That May Be

I can explain the structure of this paper by comparing it to a Soviet-era joke. Worker to party official: Tell me, what is the difference between capitalism and communism? Official: Good question, comrade. It's this. Under capitalism, man dominates man. Under communism, it's completely the other way around.

The joke operates on several levels. First and most obviously, "the other way around" presumably means that under communism, man dominates man; and that sounds exactly the same as what was said about capitalism. Second, though, it's not as though we can't hear a difference between the first "man dominates man" and the second; one has the sense it's different men in the two cases. Third, though, that difference overlays a deeper similarity; one group dominating another makes for a lot of the same unpleasant dynamics as the other group dominating the first.

Now let's look at the title sentence: That may be. On the one reading, it indicates that a certain state of affairs, let's call it, is not ruled out. On the other reading, it also indicates that is not ruled out. This is a distinction? Just as in the domination joke, it's hard to see what difference there could be between 's not being ruled out and, well, 's not being ruled out?

But, and this is the second point of analogy, it's not as though we can't hear a difference between two readings of "Such and such is not ruled out." Compare "Sabotage is not ruled out," said by an FAA investigator after a crash, and "Sabotage is not ruled out" said by a rebel leader before the crash. The investigator is saying that sabotage is not ruled out descriptively, as it would be if she'd asserted "There was no sabotage." The rebel leader is saying that it is not ruled out prescriptively, as it would be if she'd commanded her underlings not to engage in sabotage.
But that having been said (here comes the third point of analogy), just as the things it can mean for man to dominate man have at a deeper level a lot in common, the things "That may be" can mean have at a deeper level a lot in common. The descriptive reading, "That may be so," raises similar interpretive problems to the deontic reading, "That may be done," and the problems have similar solutions. That anyway is what I'll be urging in this paper.

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What does it matter if the one "may" has properties in common with the other? It matters because descriptive (sometimes called epistemic) "may" is extremely confusing; and the questions we are driven to as we attempt to understand it are questions that, as it happens, have been much discussed in connection with deontic "may."

The usual, traditional, semantics for "ϕ may be so" (that's the descriptive reading) has it expressing something in the vicinity of the speaker's failing to know that ~ϕ. There is undoubtedly something right about this. But there are things wrong with it too. Let me run quickly through five prima facie problems for the ignorance theory.

One problem is that it gets the subject matter wrong. When I say, "Bob might be in his office," I am talking about Bob and his office, not the extent of my information.

Two, the traditional account is too weak; my own personal ignorance is not enough to make it true in my mouth that "It might be that ϕ." The mere fact that I don't myself know whether Mt St Helens is about to blow doesn't let me say that Mt St Helens might be about to blow. A geologist overhearing me will tell me I'm wrong about that, and it would be silly for me to disagree by insisting that my information really is as limited as I have suggested; I really can't rule out that Mt St Helens is about to blow. (People have attempted to fix this by saying "it might be that ϕ" is true in my mouth if information "in my epistemic reach" is
consistent with \( \varphi \). But it turns out to be extremely difficult to explain epistemic reach so as to make this plausible. Eavesdropper cases.)

Three, the ignorance reading is too strong. It can be correct to say "It might be that \( \varphi \)" even when we are not unaware of the fact that \( \sim \varphi \), either because we are aware or because there is nothing to be unaware of. **Aware**: I have a thing about the sanctity of the ballot box, imagine, so if you ask me whether I am going to vote Green, I will say, "I might or I might not," despite knowing perfectly well what I've decided, and not trying to hide the fact that I know. **Or suppose that I run into my creditor who demands that I put a check in the mail by the end of the day. I know perfectly well I'm going to do this – I have the check in my pocket – but still I say, "I might or I might not; it might have to wait until tomorrow." This is not because I don't know I'll send the check today but because I reserve the right not to; it's not a limit in my information I'm indicating, but rather a limit in what I'm prepared to commit to. **Nothing to be unaware of**: I'm pitching a story line to a Hollywood mogul:

- "Raskolnikov brutally murders a pawnbroker"
- "Not if we want PG-13"
- "OK, so he might just rough her up a bit."

The "might" here is not to indicate the limits of my knowledge – there isn't anything to know in this case – it's to indicate the limits of my proposal. Or suppose I am explaining the concept of "lawlike" to a student.

- "\( L \) is a law iff it's lawlike and true"
- "So if \( L \) is lawlike, it's true?"
- "It might be true and it might not"

Again there's no fact of arbitrary \( L \)'s being true or not true; so whatever the "might" is doing in that last sentence, it isn't there to indicate that I am unaware that \( L \) is true and unaware that it is not true.
A fourth problem is more logical in nature. Suppose $\varphi$ is consistent with the available information. Then so is everything $\varphi$ entails, whence it might be that $\varphi$" ought to entail "it might be that $\psi$" whenever $\varphi$ entails $\psi$. If that is right, then, "Bob might be in Paris" should entail "Bob might be in Paris or Nairobi." And it seems if anything to be the other way around; "Bob might be in Paris or Nairobi " seems o make a stronger claim than "Bob might be in Paris," roughly to the effect that Bob might be in the one place and he might be in the other. (There is a similar puzzle about permission. How is that "you can go or stay," far from being entailed by "you can go," entails it, while also entailing that you can stay.)

