Are CDS Auctions Biased?*

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Abstract

We study the design of settlement auctions for credit default swaps (CDS). We find that the one-sided design of CDS auctions currently used in practice gives CDS buyers and sellers strong incentives to distort the final auction price, in order to profit from existing CDS positions. In the absence of frictions, the current auction mechanism tends to overprice defaulted bonds given an excess supply and underprice them given an excess demand. We propose a double auction to mitigate price biases and provide robust price discovery. The predictions of our theory on bidding behavior are consistent with CDS auction data.

Keywords: credit default swaps, credit event auctions, price bias, double auction

JEL Classifications: G12, G14, D44

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1 Introduction

This paper studies the design of settlement auctions for credit default swaps (CDS). A CDS is a default insurance contract between a buyer of protection ("CDS buyer") and a seller of protection ("CDS seller"), and is written against the default of a firm, loan, or sovereign country. The CDS buyer pays a periodic premium to the CDS seller on a given notional amount of bonds until a default occurs or the contract expires—whichever is first. If a default occurs before the CDS contract expires, then the CDS seller compensates the CDS buyer for the default loss, that is, the face value of the insured bonds less the realized bond recovery value. Because the realized recovery value is unobservable at the time of default, the market uses CDS auctions, also known as "credit event auctions," to determine the "fair" recovery value of the defaulted bonds and thus the settlement payments on CDS. In doing so, CDS auctions constitute a critical part of the markets for CDS, which, as of June 2011, have a notional outstanding of more than \$32 trillion and a gross market value of more than \$1.3 trillion.[1] Fair and unbiased prices from CDS auctions are therefore important for the proper functioning of CDS markets, whose primary economic purposes include hedging credit risk and providing price discovery on the fundamentals of companies and sovereigns.

First used in 2005, the current protocol for CDS auctions was hardwired in 2009 as the standard method used for settling CDS contracts after default (International Swaps and Derivatives Association 2009). From 2005 to June 2012, more than 120 CDS auctions have been held for the defaults of companies (such as Fannie Mae, Lehman Brothers, and General Motors) and sovereign countries (Ecuador and Greece).

A CDS auction consists of two stages, as described in detail in Section 2. In the first stage, dealers and market participants submit "physical settlement requests," which are price-insensitive market orders used for buying or selling the defaulted bonds. The sum of

[1] See the semiannual OTC derivatives statistics, Bank for International Settlements, June 2011.

these physical settlement requests is the "open interest." The first stage also produces the "initial market midpoint," which is effectively an estimate of the price at which dealers are willing to make markets in the defaulted bonds. The second stage is a uniform-price auction, in which participants submit limit orders (on the defaulted bonds) to match the first-stage open interest. The price at which the total of the second-stage bids equal the first-stage open interest is determined as the final auction price. After the final price is announced, CDS sellers pay CDS buyers the face value of the defaulted bonds less the final auction price in cash—a process called "cash settlement." Bond buyers and bond sellers, who are matched in the auction, trade the physical bonds at the final auction price—a process called "physical settlement."[2]

We show that the CDS auction procedure currently used in practice encourages price manipulation and tends to produce a biased final price, relative to the fair recovery value of the defaulted bonds. To see the intuition, suppose that everyone is risk neutral, and the first-stage open interest is to sell $200 million notional of bonds, whose recovery value is commonly known to be $50 per $100 face value. We consider a CDS seller, say bank A, who has sold protection on $100 million notional of bonds. Because bank A pays the loss on the defaulted bonds to its CDS counterparty, the higher the final auction price, the less bank A must pay. For example, if the final auction price is the true recovery value of $50, then bank A pays $50 million. If, however, the final auction price is $100, then bank A pays nothing. Thus, bank A has a strong incentive to increase the CDS final price in order to reduce the bank's payments to its CDS counterparty. The same incentive applies to other CDS sellers. Therefore, CDS sellers aggressively bid in the second stage of the auction.

[2] In conventional terms, physical settlement refers to the process in which the CDS buyer delivers the defaulted bonds to the CDS buyer, who, in turn, pays the bond's face value to the CDS buyer. Under this older physical settlement method, a defaulted bond may need to be "recycled" several times before all CDS claims are settled, which can artificially increase the bond price and create the risk that the same CDS may be settled at different prices at different times. The current CDS auction design is partly motivated by these concerns (Creditex and Markit 2009).

Barring restrictions on the final auction price, and for most plausible cases, the final auction price is strictly higher than the true recovery value of the defaulted bonds.

Why do arbitrageurs and CDS buyers not correct this price bias? After all, CDS buyers are adversely affected by high settlement prices because a high final price reduces the payments they receive from CDS sellers. The answer is that the one-sided auction prevents price correction. For example, conditional on an open interest to sell, bidders in the second stage can only submit limit orders to buy. No one is allowed to submit sell orders in this case, so CDS buyers and arbitrageurs have no choice but to buy nothing in the auction and to have no say in the final price. By symmetry, conditional on an open interest to buy, CDS buyers submit aggressive sell orders and push the final auction price below the fair recovery value of bonds. As we show, this intuition of price biases generalizes to risk-averse traders as well. With risk aversion, a trader's "fair" valuation of a bond becomes the expected recovery value weighted by the trader's marginal utility.

Our analysis strongly suggests that a double auction can greatly reduce, if not eliminate, price biases. Under a double auction design, limit orders in the second stage can be submitted in both directions, buy and sell, regardless of the open interest. With a double auction, if bank A—from our earlier example—pushes the final price from its fair value \$50 to a higher level, say \$60, CDS buyers and arbitrageurs can submit sell orders at \$60, making a profit of \$10 and simultaneously pushing the price back toward its true value. Under general conditions, a double auction exactly pins down the final price at the bond's fair recovery value.

In addition to correcting price biases, a double auction provides robust and effective price discovery. In a setting where dealers receive private signals regarding the fair value of the defaulted bonds, we show that the double auction aggregates private information dispersed across dealers. The price-discovery benefit of a double auction further calls into question the rationale of the one-sided design used in CDS auctions today.

Our theoretical results yield testable predictions of bidding behavior. For example, because a dealer[3] who submits a physical buy request in the first stage is more likely to be a net CDS seller than a net CDS buyer, our results predict that such a dealer would aggressively bid in the second stage in order to raise the final auction price. Using data from 94 CDS auctions between 2006 and 2010, we indeed find that, conditional on a sell open interest, dealers with physical buy requests submit more aggressive limit buy orders in the second stage. Conversely, conditional on an open interest to buy, we find that dealers with physical sell requests submit more aggressive limit sell orders in the second stage. These empirical findings are consistent with our theoretical predictions.

Our paper contributes to and complements the existing literature on CDS auctions. Theoretically, we focus on the design of CDS auctions, whereas Chernov, Gorbenko, and Makarov (2012) focus on frictions in bond markets. For example, Chernov, Gorbenko, and Makarov (2012) show that, for an open interest to sell, the CDS auction prices can be lower than the fair values of defaulted bonds if frictions prevent some aggressive bidders (e.g. CDS sellers) from buying bonds. While these frictions are important in practice, they are separate from our focus on manipulative bidding incentives and associated market-design perspective on CDS auctions. Moreover, our information aggregation result under a double auction goes one step beyond the symmetric-information model of Chernov, Gorbenko, and Makarov (2012).

Empirically, we exploit the novel CDS auction data to test bidding behavior predicted by our theory, whereas existing empirical studies predominately compare bond prices with CDS auction prices. For example, Chernov, Gorbenko, and Makarov (2012) and Gupta and Sundaram (2011) find that the final prices from CDS auctions tend to be lower than corresponding bond transaction prices several days before and after the auction. In an earlier

[3]For brevity, by a dealer we mean a dealer and his customers who submit bids through him. In the data, a dealer aggregates his own bids with his customers' bids and report them together under the dealer's name.

sample, Helwege, Maurer, Sarkar, and Wang (2009) find that CDS auction prices and bond prices are close to each other. Coudert and Gex (2010) provide a detailed discussion on the performance of a few large CDS auctions. As we elaborate in Section 7, the time-series pattern of bond prices could be affected by many factors, including the price caps and floors, risk premia, illiquidity, capital immobility, and the cheapest-to-deliver option, among others. For this reason, the fair price in our baseline model may *not* be accurately measured by the bond prices near the auction dates or by the realized recovery values in the future. Compared with bond transaction data, the CDS auction data provide a cleaner test of our theory. To the extent that all participant in CDS auctions are subject to similar market imperfections, the cross-section of bidding behaviors in the auction-level data can better isolate the effect of manipulative bidding incentives from that of market frictions. Since our empirical strategy and those of existing studies focus on different aspects of CDS auctions, our results and theirs are complementary.

2 The Two-Stage CDS Auctions

This section provides an overview of CDS auctions. Detailed descriptions of the auction mechanism are also provided by Creditex and Markit (2009).

The CDS auction consists of two stages. In the first stage, the participating dealers[4] submit "physical settlement requests" on behalf of themselves and their clients. These physical settlement requests indicate if they want to buy or sell the defaulted bonds as well as the quantities of bonds they want to buy or sell. Importantly, only market participants with nonzero CDS positions are allowed to submit physical settlement requests, and these requests must be in the opposite direction of, and not exceeding, their net CDS positions.

[4] In the auctions between 2006 and 2010, participating dealers include ABN Amro, Bank of America Merrill Lynch, Barclays, Bear Stearns, BNP Paribas, Citigroup, Commerzbank, Credit Suisse, Deutsche Bank, Dresdner, Goldman Sachs, HSBC, ING Bank, JP Morgan Chase, Lehman Brothers, Merrill Lynch, Mitsubishi UFJ, Mizuho, Morgan Stanley, Nomura, Royal Bank of Scotland, Société Générale, and UBS.

For example, suppose that bank A has bought CDS protection on $100 million notional of General Motors bonds. Because bank A will deliver defaulted bonds in physical settlement, the bank can only submit a physical *sell* request with a notional between 0 and $100 million. Similarly, a fund that has sold CDS on $100 million notional of GM bonds is only allowed to submit a physical *buy* request with a notional between 0 and $100 million.[5] Participants who submit physical requests are obliged to transact at the final price, which is determined in the second stage of the auction and is thus unknown in the first stage. The net of total buy physical request and total sell physical request is called the "open interest."

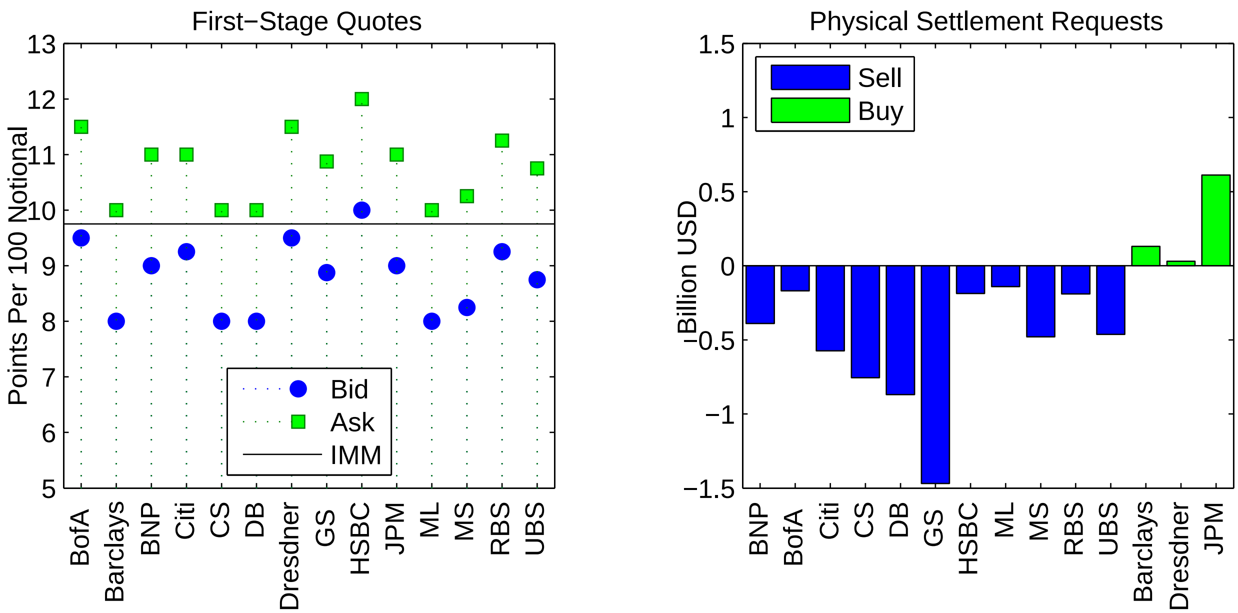
Also, in the first stage, but separately from the physical settlement requests, each dealer submits a two-way quote, that is, a bid and an offer. The quotation size (say $5 million) and the maximum spread (say $0.02 per $1 face value of bonds) are predetermined in each auction. Bids and offers that cross each other are eliminated. The average of the best halves of remaining bids and offers becomes the "initial market midpoint" (IMM), which serves as a benchmark for the second stage of the auction. A penalty called the "adjustment amount" is imposed on dealers whose quotes are off-market.

