An Efficient and Incentive Compatible Mechanism for Wholesale Electricity Markets

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Abstract—Being widely used in many deregulated wholesale electricity markets, the locational marginal pricing (LMP) mechanism is known to achieve social optimality in a competitive market. When profit-maximizing generators act strategically to manipulate prices; however, LMP may lead to high loss of economic efficiency. In this paper, we apply the Vickrey–Clarke–Groves (VCG) mechanism to wholesale electricity markets. We show that the VCG mechanism minimizes the total cost at a truth-telling dominant strategy equilibrium. We establish an important comparative result that the VCG mechanism always results in higher per-unit electricity prices than the LMP mechanism under any given set of reported supply curves. Numerical results show that the difference between the per-unit prices resulting from the two mechanisms is negligibly small (about 4%) in the IEEE 14-bus and 30-bus test systems. Finally, we apply the VCG mechanism to a day-ahead setting with start-up cost (of conventional generators) and intermittent renewable generation. We show that the VCG mechanism induces the truth-telling behavior of conventional generators in dominant strategies and yields each conventional generator a non-negative expected profit.

Index Terms—Locational marginal pricing (LMP), power networks, Vickrey–Clarke–Groves (VCG) mechanism, wholesale electricity markets.

I. INTRODUCTION

SINCE THE 1990s, electricity markets in many countries have moved from being a vertically integrated regulated monopoly to a deregulated market structure with the objectives of introducing fair competition and improving economic efficiency. Locational marginal pricing (LMP) is widely used in many deregulated wholesale electricity markets, for example, in the PJM Interconnection, Electric Reliability Council of Texas, California, New York, Midwest, and New England markets. In such a market, each individual generator submits a supply curve that specifies the amount of electricity it would like to supply at a given price. Collecting the supply curves reported by all generators, an independent system operator (ISO) adopts the LMP mechanism to set nodal electricity prices that clear the market.

It is well known that the LMP mechanism achieves social optimality in a competitive market, where price-taking market participants do not anticipate the influence of their submitted supply curves on nodal prices [1]. However, not much is understood about the LMP mechanism if generators act strategically to manipulate electricity prices so as to maximize their own profit. There are growing concerns about the exercise of market power under the LMP mechanism, as empirical evidence shows that generators have been able to raise prices above competitive levels [2]. More recently, a few theoretical works show that large generators can significantly raise electricity prices by withholding capability or by marking up marginal costs, and such strategic behavior may result in considerable loss of economic efficiency [3]–[5].

Motivated by the potential loss of economic efficiency resulting from the LMP mechanism, we seek to design an efficient market mechanism that mitigates the market power of generators. In a wholesale electricity market, each generator submits to the system operator its marginal cost function. Collecting the marginal costs reported by all market participants, an operator calculates a payment function that determines the payment of each generator as a function of its supply, and dispatches each generator to supply a socially optimal supply quantity. How the operator maps reported supply curves to a set of payment functions, i.e., the mechanism, is publicly announced to all market participants before they report their supply curves.

In this paper, we apply the well-known Vickrey–Clarke–Groves (VCG) mechanism [6] to wholesale electricity markets. The VCG payment is designed to endogenize the social welfare function into each generator’s profit function, i.e., a generator is paid the difference between the total (reported) cost of the other generators, at two optimal dispatches without and with the power injection from generator $i$. The VCG mechanism completely eliminates generators’ market power, in that no generator has the ability to profitably alter the price it faces. As a result, it is profit-maximizing for each generator to truthfully reveal its supply curve, regardless of the actions taken by other generators. The total cost is shown to be minimized
at the dominant strategy equilibrium where everyone is truth-telling.

For real-time wholesale electricity markets (where generators’ start-up costs have been paid and can be regarded as “sunk cost”), we establish an important comparative result on the prices resulting from the LMP and the VCG mechanisms. Under any given set of reported supply curves, the VCG mechanism always results in higher per-unit electricity prices than the LMP mechanism. For the IEEE 14-bus and 30-bus test systems, if all generators truthfully reveal their marginal costs, numerical results show that the difference between the per-unit electricity prices resulting from the LMP and the VCG mechanisms is negligibly small (about 4%). It is worth noting that since generators have incentives to report false supply curves under the LMP mechanism, an equilibrium induced by the LMP mechanism can yield arbitrarily high efficiency loss, as well as nodal prices that are significantly higher than the per-unit price resulting from the VCG mechanism (see Section VI-A).

We further construct a framework that applies the VCG mechanism in day-ahead electricity markets with intermittent renewable generation. In the day ahead market, commitment decisions of conventional generation needs to be made before the realization of random renewable generation. Our model incorporates conventional generators’ start-up cost, which makes the cost of conventional generators nonconvex. In the day-ahead market, the system operator makes the unit commitment decisions so as to minimize the expected total cost and applies the VCG mechanism to incentivize (conventional) generators truthfully reveal their supply curves. We show that the VCG mechanism yields each conventional generator a non-negative payment that is no less than its expected cost.

The nonconvexity of generators’ cost functions imposes challenges to the proper pricing of electricity under the LMP mechanism [7], [8]. Under LMP, a competitive equilibrium may not exist due to the nonconvex structure of supplier cost [9]. Assuming that each generator has a linear cost function (plus a fixed start-up cost), O’Neill et al. [10] and Ruiz et al. [11] constructed a set of linear prices that can support a competitive equilibrium, through a primal–dual approach on mixed-integer linear programs. For general settings with nonlinear generation cost, a variety of pricing schemes are proposed to support an efficient competitive equilibrium and to provide generators non-negative profits, through the introduction of an uplift charge [12] and a semi-Lagrangian approach [13]. We note that these aforementioned works do not consider the strategic behavior of (conventional) generators or the impact of intermittent renewable generation.

A majority of the literature on wholesale electricity markets, on the other hand, assumes the convexity of generation cost. There is a substantial literature on the analysis of generators’ behavior in wholesale electricity markets, under the assumption that generators are price-taking [14], [15], or through numerical approaches that aim to solve game theoretic equilibria [16]–[19]. Closer to this paper, a few works apply the standard VCG mechanisms to both the demand and supply sides of electricity markets. Samadi et al. [20] applied the VCG mechanism on demand-side management, so as to efficiently coordinate the demand response of multiple consumers. Schummer and Vohra [21] applied the VCG mechanism to wholesale electricity markets, without taking into account the power network constraints and intermittent renewable generation. Tang and Jain [5] proposed a one-shot variant of the standard VCG mechanism for wholesale electricity markets and showed the existence of an efficient Nash equilibrium at which every market participant is truth-telling. In this paper, for both the real-time and day-ahead markets, we show that the VCG mechanism guarantees incentive compatibility and efficiency at a dominant strategy equilibrium, a game theoretic concept that is stronger than the notion of Nash equilibrium.\footnote{It is optimal for an agent to take a dominant strategy, regardless of the actions taken by its rivals. A Nash equilibrium strategy, on the other hand, maximizes the agent’s payoff only when all the other agents follow the Nash equilibrium strategies.}

The rest of this paper is organized as follows. In Section II, we introduce our market model that incorporates dc power network constraints and prove the economic efficiency and incentive compatibility of the VCG mechanism. In Section III, we establish an important comparative result that the VCG mechanism always results in higher payment (for generators) than the LMP mechanism. In Section IV, we generalize the VCG mechanism to energy and reserve co-optimized markets and to a two-sided market model with both supplier and consumer bids. In Section V, we consider an alternative model (for day-ahead markets) where the system operator makes unit commitment decisions before the realization of random renewable generation. Numerical results are presented in Section VI. Finally, in Section VII, we make some brief concluding remarks.