A fifth problem I learned from MIT grad student Seth Yalcin. An advantage sometimes claimed for the standard theory is that it explains the paradoxicality of "$\varphi$ and it might be that $\sim \varphi$." To say that is to say in effect that $\varphi$ but I don't know it. And that's just Moore's paradox. The usual line on Moore's paradox is that here's a sentence that's unassertable even though there's no reason it can't be true. The proof it's a problem of unassertability rather than truth is that there's nothing to stop me from supposing, say in the antecedent of a conditional, that $\varphi$ and I don't know it; if I've lost my wallet and don't know it, then I'm in trouble. One would expect, then, that "$\varphi \&$ it might be that $\sim \varphi$" would be supposable too. And it isn't. "If I've lost my wallet but might not have lost it, ..." makes no sense. The fifth problem is that "$\varphi \&$ it might be that $\sim \varphi$" is not coherently supposable, and the traditional semantics offers no explanation of this.

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So the traditional semantics for epistemic modals has its problems. They has led some to propose a dynamic semantics, in which the meaning of "might $\varphi$" is given not by its truth-conditions but its effect on context or shared information. The most popular version of this is Frank Veltman's default semantics. Veltman says that "might $\varphi$" uttered in information state S returns that same information state S if S is consistent with $\varphi$, and returns the null information state if S is not consistent with $\varphi$. Both parts of this seem hard to square with the way "might" is used.
Consider first the idea that "might \( \varphi \)" returns the null state when \( S \) is inconsistent with \( \varphi \). Suppose it's agreed all around that John, Paul, George, and Ringo will be at the party. Then someone runs in with the news that Ringo might not be able to make it. Ringo's not making it is inconsistent with John, Paul, George, and Ringo being there. All our information is demolished, then, including the information that John, Paul, and George will be at the party!

The idea that "might \( \varphi \)" returns \( S \) if \( \varphi \) is consistent with \( S \) seems wrong too. It's consistent with John, Paul, George, and Ringo being at the party that Yoko's enemies might stay away; for Yoko's enemies might be Paula Abdul and Olivia Newton John. Nevertheless, if someone runs in with the news that Yoko's enemies might be going to a different party, we don't say: you haven't told us a damned thing. Rather our shared information is weakened to: those of John, Paul, George, and Ringo who aren't Yoko's enemies will be at the party.

It seems from these examples that "might \( \varphi \)" uttered in information state \( S \) has or can have the effect of cutting \( S \) down to a weaker information state \( S' \); and it can have this effect both when \( \varphi \) is consistent with \( S \) and when \( \varphi \) is inconsistent with \( S \). If we model information states with sets of worlds, then the effect of "might \( \varphi \)" is to add on additional worlds. The question of course is which additional worlds. Our reason for looking at deontic modals is that this same question (which additional worlds?) was raised in a deontic context many years ago by David Lewis, in a paper called "A Problem about Permission."

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Lewis starts by describing a simple language game. The players are Master, Slave, and Kibitzer, though we'll ignore Kibitzer (he's used to it). Master issues commands and permissions to Slave, thereby shrinking and expanding what Lewis calls the sphere of permissibility, the set of worlds where Slave behaves as he's supposed to. Behaving as he's supposed to
is Slave's only purpose in this game, and given how we defined the sphere of permissibility, that comes to behaving so that the actual world lies within that sphere. Slave can't do that though unless he knows where the sphere is. Let's try to help him with this: how does the sphere evolve?

When the game begins, let's assume, all worlds are in the sphere of permissibility. Now Master begins issuing commands and permissions. Our job is to figure out the function that takes a given sequence of commands (written !_ϕ_) and permissions (written i_ϕ_) to set of worlds permissible after all those commands and permissions have been given. That fortunately boils down to two simpler-seeming sub-tasks: first, figure out the effect of a command on the sphere of permissibility, second, figure out the effect of a permission on the sphere of permissibility.

You might think the second sub-task would be the easier one: after all, a sphere of permissibility would seem to be more directly responsive to permissions than commands. But it's actually the first sub-task that's easier. Suppose the going sphere of permissibility is S and Master says "Mop that floor!." Then clearly the new sphere S' is the old one S, restricted to worlds where the floor gets mopped. The rule stated generally is

!_ϕ_: \( S \rightarrow S \cap \mid_ϕ \mid, \)

or to state it as an identity:

\[(C) \quad !_ϕ(S) = S \cap \mid_ϕ \mid. \]

The left-to-right inclusion here (\( !_ϕ(S) \subseteq S \cap \mid_ϕ \mid \)) follows from two extremely plausible assumptions:

- (c1) commands shrink (ie. don't expand) the sphere, and
- (c2) commands to \( ϕ \) make all \( \sim_ϕ \)-worlds impermissible
The right-to-left inclusion ($!_\varphi(S) \supseteq S \cap !_\varphi(\varphi)$) follows from (c1) and another plausible assumption

(c3) commands to $\varphi$ make only $\sim\varphi$-worlds impermissible.