The first stage of the auction concludes with the simultaneous publications of (i) the initial market midpoint, (ii) the size and direction of the open interest, and (iii) adjustment amounts, if any.

Figure 1 plots the first-stage quotes (left-hand panel) and physical settlement requests (right-hand panel) of the Lehman Brothers auction in October 2008. The bid-ask spread quoted by dealers was fixed at 2 per 100 face value, and the initial market midpoint was 9.75. One dealer whose bid and ask were on the same side of the IMM paid an adjustment amount. Of the 14 participating dealers, 11 submitted physical sell requests and 3 submitted physical buy requests. The open interest to sell was about $4.92 billion.

[5]There are no formal external verifications that one's physical settlement request is consistent with one's net CDS position.

Figure 1: Lehman Brothers CDS Auction, First Stage



In the second stage of the auction, all dealers and market participants—including those without any CDS position—can submit limit orders to match the open interest. Nondealers must submit orders through dealers, and there is no restriction regarding the size of limit orders one can submit. If the first-stage open interest is to sell, then bidders must submit limit orders to buy. If the open interest is to buy, then bidders must submit limit orders to sell. Thus, the second stage is a one-sided market. The final price, p^*, is determined as in a uniform-price auction. Without loss of generality, we consider an open interest to sell, in which bidders submit limit orders to buy. Higher-priced limit orders are matched against the open interest before lower-priced limit orders are matched. If the limit orders are sufficient in matching the open interest, then the final price is set at the limit price of the last limit order used. Limit orders with prices superior to the final price are all filled, whereas limit orders with prices equal to the final price are allocated pro-rata, if necessary. If the limit orders are insufficient in matching the open interest, then the final price is 0. The determination of final price for a buy open interest is symmetric. Finally, the auction protocol imposes the restriction that the final price cannot exceed the IMM plus a predetermined "cap amount," usually \$0.01 or \$0.02 per \$1 face value. Therefore, for an open sell interest, the final price is set at

$$p^* = \min\left(M + \Delta, \max(p_b, 0)\right), \tag{1}$$

where M is the initial market midpoint, Δ is the cap amount, and p_b is the limit price of the last limit buy order used. Symmetrically, for an open buy interest, the final price is set at

$$p^* = \max\left(M - \Delta, \min(p_s, 1)\right), \tag{2}$$

where p_s is the limit price of the last limit sell order used. If the open interest is zero, then the final price is set at the IMM. The announcement of the final price, p^*, concludes the auction.

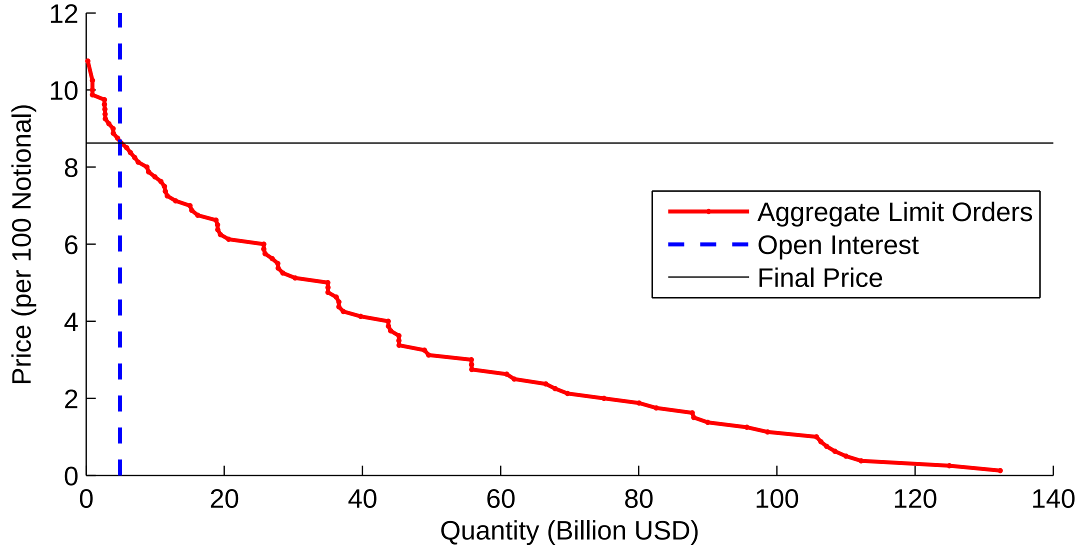
After the auction, bond buyers and sellers that are matched in the auction trade the bonds at the price of p^*; this is called "physical settlement." In addition, CDS sellers pay CDS buyers $1 - p^*$ per unit notional of their CDS contract; this is called "cash settlement."

Figure 2 plots the aggregate limit order schedule in the second stage of the Lehman auction. For any given price p, the aggregate limit order at p is the sum of all limit orders to buy at p or above. The sum of all submitted limit orders was over \$130 billion, with limit prices ranging from 10.75 (the price cap) to 0.125 per 100 face value. The final auction price was 8.625. CDS sellers thus pay CDS buyers 91.375 per 100 notional of CDS contract.

3 Price Biases in CDS Auctions

In this section we model bidding behavior in CDS auctions and associated equilibrium final price. In Sections 3.1–3.3, we characterize the optimal bidding strategies and price biases in the second stage "subgame," taking the first-stage open interest and physical requests

Figure 2: Lehman Brothers CDS Auction, Second Stage



as given. In Section 3.4 we endogenize the first-stage strategies and show that price biases persist. Finally, in Section 3.5 we show that the key intuition of price bias carries through to risk-averse traders. For simplicity, we do not model dealers' quotes in the first stage or the price cap or floor in the second stage. The potential effect of price caps and floors are discussed in Section 7.

3.1 Model

There are n risk-neutral dealers, each with a CDS position Q_i, $1 \leq i \leq n$, where Q_i is dealer i's private information. Because CDS contracts have zero net supply, we have

$$\sum_{i=1}^{n} Q_i = 0. \tag{3}$$

If dealer i is a CDS buyer, then $Q_i > 0$. If dealer i is a CDS seller, then $Q_i < 0$. If dealer i has a zero CDS position, then $Q_i = 0$. For a realization of CDS positions $\{Q_i\}_{i=1}^{n}$, let $B = \{i : Q_i > 0\}$ be the set of CDS buyers and $S = \{i : Q_i < 0\}$ be the set of CDS sellers.

The defaulted bonds on which the CDS are written have an uncertain recovery rate, v, whose probability distribution on $[0, 1]$ is commonly known by all dealers. (We consider asymmetric but interdependent valuations in Section 5.) Thus, all dealers assign a common value of $\mathbb{E}(v)$ to each unit face value of the defaulted bonds. Since all dealers have a commonly known valuation for the bonds, a dealer's existing bond position merely adds a constant to his total profits. Thus, we do not need to model dealers' existing bond positions.

We denote by r_i the physical settlement request submitted by dealer i in the first stage of the auction. As described in Section 2, r_i has the opposite sign as Q_i, and $|r_i| \leq |Q_i|$. A CDS seller (with $Q_i < 0$) can submit a physical buy request ($r_i \geq 0$), and a CDS buyer (with $Q_i > 0$) can submit a physical sell request ($r_i \leq 0$). A dealer who has zero CDS position is not allowed to submit physical settlement request. All physical settlement requests are summed to form the open interest

$$R = \sum_{i=1}^{n} r_i. \tag{4}$$

The physical settlement requests $\{r_i\}_{i=1}^n$ are published at the end of the first stage of the auction.[6] Conditional on $\{r_i\}_{i=1}^n$, CDS positions $\{Q_i\}_{i=1}^n$ have the joint distribution function F.

As described in Section 2, the second stage is a uniform-price auction, conditional on the open interest R. For an open interest to sell ($R < 0$), every dealer simultaneously submits a demand schedule $x_i : [0, 1] \times \mathbb{R} \to [0, \infty)$ that is contingent on his CDS position. Note that for $R < 0$, x_i must be nonnegative because the second stage of the auction only allows buy limit orders. The value $x_i(p; Q_i)$ specifies the maximum amount of bonds that dealer i with CDS position Q_i is willing to buy at the price p. For simplicity, suppose that $x_i(\,\cdot\,; Q_i)$ is strictly decreasing and differentiable, so $x_i'(p; Q_i) < 0$ whenever $x_i(p; Q_i) > 0$. The monotonicity

[6]This modeling choice is made for notational simplicity and does not affect our results. In practice, only the open interest R is published at the end of the first stage. In this case, dealer i's demand schedule $x_i(\,\cdot\,; Q_i, r_i)$ is contingent on *both* his CDS position and physical settlement request, and subsequent analysis, including the proof of Proposition 1, still goes through.

and differentiability of the demand schedules allow a simple analytical characterization of the equilibria without changing their qualitative nature.[7] The final auction price $p^*(Q)$ clears the market and is implicitly defined by

$$\sum_{i=1}^{n} x_i(p^*(Q); Q_i) = -R, \tag{5}$$

for every realization of $Q = \{Q_i\}_{i=1}^n$.

To rule out trivialities, we restrict modeling attention to demand schedules for which the market-clearing price $p^*(Q)$ defined by (5) exists. Since dealer i values the asset at $\mathbb{E}(v)$, his payoff, given a realization of Q, is

$$\Pi_i(Q) = (r_i + x_i(p^*(Q); Q_i))(\mathbb{E}(v) - p^*(Q)) + Q_i(1 - p^*(Q)), \tag{6}$$

where the first term represents the dealer's profit or loss from trading the bonds, and the second term represents the dealer's payoff (either positive or negative) from his outstanding CDS position.

Symmetrically, for an open interest to buy ($R > 0$), every dealer submits a supply schedule $x_i : [0,1] \times \mathbb{R} \to (-\infty, 0]$, with the property that $x_i'(p; Q_i) < 0$ whenever $x_i(p; Q_i) < 0$. Note that we use *negative* numbers $\{x_i(p)\}$ to describe sell orders. Because $x_i'(p; Q_i) < 0$, a higher price p implies a more negative $x_i(p; Q_i)$, that is, dealer i wants to sell more bonds at a higher price.[8] Under our sign convention, a dealer's payoff for a buy open interest is still given by (6).

[7]For example, Kastl (2011) shows that in Wilson's divisible auction model, if bidders are restricted to submit at most K bids (so that the demand schedule is a K-step function), the resulting equilibrium converges in K to an equilibrium that consists of differentiable demand schedules. If a large number of limit orders is allowed, discrete demand schedules are well approximated by differentiable ones. Kremer and Nyborg (2004a) show that continuous demand schedules naturally arise when allocation rule is "pro-rata on the margin," as in CDS auctions.

[8]This is equivalent to the conventional notion in which supply schedules are upward-sloping.