II. Formulation

In Section II-A, we introduce the dc power-flow model and the economic dispatch problem faced by the operator. In Section II-B, we formulate the set of market mechanisms considered in this paper, i.e., the LMP and VCG mechanisms. In Section II-C, we introduce the game-theoretic equilibrium concepts used in this paper.

A. Power Network and Economic Dispatch

Consider a transmission network consisting of \( n \) buses and \( m \) lines, whose bus susceptance matrix is given by \( Y = \{y_{ij}\} \in \mathbb{R}^{n \times n} \). We will adopt the model considered in [1], which is based on the standard dc power flow assumptions of lossless lines, constant bus voltages, and small phase angle differences between neighboring buses. Let \( x_{i} \) and \( \theta_{i} \) denote the power injection and the phase angle at bus \( i \), respectively; we obtain a set of linear power flow equations

\[
\begin{align*}
\sum_{j=1}^{n} y_{ij} (\theta_{i} - \theta_{j}) = D_{i}, & \quad i = 1, \ldots, n 
\end{align*}
\]

where \( D_{i} \geq 0 \) is the demand at bus \( i \). Clearly, the summation of the \( n \) linear equations in (1) yields \( \sum_{i=1}^{n} x_{i} = \sum_{i=1}^{n} D_{i} \).

The real power flow \( p_{ij} \) from node \( i \) to \( j \) is limited by

\[
\begin{align*}
\text{subject to } p_{ij} = y_{ij} (\theta_{i} - \theta_{j}) \leq C_{ij}, & \quad \forall (i, j) \in M
\end{align*}
\]

It is optimal for an agent to take a dominant strategy, regardless of the actions taken by its rivals. A Nash equilibrium strategy, on the other hand, maximizes the agent’s payoff only when all the other agents follow the Nash equilibrium strategies.
where \( \mathcal{M} \) is the set of \( 2m \) (directed) transmission lines, and \( C_{ij} = C_{ji} \geq 0 \) represents the thermal loading limit on the line connecting nodes \( i \) and \( j \).

Though widely used in the literature, the above dc power-flow model has its limitations. We note, however, since locational marginal prices are typically calculated using linearized dc power-flow models [22]–[24], it is a reasonable model for the purpose of studying generators’ strategic bidding behavior in wholesale electricity market.

Without loss of generality, we assume that at each bus \( i \) there is only one generator, which will be (sometimes) referred to as agent \( i \). For each generator \( i \), we use a nondecreasing, convex, and continuously differentiable function \( C_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) to measure its generation cost.

Collecting the supply curves reported by all generators, a system operator solves the following economic dispatch problem to minimize the aggregate cost:

\[
\text{minimize} \quad \sum_{i=1}^{n} C_i(x_i) \\
\text{subject to} \quad (1), \quad (2) \\
\quad x_i \in [X_i, X_i^*], \quad i = 1, \ldots, n \tag{3}
\]

where \( \mathbf{x} = (x_1, \ldots, x_n) \), \( \mathbf{\theta} = (\theta_1, \ldots, \theta_n) \), and \( X_i \) and \( X_i^* \) are the minimum and maximum output of generator \( i \), respectively. We assume that there exists a feasible dispatch when one generator is down. We note that this assumption is practical in real power systems and holds for almost all IEEE test systems [25], [26].

**B. Market Mechanisms**

Each agent \( i \) is asked to submit an inverse supply curve \( f_i : \mathbb{R} \rightarrow \mathbb{R}_+ \) to the system operator. The submitted curve describes the marginal cost of agent \( i \) at a certain generation level. Since we have assumed that each \( C_i(\cdot) \) is convex, generators are restricted to submit nondecreasing marginal cost curves. We let \( \mathbf{f} \) denotes the supply curves reported by all agents, and let \( \mathbf{f}_{-i} \) denotes the supply curves reported by all agents excluding \( i \).

We focus on market mechanisms that always choose the cost minimizing dispatch, i.e., \( \mathbf{x}^*(\mathbf{f}) = (x_1^*(\mathbf{f}), \ldots, x_n^*(\mathbf{f})) \) which is an optimal solution to (3) (given the supply curves reported by all agents \( \mathbf{f} \)). In this paper, a set of payment schemes \( \mathbf{v} = (v_1, \ldots, v_n) \) is referred to as a mechanism, where \( v_i \) maps \( \mathbf{f} \) into the payment made to agent \( i \). A mechanism \( \mathbf{v} \) (that is commonly known by all agents before they act) defines a game among the \( n \) agents.

We first introduce the widely used LMP mechanism. Let \( \lambda_i \) be the Lagrange multiplier associated with the constraint in (1), for every bus \( i \). The Karush–Kuhn–Tucker conditions of the optimization problem (3) yield

\[
\begin{align*}
(C_i'(x_i) - \lambda_i)(x_i - X_i) &= 0, \quad C_i'(x_i) - \lambda_i \geq 0, \quad X_i \leq x_i \leq X_i^* \tag{4} \\
(C_i'(x_i) - \lambda_i)(x_i - X_i) &= 0, \quad C_i'(x_i) - \lambda_i \geq 0, \quad X_i < x_i \leq X_i^*.
\end{align*}
\]

\( ^2 \)It is straightforward to add virtual buses to incorporate the case where multiple consumers and suppliers locate at a single physical bus.

**Definition 1 (LMP Mechanism):** Collecting a set of supply curves reported by all agents \( \{f_i\}_{i=1}^{n} \), the operator solves the optimization problem (3) defined by

\[
C_i(x_i) = \int_{0}^{x_i} f_i(y)dy, \quad i = 1, \ldots, n.
\]

Let \( \{\lambda_i\}_{i=1}^{n} \) be a set of Lagrange multipliers associated with an optimal solution. The payment scheme is defined by

\[
v_i(f) = \lambda_i x_i^*(f), \quad i = 1, \ldots, n \tag{5}
\]

where \( x_i^*(f) \) is the social optimal dispatch of generator \( i \).