All this is treated by Lewis as relatively undebatable, and nothing will be said against it here; it serves as the background to the problem to come.

That problem concerns permission. If commands go with intersection, the obvious first thought about permissions is that they would go with unions:

$$j_\varphi: S \rightarrow S \cup !_\varphi,$$

or, the corresponding identity

$$(P) \quad j_\varphi(S) = S \cup !_\varphi.$$

The left to right inclusion ($j_\varphi(S) \subseteq S \cup !_\varphi$) is hard to argue with; it follows from

(p1) permissions expand (i.e., do not shrink) the sphere, and
(p2) permission to $\varphi$ renders only $\varphi$-worlds permissible.

But the right to left direction requires along with (p1) the principle

(???) permission to $\varphi$ renders all $\varphi$-worlds permissible.

And while it is hard to argue with
(p3) permission to \(q\) renders some \(q\)-worlds permissible,

(??) seems clearly wrong. Lewis explains why:

Suppose the Slave had been commanded to carry rocks every day of the week, but on Thursday the Master relents and says to the Slave, '¡The Slave does no work tomorrow'....He has thereby permitted a holiday, but not just any possible sort of holiday...[not] a holiday that starts on Friday and goes on through Saturday, or a holiday spent guzzling in his wine cellar...(27).

So (??) allows in too much. (p3) on the other hand, although correct, can't be the whole story. Not any old expanded sphere which contains \(q\)-worlds will do, for the one whose sole \(q\)-world has Slave staying on holiday through Saturday won't do. So the situation is this:

Some worlds where the Slave does not work on Friday have been brought into permissibility, but not all of them. The Master has not said which ones. He did not need to; somehow, that is understood (27).

If it's understood, there must be a way we understand it: there must be a rule or principle of sphere-evolution that captures our shared implicit understanding of how permissions work.

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Now we reach the problem of Lewis's paper. What is that rule? Or to put negatively, what exactly is wrong with a rule \(R\) that tells us that having been permitted to take Friday off, Slave can take that and other days off. Lewis looks at five answers.

(1) \(R\) lets in more worlds than necessary
Putting in a Saturday-off world enlarges the sphere more than necessary to allow Friday-off worlds. It's a "gratuitous enlargement" in the sense of adding more worlds than necessary.

Lewis replies that any reasonable enlargement will be gratuitous in that sense, since the only non-gratuitous enlargement adds in just a single world. Fair enough – but if that were the "real" problem, then limiting ourselves to non-gratuitous (single-world) enlargements would address it, and it doesn't. After all we could pick as our single world a world where Slave takes Saturday off too.

(2) R lets in worlds more remote than necessary

Putting the Saturday-off world in is a gratuitous enlargement in a qualitative sense. We should allow in only the closest worlds where the permitted action is done.

But this, Lewis says, is too restrictive. Suppose Slave has previously been ordered to carry rocks around. Then he is forced to spend his vacation lifting weights! For weight-lifting worlds are closer to rock-carrying worlds than lying around-at-the-beach worlds are to rock-carrying worlds.

One can put the problem like this. A permission should cleanly cancel relevant earlier commands; but on the present approach supposedly cancelled commands continue, from beyond the grave as it were, to exert an effect. This kind of problem will come up again; let's refer to it as the clean cancellation problem.

(3) R lets in worlds more impermissible than necessary

Putting the Saturday-off world in is a gratuitous enlargement not in a qualitative but a prescriptive sense. We should put in the least impermissible worlds where the permitted action is done. Taking Friday and Saturday off was more impermissible than taking Friday off so the two-day-off worlds aren't added in.

The objection Lewis makes is that this "solution" just restates, indeed aggravates, the problem: figuring out how comparative impermissibility evolves under the impact of commands and permissions is if anything even harder than figuring out how straight permissibility does.
But there's again a prior worry, I think – a version of the clean cancellation problem. Suppose Master first says not to eat any animals, then relents and permits eating lobster. Before the permission it was less impermissible to nibble on lobster than to eat a whole one. So afterward is it only permissible to nibble on lobster?

(4) \( R \) lets in worlds more disagreeable to Master than necessary

Putting the Saturday-off world frustrates Master's known or guessable purposes.

Lewis objects that either Slave knows Master’s purposes or he doesn’t. If he does, there’s no need for commands; he can work unsupervised. If he doesn’t, then the principle cannot be what’s guiding him.

That is fair enough but once again there’s a prior worry. Let’s say Master ordered Slave to carry rocks up the hill. Presumably she did this because she wants the rocks up the hill. But the Friday-off worlds that best serve the purpose of getting the rocks up the hill are ones where Slave invites his friends to play a game where two teams compete to see who can carry more rocks up the hill. This is again a version of the clean cancellation problem.