3.2 Characterizing Equilibria in the Second Stage

We now characterize Bayesian Nash equilibria of the second-stage auction. In a Bayesian Nash equilibrium $\{x_i\}_{i=1}^n$, each dealer i's demand schedule $x_i(\,\cdot\,; Q_i)$ is optimal, given his conditional belief $F(\,\cdot \mid Q_i)$ about others dealers' CDS positions, $\{Q_j\}_{j\neq i}$, and other dealers' demand schedules, $\{x_j\}_{j\neq i}$.

Proposition 1. *Suppose that the first-stage open interest is to sell. Then, in any Bayesian Nash equilibrium of the one-sided auction in the second stage:*

(i) The final price satisfies $p^*(Q) \geq \mathbb{E}(v)$ *for every realization of* $Q = \{Q_i\}_{i=1}^n$.

(ii) All dealers with positive or zero CDS positions receive zero share of the open interest. That is, for every realization of Q, $x_i(p^*(Q); Q_i) = 0$ *if* $i \notin S$.

(iii) For every realization of Q, *the final price* $p^*(Q) > \mathbb{E}(v)$, *unless all CDS buyers submit full physical settlement requests (i.e.,* $r_i = -Q_i$ *for all* $i \in B$*).*

Symmetrically, suppose that the first-stage open interest is to buy. Then, in any Bayesian Nash equilibrium of the one-sided auction in the second stage:

(i) The final price satisfies $p^*(Q) \leq \mathbb{E}(v)$ *for every realization of* $Q = \{Q_i\}_{i=1}^n$.

(ii) All dealers with negative or zero CDS positions receive zero share of the open interest. That is, for every realization of Q, $x_i(p^*(Q); Q_i) = 0$ *if* $i \notin B$.

(iii) For every realization of Q, *the final price* $p^*(Q) < \mathbb{E}(v)$, *unless all CDS sellers submit full physical settlement requests (i.e.,* $r_i = -Q_i$ *for all* $i \in S$*).*

Proof. The proof is provided in Appendix A. □

Proposition 1 reveals that, under fairly general conditions, the final auction price is either strictly above or strictly below the fair value of the bond. Moreover, this bias is in the *opposite* direction of the open interest: an open interest to sell produces too high a price, and an open interest to buy produces too low a price.

The intuition of Proposition 1 is simple. Given a sell open interest, CDS sellers have strong incentives to increase the final auction price in order to reduce payments to CDS buyers. The open interest cannot be larger than the CDS positions of CDS sellers, so the expected benefit of reducing CDS payments dominates the expected cost associated with buying bonds at an artificially high price. Thus, CDS sellers bid aggressively in order to increase the final auction price. Because of the one-sided nature of the auction, CDS buyers and arbitrageurs can only decrease the auction price by reducing the price and quantity of their buy orders. Once their demands reach zero, it is impossible for CDS buyers and arbitrageurs to further counteract the upward price distortion by CDS sellers. An artificially high price is thus sustained in equilibrium. The intuition for a buy open interest is symmetric: CDS buyers have strong incentives to suppress the bond price, and CDS sellers and arbitrageurs cannot counteract this price suppression because of the one-sided nature of the auction. We further illustrate the intuition of Proposition 1 in Section 3.3.

Proposition 1 implies that prices are strictly biased in equilibrium unless (a) every CDS buyer submits a full physical sell request, given a sell open interest or (b) every CDS seller submits a full physical buy request, given a buy open interest. Full physical settlement requests are, however, unlikely to apply to everyone. For example, for CDS buyers who do not own the underlying bonds and CDS sellers who do not wish to receive the defaulted bonds, cash settlement is more natural than physical settlement. In Section 3.4 we explicitly construct an equilibrium in which dealers submit zero physical requests with positive probabilities.

The equilibria of Proposition 1 differ from "underpricing" equilibria characterized by Wilson (1979) and Back and Zender (1993), who study divisible auctions with a supply to sell. In these models, the flexibility of bidding with demand schedules produces equilibria in which buyers tacitly collude and drive the final auction price below the commonly known value of the asset. However, in CDS auctions with open interests to sell, these underpricing equilibria do not exist because CDS sellers bid high prices in order to reduce CDS payments.[9]

The derivative externality in CDS auctions complements other forms of auction externalities documented in the literature. In Nyborg and Strebulaev (2004), for example, traders who have pre-established short positions in the auctioned asset bid differently from those who have long positions because the latter may short-squeeze the former after the auction. Bulow, Huang, and Klemperer (1999) and Singh (1998) consider takeover contests with toeholds (i.e. existing positions in the firm to be acquired). They find that toeholders behave differently from outside bidders because a bid from a toeholder is also an offer for his existing position. Jehiel and Moldovanu (2000) study a single-unit second price auction in which a bidder's utility directly depends on the value of the other bidder. In a multi-unit auction setting, Aseff and Chade (2008) characterize the revenue-maximizing mechanism when buyers' values depend on who else win the goods.

3.3 Commonly Known CDS Positions

The objective of this subsection is to further illustrate the intuition of price biases. To reduce technical complication, we sketch the proof for a special case of Proposition 1, namely when the CDS positions $Q = \{Q_i\}_{i=1}^n$ are commonly known by the dealers. Since $\{Q_i\}_{i=1}^n$ are common knowledge, we write the final price $p^*(Q)$ as p^* and the demand schedule $x_i(p; Q_i)$ as $x_i(p)$. Without loss of generality, we consider an open interest to sell ($R < 0$).

[9] Several studies examine how underpricing in the Wilson (1979) model may be reduced or eliminated. For example, Back and Zender (1993) generalize Wilson's result and suggest that discriminatory auctions can reduce underpricing. Kremer and Nyborg (2004a) demonstrate that an alternative pro-rata allocation rule can encourage aggressive bidding and eliminate underpricing. Kremer and Nyborg (2004b) show that underpricing can also be made arbitrary small if, among other restrictions, bidders can only submit a finite number of bids, or there is a tick size or quantity multiple. Finally, Back and Zender (2001), LiCalzi and Pavan (2005), and McAdams (2007) show that underpricing can be reduced if the seller is allowed to adjust the supply after bids are submitted.

We can rewrite (6) as

$$\begin{aligned}\Pi_i(p^*) &= (r_i + x_i(p^*))(\mathbb{E}(v) - p^*) + Q_i(1 - p^*) \\ &= \left(r_i - R - \sum_{j \neq i} x_j(p^*)\right)(\mathbb{E}(v) - p^*) + Q_i(1 - p^*). \qquad (7)\end{aligned}$$

In equilibrium, each dealer i submits an $x_i(\cdot)$ that maximizes his payoff Π_i, given $\{x_j(\cdot)\}_{j \neq i}$. In equilibrium, $x_i(p^*) + \sum_{j \neq i} x_j(p^*) = -R$ and each x_j is strictly downward-sloping, so there is a one-to-one mapping between $x_i(p^*)$ and p^*. Thus, we can write the first-order condition of dealer i in terms of the market-clearing price p^* (instead of quantity x_i):

$$\begin{aligned}\Pi_i'(p^*) &= -\left(r_i - R - \sum_{j \neq i} x_j(p^*)\right) - \left(\sum_{j \neq i} x_j'(p^*)\right)(\mathbb{E}(v) - p^*) - Q_i \\ &= -(r_i + x_i(p^*) + Q_i) - \left(\sum_{j \neq i} x_j'(p^*)\right)(\mathbb{E}(v) - p^*). \qquad (8)\end{aligned}$$

Since $\sum_{i=1}^n [r_i + x_i(p^*) + Q_i] = R + \sum_{i=1}^n x_i(p^*) + \sum_{i=1}^n Q_i = 0$, we can always find a dealer i such that $-r_i - x_i(p^*) - Q_i \geq 0$. By downward-sloping demand schedule, we have $\sum_{j \neq i} x_j'(p^*) > 0$, so it must be that $\mathbb{E}(v) - p^* \leq 0$; otherwise, $\Pi_i'(p^*) > 0$ and dealer i would increase the price by bidding more at p^*. Thus, in equilibrium $p^* \geq \mathbb{E}(v)$. In Appendix A, we show that under partial physical request the equilibrium price $p^* > \mathbb{E}(v)$.

Example 1. For concreteness, we now explicitly construct an equilibrium in which, under partial physical sell requests, and given an open interest to sell, the final price is $p^* = 1$. Specifically, for all $i \in S$, we let

$$a_i = \frac{|Q_i + r_i|}{\sum_{j \in S} |Q_j + r_j|} |R|. \qquad (9)$$

This a_i is the quantity received by CDS seller i in the equilibrium we are constructing. Because at least one CDS buyer has submitted a partial physical settlement request, we must have $\sum_{j\in S}|Q_j+r_j| > |R| > 0$, and hence $a_i < |Q_i+r_i|$ whenever $a_i > 0$. For each $i \in S$ with $a_i = 0$, we set $b_i = 0$. For each $i \in S$ with $a_i > 0$, we choose sufficiently small $b_i > 0$ with the property that

$$|Q_i + r_i| - a_i > (1 - \mathbb{E}(v)) \sum_{j\neq i, j\in S} b_j. \tag{10}$$

Finally, for each $i \in S$, we set

$$x_i(p) = a_i + b_i(1-p).$$

For $k \notin S$, we arbitrarily set $x_k(p)$, under the restriction that $x_k(p) = 0$ in a neighborhood of $p = 1$. For any CDS seller i with $a_i > 0$, (10) implies that his first-order condition (8) at $p^* = 1$ satisfies $\Pi_i'(1) > 0$. For any CDS seller j with $a_j = 0$, we have $\Pi_j'(1) < 0$. For any $k \notin S$, we also have $\Pi_k'(1) < 0$. Thus, $p^* = 1$ is supported as an equilibrium by strategy $\{x_i\}_{i=1}^n$. In this equilibrium, CDS sellers submit limit orders with sufficiently "flat" slopes, so it is inexpensive to push the final price to 1. For each CDS seller involved in this manipulation (those with $a_i > 0$), the reduction in settlement payments outweighs the cost of buying the bonds at par. All other dealers have no influence on the final price.

3.4 Endogenizing First-Stage Strategies

In this subsection, we endogenize the choice of physical settlement requests in the first stage and show that price biases can persist in equilibrium. In the first stage, each dealer i selects the optimal physical settlement request r_i, taking other dealers' strategies as given. We characterize a mixed-strategy equilibrium in which every dealer is indifferent between submitting a full physical settlement request and a zero physical settlement request. This equilibrium captures dealers' uncertainty regarding the impact of their physical requests on the open interest and hence the direction of the price bias.

To see the intuition of the mixed-strategy equilibrium, we consider a CDS buyer, who can only submit a physical request to sell. On the one hand, by submitting a zero physical request, the CDS buyer maximizes the likelihood that the open interest is to buy (i.e. $R > 0$), which allows him to submit aggressive sell orders in the second stage and benefit from the low (and biased) final price. On the other hand, by submitting a full physical request, the CDS buyer eliminates the risk of having to cash settle at the high (and biased) final price in the event that the open interest is to sell. In the mixed-strategy equilibrium, these two incentives exactly offset each other.

Formally, we follow the setting of Section 3.3 and suppose that the CDS positions are common knowledge. We conjecture that each dealer i chooses a full physical request (i.e. $r_i = -Q_i$) with probability $q_i \in (0, 1)$ and chooses a zero physical request (i.e. $r_i = 0$) with probability $1 - q_i$.

Without loss of generality, we analyze the strategy of dealer 1, who is a CDS buyer with $Q_1 > 0$. First, recall that dealer 1 makes a profit of

$$Q_1(1 - \mathbb{E}(v)) + (Q_1 + r_1 + x_1)(\mathbb{E}(v) - p^*). \tag{11}$$

So, by submitting $r_1 = -Q_1$ and setting $x_1 = 0$, dealer 1 makes a fixed profit of $Q_1(1-\mathbb{E}(v))$, regardless of the open interest and the final price.