There may exist multiple sets of Lagrange multipliers [associated with one or more optimal solutions to (3)]. As a result, there may exist multiple different LMP mechanisms satisfying Definition 1. We note that under an LMP mechanism (see Definition 1), a nodal price \( \lambda_i \) at bus \( i \) depends on the curves submitted by all agents, including agent \( i \) who will be paid this price.

Under an LMP mechanism, it follows from conditions in (4) and the payment scheme in (5) that being truth-telling maximizes the payoff of a price-taking agent \( i \) (who treats the nodal price \( \lambda_i \) as fixed). For a strategic agent who can anticipate the impact of its bid on nodal prices, however, it may benefit from reporting false marginal cost curves. As a result, the LMP mechanism is not incentive compatible; indeed, the LMP mechanism may result in arbitrarily high efficiency loss at an equilibrium, as demonstrated in Section VI-A.

We now introduce the incentive compatible VCG mechanism. Given the reported supply curves, let \( G(\mathbf{f}) \) denote the minimum aggregate cost [i.e., the optimal value of the optimization problem (3)]. We use \( G_{-i}(\mathbf{f}_{-i}) \) to denote the minimum aggregate cost without generator \( i \), i.e., the optimal value of the optimization problem (3) with \( x_i = 0 \). The payment made to each generator \( i \) reflects the (positive) externality it causes to the system

\[
v_i(f) = G_{-i}(\mathbf{f}_{-i}) - \left( G(f) - \int_{0}^{x_i^*(f)} f_i(x)dx \right) \tag{6}
\]

which is the difference between the total cost of the other generators, without and with the optimal power injection of generator \( i \).

**C. Incentive Compatibility in Dominant Strategies**

In this section, we first introduce the concept of dominant strategy equilibrium and then show that the VCG mechanism define in (6) induces generators’ truth-telling behavior in a dominant strategy equilibrium.

A standard solution concept is Nash equilibrium. A Nash equilibrium is a set of strategies \( \mathbf{f} = (f_1, \ldots, f_n) \) such that

\[
v_i(f_i, \mathbf{f}_{-i}) - C_i(x_i^*(f_i, \mathbf{f}_{-i})) \geq v_i(f^*_i, \mathbf{f}_{-i}) - C_i\left(x_i^*(f^*_i, \mathbf{f}_{-i})\right)
\]

for every \( f^*_i \neq f_i \) and every agent \( i \). The concept of Nash equilibrium requires each agent \( i \) to form correct belief about the strategies used by other agents \( f_{-i} \). In an electric power grid, it may be hard for a generator to form correct beliefs...
about the actions taken by other generators, or even about the other generators’ cost functions.

In this paper, we therefore use a stronger equilibrium concept—dominant strategy equilibrium, at which the strategy used by each agent is optimal regardless of its belief on the actions taken by other agents. Formally, reporting a supply curve \( f_i \) is a dominant strategy for agent \( i \), if

\[
v_i(f_i, f_{-i}) - C_i(x^i_i(f_i, f_{-i})) \geq v_i(\tilde{f}_i, f_{-i}) - C_i(x^i_i(\tilde{f}_i, f_{-i}))
\]

for any \( \tilde{f}_i \neq f_i \), and for any possible actions taken by other agents \( f_{-i} \). A strategy profile \( f = (f_1, \ldots, f_n) \) is a dominant strategy equilibrium, if for every \( i \), \( f_i \) is a dominant strategy for agent \( i \).

**Theorem 1 (Incentive Compatibility):** Under the VCG mechanism, there exists a dominant strategy equilibrium at which every agent \( i \) reports her true supply curve.

**Proof:** Under the payment scheme defined in (6), generator \( i \) which reports \( \tilde{f}_i \) obtains a profit of

\[
G_{-i}(\tilde{f}_{-i}) - \left( G(\tilde{f}_i, f_{-i}) - \int_0^{x^i_i}(\tilde{f}_i, f_{-i}) f_i(x)dx \right) - \int_0^{x^i_i}(\tilde{f}_i, f_{-i}) f_i(x)dx.
\]

We note that the term

\[
G(\tilde{f}_i, f_{-i}) - \int_0^{x^i_i}(\tilde{f}_i, f_{-i}) f_i(x)dx + \int_0^{x^i_i}(\tilde{f}_i, f_{-i}) f_i(x)dx
\]

is the aggregate cost of a system with supply curves \((f_i, f_{-i})\) achieved at the dispatch \( x^i_i(\tilde{f}_i, f_{-i}) \), where \( f_i \) is generator \( i \)'s true supply curve. Since the optimal dispatch \( x^i_i(f_i, f_{-i}) \) minimizes the aggregate cost of the system [with supply curves \((f_i, f_{-i})\)], it is profit-maximizing for each generator \( i \) to report its true supply curve \( f_i \), regardless of the curves reported by other generators \( f_{-i} \).

**Remark 1:** We note that the VCG mechanism naturally extends to a setting where a generator is restricted to choose from a set of supply curves (e.g., piece-wise linear ones). In this setting, it is straightforward to show that the incentive compatibility and efficiency results still hold. Since each generator’s profit is social welfare (the negative of the aggregate cost) plus some constant, it is a dominant strategy for each generator to report a supply curve that is the closest to the generator’s true supply curve such that the supply curves reported by all generators minimize the aggregate cost among the set of feasible supply curves that could be reported.

**III. VCG AND LMP PAYMENTS**

In this section, we show an important comparative result that the VCG mechanism yields each generator a higher payment than the LMP mechanism. We start by introducing a functional that will be useful in our proof. Given the supply curves reported by all agents other than \( i \), the following functional denotes the aggregate cost of the \( n - 1 \) agents (excluding \( i \)), given that agent \( i \)'s injection at bus \( i \) is \( x_i \):

\[
H_i(f_{-i}, x_i) = \min_{(\mathbf{x}, \theta)} \sum_{j \neq i} \int_0^{\bar{x}_j} f_j(y)dy
\]

subject to (1), (2)

\[
x_j \in [X_j, \bar{X}_j], \quad \forall j \neq i
\]

where \( x_{-i} = \{x_j\}_{j \neq i} \), and the parameter \( x_i \) is implicitly incorporated by the constraint (1), i.e., the net injection at bus \( i \) is \( x_i \). It is straightforward to check that for every \( i \), there exists some \( \bar{x}_i \leq \bar{x}_i \leq X_i \) such that the functional \( H_i(f_{-i}, x_i) \) is well defined for \( x_i \in [\bar{x}_i, \bar{x}_i] \). That is, \( \bar{x}_i (\bar{x}_i) \) is the minimum (maximum, respectively) generation at bus \( i \) that guarantees the existence of a feasible dispatch. Since we have assumed the existence of a feasible dispatch without generator \( i \), we have \( \bar{x}_i = 0 \) for every \( i \).