(5) \( R \) lets in worlds violating more commands than necessary

This takes a bit more explanation. It's a given that Master doesn't issue commands and permissions unless she needs to; she doesn't issue the command to \( \varphi \) if it is already impermissible for Slave not to \( \varphi \); and she doesn't issue permission to \( \varphi \) if it is already permissible for Slave to \( \varphi \). In particular then Master would not have permitted Slave to take Friday off unless taking Friday off would otherwise have been an act of disobedience, an act in violation of some explicit or understood command. So, proposal: the effect of permitting \( \varphi \) should be to invalidate any commands that forbid \( \varphi \)’ing – that are inconsistent with \( \varphi \) – while leaving other commands in place. The problem with an update rule that lets Saturday-off worlds into the sphere is that it invalidates more commands than necessary. To make Slave's taking Friday off permissible, it's
enough to invalidate the work-Friday command; the work-Saturday command doesn't care if Slave takes Friday off so it should be left in place.

Call this update rule the **remainder** rule because it defines $S^-$ as the set of worlds satisfying the commands that remain when the $\varphi$-inconsistent commands are knocked out. Lewis doesn't like this rule either; here's why. Clearly, to apply the remainder rule, we need there to be a list of commands $\psi_1, \ldots, \psi_k$ such that a world is permissible iff it complies with all of them, that is,

$$S = h\psi_1 \cap h\psi_2 \cap \ldots \cap h\psi_k.$$

For the way the rule works is we delete from this list all the $\psi_i$s inconsistent with $\varphi$, and let the commands that remain define $S^+$. So if the $\varphi$-incompatible commands are $\psi_{j+1}, \psi_{j+2}, \ldots, \psi_k$; then the new sphere would be

$$S^+ = h\psi_1 \cap h\psi_2 \cap \ldots \cap h\psi_j.$$

Where is the initial set of commands supposed to come from, though, the one we thin out to arrive at the reduced command-set that defines the $S^+$-worlds? It would be one thing if "$i\varphi" were the first permission uttered; for then Master's earlier utterances were all commands and we can let the $\psi_i$s be those commands. Ordinarily though "$i\varphi" is preceded by commands and other permissions. One could try considering just the commands that have already been given, ignoring the permissions, but these will not define the current sphere of permissibility, because the update effects of the permissions will have been ignored.

It seems then that we are driven to contriving, reverse-engineering if you like, a package of commands that define the current sphere, a set of $\psi_i$s that together define $S$. Unfortunately the relation between $S$ and commands defining $S$ is one-many; lots of packages will issue in the same sphere of permissibility. How does Slave know which package to use? It makes a difference, because the effect on $S$ of permitting $\varphi$ varies enormously with our choice of implicit commands $\psi_i$. 
So, for instance, suppose that the current sphere $S = \text{the worlds where Slave works all day every day};$ but we arrived at that sphere by a complicated series of enlargements and contraction that offers no clues to what the right $\psi_s,$ the right implicit commands, are. Slave might think that initially, before he is given Friday off, the commands in effect are

\[\psi_1: \text{Slave carries rocks on Monday.}\]
\[\psi_2: \text{Slave carries rocks on Tuesday.}\]
\[\psi_3: \text{Slave carries rocks on Wednesday.}\]
\[\psi_4: \text{Slave carries rocks on Thursday.}\]
\[\psi_5: \text{Slave carries rocks on Friday,}\]
\[\psi_6: \text{Slave carries rocks on Saturday}\]
\[\psi_7: \text{Slave carries rocks on Sunday.}\]

The one command here inconsistent with "Slave takes Friday off" is "Slave carries rocks on Friday." Suspending that one command leaves the commands to work other days still in place. Clearly on this way of doing it, Slave has not been permitted to take other days off, which was the desired result. But Slave might also think that the implicit demands are

\[\chi_1: \text{Slave carries rocks on weekdays.}\]
\[\chi_2: \text{Slave carries rocks on the weekend.}\]

Now the $\varphi$-inconsistent rule rule, the one to be cancelled on the present hypothesis, is "Slave carries rocks weekdays." But then the sphere of permissibility expands to include all worlds where Slave works on the weekend. And that seems crazy. Master meant to give Slave Friday off, but not Monday - Thursday as well!

Lewis's objection in a nutshell is that the implicit commands are too unconstrained for the remainder rule to be of any use. I suspect,
however, that there are constraints he is missing, that don't come into view until you raise his sort of problem in the starkest possible. Let’s look then at the most extreme cases of badly chosen implicit commands. At the one extreme we have commands each of which is inconsistent with \( \varphi \); at the other we have ones none of which is inconsistent with \( \varphi \). An example of all-inconsistent is

\[
\theta_1: \text{ Slave carries rocks every morning of the week.} \\
\theta_2: \text{ Slave carries rocks every afternoon of the week.}
\]

Neither of these is compatible with Slave taking Friday off. Cancelling the \( \varphi \)-inconsistent commands, then, is cancelling all commands whatsoever. If all commands are cancelled, then everything is permitted. Master wanted to let Slave take Friday off but winds up giving Slave his freedom.