Next, we calculate dealer 1's profit from submitting $r_1 = 0$. Among multiple equilibria in the second stage, we select the one characterized in Example 1:

1. If $R < 0$, then CDS sellers push the final price up to $p^* = 1$. CDS seller i buys

$$\frac{|Q_i + r_i|}{\sum_{j \in S} |Q_i + r_i|} |R|$$

units of the bonds, where S denotes the set of CDS sellers. CDS buyers receive zero share of the open interest.

2. If $R > 0$, then CDS buyers push the final price down to $p^* = 0$. CDS buyer i sells

$$\frac{Q_i + r_i}{\sum_{j \in B}(Q_i + r_i)} R$$

units of the bonds, where B denotes the set of CDS buyers. CDS sellers receive zero share of the open interest.

3. If $R = 0$, then $p^* = \mathbb{E}(v)$.

Therefore, by submitting $r_1 = 0$, dealer 1 makes an expected profit of

$$\begin{aligned}
&Q_1(1 - \mathbb{E}(v)) + \mathbb{E}\left[-\mathbb{I}_{R<0}\, Q_1(1-v) + \mathbb{I}_{R>0} \left(Q_1 - \frac{Q_1}{\sum_{j \in B}(Q_j + r_j)} R \right) v \,\middle|\, r_1 = 0 \right] \\
= &Q_1(1 - \mathbb{E}(v)) + Q_1\, \mathbb{E}\left[-(1-v) + \mathbb{I}_{R>0} \left(1 - \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B}(Q_j + r_j)} v \right) \,\middle|\, r_1 = 0 \right], \qquad (12)
\end{aligned}$$

where $\mathbb{I}$ is the indicator function, and the expectation $\mathbb{E}$ takes into account the distribution of other dealers' physical requests $\{r_j\}_{j \neq 1}$. Therefore, for dealer i to mix between $r_i = -Q_i$ and $r_i = 0$, we must have

$$1 - \mathbb{E}(v) = \mathbb{E}\left[\mathbb{I}_{R>0} \left(1 - \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B, j \neq 1}(Q_j + r_j) + Q_1} v \right) \,\middle|\, r_1 = 0 \right], \qquad (13)$$

which is a polynomial equation in other dealers' mixing strategies, $\{q_j\}_{j \neq 1}$.

For each other dealer $i \neq 1$, we can apply the same argument and obtain an indifference condition similar to (13). Since there are n polynomial equations and n unknowns, we expect a solution $\{q_i\}_{i=1}^n$ to exist. In Appendix A, we specify the off-equilibrium strategies and verify that dealers do not wish to deviate from their equilibrium strategy of mixing between $r_1 = -Q_1$ and $r_1 = 0$. This completes the characterization of the mixed-strategy equilibrium.

For concreteness and further illustration of the intuition, we now explicitly characterize a mixed-strategy equilibrium in a symmetric market where all CDS buyers have the same CDS position, and all CDS sellers have the same CDS position.

Example 2. Suppose that there are $k = n/2$ symmetric CDS buyers and k symmetric CDS sellers. Their CDS positions satisfy $Q_1 = \cdots = Q_k > 0$ and $Q_{k+1} = \cdots = Q_{2k} = -Q_1 < 0$. The following proposition explicitly characterizes a subgame-perfect mixed-strategy equilibrium, in which the mixing probability $q_i = q_B$ for all $i \in B$ and $q_i = q_S$ for all $i \in S$, for some $q_B, q_S \in (0, 1)$.

Proposition 2. *In the market with symmetric dealers, there exists a mixed-strategy equilibrium of the two-stage auction, in which the first-stage mixing probabilities $q_B \in (0, 1)$ and $q_S = 1 - q_B$ solve*

$$\begin{aligned} &\sum_{l=0}^{k} \sum_{j=0}^{l-1} \binom{k-1}{j} q_B^j (1-q_B)^{k-1-j} \binom{k}{l} q_S^l (1-q_S)^{k-l} \frac{k-l}{k-j} \mathbb{E}(v) \\ =&\sum_{l=0}^{k} \sum_{j=l+1}^{k-1} \binom{k-1}{j} q_B^j (1-q_B)^{k-1-j} \binom{k}{l} q_S^l (1-q_S)^{k-l} (1-\mathbb{E}(v)). \end{aligned} \tag{14}$$

Proof. See Appendix A. □

The mixed-strategy equilibrium of the first stage differs from the first-stage equilibrium of Chernov, Gorbenko, and Makarov (2012). In the model of Chernov, Gorbenko, and Makarov (2012), one side of the market, say CDS buyers, submit full physical requests, whereas the other side of the market, say CDS sellers, submit zero physical requests. Thus, the equilibrium of Chernov, Gorbenko, and Makarov (2012) predicts that $|R| = \sum_{i \in B} |Q_i|$ with probability 1, that is, the open interest in a CDS auction is the same as the total notional outstanding on one side of the market. By contrast, in the mixed-strategy equilibrium of this section, both sides of the market submit partial physical requests with strictly positive probability. Partial physical requests imply that $|R| < \sum_{i \in B} |Q_i|$ with strictly positive probability and prices are strictly biased (see Proposition 1).

3.5 Risk Aversion

In this short subsection, we show that our basic intuition on the price bias generalizes to risk-averse dealers.

As in Section 3.3, we suppose that CDS positions $\{Q_i\}_{i=1}^n$ are common knowledge and that the fair recovery value v of defaulted bonds has a commonly known probability distribution. We relax the risk-neutrality assumption and let dealer i to have utility function $u_i(m_i)$, where

$$m_i \equiv (v - p)(x_i + r_i) + (1 - p)Q_i \tag{15}$$

is dealer i's profit. In the above expression, x_i is the quantity of bonds allocated to dealer i, r_i is dealer i's physical settlement request, and the expectation is taken over all realizations of v. The utility functions $\{u_i\}_{i=1}^n$ can be distinct, and they satisfy $u_i'(\,\cdot\,) > 0$.

Given an equilibrium price of p, dealer i has the expected utility

$$U_i(p) = \mathbb{E}[u_i((v - p)(x_i + r_i) + (1 - p)Q_i)]. \tag{16}$$

Each dealer i maximizes the expected utility $\mathbb{E}[u_i(m_i)]$ by choosing the optimal downward-sloping demand schedules $x_i(p)$. With risk aversion, the "fair" price for dealer i is no longer $\mathbb{E}(v)$, but

$$\frac{\mathbb{E}[u_i'(m_i)v]}{\mathbb{E}[u_i'(m_i)]},$$

that is, the expected recovery value v weighted by his marginal utility $u_i'(m_i)$.

Proposition 3. *Suppose that the CDS positions $\{Q_i\}$ are common knowledge and that dealers are risk averse. Then, in any equilibrium of the one-sided auction in the second stage:*

(a) If the open interest is to sell, then for any CDS seller i, such that $r_i + x_i(p^*) + Q_i < 0$, we have

$$p^* > \frac{\mathbb{E}[u_i'(m_i)v]}{\mathbb{E}[u_i'(m_i)]}. \tag{17}$$

(b) If the open interest is to buy, then for any CDS buyer i, such that $r_i + x_i(p^*) + Q_i > 0$, we have

$$p^* < \frac{\mathbb{E}[u_i'(m_i)v]}{\mathbb{E}[u_i'(m_i)]}. \tag{18}$$

To see the intuition of Proposition 3, consider the case of a sell open interest. Dealer i's marginal utility at the equilibrium price p^* is

$$\begin{aligned} U_i'(p^*) &= \mathbb{E}\left[u_i'(m_i) \cdot \left(-r_i - x_i(p^*) - Q_i - \sum_{j \neq i} x_j'(p^*)(v - p^*)\right)\right] \\ &= (-r_i - x_i(p^*) - Q_i)\, \mathbb{E}\left[u_i'(m_i)\right] - \sum_{j \neq i} x_j'(p^*) \mathbb{E}\left[u_i'(m_i)(v - p^*)\right], \end{aligned} \tag{19}$$

where the second equality follows because p^* is fixed here, and the expectation is taken over realizations of v. We claim that if dealer i's net position $r_i + x_i(p^*) + Q_i$ is negative, then $\mathbb{E}[u_i'(m_i)(v - p^*)]$ must be negative; otherwise, because $x_j'(p^*) < 0$, we have $\Pi_i'(p^*) > 0$, and dealer i would increase the equilibrium price by increasing his bids. Therefore, $p^* > \mathbb{E}[u_i'(m_i)v]/\mathbb{E}[u_i'(m_i)]$. In other words, the bond is "overpriced" for CDS seller i, relative to the expected recovery value v weighted by his marginal utility. The CDS seller is nonetheless willing to purchase the overpriced bond in order to increase its price and reduce his CDS liabilities. Moreover, as long as some CDS buyer submits a partial physical settlement request, there always exists some CDS seller i with $r_i + x_i(p^*) + Q_i < 0$, as shown in Proposition 1. The intuition for an open interest to buy is symmetric.

4 A Double Auction Proposal

As we show in Section 3, price biases occur because the second stage of CDS auctions is one-sided. In this section, we propose a double auction, in which dealers can submit *both* buy and sell limit orders, regardless of the open interest from the first stage. Under a double auction, an artificially high price is corrected by sell orders, and an artificially low price is corrected by buy orders. As a result, a double auction produces an unbiased price.

Formally, for each i, we allow dealer i's demand schedule $x_i : [0,1] \times \mathbb{R} \to \mathbb{R}$ to take both positive and negative values. Demand schedules are differentiable and strictly decreasing in price p. The double auction executes orders in accordance with price priority. Because the first-stage open interest consists of price-independent market orders, the open interest has higher execution priority than do limit orders on the same side of the market. For example, if the open interest is to sell ($R < 0$), then limit buy orders are first used to match the open interest before they are used to match limit sell orders. With the exception that the double auction replaces the one-sided auction, the model here is identical to that in Section 3. The final market-clearing price $p^*(Q)$ still satisfies $\sum_{i=1}^{n} x_i(p^*(Q); Q_i) + R = 0$.

Proposition 4. *For either direction of the open interest and in any Bayesian Nash equilibrium of the double auction:*

(i) The final price satisfies $p^*(Q) = \mathbb{E}(v)$ for every realization of $Q = \{Q_i\}_{i=1}^{n}$.

(ii) Every dealer i clears his CDS position. That is, $x_i(\mathbb{E}(v); Q_i) + r_i + Q_i = 0$ for every realization of Q and for all i.

Proof. The proof is similar to that of Proposition 1 and is omitted. □

The double auction corrects price biases by allowing buyers and sellers to jointly determine the auction final price. For example, when CDS sellers try to increase the final auction price above $\mathbb{E}(v)$, arbitrageurs can submit sell orders at prices higher than $\mathbb{E}(v)$, making a profit and simultaneously correcting the overpricing. Similarly, attempts by CDS buyers to decrease the auction final price below $\mathbb{E}(v)$ are counterbalanced by arbitrageurs who submit buy orders at prices lower than $\mathbb{E}(v)$.

The double auction corrects price biases in CDS auctions precisely because of the outstanding CDS positions. Without a similar externality, a double auction need not correct price biases. For example, in the "collusive" equilibria studied by Wilson (1979), buyers coordinate to bid low prices, which drives the final sale price below the fair value of the auctioned asset. Adding a double auction in Wilson's model does not correct the underpricing because no one wishes to sell the asset at a price below its fair value.

5 Price Discovery in Double Auctions

In addition to correcting price distortions, a double auction has the advantage of aggregating dispersed information regarding the fair value of defaulted bonds. In this section we formally demonstrate the price-discovery property of the double auction in the CDS setting. Our analysis in this section extends the divisible auction model of Du and Zhu (2012).

