + v_L)/2$.

As in the proof for the previous proposition, define

$$b_i' = b_i - p^{CDS} - c + \pi\left(1 - \frac{v_H + v_L}{2}\right). \tag{A18}$$

Similar to the proof of Proposition 4, we solve Problem (A17) conditional on three possible ranges of Q_i.

First, conjecture $Q_i \leq \frac{p_B^* - v_L}{\lambda}$. The problem (A17) reduces to

$$\max_{0 \leq Q_i \leq (p_B^* - v_L)/\lambda}\left(b_i' - \frac{\pi m}{2}(v_L - p_S^*)\right)Q_i - \left(\frac{\gamma}{2} + \frac{\pi m\lambda}{4}\right)(Q_i)^2, \tag{A19}$$

whose solution is

$$Q_i = \frac{\left(b_i' + (p_S^* - v_L)\pi m/2\right)^+}{\gamma + \pi m\lambda/2} \leq \frac{p_B^* - v_L}{\lambda}. \tag{A20}$$

Next, we conjecture $\frac{p_B^* - v_L}{\lambda} < Q_i \leq \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}$. The problem (A17) reduces to

$$\max_{(p_B^* - v_L)/\lambda < Q_i \leq (mp_S^* + (1-m)p_B^* - v_L)/\lambda}\left(b_i' - \frac{\pi m}{2}(v_L - p_S^*) - \frac{\pi(1-m)}{2}(v_L - p_B^*)\right)Q_i$$
$$-\left(\frac{\gamma}{2} + \frac{\pi m\lambda}{4} + \frac{\pi(1-m)\lambda}{4}\right)(Q_i)^2, \tag{A21}$$

whose solution is

$$Q_i = \frac{b_i' + (mp_S^* + (1-m)p_B^* - v_L)\pi/2}{\gamma + \pi\lambda/2} \in \left(\frac{p_B^* - v_L}{\lambda}, \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}\right], \tag{A22}$$

where in the solution above the condition $Q_i > \frac{p_B^* - v_L}{\lambda}$ is equivalent to

$$\frac{b_i' + (p_S^* - v_L)\pi m/2}{\gamma + \pi m\lambda/2} > \frac{p_B^* - v_L}{\lambda}, \tag{A23}$$

and the condition $Q_i \leq \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}$ is equivalent to

$$\frac{b_i'}{\gamma} \leq \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}. \tag{A24}$$

Finally, we conjecture $Q_i > \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}$. The problem (A17) reduces to

$$\max_{Q_i > (mp_S^* + (1-m)p_B^* - v_L)/\lambda} b_i' Q_i - \frac{\gamma}{2}(Q_i)^2, \tag{A25}$$

whose solution is

$$Q_i = \frac{b_i'}{\gamma} > \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}. \tag{A26}$$

We see that the parameter conditions for the above three cases complement each other. Therefore, the solution to problem (A17) is

$$Q_i = \begin{cases} \frac{(b_i' + (p_S^* - v_L)\pi m/2)^+}{\gamma + \pi m\lambda/2} & \text{if } \frac{b_i' + (p_S^* - v_L)\pi m/2}{\gamma + \pi m \lambda/2} \leq \frac{p_B^* - v_L}{\lambda}, \\ \frac{b_i' + (mp_S^* + (1-m)p_B^* - v_L)\pi/2}{\gamma + \pi\lambda/2} & \text{if } \frac{p_B^* - v_L}{\lambda} \leq \frac{b_i' + (mp_S^* + (1-m)p_B^* - v_L)\pi/2}{\gamma + \pi\lambda/2} \leq \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda}, \\ \frac{b_i'}{\gamma} & \text{if } \frac{mp_S^* + (1-m)p_B^* - v_L}{\lambda} \leq \frac{b_i'}{\gamma}. \end{cases} \tag{A27}$$

When $b_i < 0$, the optimal Q_i is computed symmetrically. To clear all CDS positions $\{Q_i\}$, we must have

$$p^{CDS} = \pi \left(1 - \frac{v_H + v_L}{2}\right). \tag{A28}$$

Given the strategy in equation (A27) and the counterpart when $b_i < 0$, the market-clearing condition for the auction price p_B^* in the high state is equation (28).[17] Clearly, the left-hand side of equation (28) is strictly decreasing in p_B^*, is positive when $p_B^* = v_L$, and is negative when $p_B^* = v_H$. Thus, it has an unique solution in p_B^*. Moreover, this solution p_B^* is larger than $p_S^* = v_H + v_L - p_B^*$ if and only if the left-hand side of equation (28) is positive when $p_B^* = (v_H + v_L)/2$, that is, condition (27).

Appendix B: A More General Double Auction Model

In this appendix, we generalize the double auction model in Section VII by allowing a finite number n of traders and general distributions of values and CDS positions. We show that the double auction still has an equilibrium that produces the competitive price, even though each trader has a price impact.