Now consider commands none of which individually requires Slave to work Friday, but whose joint effect is to require Slave to work every day of the week.

\[
\sigma_1: \text{ Slave carries rocks every morning if any afternoon.} \\
\sigma_2: \text{ Slave carries rocks every afternoon if every morning.} \\
\sigma_3: \text{ Slave carries rocks some afternoon.}
\]

\( \sigma_1 \) allows Slave to take Friday off, provided he never carries rocks in the afternoon. \( \sigma_2 \) allows him to take Friday off, provided he omits to carry rocks some morning. \( \sigma_3 \) allows him to take Friday off, provided he carries rocks some afternoon. Each of the \( \sigma_i \)s is consistent with \( \varphi \) so none of them is cancelled on the present rule. But then the sphere of impermissibility never changes. Master tried to give Slave permission to take Friday off, but it turns out he still has to work on Friday!
What can we conclude from this? The remainder rule – the one that says to cancel all and only preexisting commands that forbid $\varphi$ – can give very silly results. But we can make that work in our favor, by using the silliness of these results as a way of tightening up the rule. Call a command-list admissible if running it through the remainder rule yields an expansion satisfying (p1)-(p3) above.

(p1) permissions (expand) do not shrink the sphere, and  
(p2) permission to $\varphi$ renders only $\varphi$-worlds permissible.  
(p3) permission to $\varphi$ renders some $\varphi$-worlds permissible.

It is not hard to establish the following (proof in appendix)

FACT: If $S$ is defined by an admissible list of commands, then $S = \lnot \varphi \cap \psi$ for some $\psi$ consistent with $\varphi$. Equivalently, any admissible command list is rewritable as one command "You must not $\varphi$" that disallows $\varphi$-ing followed by a second command "You must $\psi$" that allows $\varphi$-ing.

This transforms the problem in a helpful way. Before we had one equation in several unknowns (corresponding to the several choices of implicit commands $\psi_i$). Now we have something very like one equation in one unknown. For we know what $S$ is; that's just the present, pre-permission-to-$\varphi$, sphere of permissibility. And we know what $\lnot \varphi$ is; that's just the worlds where the permitted behavior does not occur. The one unknown here is $\psi$, that is, the new sphere of permissibility $S^+$.  

So, to review what we know. Whenever a permission to $\varphi$ is issued, it's as though the initial command list had consisted of two commands:

first, one saying (precisely) do not $\varphi$.
second, a command $\psi$ that allows $\varphi$-ing,
Our job as sphere-redrawers is to throw out the do not $\psi$ command and form the set of worlds allowed by the command that remains. This is nothing like an algorithm, because there's more than one way of choosing the command $\psi$ that remains. (There are many sets whose intersection with $\lnot \phi$ is $S$.) But let's see what we can do with it.

It helps to conceive the task diagrammatically. We are given the striped region – that's the worlds where Slave works Friday – and its complement the dotted region – that's the worlds where Slave takes Friday off. We are given the yellow striped region – that's the set of initially permissible worlds $S$, the worlds where Slave works all week. Our job is to extrapolate the yellow region beyond the the bounds imposed by the striped region, thus arriving at the set $\psi$ of worlds that are permissible after Master cancels the command to work Friday.

### Diagram

- $\lnot \phi$ (works friday)
- $\phi$ (doesn't work friday)

- $S = \lnot \phi \cap \psi$ (works all week)
- $S^+ = \psi$ (works except friday)

Three observations about this diagram. **First,** the diagram helps us to see what it means to say that $S^+$ "solves the equation" $S = \lnot \phi \cap S^+$. It means that $S$ is the part of $S^+$ where the newly permitted behavior
doesn't occur, or to run it the other way around, \( S^+ \) is the result of extending the \( S \)-region into the region where the newly permitted behavior does occur.

A second thing the diagram helps us to see is that the extrapolation approach is no panacea, for there is not going to be just a single way of extending \( S \) into the \( \varphi \)-region, the region where Slave takes Friday off. One could in principle let \( S^+ \) suddenly triple in width as it enters the \( \varphi \)-region, so that Slave is allowed to take the weekend off too.

A third thing that the diagram helps us to see, however, is that some ways of extrapolating are more natural than others. Extending \( S \) "directly" into the \( \varphi \)-region, ignoring the \( \varphi/\sim\varphi \) boundary, is better than taking notice of that boundary. This is the geometrical version of our earlier worries about clean cancellation. To the extent an extrapolation takes account of the \( \varphi/\sim\varphi \) boundary, the supposedly cancelled prohibition against \( \varphi \)-ing is still exerting some influence from beyond the grave. Another way to put is that we want our remainder command \( \psi \) to be free of any taint of the cancelled command \( \sim\varphi \).