5.1 A Double-Auction Model with Interdependent Values

As in Section 4, $n \geq 2$ dealers participate in the double auction, which permits both buy and sell orders. For simplicity, we study in this section an one-stage double auction in which dealers submit limit orders. Modeling only limit orders is without loss of generality because physical settlement requests are market orders and can be modeled as limit orders with extreme prices. And in contrast with Section 4, we allow heterogeneity in dealers' information about the value of defaulted bonds. This information heterogeneity is necessary for price discovery.

Specifically, each dealer i receives a signal, $s_i \in [0, 1]$, which is observed by dealer i only.

Given the profile of signals $(s_1, \ldots, s_n)$, dealer i values the defaulted bond at a weighted average of all signals:

$$v_i = \alpha s_i + \beta \sum_{j \neq i} s_j, \tag{20}$$

where α and β are positive constants that, without loss of generality, sum up to one:

$$\alpha + (n-1)\beta = 1.$$

Thus, dealers have *interdependent values*, and price discovery would depend on how the market-clearing price in the double auction aggregates information contained in the profile of signals $(s_1, \ldots, s_n)$. Potentially different weights α and β introduce a private component into the valuation and generates trades. We emphasize that v_i is unobservable to dealer i because other dealers' signals $\{s_j\}_{j \neq i}$ are unobservable to dealer i.

In addition to receiving a private signal, each dealer i holds a *private* inventory z_i of defaulted bonds and a *private* CDS position Q_i before the auction. The total inventory, $Z = \sum_{i=1}^{n} z_i$, is common knowledge. For example, the total supply of defaulted bonds of a firm is often public information. Inventories $\{z_i\}$ matter in the price-discovery model of this section because dealers face uncertainties regarding their valuations $\{v_i\}$. In the settings of Section 3 and Section 4, inventories do not matter because valuations there are commonly known to be $\mathbb{E}(v)$.

Finally, bidder i's utility after acquiring q_i unit of the defaulted bonds at the price of p is

$$U(q_i, p; v_i, z_i, Q_i) = v_i z_i + (v_i - p) q_i + (1-p) Q_i - \frac{1}{2}\lambda (q_i + z_i)^2, \tag{21}$$

where $\lambda > 0$ is a commonly known constant. The last term $-\frac{1}{2}\lambda(q_i + z_i)^2$ captures funding costs, risk aversion or other frictions that make it increasingly costly for dealers to hold larger positions in the defaulted bonds. This quadratic cost is also used by Vives (2011) and

Rostek and Weretka (2012) in models of auctions and trading. As before, the second term $(v_i - p)q_i$ on the right-hand side of (21) captures the profits of trading the bonds, and the third term $(1-p)Q_i$ captures the net payments on the CDS contracts. Because dealer i does not observe his valuation v_i before the auction, the value of his existing bond positions, $v_i z_i$, also enters his utility function.

5.2 An Ex Post Equilibrium

Now we proceed to the equilibrium analysis of the double auction. We denote by $x_i(p; s_i, z_i, Q_i)$ bidder i's demand schedule. At a potential market-clearing price of p, dealer i who has a signal of s_i, an inventory of z_i, and a CDS position of Q_i is willing to buy $x_i(p; s_i, z_i, Q_i)$ units of the defaulted bonds. As before, a negative $x_i(p; s_i, z_i, Q_i)$ represents sell orders. The market-clearing price p^* is determined by

$$\sum_{i=1}^{n} x_i(p^*; s_i, z_i, Q_i) = 0, \tag{22}$$

where for ease of notation we suppress the dependence of p^* on $\{s_i\}_{i=1}^n$, $\{z_i\}_{i=1}^n$, and $\{Q_i\}_{i=1}^n$.

Our object is to find an *ex post equilibrium*. In an ex post equilibrium, a dealer's strategy, which only depends on his private information (s_i, z_i, Q_i), is optimal even if he observes other dealers' private information, which consists of $\{s_j\}_{j\neq i}$, $\{z_j\}_{j\neq i}$, and $\{Q_j\}_{j\neq i}$. Thus, an ex post equilibrium is a Bayesian Nash equilibrium given *any* joint probability distribution of signals, inventories, and CDS positions. We now characterize an ex post equilibrium, in which the equilibrium price aggregates private information from all dealers.

Proposition 5. *Suppose that $n\alpha > 2$. In the double auction with interdependent values, private inventories, and private CDS positions, there exists an ex post equilibrium in which* *dealer i submits the demand schedule*

$$x_i(p; s_i, z_i, Q_i) = \frac{n\alpha - 2}{\lambda(n-1)}(s_i - p) - \frac{n\alpha - 2}{n\alpha - 1} z_i + \frac{(1-\alpha)(n\alpha - 2)}{(n-1)(n\alpha - 1)} Z - \frac{1}{n\alpha - 1} Q_i, \tag{23}$$

and the equilibrium price is

$$p^* = \frac{1}{n}\sum_{i=1}^{n} s_i - \frac{\lambda}{n} Z. \tag{24}$$

Moreover, if $n > 3$ this is the unique ex post equilibrium.

Proof. See Appendix A. □

The equilibrium demand schedule (23) confirms our results in Section 3 that the bidding aggressiveness of a dealer depends on his CDS position. As suggested by the term $-\frac{1}{n\alpha-1}Q_i$ in (23), CDS sellers (with negative Q_i) send more aggressive buy orders (or less aggressive sell orders) because they benefit from a higher final price p^*. Conversely, CDS buyers (with positive Q_i) send more aggressive sell orders (or less aggressive buy orders) because they benefit from a lower final price. Since the net supply of CDS is zero, the incentives of CDS buyers and CDS sellers to affect the price offset each other, and the equilibrium price p^* does not depend on $\{Q_i\}_{i=1}^n$. Moreover, we see that more aggressive buy orders are sent by dealers with higher signals and dealers with higher existing bond inventories.

The equilibrium of Proposition 5 also reveals that the equilibrium price (24) aggregates diverse information from the dealers. The equilibrium price p^* is equal to the average signal, $\sum_{i=1}^n s_i/n$, adjusted for the average marginal holding cost, $-\frac{\lambda}{n}Z$. This ex post equilibrium price (24) coincides with the competitive equilibrium price if there is no private information (that is, when signals, inventories, and CDS positions are all commonly known). To see this, recall that the competitive equilibrium price, p^c, is equal to the marginal valuation of each bidder at the competitive equilibrium allocation, $\{q_i^c\}_{i=1}^n$. That is,

$$p^c = v_i - \lambda(z_i + q_i^c).$$

Averaging the above equation across all the dealers, we have

$$p^c = \frac{1}{n}\sum_{i=1}^n v_i - \frac{\lambda}{n}\left(\sum_{i=1}^n z_i + \sum_{i=1}^n q_i^c\right) = \frac{1}{n}\sum_{i=1}^n s_i - \frac{\lambda}{n}Z = p^*.$$

In addition to revealing the bidding strategies of dealers and the associated price behavior, the equilibrium in Proposition 5 has a couple of desirable properties in itself. First, because it is ex post optimal, the equilibrium is robust to distribution assumptions of the signals, the inventories, and the CDS positions, as well as implementation details of the double auction. In particular, the equilibrium does not rely on the normal distribution that is used extensively in existing trading models, such as Grossman (1976) and Kyle (1985), among many others. Nor does the ex post equilibrium depends on the transparency of the auction, namely whether the demand schedules $\{x_i\}$ are observable. Second, because the ex post optimality is difficult to satisfy, it serves as a natural and powerful equilibrium selection criterion. It is particular useful for uniform-price auctions of divisible assets, which in many cases admit a continuum of Bayesian Nash equilibria (Wilson 1979). In the price-discovery model of this section, the ex post optimality implies uniqueness of the equilibrium for $n > 3$. Additional theories and properties of ex post equilibria are developed in Du and Zhu (2012).

6 Testing Bidding Behavior in CDS Auction Data

In this section we test our theory of bidding behavior in CDS auction data. Our empirical strategy is to use the first-stage physical settlement requests to infer the directions of dealers' CDS positions, and then test whether, as the theory predicts, CDS sellers (resp. CDS buyers) are more aggressive buyers (resp. sellers) in the second stage.

6.1 Data

We use data from 87 credit events (bankruptcy, failure to pay, and restructuring, etc.) from 2006 to 2010. Because some credit events, such as the defaults of Fannie Mae and Freddie Mac, involve multiple classes of debt, we have a total of 94 auctions. For each auction we observe

- Dealers' first-stage quotes, which determine the initial market midpoint.
- First-stage physical settlement requests, which form the open interest.
- Second-stage limit orders, which clear the open interest and determine the final price.

We emphasize that these auction data are based on dealers, who bid for both themselves and their clients. For brevity, however, we will refer to "dealers and their clients" simply as "dealers," keeping in mind that dealers' bids and clients' bids are not separately observable.

Table 1 shows the number of CDS auctions by type of the underlying debt, year, currency, and open interest. About two-thirds of the auctions are on CDS, and about one-third are on loan CDS. The most recent three years account for the vast majority of defaults, with the year 2009 accounting for more than a half of the total. About two-thirds of the auctions are in U.S. dollars, and the remaining majority are in euros. Finally, about 70% of the auctions have open interests to sell, and the rest, with the exception of seven auctions, have open interests to buy.

Table 2 summarizes the final price and open interests of the auctions. In this calculation, we exclude the seven auctions in which the second stage had no limit orders. The average final price of all auctions is 37 per 100 face value. Overall, the final price of the auction is close to the price cap or floor determined by the first stage, with a median difference of 2 point per 100 face value.

Table 1: Credit Event Auctions by Types

Type		Year		Currency		Open Interest	
CDS Senior	53	2006	2	USD	62	Sell	67
CDS Subordinate	10	2007	1	EUR	25	Buy	20
CDS Senior/Sub	1	2008	16	JPY	3	Zero	7
Loan CDS (LCDS)	22	2009	59	GBP	4		
European LCDS	8	2010	16				
Total	94	Total	94	Total	94	Total	94

Table 2: Summary Statistics of Credit Event Auctions

	Mean	Std. Dev.	Median
Final Price	37.28	33.22	23.94
| Final Price − Price Cap/Floor |	3.01	3.84	2.00
Sell Open Interest	254.92	633.95	84.71
Buy Open Interest	140.78	192.90	51.00

Prices are per 100 face value, and open interests are in million USD. When calculating the difference between the final price and the price cap or floor, we exclude the seven auctions in which the second stage had no limit orders. All other summary statistics are calculated from all 94 auctions.

In the remainder of the section, we test predictions of our theory on bidding aggressiveness. Table 3 provides a glossary of the variables we use. In all regressions, we assume that the errors are uncorrelated with the right-hand side variables so that the estimates are consistent.

6.2 Physical Requests and the Aggressiveness of Limit Orders

Prediction 1. *If the open interest is to* sell*, then in the second stage, dealers with physical* buy *requests bid more aggressively than do dealers with physical sell requests. Conversely, if the open interest is to* buy*, then in the second stage, dealers with physical* sell *requests bid more aggressively than do dealers with physical buy requests.*

The rationale for this prediction comes directly from our model: a dealer who has sub-

Table 3: Variables Used in Empirical Analysis of Section 6

Variable	Description
$AvgP_{i,t}$	Average price of filled limit orders in the second stage
$FinalP_t$	Final auction price
$FracOI_{i,t}$	Fraction of open interest won by dealer i in auction t
$Opposite_{i,t}$	Dummy variable that takes the value 1 if dealer i's physical settlement request is opposite in direction to the open interest, and is otherwise 0
$FracReq_{i,t}$	Dealer i's signed physical settlement request as a fraction of the sum of unsigned physical settlement requests from all dealers
OI_t	Signed open interest. A positive OI_t represents a buy open interest, and a negative OI_t represents a sell open interest
d_i	Dealer dummy
$quarter_{q(t)}$	Quarter dummy

An auction is denoted by t, and a dealer is denoted by i.

mitted a physical buy request in the first stage is more likely to be a CDS seller, or represent customers who are CDS sellers on average. Thus, in the case of an open interest to sell, this dealer, as well as his customers, would aggressively bid in order to increase the final price in the second stage. On the other hand, a dealer who has submitted a physical sell request in the first stage is more likely to be a net CDS buyer; in the case of a buy open interest, he would aggressively bid in order to decrease the final price. We note that dealer aggregation of customer orders introduce noises into observed physical requests and limit orders.