The VCG payment made to each agent \( i \) [see (6)] can be written as

\[
v_i(f) = H_i(f_{-i}, 0) - H_i(f_{-i}, x^i_i(f)), \quad i = 1, \ldots, n
\]

where \( x^i_i(f) \) is the optimal dispatch level of generator \( i \) under the reported curves \( f \).

The following lemma will serve as a basis in the establishment of the main result.

**Lemma 1:** For every \( i \) and any given \( f_{-i} \), \( H_i(f_{-i}, x_i) \) is convex in \( x_i \), over the interval \([0, \bar{x}_i]\).

**Proof:** Within the proof, we fix an agent \( i \) and the supply curves reported by other agents \( f_{-i} \). We consider two levels of generation provided by agent \( i \) such that \( 0 \leq x^1_i < x^2_i \leq \bar{x}_i \). Let \((\mathbf{x}^1_i, \theta^1)\) and \((\mathbf{x}^2_i, \theta^2)\) be an optimal solution to (8) with parameter \( x^1_i \) and \( x^2_i \), respectively. It is straightforward to check that

\[
\left(\frac{x^1_i + x^2_i}{2}, \frac{\theta^1 + \theta^2}{2}\right)
\]

is a feasible solution to the optimization problem (8) with parameter \((x^1_i + x^2_i)/2\). Since the objective function of (8) is convex in \((x_i, \mathbf{x}_{-i})\), we conclude that the objective of (8) with parameter \((x^1_i + x^2_i)/2\) and the solution in (10) is no greater than

\[
\frac{H_i(f_{-i}, x^1_i) + H_i(f_{-i}, x^2_i)}{2}
\]

Since the solution in (10) is feasible to the optimization problem (8) with parameter \((x^1_i + x^2_i)/2\), we have

\[
H_i(f_{-i}, \left(\frac{x^1_i + x^2_i}{2}\right)) \leq \frac{H_i(f_{-i}, x^1_i) + H_i(f_{-i}, x^2_i)}{2}
\]

which implies the desired result.

**Lemma 1** implies the existence of right and left derivatives of \( H_i(f_{-i}, x_i) \), with respect to \( x_i \). It also implies that the aggregate cost (given the curves reported by all agents \( f \))

\[
H_i(f_{-i}, x_i) + \int_0^{x_i} f_i(y)dy
\]

is convex in \( x_i \), since \( f_i \) is restricted to be nondecreasing. For every bus \( i \), given the curves \( f \) submitted by all agents,
an optimal \( x_i \) that minimizes the aggregate cost in (20) can be therefore given by

\[
x_i^*(f) = \min \left\{ x \geq x_i : f_i(x) \geq -\frac{\partial^+ H_i(f_{-i}, x)}{\partial x} \right\}.
\] (12)

It is straightforward to check that \( \{x_i^*(f)\}_{i=1}^n \) is an optimal solution to (3) defined by the reported curves \( f \). For notational convenience, for every \( x \geq 0 \), we let

\[
\phi_i(f_{-i}, x) \triangleq -\frac{\partial^+ H_i(f_{-i}, x)}{\partial x}
\] (13)
denote the negative of the partial derivative of \( H_i \) with respect to \( x \).

It follows from (12) and (13) that \( H_i(f_{-i}, x_i) \) must be decreasing in \( x_i \) over \([X_i, x_i^*(f)]\). It is then straightforward to check [through the conditions in (4) and (12)] that the following non-negative \( n \)-dimensional vector:

\[
\left( \phi_1(f_{-1}, x_1^*(f)), \ldots, \phi_n(f_{-n}, x_n^*(f)) \right)
\] (14)
is a set of Lagrange multipliers associated the constraints in (1), at the optimal dispatch \( \{x_i^*(f)\}_{i=1}^n \). Therefore, the set of payment schemes

\[
v_i(f) = x_i^*(f) \cdot \phi_i(f_{-i}, x_i^*(f)), \quad i = 1, \ldots, n
\]
is an LMP mechanism (see Definition 1). It is important to note that the nodal prices in (14) depend only on \( f \), but not on the supply vector \( x \). As we have noted before, this LMP mechanism is not incentive compatible, in that a strategic agent \( i \) could report a false curve \( \tilde{f}_i \) to manipulate the nodal price.

We are now ready to compare the electricity prices under the proposed mechanism and the LMP mechanism characterized in the above.

**Proposition 1:** The VCG payment made to each agent \( i \) can be written as

\[
v_i(f) = H_i(f_{-i}, 0) - H_i(f_{-i}, x_i^*(f)) = x_i^*(f) \cdot \phi_i(f_{-i}, x_i^*(f)) (1 + \alpha_i(f))
\] (15)

where \( \phi_i(f_{-i}, x_i^*(f)) \) is the nodal price at bus \( i \) (according to an LMP mechanism), and the price distortion factor is given by

\[
\alpha_i(f) \triangleq \frac{H_i(f_{-i}, 0) - H_i(f_{-i}, x_i^*(f))}{\phi_i(f_{-i}, x_i^*(f)) x_i^*(f)} - 1.
\] (16)

**Remark 2:** Given any \( f_{-i} \), we have shown in Theorem 1 that it is optimal for agent \( i \) to truthfully reveal its marginal cost. We have also shown that at the second stage, each agent \( i \) would like to take the action \( x_i^*(f) \), which maximizes its own payoff [as well as the aggregate welfare in (20)]. As shown in the following Theorem 2, the convexity of \( H_i(f_{-i}, \cdot) \) implies that:

\[
\alpha_i(f) = \frac{H_i(f_{-i}, 0) - H_i(f_{-i}, x_i^*(f))}{\phi_i(f_{-i}, x_i^*(f)) x_i^*(f)} - 1 \geq 0.
\] (17)

That is, electricity prices under the proposed mechanism cannot be lower than the nodal prices resulting from LMP. The price distortion factor defined in (16) increases with the “convexity” of the function \( H_i(f_{-i}, \cdot) \). When \( H_i(f_{-i}, \cdot) \) is linear, \( \alpha_i(f) = 1 \) and agent \( i \) faces the same per-unit electricity price under both VCG and LMP mechanisms.

It is worth noting that since the price distortion factor is defined for the same set of reported curves \( f \) [see (16)], it essentially compares the prices resulting from the two mechanisms when all agents are truth-telling. Since the LMP mechanism is not incentive compatible, it is possible that generators can raise the nodal prices by lying. A numerical example in Section VI-A shows that the electricity price induced by the VCG mechanism (at the truth-telling dominant strategy equilibrium) can be lower than that resulting from the LMP mechanism at a Nash equilibrium.