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So our problem boils down to this: what could it mean for one proposition (in this case, \( |\psi| \)) to be "free" of another (\( |\sim\varphi| \)) in the sense of indifferent to whether that other proposition is true or false. There is in fact a bit of a literature on this, The answer I propose:

\[(F) \text{ A is B-free iff} \]

in all (not too remote?) worlds where \( A \) is true, it is true for reasons compatible with \( B \)'s falsity, and in all (not too remote?) worlds where \( A \) is false, it is false for reasons compatible with \( B \)'s truth

This notion of "true for the same reason" will have to be left at an intuitive level for now, but it basically means true with the same truth-
maker – here one wheels in an appropriate theory of truth-makers. The proposed update rule for permissions (it's only a partial rule, but enough for our purposes) is this:

\[(URP)\]

Suppose that \(S\) is the present sphere of permissibility and that \(\varphi\) holds in no \(S\)-worlds. \(S^+ = S + \Box \varphi\) iff four conditions are met:

- **Difference** \(S^+ - S\) is non-empty
- **Equality** \(S = \neg \varphi \cap S^+\)
- **Freedom** \(S^+\) is \(\neg \varphi\)-free
- **Extremal** No competitor is as small/natural/....?

Suppose, for instance, that \(S\), our initial sphere of possibility, is the proposition that Slave works all week, and \(\neg \varphi\) is the proposition that Slave works on Friday. How good a candidate is the proposition that Slave works Monday-Thursday and the weekend for the role of what's permitted after Slave is given Friday off?

Is the difference condition met? Sure – not all worlds where Slave works Monday-Thursday and the weekend are worlds where Slave works Monday thru Sunday.

Is the equality condition met? Yes, for the work-all-week worlds are the intersection of the work-every-day-but-Friday worlds and the work-Friday worlds.

Is the freedom condition met? In worlds where \(S^+\) is true, it is made true by Slave's rock-carrying activities Monday-Thursday and the weekend. Those activities whatever exactly they may be are compatible with Slave taking Friday off. In worlds where \(S^+\) is false, it is made false by Slave's
non-rock-carrying activities on some day or days other than Friday. Those activities whatever they may be are compatible with Slave carrying rocks on Thursday. "Slave works Monday-Thursday and the weekend" is thus free of any taint of "Slave works Friday." That and the equality fact already mentioned are what makes "Slave works Monday-Thursday and the weekend" a good candidate for what is left over when we subtract Slave's working Friday from his working every day of the week.

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Now let's consider some other hypothesis for what is ultimately required of Slave when we first command him to work Monday-Sunday and then relent, letting him take Friday off. Harsh hypothesis: Slave still has to work Monday-Sunday, exactly as before. Lenient hypothesis: Slave only has to work Monday-Thursday; after his Friday holiday he can stay off through the weekend. Intermediate hypothesis, Slave has to work Monday-Thursday, the weekend, and Master's birthday.

The harsh hypothesis – Slave still has to work every day – falls afoul of the Difference condition, which says permission to φ, issued in a context where φ was previously impermissible, must result in the addition of at least one φ-world to the previous sphere. Obviously no Friday-off worlds have been added if S+ is the set of work-all-week worlds.

The lenient hypothesis – Slave only has to work Monday-Thursday – falls afoul of the Equality condition. The work-every-day worlds are not identical to but a proper subset of S+ = the work-Monday-Thursday worlds intersected with I∼φI = the work-Friday worlds.

The intermediate hypothesis – Slave works Monday-Thursday, the weekend, and Master's birthday – falls afoul of the Freedom condition, by which I mean that the third hypothesis is not free of any taint of the cancelled work-Friday command. A is B-free, recall, iff A is true, in an A-worlds w, for reasons compatible with B's falsity, and false, in ~A-worlds w, for reasons compatible with B's truth.
But consider a world $w$ where Master's birthday falls on a Friday, and Slave works Monday-Thursday, the weekend, and Master's birthday, as the intermediate hypothesis requires him to. The hypothesis's truth-maker involves in part that Slave works Friday and Friday is Master's birthday. And this is not compatible with $\neg \varphi$ being false, in other words with Slave's taking Friday off. Alternatively consider worlds where Master's birthday falls on a Friday, and the third hypothesis doesn't hold, because Slave takes Friday off. Is the falsity-maker here compatible with $\neg \varphi$'s truth, that is, with Slave's working Friday? Clearly not. So our update rule predicts that Slave is not required to work Monday-Thursday, the weekend, and Master's birthday, even where that birthday is not on Friday, because in some not-too-distant worlds the command would be obeyed by working on Friday, or disobeyed by taking Friday off; and what Slave does on Friday should not make him either obedient or disobedient, once he has been freed of all Friday-related obligations.