To test Prediction 1, we use two measures of bidding aggressiveness for each dealer: (i) the average price of the limit orders that are filled, and (ii) the fraction of open interest won by the dealer.

6.2.1 Average Price of Filled Limit Orders

Our first proxy of aggressiveness in bidding is the average price $AvgP_{i,t}$ of filled limit orders for dealer i in auction t. By definition, the average price $AvgP_{i,t}$ of filled limit orders must be above the final price $FinalP_t$ in the case of a sell open interest and must be below the final price FinalP_t in the case of a buy open interest. The absolute difference $|\log(\mathit{AvgP}_{i,t}) - \log(\mathit{FinalP}_t)|$ is an indication of the aggressiveness of the limit orders. The more aggressive are dealer i's limit orders, the larger is $|\log(\mathit{AvgP}_{i,t}) - \log(\mathit{FinalP}_t)|$. Consequently, we run the regression

$$\begin{aligned} |\log(\mathit{AvgP}_{i,t}) - \log(\mathit{FinalP}_t)| = \alpha + \gamma\, \mathit{Opposite}_{i,t} + \beta\, \mathit{FracReq}_{i,t} \cdot (-\operatorname{sign}(\mathit{OI}_t)) \\ + d_i + \mathit{quarter}_{q(t)} + \epsilon_{i,t}. \end{aligned} \tag{25}$$

Variable $\mathit{Opposite}_{i,t}$ is a dummy that takes the value 1 if dealer i's physical settlement request is opposite in direction to the open interest and is otherwise the value 0. Our theory predicts that a dealer on the opposite side of the open interest bids more aggressively, and we expect the coefficient γ to be positive. We use control variable $\mathit{FracReq}_{i,t}$, defined as dealer i's signed physical settlement request as a fraction of the total physical requests (sum of the unsigned buy requests and sell requests). A negative (resp., positive) $\mathit{FracReq}_{i,t}$ indicates a physical sell (resp., buy) request from dealer i in auction t. We multiply $\mathit{FracReq}_{i,t}$ by the negative open interest, $-\operatorname{sign}(\mathit{OI}_t)$, so that a dealer i with physical settlement request opposite (resp., same) in direction to the open interest always has a positive (resp., negative) $\mathit{FracReq}_{i,t} \cdot (-\operatorname{sign}(\mathit{OI}_t))$. We also control for dealer dummy d_i and quarter dummy $\mathit{quarter}_{q(t)}$.

We consider four variants of regression (25), with (1) no fixed effect, (2) only dealer fixed effect, (3) only quarter fixed effect, and (4) both dealer and quarter fixed effects. Table 4 summarizes the results of regression (25). As the model predicts, the coefficient γ on the dummy $\mathit{Opposite}_{i,t}$ is significantly positive for all four specifications. Table 4 reveals that dealers whose physical requests are opposite in direction to the open interest are willing to pay an average price that is approximately 5% further from the final auction price, compared with dealers whose physical requests are on the same side as the open interest. That is, on average, CDS sellers are more aggressive buyers, and CDS buyers are more aggressive sellers.

Conditioning on the direction of the physical requests, however, the size of the physical request does not significantly correlate with the average price of filled limit orders. In fact, the lack of statistical significance of $FracReq_{i,t} \cdot (-\operatorname{sign}(OI_t))$ is not inconsistent with our model. For example, in Example 1 of Section 3.3, all filled limit buy orders have the same price of 1, even though CDS sellers may have different physical requests.

Table 4: Estimation Results of Regression (25), with Dependent Variable $|\log(AvgP_{i,t}) - \log(FinalP_t)|$

	(1)	(2)	(3)	(4)
$Opposite_{i,t}$	0.043*	0.048*	0.061**	0.071**
	(0.034)	(0.034)	(0.036)	(0.034)
$FracReq_{i,t} \cdot (-\operatorname{sign}(OI_t))$	-0.032	-0.061	-0.037	-0.071^{*}
	(0.048)	(0.051)	(0.047)	(0.05)
Constant	0.121***	0.115***	0.036	0.045
	(0.023)	(0.03)	(0.028)	(0.037)
Dealer FE	No	Yes	No	Yes
Quarter FE	No	No	Yes	Yes
N	611	611	611	611
R^2(%)	0.4	6	8.7	13.5

Robust standard errors in parentheses are clustered by auctions. Statistical significance at 10%, 5%, and 1% levels are, in accordance with one-tailed tests, denoted by *, **, and ***, respectively. We drop dealer i in auction t if dealer i won zero amount of the open interest in auction t.

6.2.2 Shares of Open Interest Won

Our second proxy of aggressiveness in bidding is the share of open interests won by dealers. Naturally, a dealer who wins a larger fraction of the open interest is considered to be a more aggressive bidder. Thus, we run the regression

$$\begin{aligned} FracOI_{i,t} = \alpha + \gamma\, Opposite_{i,t} + \beta\, FracReq_{i,t} \cdot (-\operatorname{sign}(OI_t)) \\ + d_i + quarter_{q(t)} + \epsilon_{i,t}, \end{aligned} \tag{26}$$

where $FracOI_{i,t}$ is the unsigned fraction of open interests won by dealer i in auction t. Other right-hand side variables in (26) are the same as those in the regression (25). Our theory predicts that the coefficient γ is positive.

Table 5 summarizes the results of regression (26). Without the dealer fixed effect (in the first and third columns), dealers with physical requests opposite to the open interest win about 3% more of the open interest than do dealers with physical requests on the same side as the open interest. After introducing the dealer fixed effect (the second and fourth columns), however, the estimated γ shrinks in magnitude by about a half and becomes statistically insignificant. This result suggests that certain dealers (and their customers) persistently sit on one side of the CDS market and also obtain larger shares of the open interests. As in regression (25), the coefficient for $FracReq_{i,t} \cdot (-\operatorname{sign}(OI_t))$ is not statistically significant after conditioning on the direction of the physical requests.[10]

Table 5: Estimation Results of Regression (26), with Dependent Variable $FracOI_{i,t}$.

	(1)	(2)	(3)	(4)
$Opposite_{i,t}$	0.029**	0.016	0.031**	0.017
	(0.016)	(0.016)	(0.016)	(0.017)
$FracReq_{i,t} \cdot (-\operatorname{sign}(OI_t))$	-0.038	-0.013	-0.035	-0.012
	(0.036)	(0.037)	(0.036)	(0.038)
Constant	0.077***	0.119***	0.079***	0.11***
	(0.003)	(0.023)	(0.004)	(0.024)
Dealer FE	No	Yes	No	Yes
Quarter FE	No	No	Yes	Yes
N	1039	1039	1039	1039
$R^2(\%)$	0.4	5.8	0.6	5.9

Robust standard errors in parentheses are clustered by auctions. Statistical significance at 10%, 5%, and 1% levels are, in accordance with one-tailed tests, denoted by *, **, and ***, respectively.

[10] Using a method similar to that in Example 1 of Section 3.3, one can construct a second-stage equilibrium in which all dealers who win positive shares of the open interest win equal shares of the open interest.

7 Discussion: Price Caps, Frictions, and Bond Prices

So far, our model of CDS auctions does not have price caps or floors. In practice, price caps and floors are imposed. Although price caps and floors could sometimes limit price biases, they may not always work. For example, given an open interest to sell, if the price cap, p_{cap}, is set below $\mathbb{E}(v)$, then the final auction price $p^* = p_{cap}$, which is too low. If $p_{cap} \geq \mathbb{E}(v)$, then our model predicts that $\mathbb{E}(v) \leq p^* \leq p_{cap}$. That is, the final price is too high. Therefore, for the final auction price to be unbiased, the price cap has to be *exactly* right, which is unlikely to obtain in practice because dealers typically face uncertainty regarding the recovery value v. Moreover, the existence of manipulative bidding and price bias in a *simpler* market (without caps and floors) calls into question whether the current design of CDS auctions can deliver fair prices in more sophisticated markets.

The price caps and floors can also obscure the empirical relations between $\mathbb{E}(v)$ and p^*. For example, the expected final auction price, given an open interest to sell, is

$$\begin{aligned}\mathbb{E}(p^* \,|\, R < 0) = \mathbb{P}\left[p_{cap} \geq \mathbb{E}(v)\right] \cdot \mathbb{E}\left[p^* \,|\, \mathbb{E}(v) \leq p^* \leq p_{cap}, R < 0\right] \\ + \mathbb{P}\left[p_{cap} < \mathbb{E}(v)\right] \cdot \mathbb{E}[p_{cap} \,|\, p_{cap} < \mathbb{E}(v)]. \end{aligned} \tag{27}$$

Given (27), it is not clear whether, ex ante, $\mathbb{E}(p^* \,|\, R < 0)$ is higher or lower than $\mathbb{E}(v)$. This inference is made more difficult by the multiplicity of equilibria in the second stage of the auction. A similar ambiguity applies to the case where $R > 0$ and a price floor is imposed. Thus, predicting the relations between $\mathbb{E}(v)$ and p^* requires, at a minimum, a model of dealers' quotes in the first stage. Under the current CDS auction mechanism, dealers' quoting strategies are important but separate from our main point of manipulation bidding incentives. Under a double auction, dealers' quotes would be less important because price caps and floors would be nonessential.

The empirical relation between CDS auction prices and bond prices has been the focus of existing studies on CDS auctions. For example, Helwege, Maurer, Sarkar, and Wang (2009) find that CDS auction prices appear to be close to bond prices near the auction dates. More recently, Chernov, Gorbenko, and Makarov (2012) and Gupta and Sundaram (2011) find that bond prices tend to be higher than CDS auction prices before and after the auction if there is an open interest to sell.

Several factors may potentially explain this bond price pattern. First, risk-averse investors could demand a risk premium for buying the bonds in the auction and produce an auction price that is lower than bond transaction prices after the auction. Second, to the extent that investors have limited risk budgets to absorb shocks in the open interests, the final auction price can reflect investors' capital constraints in addition to their estimates of the fair recovery values of defaulted bonds. Such "capital mobility" friction can cause sharp price reactions to demand and supply shocks and subsequent reversals (Duffie 2010). Third, as indicated by (27), the auction prices would be lower than bond prices if the dealers' quotes and associated price caps are set too low. Low quotes can occur in equilibrium because dealers' quotes enter the second stage as limit orders, and dealers could demand a risk premium as investors do. Fourth, Chernov, Gorbenko, and Makarov (2012) suggest that, if some aggressive bidders (e.g. CDS sellers) are forbidden from buying bonds, then the remaining bidders may bid strategically as in Wilson (1979) and lead to too low an auction price. All four potential explanations rely on frictions and are separate from manipulative bidding incentives. As we illustrate in Section 3.5 and Section 5, manipulative bidding incentives also exist in the presence of frictions such as risk aversion, asymmetric information, and inventory cost. Therefore, we view our results as complementary to the empirical evidence on bond prices, and the relation between bond prices and auction prices in the time-series does not necessarily confirm or contradict our theoretical or empirical analysis.

In addition to these four factors that affect bond prices, three measurement issues make it challenging to approximate the fair recovery values from bond transaction data. One issue is the cheapest-to-deliver (CTD) option—the option of CDS buyers to deliver the cheapest bond for physical settlements.[11] Because rational CDS buyers would deliver the cheapest bonds available, the fair auction price should reflect the volume-weighted average price (VWAP) of *delivered* bonds, not all bonds. A proper measure of the average price of delivered bonds requires, at a minimum, detailed data on which bonds are delivered for physical settlement and the number of times. A second measurement issue is the illiquidity of the corporate bond markets, as demonstrated by Bessembinder and Maxwell (2008) and Bao, Pan, and Wang (2011), among others. The third measurement issue is the lack of reliable transaction data for non-US corporate bonds, structured bonds, and loans. The lack of bond data cuts the sample size substantially. For example, in our sample of 94 CDS auctions, only 25 have TRACE data on the corresponding bonds.