**Theorem 2:** Given any set of reported curves \( f \) and for every bus \( i \), the per-unit electricity price induced by the proposed mechanism is higher than or equal to the nodal price resulting from an LMP mechanism.

**Proof:** Lemma 1 shows that \( H_i(f_{-i}, x_i) \) is convex in \( x_i \). It follows from (12) and (13) that \( H_i(f_{-i}, x_i) \) is decreasing in \( x_i \) over \([X_i, x_i^*(f)]\). We conclude that the inequality in (17) holds, and the desired result follows from Proposition 1.

Theorem 2 shows that the proposed mechanism always leads to higher payment than the LMP mechanism. This result implies that if a generator benefits from participating in the LMP market, then it must voluntarily participate in the proposed market (under the VCG mechanism). We note, however, that under the LMP mechanism, a generator \( i \) may receive a negative payment if it is dispatched to operate at its minimum output \( X_i \) (see the Karush–Kuhn–Tucker conditions in (4)). Analogously, it is possible that a generator \( i \) (which is scheduled to supply \( X_i \)) receives a negative VCG payment, if \( x_i = 0 \) (i.e., generator \( i \) is shut down) results in a lower aggregate cost compared with the optimal solution \( x_i^*(f) = X_i \) [see the expression of VCG payment in (9)].

**IV. Extensions**

In Section IV-A, we extend the proposed mechanism to electricity markets where energy and reserve are co-optimized. In Section IV-B, we consider a two-sided market where consumers also report their demand curves. We note that the main results derived in Theorems 1 and 2 still hold in these two extended settings.

**A. Extension to Energy-Reserve Co-Optimized Markets**

In many wholesale electricity markets, energy and reserve market are co-optimized [29]. The proposed market mechanism naturally extends to this setting. We first write down the energy-reserve co-optimization problem faced by the operator

\[
\text{minimize} \sum_{i=1}^{n} C_i(x_i, r_i)
\]

subject to \( x_i \geq X_i \), \( x_i + r_i \leq X_i \), \( i = 1, \ldots, n \)

\[
\sum_{i=1}^{n} r_i \geq R
\] (18)

where \( r_i \) is the regulation reserve\(^3\) provided by generator \( i \), and \( R > 0 \) is the minimum requirement for regulation reserve.

\(^3\) Other type of system reserves can be modeled analogously.
In this setting, each generator \( i \) submits its marginal cost function \( f_i(x_i, r_i) \), and the payment function can be determined in a way that is analogous to (6). We let \( H_i(f_{-i}, x_i, r_i) \) denote the aggregate cost of all agents other than \( i \), given that agent \( i \)'s supply (of energy and reserve) at bus \( i \) is \((x_i, r_i)\), and the cost curves reported by other agents are \( f_{-i} \). It can be shown that under the following payment scheme:

\[
v_i(f) = H_i(f_{-i}, 0, 0) - H_i(f_{-i}, x_i^*(f), r_i^*(f)), \quad i = 1, \ldots, n
\]

(19)

the incentive compatibility result derived in Theorem 1 still holds. If the cost function \( C_i \) is convex in \((x_i, r_i)\) for every \( i \), then the proposed mechanism will yield higher energy and reserve prices than the LMP mechanism.

### B. Extension to a Two-Sided Market

In this section, we extend the proposed market mechanism to a two-sided market model where each consumer (e.g., an aggregator) submits a concave utility function (or equivalently, a nonincreasing demand curve) to the system operator. Let \( u_i \) denote the demand curve submitted by the consumer at bus \( i \). The operator solves the following modified version of the aggregate cost minimization problem in (3):

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{n} \int_{0}^{x_i} f_i(x)dx - \sum_{i=1}^{n} \int_{0}^{d_i} u_i(x)dx \\
\text{subject to} & \quad x_i - d_i = \sum_{j=1}^{n} y_{ij}(\theta_i - \theta_j), \quad \forall \ i \\
& \quad p_{ij} = y_{ij}(\theta_i - \theta_j) \leq C_{ij}, \quad \forall \ (i,j) \\
& \quad x_i \in [X_i, X_i], \quad i = 1, \ldots, n
\end{align*}
\]

(20)

where \( d_i \) is the dispatched demand at bus \( i \), and the objective function is the difference between the aggregate supplier cost and aggregate consumer utility.

Given the reported curves \((f, u)\), let \( \tilde{\mathcal{G}}_i(f, u) \) denote the minimum aggregate cost [i.e., the optimal value of the optimization problem (20)]. We use \( \tilde{\mathcal{G}}_{i-}(f, u) \) to denote the minimum aggregate cost without generator \( i \) [i.e., the optimal value of the optimization problem (20) with \( x_i = 0 \)], and \( \tilde{\mathcal{G}}_{i-}(f, u_{-i}) \) to denote the minimum aggregate cost without consumer \( i \) [i.e., the optimal value of the optimization problem (20) with \( d_i = 0 \)]. Let \((x^*(f, u), d^*(f, u))\) be an optimal solution to (20). Each consumer pays the externality it causes to the society

\[
\left( \tilde{\mathcal{G}}_i(f, u) + \int_{0}^{d_i} u_i(x)dx \right) - \tilde{\mathcal{G}}_{i-}(f, u_{-i})
\]

(21)

and the payment made to each generator \( i \) is analogous to that defined in (6)

\[
\tilde{\mathcal{G}}_{i-}(f_{-i}, u) - \left( \tilde{\mathcal{G}}_i(f, u) - \int_{0}^{r_i^*} f_i(x)dx \right).
\]

(22)

It is straightforward to check that the incentive compatibility result established in Theorem 1 applies to all generators and consumers. Still, the VCG payment made to generators is no less than that under the LMP mechanism.

Before ending this section, we make some discussion on the revenue adequacy of the proposed mechanism and its effect on the financial transmission right market. Since the proposed mechanism yields higher payments to generators, it may result in insufficient revenue for transmission owners. A plausible solution is to add negative terms (e.g., negative constants) to some generators’ payment functions in (22), and/or to add positive terms (e.g., positive constants) to some consumers’ payment functions in (21). The added term to a generator’s (consumer’s) payment is designed to be independent from its reported supply (demand, respectively) curve, and therefore does not violate the incentive compatibility of the VCG mechanism. In theory, these added terms could violate the individual rationality of market participants. Our numerical results on the IEEE 14-bus and 30-bus test systems show that since the total revenue deficit resulting from the VCG mechanism amounts to only a few percent of consumers’ total payment and is much less than generators’ total profit, it is feasible to allocate the deficit among generators and consumers without violating their individual rationality (see Section VI-B).