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So there's my first-pass story about how to understand the semantic effect of permissions. There are gaps in it to be sure. One large gap traces back to how Lewis sets the problem up. Lewis assumes that permission to $\varphi$ "makes a difference" if, but only if, the sphere has to be enlarged for there to be permissible worlds in which $\varphi$. If there were any permissible $\varphi$-worlds beforehand, then the permission accomplishes exactly nothing.

This may seem only reasonable. The first day of camp, the counselor says: “It is forbidden to climb trees.” The director then whispers something in his ear, and he adds, "except you may climb trees to rescue a kitten.” Here “you may climb trees to rescue a kitten” deletes something from the list of what was ruled off limits by “It is forbidden to climb that tree.” It might seem at first that all permission statements are like that. What would be the point of allowing something which had never been forbidden in the first place?

And yet we do it all the time. I had the counselor forbidding tree climbing first and then relenting a bit: “It is OK to climb trees to rescue a kitten.” But she might just as well have granted the permission first, before other rules were decided. There would be a clear point to doing this. If the counselor says
nothing, her campers don't know if tree climbing is forbidden; perhaps she hasn't got around to announcing it yet. If she says tree climbing after kittens is OK, that tells them she has no plans to announce it.

There are also intermediate cases where a permission "softens" earlier commands, even though the earlier commands did not strictly rule out behavior of the kind now permitted. On day one counselor says, "You must never climb the tree." On day two she says, "You may do whatever you like on your birthday." Lewis would it seems have to say that the permission on day two leaves the command on day one entirely in place, since there are plenty of worlds where you never climb the tree and do whatever you like, for instance worlds where you have no desire to climb it. Intuitively though the day one command is weakened. More on this in a moment.

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Now I want to explore the epistemic analogue of Lewis's Master-Slave game. Here is how I understand the new game to work.

(1) The players this time are Teacher and Student, and the sphere of permissibility becomes the sphere of believability.

(2) The old game had Slave constantly adjusting his plans to fit with changes in what was permissible; the new one has Student constantly adjusting his theory to fit with changes in what is believable.

(3) It contracted the sphere of permissibility when Master said "Do $\psi$"; the sphere expanded when Master said "You may do $\varphi$." Likewise it contracts the sphere of believability when Teacher says "$\psi$ is so"; the sphere expands when Teacher says "$\varphi$ may be so."

(4) There was no great mystery about the kind of contraction brought on by "Do $\psi$" shrunk the sphere of permissibility; one simply rejected as impermissible worlds where $\psi$ failed. Similarly there is no great mystery about the kind of contraction brought on by "$\psi$ is so"; worlds where $\psi$ fails are rejected as unbelievable.
(5) It was initially mysterious how "You may do \( \varphi \)" enlarged the sphere of permissibility. Similarly it is now mysterious (to begin with) how "\( \varphi \) may be so" enlarges the sphere of believability.

It'll be simplest if we continue the pretense started above that "\( \varphi \) may be so" has no effect on a sphere of believability that contains \( \varphi \)-worlds; it's only when all believable worlds are \( \sim \varphi \) that we get an expansion. The question is, what expansion do we get? I think you will not die of shock if I now propose that that the update rule is pretty much as before.

(URM)

Suppose that \( S \) is the present sphere of permissibility and that \( \varphi \) holds in no \( S \)-worlds. \( S^{+} \) is \( S + \Diamond \varphi \) iff four conditions are met:

- **Difference** \( S^{+} - S \) is non-empty
- **Equality** \( S = |\sim \varphi| \cap S^{+} \)
- **Freedom** \( S^{+} \) is \( |\sim \varphi| \)-free
- **Extremal** No competitor is so small/natural/...?

Imagine for instance that Teacher starts by saying it will rain all week, Monday-Sunday. Student banishes from the sphere of believability all worlds where this doesn't happen, where the rain lets up one or more days. When Teacher learns that her evidence as regards Friday was shaky, she says, "Hold on, it might not rain Friday after all." Which worlds is Student to put back into the sphere of believability? Or to put it in more intuitive terms, what remains of the Teacher's original prediction of rain all week, once she has conceded it might not rain Friday.

One hypothesis, the strong hypothesis call it, has the Teacher still predicting that it rains every day. That is not allowed by our update rule, however for it violates Difference. Difference says that new worlds have to be added to accommodate a previously unbelievable hypothesis. And it is clear from Equality that the added worlds have to be \( \varphi \)-worlds; for if a \( \sim \varphi \)-world \( w \) is
added, then $S$ is a proper subset of $l_{\neg \varphi} \cap S^+$, not equal to it as required by Equality. Teacher's announcement that it might not rain Friday forces the addition of at least one dry-Friday world to the sphere of believability, so the strong hypothesis is mistaken.