8 Conclusion

CDS auctions settle CDS claims after the defaults of firms, loans, and sovereigns. We find that under the current design of CDS auctions, CDS buyers and sellers have strong incentives to—and are able to—distort the final auction price, in order to achieve favorable settlement payouts. The one-sidedness of the second stage of the auction prevents arbitrageurs from correcting the price bias. Our results suggest that a double auction can correct price biases and provide effective price discovery regarding the value of defaulted bonds. The predictions of our model on bidding behavior are broadly consistent with CDS auction data.

[11] Ammer and Cai (2011) provide evidence that the CTD option is priced in sovereign CDS basis—the difference in credit qualities that are implied by bond spreads and CDS spreads, respectively. Similarly, Packer and Zhu (2005) find that CDS spreads on corporate bonds and sovereign bonds tend to be higher if the CDS contracts allow a broader set of deliverable obligations (i.e., higher CTD option). Longstaff, Mithal, and Neis (2005) and Pan and Singleton (2008) discuss the potential pricing implications of the CTD option, although they do not explicitly model it.

Appendix

A Proofs

A.1 Proof of Proposition 1

We prove the proposition for the case of a sell open interest ($R < 0$). The case for a buy open interest is symmetric.

Part (i). Given the demand schedules, $x_j, 1 \leq j \leq n$, of all dealers, dealer i's expected profit at position Q_i is

$$\Pi_i(Q_i) = \int_0^1 ((x_i(p;Q_i) + r_i)(\mathbb{E}(v) - p) + Q_i(1-p)) \frac{d}{dp} (H(p, x_i(p;Q_i) \mid Q_i))\, dp. \tag{28}$$

Following Wilson (1979), we define $H(p, x \mid Q_i) = F\left(\sum_{j \neq i} x_j(p;Q_j) + x \leq -R \mid Q_i\right)$, which is the probability that the final price is less than or equal to p if dealer i bids x at a price of p and everyone else bids in accordance with the demand schedule $x_j(\,\cdot\,;Q_j)$, $j \neq i$.

Rewriting (28) by integration by parts, we have

$$\begin{aligned}\Pi_i(Q_i) =&(x_i(1;Q_i) + r_i)(\mathbb{E}(v) - 1) - ((x_i(0;Q_i) + r_i)\mathbb{E}(v) + Q_i)H(0, x_i(0;Q_i) \mid Q_i) \\ &- \int_0^1 (-(x_i(p;Q_i) + r_i + Q_i) + x_i'(p;Q_i)(\mathbb{E}(v) - p))H(p, x_i(p;Q_i) \mid Q_i)\, dp.\end{aligned} \tag{29}$$

Thus, dealer i chooses $x_i(\,\cdot\,;Q_i)$ in order to maximize (29), subject to

(i) If $x_i(p;Q_i) > 0$, then $x_i'(p;Q_i) < 0$;

(ii) If $x_i(p;Q_i) = 0$, then $x_i(p';Q_i) = 0$ for any $p' \geq p$.

Let us fix a Bayesian Nash equilibrium $\{x_i\}_{i=1}^n$. Then for every dealer i and position Q_i, the optimality of $x_i(\,\cdot\,; Q_i)$ implies that $x_i(\,\cdot\,; Q_i)$ must satisfy the first-order condition, which is known as the Euler equation,[12] that is,

$$H_x(p, x_i(p; Q_i) \mid Q_i)(r_i + x_i(p; Q_i) + Q_i) + H_p(p, x_i(p; Q_i) \mid Q_i)(\mathbb{E}(v) - p) = 0, \qquad p \in [0, \bar{p}(Q_i)), \tag{30}$$

where $\bar{p}(Q_i) = \inf\{p : x_i(p; Q_i) = 0\}$, $H_x(p, x \mid Q_i) = \frac{\partial H}{\partial x}(p, x \mid Q_i) \le 0$, and $H_p(p, x \mid Q_i) = \frac{\partial H}{\partial p}(p, x \mid Q_i) \ge 0$. Notice that (30) is the direct analogue of the first-order condition in (8), where Q is common knowledge.

We now show that for any realization of CDS positions $Q = \{Q_i\}_{i=1}^n$, the final price $p^*(Q)$, given by the equilibrium demand schedules $\{x_i(\,\cdot\,; Q_i)\}_{i=1}^n$, is least $\mathbb{E}(v)$. For the sake of contradiction, suppose that a realization $Q = \{Q_i\}_{i=1}^n$ satisfies $p^*(Q) > \mathbb{E}(v)$. At $p^*(Q)$, we have $\sum_{i=1}^n r_i + x_i(p^*(Q); Q_i) + Q_i = 0$. Thus, there exists a dealer i for whom $r_i + x_i(p^*(Q); Q_i) + Q_i \le 0$. Let us fix this dealer i. We will show that dealer i, who knows only his CDS position Q_i but not $\{Q_j\}_{j \ne i}$, has the incentive to deviate from $x_i(\,\cdot\,; Q_i)$ by bidding for more shares at the price $p^*(Q)$. This deviation would contradict the fact that $\{x_i\}_{i=1}^n$ is a Bayesian Nash equilibrium because in equilibrium dealer i should not want to change his bid $x_i(p; Q_i)$ at *any* price $p \in [0, 1]$.

There are two cases. If $x_i(p^*(Q); Q_i) > 0$, then (30) cannot hold for dealer i at $p = p^*(Q)$ because we have $H_p(p^*(Q), x_i(p^*(Q); Q_i) \mid Q_i) > 0$. Thus, the first part of (30) is weakly positive, whereas the second part of (30) is strictly positive (recall that $p^*(Q) < \mathbb{E}(v)$). This means that dealer i has an incentive to deviate from $x_i(\,\cdot\,; Q_i)$ by bidding for more shares at a price of $p^*(Q)$, which contradicts the definition of equilibrium.

The second case involves $x_i(p^*(Q); Q_i) = 0$. Dealer i's equilibrium demand schedule

[12] Let $L(x, x', p) = ((x + r_i + Q_i) - x'(\mathbb{E}(v) - p))H(p, x \mid Q_i)$. Since dealer i maximizes $\int_0^1 L(x_i(p; Q_i), x_i'(p; Q_i), p)\, dp$ by varying $x_i(\,\cdot\,; Q_i)$, the Euler equation (or the first-order condition for calculus of variation problem) is $\frac{\partial L}{\partial x}(x_i(p; Q_i), x_i'(p; Q_i), p) - \frac{d}{dp}\left(\frac{\partial L}{\partial x'}(x_i(p; Q_i), x_i'(p; Q_i), p)\right) = 0$ for every p.

$x_i(\,\cdot\,; Q_i)$ must satisfy the following necessary conditions for the bounded optimal control problem (29) (see Kamien and Schwartz 1991, pp. 185-187, for details):

$$L_{x'}(x_i(p;Q_i), x_i'(p;Q_i), p) + \lambda(p) \begin{cases} = 0 & \text{if } x_i'(p;Q_i) < 0, \\ \geq 0 & \text{if } x_i'(p;Q_i) = 0, \end{cases} \tag{31}$$

$$\lambda'(p) = -L_x(x_i(p;Q_i), x_i'(p;Q_i), p)$$

for every $p \in [0,1]$, where

$$L(x, x', p) = ((x + r_i + Q_i) - x'(\mathbb{E}(v) - p))H(p, x \mid Q_i).$$

We claim that $x_i(p;Q_i)$ cannot satisfy (31) for some $p < \mathbb{E}(v)$, which contradicts the optimality of $x_i(\,\cdot\,;Q_i)$. To see this, notice that

$$\begin{aligned} &\frac{d}{dp}(L_{x'}(x_i(p;Q_i), x_i'(p;Q_i), p) + \lambda(p)) \\ =&-H_x(p, x_i(p;Q_i) \mid Q_i)(r_i + x_i(p;Q_i) + Q_i) - H_p(p, x_i(p;Q_i) \mid Q_i)(\mathbb{E}(v) - p). \end{aligned} \tag{32}$$

Clearly, (32) is weakly negative when $x_i(p;Q_i) = 0$. Since $H_p(p, x_i(p;Q_i) \mid Q_i) > 0$ at $p = p^*(Q) < \mathbb{E}(v)$, (32) must be strictly negative in a small neighborhood of $p = p^*(Q)$ (in which we still have $x_i(p;Q_i) = 0$). Thus, if we have

$$L_{x'}(x_i(p;Q_i), x_i'(p;Q_i), p) + \lambda(p) = 0$$

for $x_i(p;Q_i) > 0$ (recall that $x_i'(p;Q_i) < 0$ whenever $x_i(p;Q_i) > 0$), then we must have

$$L_{x'}(x_i(p;Q_i), x_i'(p;Q_i), p) + \lambda(p) < 0$$

for some p at which $x_i(p; Q_i) = 0$.

Therefore, $p^*(Q) < \mathbb{E}(v)$ cannot occur in equilibrium.

For the proofs of part (ii) and (iii), we fix a Bayesian Nash equilibrium $\{x_i\}_{1 \le i \le n}$ and a realization of $Q = \{Q_i\}_{i=1}^n$.

Part (ii). From the previous part we know that $p^*(Q) \ge \mathbb{E}(v)$. For the sake of contradiction, suppose that dealer i has a nonnegative CDS position $Q_i \ge 0$ and receives a positive share $x_i(p^*; Q_i) > 0$. Then we have

$$
\begin{aligned}
&H_x(p^*(Q), x_i(p^*(Q); Q_i) \mid Q_i)(r_i + x_i(p^*(Q); Q_i) + Q_i) \\
&+H_p(p^*(Q), x_i(p^*(Q); Q_i) \mid Q_i)(\mathbb{E}(v) - p^*(Q)) < 0
\end{aligned}
$$

because $r_i + x_i(p^*(Q); Q_i) + Q_i > 0$ and $H_x(p^*(Q), x_i(p^*(Q); Q_i) \mid Q_i) < 0$. Therefore, by the reasoning in Part (i), dealer i would deviate by bidding less at price $p^*(Q)$, which is a contradiction.

Part (iii). Suppose that $p^*(Q) = \mathbb{E}(v)$. Then (32) implies that

$$
r_i + x_i(p^*(Q); Q_i) + Q_i = 0,
$$

for every $i \in S$. Summing the above equation over $i \in S$ gives

$$
\sum_{i \in S} r_i - R + \sum_{i \in S} Q_i = -\sum_{i \in B} r_i - \sum_{i \in B} Q_i.
$$

Hence $r_i + Q_i = 0$ for every $i \in B$, that is, every CDS buyer has submitted a full physical settlement request.

A.2 Off-equilibrium strategies of the mixed-strategy equilibrium in the first stage

In this appendix we specify the off-equilibrium strategies in the mixed-strategy equilibrium characterized in Section 3.4. Given these off-equilibrium strategies, we verify that dealers do not wish to deviate from the equilibrium strategy of mixing between $r_i = 0$ and $r_i = -Q_i$, for all i.

Again, we consider dealer 1, a CDS buyer. Suppose that dealer 1 deviates to submitting $r_1 \in (-Q_1, 0)$, we select the following second-stage equilibrium:

1'. If $R < 0$, then $p^* = 1$, and CDS seller j's purchase of the open interest is proportional to $|Q_j + r_j|$.