### V. DAY-AHEAD MARKETS WITH INTERMITTENT SUPPLY

In this section, we generalize the energy-reserve co-optimized market model formulated in Section IV-A by incorporating intermittent (renewable) suppliers as well as the start-up cost of conventional generators. We use \( \Phi_i \) to denote the realized intermittent generation available at bus \( i \). In the day-ahead market, the renewable generation available at bus \( i \) is represented by a random variable \( \Phi_i \), whose distribution is known to the system operator.\(^4\) We let \( \Phi = (\Phi_1, \ldots, \Phi_n) \).

Collecting the supply curves reported by conventional generators \( f \), the system operator solves the following two-stage stochastic program to minimize the expected cost:

\[
\begin{align*}
\min_{\mathbf{z}} & \quad \sum_{i=1}^{n} S_i z_i + E_{\Phi} \{ Q(z, \Phi) \} \\
\text{subject to} & \quad z_i \in [0, 1], \quad i = 1, \ldots, n
\end{align*}
\]

(23)

where \( S_i \geq 0 \) is the start-up cost of (conventional) generator \( i \), and the expectation is over the renewable generation \( \Phi \). In (23), \( Q(z, \Phi) \) is the ex-post cost incurred under the ex-ante unit commitment decision \( z \) and the realized renewable generation \( \Phi \), which is given by

\[
Q(z, \Phi) = \min_{(x, \Phi, r, \omega)} \sum_{i=1}^{n} \int_{0}^{x_i} \int_{0}^{r_i} f_i(x, r)drdx
\]

subject to

\[
\begin{align*}
& x_i + \omega_i - D_i = \sum_{j=1}^{n} y_{ij}(\theta_i - \theta_j), \quad \forall \ i \\
& p_{ij} = y_{ij}(\theta_i - \theta_j) \leq C_{ij}, \quad \forall \ (i,j) \\
& x_i + z_i r_i \leq C_i, \quad \forall \ i \\
& \sum_{i=1}^{n} r_i \geq R, \quad 0 \leq \omega_i \leq \phi_i, \quad \forall \ i
\end{align*}
\]

(24)

\(^4\)We note that many ISOs have centralized wind power forecasting programs that provide renewable generation forecast for the scheduling and commitment of conventional generation [27], [28].
where \( o_i \in [0, \phi_i] \) is the amount of renewable generation injected to bus \( i \).\(^5\) We note that the economic dispatch problem in (24) without renewable generation (\( \phi_i = 0 \) for every \( i \)) is the same as the economic dispatch problem in (18) (over all conventional generators \( i \) with \( z_i = 1 \)).

Given the supply curves reported by all conventional generators \( f \), we use \( J(f) \) to denote the minimum expected cost, i.e., the optimal value of the two-stage stochastic program in (23). We let \( J^C_i(f_{-i}) \) denote the minimum expected cost of a system without conventional generator \( i \) (i.e., \( z_i \) is fixed to be zero), and \( J^R_i(f) \) denotes the minimum expected cost of a system without intermittent generator \( i \) (i.e., \( \phi_i \) is fixed to be constantly zero).

Before formally introducing the VCG payment (made to conventional and renewable generators), we first write down the expected cost incurred to conventional generator \( i \) according to the optimal commitment and dispatch. Given the supply curves \( f \) reported by conventional generators, let \( z_i^*(f) \) denote the optimal commitment of generator \( i \) [i.e., an optimal solution to (23)]. Under the commitment decision \( z \) and realized renewable generation \( \phi \), let \( (x_i^*(f, z, \phi), r_i^*(f, z, \phi)) \) denote the optimal dispatch level of generator \( i \) [i.e., an optimal solution to (24)].

Generator \( i \)'s expected cost is given by 

\[
\tilde{C}_i(f) = S_i z_i^*(f) + \mathbb{E}_{\Phi} \left[ C_i(x_i^*(f, z_i^*(f, \phi), r_i^*(f, z_i^*(f, \phi))) \right].
\]

(25)

The VCG payment made to conventional generator \( i \) is given by 

\[
v_C^i(f) = J^C_i(f_{-i}) - (J(f) - \tilde{C}_i(f)), \quad i = 1, \ldots, n
\]

(26)

which is the difference between the (expected) aggregate cost of the other conventional generators without and with the supply provided by generator \( i \). Analogously, the VCG payment made to renewable generator \( i \) is 

\[
v_R^i(f, \Phi) = J^R_i(f) - J(f), \quad i = 1, \ldots, n.
\]

(27)

Following an approach that is similar to the proof of Theorem 1, it is straightforward to check that the mechanism defined in (26) is incentive compatible, i.e., it is a dominant strategy for each conventional generator to report its true marginal cost. This is because the payment rule (26) endogenizes the expected aggregate cost [the objective of optimization problem (23)] into each generator's expected profit function. We note that each conventional generator does not need the knowledge on the distribution of renewable generation, because reporting its true supply curve maximizes the conventional generator's expected profit, regardless of the distribution of the random variables \( \Phi \).

We now discuss the cost recovery of conventional generators under the proposed mechanism. We first note that the payment define in (26) and (27) must be non-negative. Indeed, at the dominant strategy equilibrium where each conventional generator truthfully reveals its cost, the VCG payment made to a conventional generator is large enough to cover its expected cost, as shown in the following theorem.

Theorem 3 (Individual Rationality): Suppose \( C_i(0, 0) = 0 \) for every conventional generator \( i \). At the dominant strategy equilibrium where each conventional generator truthfully reveals its cost, the VCG payment made to each conventional generator is no less than the generator's expected cost.

**Proof:** We argue that the VCG mechanism yields each conventional generator a non-negative expected profit, that is 

\[
v_C^i(f) - \tilde{C}_i(f) \geq 0, \quad \forall i, \forall f
\]

where \( f \) is the set of true marginal cost functions reported by conventional generators. It follows from (25) and (26) that conventional generator \( i \) obtains an expected profit of:

\[
J^C_i(f_{-i}) - J(f).
\]

The desired result follows from the fact that without conventional generator \( i \), the system cannot incur an expected aggregate cost that is lower than \( J(f) \).

**Remark 3:** We note that given a certain commitment decision, both the LMP and the VCG mechanisms may yield a (conventional) generator a negative payment, if it is dispatched to operate at its minimum output level \( X_i \) (see the discussion at the end of Section III). On the other hand, Theorem 3 shows that in a day-ahead market that incorporates conventional generators’ start-up costs, each conventional generator must receive a non-negative VCG payment that is enough to cover its (ex-ante) expected cost. Interestingly, these results coincide with the observation that under the LMP mechanism, negative prices occur much more frequently in the real-time market than in the day-ahead market [30].