[17] Here we also need the fact that

$$\frac{b_i' + (p_S^* - v_L)\pi m/2}{\gamma + \pi m\lambda/2} \leq \frac{p_B^* - v_L}{\lambda} \quad \text{iff} \quad \frac{b_i' + (p_S^* - v_L)\pi m/2}{\gamma + \pi m\lambda/2} \leq \frac{b_i' + (mp_S^* + (1-m)p_B^* - v_L)\pi/2}{\gamma + \pi\lambda/2}.$$

Although the allocations in this equilibrium are not fully efficient, we show that the difference between the equilibrium allocation and the efficient allocation for each trader is on the order of $O(1/n)$, where n is the total number of traders.

Suppose there are $n \geq 3$ traders. We focus on the date $t = 1$ after the default of the bond. As in Section II, each trader i has a private valuation v_i for owning the defaulted bond and a CDS position Q_i, for $1 \leq i \leq n$. We allow for any joint probability distribution of values $\{v_i\}_{1\leq i\leq n}$ and CDS positions $\{Q_i\}_{1\leq i\leq n}$ (with the condition that path-by-path, $\sum_{i=1}^{n} Q_i = 0$). Trader i's utility is still given by equation (5). The double auction rule is described in Section VII. The final price p^* is determined to clear the market:

$$\sum_{i=1}^{n} x_i(p^*) + \sum_{i=1}^{n} r_i = 0. \tag{B1}$$

Notice that with a finite number of traders, each one can affect the final price p^* by changing his physical request r_i or his demand schedule $x_i(p)$.

PROPOSITION B1: *In the double auction with $n \geq 3$ traders, there exists an equilibrium in which trader i submits a (arbitrary) physical request r_i between zero and $-Q_i$ in the first stage, and in the second stage submits the demand schedule*

$$x_i(p) = -r_i + \frac{n-2}{\lambda(n-1)}(v_i - p) - \frac{1}{n-1}Q_i. \tag{B2}$$

The equilibrium price is

$$p^* = \frac{1}{n}\sum_{i=1}^{n} v_i. \tag{B3}$$

PROOF: Without loss of generality, suppose that $r_i = 0$.

Fix a strategy profile $(x_1, x_2, \ldots, x_n)$. Given that all other traders use this strategy profile and for a fixed profile of values $(v_1, v_2, \ldots, v_n)$, the payoff of trader i at the price p is

$$\Pi_i(p) = (1-p)Q_i + (v_i - p)\left(-\sum_{j\neq i} x_j(p)\right) - \frac{1}{2}\lambda\left(-\sum_{j\neq i} x_j(p)\right)^2.$$

We can see that trader i is effectively selecting an optimal price p. Taking the first-order condition of $\Pi_i(p)$ at $p = p^*$, we have, for all i,

$$0 = \Pi_i'(p^*) = -Q_i - x_i(p^*) + \big(v_i - p^* - \lambda x_i(p^*)\big)\left(-\textstyle\sum_{j\neq i} \frac{\partial x_j}{\partial p}(p^*)\right). \tag{B4}$$

We conjecture a symmetric linear demand schedule,

$$x_j(p) = a v_j - b p + c Q_i, \tag{B5}$$

where a, b, and c are constants. Given this conjecture, trader i's first-order condition (B4) becomes

$$x_i(p^*) = \frac{(n-1)b(v_i - p^*) - Q_i}{1 + \lambda(n-1)b}, \tag{B6}$$

that is, when trader i uses the demand schedule in equation (B5) with

$$a = b = \frac{n-2}{\lambda(n-1)}, \; c = -\frac{1}{n-1}, \tag{B7}$$

trader i's first-order condition is always satisfied for every realization of $(v_1, v_2, \ldots, v_n)$.

It is easy to verify that, under this linear strategy, $\Pi_i''(\cdot) < 0$ if $n > 2$. □

A notable feature of the equilibrium in Proposition B1 is that it is independent of (and hence robust to) assumptions about values and CDS positions.

The equilibrium strategy in Proposition B1 clearly converges to the competitive equilibrium strategy in equation (7) as the number n of traders tends to infinity. The factor $\frac{n-2}{n-1}$ in equation (B2) captures the equilibrium amount of "demand reduction" due to traders' price impact in the finite market. This factor is canceled out in the determination of the equilibrium price in equation (B3), and hence the equilibrium price is the same as the competitive price. The final allocation from this equilibrium for trader i is

$$r_i + x_i(p^*) = \frac{n-2}{\lambda(n-1)}(v_i - p^*) - \frac{1}{n-1}Q_i, \tag{B8}$$

and the efficient allocation for trader i is

$$\frac{1}{\lambda}(v_i - p^*). \tag{B9}$$

The difference between the two allocations is on the order of $O(1/n)$.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.