2'. If $R > 0$, then $p^* = 0$. Dealer 1, the deviator, sells

$$(Q_1 + r_1) \cdot \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B, j \neq 1}(Q_j + r_j) + Q_1}$$

units of the bonds. It is easy to check that dealer 1's sale is larger than the proportional allocation, $\frac{Q_1+r_1}{\sum_{j\in B}(Q_j+r_j)} R$. Intuitively, dealer 1 is "punished" in the second stage by being forced to sell a larger quantity of bonds at the price 0. Each CDS buyer $i \neq 1$ who submits a zero physical request sells

$$\frac{Q_i + r_i}{\sum_{j \in B, j \neq 1}(Q_j + r_j)} \left(R - (Q_1 + r_1) \cdot \frac{\sum_{j \neq 1} r_j}{\sum_{j \in B, j \neq 1}(Q_j + r_j) + Q_1} \right)$$

units of bonds. The explicit construction of a second-stage equilibrium that supports these allocations can be done using the method in Example 1, and is omitted here.

3'. If $R = 0$, then $p^* = \mathbb{E}(v)$.

Given the off-equilibrium strategies defined in 1'–3', dealer 1's profit for submitting $r_1 \in$

$(-Q_1, 0)$ is

$$Q_1(1-\mathbb{E}(v)) + (Q_1+r_1)\mathbb{E}\left[-(1-v) + \mathbb{I}_{R>0}\left(1 - \frac{\sum_{j\neq 1} r_j}{\sum_{j\in B, j\neq 1}(Q_j+r_j) + Q_1} v\right) \middle| \, r_1\right]$$
$$< Q_1(1-\mathbb{E}(v)), \tag{33}$$

where the inequality follows from (13) and the fact that $\mathbb{I}_{R>0}$ is increasing in r_1. Thus, submitting $r_1 \in (-Q_1, 0)$ results in a lower expected profit than submitting $r_1 = 0$ or $r_1 = -Q_i$. This completes the verification that dealer 1 does not deviate from his mixed strategy.

A.3 Proof of Proposition 2

Without loss of generality, we consider dealer 1, who is a CDS buyer. We denote by j the number of CDS buyers who submit a full physical request to sell and denote by l the number of CDS sellers who submit a full physical request to buy. The open interest is $R = (l-j)Q_1$. Clearly, by submitting $r_1 = -Q_1$ and choosing $x_1 = 0$, dealer 1 gets a fixed profit of $Q_1(1-\mathbb{E}(v))$. If dealer 1 submits $r_i = 0$, then there are three possibilities:

1. If $l < j$, the open interest is to sell (i.e. $R < 0$), which happens with probability

$$\sum_{l=0}^{k}\sum_{j=l+1}^{k-1}\binom{k-1}{j} q_B^j (1-q_B)^{k-1-j}\binom{k}{l} q_S^l (1-q_S)^{k-l}. \tag{34}$$

   In this case, we select a second-stage equilibrium in which $p^* = 1$. By Proposition 1, dealer 1 receives zero open interest.

2. If $l > j$, the open interest is to buy (i.e. $R > 0$), which happens with probability

$$\sum_{l=0}^{k}\sum_{j=0}^{l-1}\binom{k-1}{j} q_B^j (1-q_B)^{k-1-j}\binom{k}{l} q_S^l (1-q_S)^{k-l}. \tag{35}$$

In this case, we select a second-stage equilibrium in which $p^* = 0$, and the $(k-j)$ CDS buyers with zero physical request (including dealer 1) evenly divide the open interest. One can show that this allocation and price can be supported in equilibrium in the second stage.

3. If $l = j$, then the open interest is zero, which happens with probability

$$\sum_{j=0}^{k-1} \binom{k-1}{j} q_B^j (1-q_B)^{k-1-j} \binom{k}{j} q_S^j (1-q_S)^{k-j}. \tag{36}$$

In this case, we set $p^* = \mathbb{E}(v)$.

Therefore, dealer 1's expected profit from submitting $r_1 = 0$ is

$$\begin{aligned} Q_1(1-\mathbb{E}(v)) + \sum_{l=0}^{k}\sum_{j=0}^{l-1} \binom{k-1}{j} q_B^j (1-q_B)^{k-1-j} \binom{k}{l} q_S^l (1-q_S)^{k-l} \left(Q_1 - \frac{l-j}{k-j} Q_1\right) \mathbb{E}(v) \\ + \sum_{l=0}^{k}\sum_{j=l+1}^{k-1} \binom{k-1}{j} q_B^j (1-q_B)^{k-1-j} \binom{k}{l} q_S^l (1-q_S)^{k-l} Q_1(\mathbb{E}(v)-1). \end{aligned} \tag{37}$$

For dealer 1 to be indifferent between $r_1 = -Q_1$ and $r_1 = 0$, we must have (14). By symmetry, (14) ensures that all CDS buyers are indifferent between setting $r_i = -Q_i$ and $r_i = 0$.

By a symmetric argument, CDS sellers are indifferent between setting $r_i = -Q_i$ and $r_i = 0$ if and only if

$$\begin{aligned} &\sum_{l=0}^{k}\sum_{j=0}^{l-1} \binom{k-1}{j} q_S^j (1-q_S)^{k-1-j} \binom{k}{l} q_B^l (1-q_B)^{k-l} \frac{k-l}{k-j} (1-\mathbb{E}(v)) \\ = &\sum_{l=0}^{k}\sum_{j=l+1}^{k-1} \binom{k-1}{j} q_S^j (1-q_S)^{k-1-j} \binom{k}{l} q_B^l (1-q_B)^{k-l} \mathbb{E}(v). \end{aligned} \tag{38}$$

We can rewrite the left-hand side of the indifference condition of CDS buyers, equation (14), as:

$$\begin{aligned}&\sum_{l=0}^{k}\sum_{j=0}^{l-1}\binom{k-1}{j}q_B^j(1-q_B)^{k-1-j}\binom{k}{l}q_S^l(1-q_S)^{k-l}\frac{k-l}{k-j}\mathbb{E}(v)\\=&\frac{1-q_S}{1-q_B}\sum_{l=0}^{k-1}\sum_{j=0}^{l-1}\binom{k}{j}q_B^j(1-q_B)^{k-j}\binom{k-1}{l}q_S^l(1-q_S)^{k-1-l}\mathbb{E}(v)\end{aligned}$$

and the right-hand side of (14) as:

$$\begin{aligned}&\sum_{l=0}^{k}\sum_{j=l+1}^{k-1}\binom{k-1}{j}q_B^j(1-q_B)^{k-1-j}\binom{k}{l}q_S^l(1-q_S)^{k-l}(1-\mathbb{E}(v))\\=&\frac{1-q_S}{1-q_B}\sum_{l=0}^{k-1}\sum_{j=l+1}^{k}\binom{k}{j}q_B^j(1-q_B)^{k-j}\binom{k-1}{l}q_S^l(1-q_S)^{k-1-l}\frac{k-j}{k-l}(1-\mathbb{E}(v)).\end{aligned}$$

Therefore, (14) is equivalent to (38), and we only need to solve (14) to find a solution (q_B, q_S).

By the previous argument, (14) can be written as

$$\frac{1-q_S}{1-q_B}\cdot\frac{\sum_{l=0}^{k-1}\sum_{j=0}^{l-1}\binom{k}{j}q_B^j(1-q_B)^{k-j}\binom{k-1}{l}q_S^l(1-q_S)^{k-1-l}}{\sum_{l=0}^{k}\sum_{j=l+1}^{k-1}\binom{k-1}{j}q_B^j(1-q_B)^{k-1-j}\binom{k}{l}q_S^l(1-q_S)^{k-l}}=\frac{1-\mathbb{E}(v)}{\mathbb{E}(v)}. \tag{39}$$

We set $q_S = 1 - q_B$. The left-hand side of (39) tends to ∞ as q_B tends to 0, and tends to 0 as q_B tends to 1. Therefore, by the Intermediate Value Theorem, there exists a solution to (14) that satisfies $q_B \in (0, 1)$ and $q_S = 1 - q_B$.

The solution to (14) ensures that every dealer is indifferent between $r_i = 0$ and $r_i = -Q_i$. The off-equilibrium strategies are the same as those in the general case in page 42 and ensure that a dealer does not have any incentive to submit a physical request strictly between 0 and $-Q_i$. Therefore, we have a mixed-strategy equilibrium of the two-stage auction.

A.4 Proof of Proposition 5

We conjecture a symmetric demand schedule $x_i(p; s_i, z_i, Q_i)$, $i \in \{1, \ldots, n\}$. Each dealer i chooses an optimal strategy given that all other dealers use this conjectured strategy profile $\{x_j\}_{j \neq i}$. For a fixed profile of signals $(s_1, \ldots, s_n)$, inventories $(z_1, \ldots, z_n)$, and CDS positions $(Q_1, \ldots, Q_n)$, the ex post profit of dealer i at the price p is

$$\Pi_i(p) \equiv v_i z_i + (v_i - p)\left(-\sum_{j \neq i} x_j(p)\right) + (1-p)Q_i - \frac{1}{2}\lambda\left(z_i - \sum_{j \neq i} x_j(p)\right), \tag{40}$$

where we have suppressed in x_j the conditional variables s_j, z_j, and Q_j, and have used the fact that at the price p bidder i gets $-\sum_{j \neq i} x_j(p)$ units of bonds.

We can see from (40) that dealer i effectively chooses the optimal price p, again for fixed $\{s_j\}_{j=1}^n$, $\{z_j\}_{j=1}^n$, and $\{Q_j\}_{j=1}^n$. Taking the first-order condition of (40) at the market-clearing price of p^*, we have

$$0 = \Pi_i'(p^*) = -(x_i(p^*) + Q_i) + \left(\alpha s_i + \beta \sum_{j \neq i} s_j - p^* - \lambda(x_i(p^*) + z_i)\right)\left(-\sum_{j \neq i} \frac{\partial x_j(p^*)}{\partial p}\right). \tag{41}$$

Finding an ex post equilibrium boils down to finding a solution to (41), in which $x_i(p)$ does not depend on $\{s_j\}_{j \neq i}$, $\{z_j\}_{j \neq i}$, and $\{Q_j\}_{j \neq i}$.

We conjecture a linear demand schedule

$$x_i(p; s_i, z_i, Q_i) = a s_i - bp + d z_i + eZ + fQ_i,$$

where a, b, d, e, and f are constants. Under the conjectured linear strategy, for all $j \neq i$, the signal s_j of dealer j can be rewritten, in equilibrium, as

$$s_j = \frac{1}{a}\left(x_j(p^*) + bp^* - dz_j - eZ - fQ_j\right).$$

Using the facts that $\sum_{j\neq i} x_j(p^*) = -x_i(p^*)$, $\sum_{j\neq i} z_j = Z - z_i$, and $\sum_{j\neq i} Q_j = -Q_i$, we have

$$\sum_{j\neq i} s_j = \frac{1}{a}\left(-x_i(p^*) + (n-1)bp^* - d(Z-z_i) - (n-1)eZ + fQ_i\right).$$

Thus, the first order condition (41) can be rewritten as

$$\begin{aligned} 0 = &- (x_i(p^*) + Q_i) + (n-1)b \cdot \Bigg[\alpha s_i - p^* - \lambda(x_i(p^*) + z_i) \\ &+ \beta \frac{1}{a}\left(-x_i(p^*) + (n-1)bp^* - d(Z-z_i) - (n-1)eZ + fQ_i\right)\Bigg]. \end{aligned} \tag{42}$$

We observe that (42) implies that $x_i(p^*)$ is a linear function of s_i, p^*, z_i, Z, and Q_i. Matching the coefficients of (42) and those of $x_i(p^*) = as_i - bp^* + dz_i + eZ + fQ_i$, we solve

$$a = b = \frac{n\alpha - 2}{\lambda(n-1)}, \quad d = -\frac{n\alpha - 2}{n\alpha - 1}, \quad e = -\frac{n\alpha - 2}{n\alpha - 1} \cdot \frac{1-\alpha}{n-1}, \quad f = -\frac{1}{n\alpha - 1}.$$

Finally, the second-order condition $\Pi_i''(p^*) = -nb(1-\alpha)$, which is nonpositive if and only if $n\alpha > 2$. This completes the construction of the equilibrium. Uniqueness of the equilibrium follows from the argument in Du and Zhu (2012) and is omitted here.

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