**Remark 4:** We note that under the LMP mechanism, a competitive equilibrium may fail to exist due to the nonconvex structure of supplier cost [9]. On the other hand, as we have demonstrated in this section, the nonconvexity of supplier cost is not an issue for the VCG mechanism. This is because of the fundamental difference between (LMP) nodal pricing and the VCG mechanism, the nodal price (paid to a certain generator) equals the marginal value of its power injection, while the VCG payment made to the generator equals the overall net benefit (saving in total cost) the generator brings to the system.

The focus of this paper is on the design of efficient and incentive compatible mechanisms that incentivize conventional generators to truthfully reveal their supply curves. Our results show that the VCG mechanism guarantees (dominant strategy) incentive compatibility and (ex-ante) individual rationality for conventional generators, even in the presence of intermittent renewable generation. In our model, the curtailment of renewable generation is determined by the system operator, and each renewable generator receives a non-negative day-ahead payment according to (27). The analysis on the behavior of (possibly strategic) renewable generators under different market mechanisms are beyond the scope of this paper.\(^6\)

---

\(^5\)Advanced control technologies have allowed wind power plants to curtail (a fraction of) their generation.

\(^6\)We note that there is a growing literature that studies the optimal trading strategy of a single or multiple renewable generator(s) under different market mechanisms [30], [31].
VI. NUMERICAL RESULTS

In this section, we present several numerical examples to complement our theoretic analysis. In Sections VI-A and VI-B, we compare the economic efficiency and electricity prices resulting from the LMP and the VCG mechanisms in the market model formulated in Section II. In Section VI-C, we consider the day-ahead market model formulated in Section V and compare the VCG payments made to conventional generators with their expected costs.

In Section VI-A, we revisit a simple two-bus network considered in [5], where the LMP mechanism can result in arbitrarily high efficiency loss at a Nash equilibrium. Although Theorem 2 shows that for any given set of reported supply curves, the VCG mechanism results in higher electricity prices than the LMP mechanism, the two-bus example considered in Section VI-A demonstrates that the electricity price resulting from the VCG mechanism can be lower, if compared with LMP nodal prices at a Nash equilibrium.

In Section VI-B, we consider the IEEE 14-bus and 30-bus test systems. In each case, we compare the per-unit electricity prices induced by the LMP and the VCG mechanisms provided that all agents truthfully reveal their marginal costs. For both the IEEE test systems, our numerical results show that the difference between the per-unit prices induced by the two mechanisms is small, which is about 4% of the nodal price induced by LMP.

A. Two-Bus Example

Consider the two-node network as shown in Fig. 1, with four generators. The line capacity is $D$. The cost functions are $C_1(x_1) = x_1$, $C_2(x_4) = 2kx_2$, $C_3(x_2) = kx_3$, and $C_4(x_3) = kx_4$, where $k > 1$ is a scalar subject to change. Suppose that there is a demand of $2D$ at node 1, i.e., $D_1 = 2D$.

We argue that under LMP, there exists a Nash equilibrium at which generators 1 and 2 submit a constant marginal cost of $2k$, generators 3 and 4 submit a constant marginal cost of $k$, and at the second stage. The system operator chooses the following optimal dispatch\(^7\):

\[
x_1 = D, \quad x_2 = 0, \quad x_3 = D, \quad x_4 = 0.
\]

\(^7\)Given the reported supply curves $\hat{f}$, there exist multiple “optimal” dispatches that minimize the aggregate cost given the false reported supply curves. Indeed, any supply vector such that $x_1 + x_3 = D$ and $x_2 + x_4 = D$ constitute an optimal dispatch. We note that the optimal dispatch in (28) yields the minimum aggregate cost among the set of “optimal” dispatches.

We note that no generator could strictly benefit from reporting a different supply curve. For example, it can be shown that given the actions taken by other generators, the marginal cost function $f_1(x) = 2k$ yields generator 1 the highest profit of $(2k - 1)D$, among all possible marginal cost functions. This is because the highest nodal price generator 1 can expect is $2k$, and its supply is at most $D$ as long as its reported marginal cost is higher than $k$.

The aggregate cost incurred at this Nash equilibrium is $(k + 1)D$. Under the VCG mechanism, there exists a dominant strategy equilibrium at which all generators are truth-telling, and generator 1 supplies the total demand $2D$. The dominant strategy equilibrium achieves the minimum total cost $2D$. The total costs resulting from the two mechanisms are presented on the left subplot of Fig. 2. Note that as $k$ grows large, the aggregate cost ratio $(k + 1)/2$ increases to infinity.

We now turn our attention to electricity prices induced by the two mechanisms. At the Nash equilibrium induced by the LMP mechanism, the per-unit electricity price at node 1 is $2k$. Suppose now that all agents are truth-telling. The LMP leads to a nodal price of 1 at bus 1, and under the VCG mechanism, the average price is given by

\[
\frac{H_1(f_{-1}, 0) - H_1(f_{-1}, 2D)}{2D} = \frac{3kD - 0}{2D} = \frac{3k}{2}
\]

where the functional $H_i(f_{-1}, x_i)$ is defined in (8). The prices resulting from the two mechanisms are compared on the right subplot of Fig. 2.

As shown in Theorem 2, under a given set of reported supply curves, the price resulting from the VCG mechanism is no less than that induced by the LMP mechanism. We note, however, the average price under the VCG mechanism is lower than the per-unit price induced by the LMP mechanism, because the generators’ strategic behavior significantly raises the electricity price at a Nash equilibrium.

B. IEEE Test Systems

In this section, we abstract away from agents’ strategic behavior and consider a setting where all agents truthfully reveal their marginal costs. As a result, the energy output level would be socially optimal under both the LMP and the VCG mechanism. We are interested in comparing the electricity prices resulting from the two mechanisms.

We first consider the IEEE 14-bus test system [25], which has five generators at buses 1–3, 6, and 8. We numerically
compare the per-unit electricity prices induced by the VCG and the LMP mechanisms in Table I.\textsuperscript{8} We observe that VCG mechanism results in slightly higher average prices than the LMP mechanism. Under the VCG mechanism, the average electricity price in the system (the ratio of the total payment made to generators to the total generation) is given by

\[
\frac{40.7096 \times 220.97 + 39.9097 \times 38.03}{220.97 + 38.03} = 40.59 \text{ $/MWh}
\]

which is about 4\% higher than the nodal price induced by the LMP mechanism. It is worth noting that the prices presented in Table I are calculated according to generators’ true supply curves; under the LMP mechanism, generators have incentives to report false marginal cost functions, and the equilibrium nodal price can be significantly higher than the prices resulting from the VCG mechanism (see the two-node example in Section VI-A).

To explore the revenue adequacy issue of the VCG mechanism, we compute the revenue deficit and compare it with generators’ total profit. There are 11 demand buses in the IEEE 14-bus test system. We calculate the payment made by a single consumer at each of these 11 buses according to the VCG payment function (21). The total payment made by all the 11 consumers is $9620. It is straightforward to check if there are more than one consumers at each bus, then according to (21) the total payment made by consumers will increase, for example, if there were four equal-sized consumers at each bus, total consumer payment would be $9983.9.

We note from Table I that under the VCG mechanism, the total payment made to generators is $10 513.4 (generator 1: $8995.5 and generator 2: $1517.9). As a result, there is $893.4 ($529.5) revenue deficit resulting from the standard VCG mechanism. Since the total generator profit under the standard VCG mechanism is $2870.8 (generator 1: $2475.2 and generator 2: $395.6), revenue adequacy could be achieved by imposing negative constants on generators’ payment functions, without violating the individual rationality of generators (i.e., each generator still earns a non-negative profit). Alternatively, (a fraction of) the total revenue deficit could be allocated to consumers by adding positive constants to their payment functions; we note that the total revenue deficit amounts to about 5.3\% of the original consumer bill (of $9983.9), when there are four equal-sized consumers at each bus.

We now consider the IEEE 30-bus test system [32], where generator locations, costs, limits, and bus areas are taken from [33]. It has six generators at buses 1, 2, 22, 27, 23, and 13. The per-unit prices paid to generators are presented in Table II. For this 30-bus system, the average electricity price resulting from the VCG mechanism is $3.94/MWh, which is again approximately 4\% higher than the nodal price induced by the LMP mechanism, provided that all generators are truth-telling.

Before ending this section, we discuss the revenue adequacy of the VCG mechanism for the IEEE 30-bus test system. There are 20 demand buses in the IEEE 30-bus test system. We calculate the payment made by a single consumer at each of these 20 buses according to the VCG payment function (21). The total payment made by the 20 consumers is $707.8. We note from Table II that under the VCG mechanism, the total payment made to generators is $745.5. As a result, there is $37.7 revenue deficit resulting from the standard VCG mechanism. Since the total generator profit under the standard VCG mechanism is $180.3, revenue adequacy could be achieved by imposing negative constants on generators’ payment functions, without violating the individual rationality of generators. Alternatively, (a fraction of) the $37.7 revenue deficit could be resolved by adding positive constants to consumers’ payment functions, as the total deficit amounts to only 5.33\% of the original consumer bill (of $707.8).

\begin{table}
\centering
\caption{Per-Unit Electricity Prices Resulting From the Two Mechanisms (IEEE 14-Bus Test System)}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Generator & 1 & 2 & 3 & 6 & 8 \\
\hline
Socially optimal energy output (MWh) & 220.97 & 38.03 & 0 & 0 & 0 \\
Per-unit price under the VCG mechanism ($/MWh) & 40.7096 & 39.9097 & 39.016 & 39.016 & 39.016 \\
\hline
\end{tabular}
\end{table}

\begin{table}
\centering
\caption{Per-Unit Electricity Prices Resulting From the Two Mechanisms (IEEE 30-Bus Test System)}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Generator & 1 & 2 & 22 & 27 & 23 & 13 \\
\hline
Socially optimal energy output (MWh) & 44.73 & 58.26 & 22.51 & 32.33 & 15.78 & 15.78 \\
Per-unit price under the VCG mechanism ($/MWh) & 3.953 & 4.01 & 3.862 & 3.948 & 3.845 & 3.845 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{8}For generator $i$, the per-unit electricity price induced by the VCG mechanism is calculated by $v_i(f) / x_i^*(f)$, where $v_i(f)$ is given in (6).

\subsection{Day-Ahead Markets}

In this section, we numerically compute the VCG payments made to conventional generators in the day-ahead market model formulated in Section V. As in Section VI-B, we consider the IEEE 14-bus test system [25], where each (conventional) generator has a quadratic generation cost plus a fixed start-up cost.

There is a renewable generator at bus 3. Empirical evidences indicate that Weibull distribution is a reasonable
approximation for the distribution of random wind power generation [34], [35]. The cumulative distribution function for the Weibull distribution is

\[ F(\phi_3) = 1 - e^{-\left(\frac{\phi_3}{\lambda}\right)^k}, \quad \phi_3 \geq 0 \]

where \( k \) is the shape parameter and \( \lambda \) is the scale parameter. The expected value of a Weibull random variable is given by

\[ \lambda \Gamma(1+1/k), \]

where \( \Gamma() \) is the Gamma function. In this section, we let \( k = 1.8 \), which lies in the range of shape parameters observed in empirical studies [36], [37]. We vary the scale parameter \( \lambda \) to explore the influence of renewable generation level on VCG payments and generation costs.

The expected cost of (conventional) generators 1 and 2, and the VCG payments made to these two generators are depicted in Fig. 3. We note that the VCG mechanism yields both generators a positive expected profit, which is in accordance with Theorem 3. We observe from Fig. 3 that both the payment and expected cost of conventional generators decrease with the level of renewable generation. Interestingly, it can be seen from Fig. 3 that the level of renewable generation has little influence on the expected profit of both (conventional) generators.

VII. CONCLUSION

Under the LMP mechanism, the strategic behavior of profit-maximizing generators may lead to nonexistence of a Nash equilibrium, and may result in high efficiency loss even if an equilibrium exists. We apply the VCG mechanism to wholesale electricity markets. We show that the VCG mechanism induces truth-telling behavior at a dominant strategy equilibrium. In other words, it is profit-maximizing for every generator to reveal its true marginal cost, regardless of the actions taken by other generators.

We show that the VCG mechanism results in higher electricity prices than the LMP mechanism, under any given set of reported marginal cost functions. For the IEEE 14-bus and 30-bus test systems, if all generators truthfully reveal their marginal costs, the VCG mechanism raises the per-unit price by only 4%, compared with the LMP mechanism. It is worth noting that since the LMP mechanism is not incentive compatible, the LMP nodal prices at an equilibrium can be significantly higher than the per-unit prices resulting from the VCG mechanism (see the numerical example in Section VI-A); as a result, the LMP mechanism may result in arbitrary high efficiency loss, compared to the minimum aggregate cost achieved at the truth-telling dominant strategy equilibrium (that is induced by the VCG mechanism).

We construct a framework that applies the VCG mechanism in day-ahead electricity markets with intermittent renewable generation. Before the realization of random renewable generation, the system operator solves a two-stage stochastic program to minimize the expected total cost. We show that the VCG mechanism guarantees (dominant strategy) incentive compatibility and (ex-ante) individual rationality for conventional generators in day-ahead markets.

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