A second hypothesis, the weak hypothesis call it, has the Teacher now predicting only that it rains Monday-Thurday. That too is not allowed by our update rule. For suppose $S^-$ is the set of worlds where it rains Monday-Thurday. Then $S^-$ intersected with $l_{\neg \varphi}$ (the set of wet-Friday worlds) contains worlds where Saturday and Sunday are dry. There are no such worlds in $S$, however; before Teacher allows it might be dry Friday, all believable worlds have it raining Monday-Sunday. It follows that $S^-$ intersected with $l_{\neg \varphi}$ contains worlds outside of $S$, which is contrary to Equality. So the weak hypothesis about what remains of the Teacher's original prediction is mistaken too.

A third hypothesis, intermediate between the first two, has the teacher now predicting that it rains Monday-Thurday, the weekend, and on Teacher's birthday. The problem with this is that "It rains on Teacher's birthday" is not free of any taint of the cancelled wet-Friday assertion; for in worlds where Teacher's birthday falls on Friday, either "It rains on Teacher's birthday" is true for reasons incompatible with the falsity of "It rains Friday," or it is false for reasons incompatible with the truth of "It rains Friday." Our update rule objects to "It will rain Monday-Thurday, the weekend, and Teacher's birthday," even if her birthday is not on Friday, because in some worlds it is on Friday. In those worlds the prediction is verified (falsified) by Friday's lack of rain; and Friday's lack of rain ought to be irrelevant given that Teacher freely admits there might be no rain on Friday.

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I want to return now to a limitation of the Lewis game noted earlier. Lewis stipulates that permission to $\varphi$ has no impact if $\varphi$ wasn't antecedently forbidden. That makes nonsense both of out-of-the-blue, discourse-initial, permissions and permissions that weaken earlier commands that didn't strictly forbid the now-permitted behavior. The exact same points apply to our epistemic analogue of the Lewis game. We have been assuming that "$\varphi$ might
be so" has no impact unless $\varphi$ was antecedently denied. But then what is going on in this conversation?

A: Where is Bob?
B: Hmmm, don't know for sure, but he might be in his office.
[A: *I never said he wasn't.]

Or this one?

A: Bob will be at the office tomorrow.
B: Not so fast, he might still have the flu tomorrow.
[A: *That's compatible!]

Interestingly Lewis confronts what might be considered the dual of this problem in "A Problem About Permission." Having laid it down that commands shrink the sphere of permissibility, he says

One sort of commanding may seem to require special treatment: commanding the impermissible. Suppose that $|\varphi|$ contains no worlds that are ...permissible...The Master may nevertheless wish to command ...that $\varphi$...Having commanded at dawn that the Slave devote his energies all day to carrying rocks, the Master may decide at noon that it would be better to have the Slave spend the afternoon on some lighter or more urgent task. If the master simply commands...that $\varphi$, then no world ....remains permissible; the Slave, through no fault of his own, has no way to play his part by trying to see to it that the the world remains permissible...Should we therefore say that in this case the sphere evolves not by intersection but in some more complicated way? I think not....What the Master should have done was first to permit and then to command that $\varphi$. (27).

He notes a possible fix: whenever $\varphi$ is impermissible, "a command that $\varphi$ is deemed to be preceded by a tacit permission that $\varphi$, and the sphere of permissibility evolves accordingly" (27). Our present concern is more serious but it can be put in similar language:

One sort of permitting may seem to require special treatment: permitting the not impermissible. Suppose that the sphere of permissibility contains $\varphi$-worlds...The Master may nevertheless wish to permit...that $\varphi$... Having at dawn permitted the Slave to take the day
off, the Master may decide at noon that the Slave should be permitted to visit his mother this week. If the Master simply permits...the Slave to visit his mother this week, then no additional worlds become permissible; for there are already permissible worlds where the Slave visits his mother, namely worlds where the Slave visits his mother today...Should we therefore say that in this case the sphere evolves not by the remainder rule but in some more complicated way?

I wonder whether we can avoid this by a fix similar to Lewis's: whenever $\varphi$ is already permissible, a permission to $\varphi$ is imagined to be preceded by a tacit command not to $\varphi$, with the sphere of permissibility evolving accordingly. Likewise whenever $\varphi$ is already believable, "it might be that $\varphi$" is imagined to be in response to the unspoken assertion that $\sim \varphi$.

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How much justice does this kind of maneuver do to our feeling of still conveying something when we permit the not previously permissible, or suggest that things might be a way that no one had ever said they weren't?

The first thing to notice is that, just as permitting and then immediately commanding that $\varphi$ can (even by our existing rules) change the sphere of permissibility, forbidding and then permitting $\varphi$ can change the sphere of permissibility too. Mathematically speaking there is no reason whatever to expect that $S^+ - S = \mathcal{I}_\mathcal{F}(\mathcal{I}_\mathcal{F}\mathcal{H}(S))$ will just be $S$ again. Indeed there is reason to expect it often won't. We know by Freedom that

$$S^+$$ is $\mathcal{I}_\mathcal{F}$-free.

It follows that whenever S (which is arbitrary, recall) is not $\mathcal{I}_\mathcal{F}$-free, $S^+$ is not $S$, which is just to say that the operation of forbidding and then permitting $\varphi$ will have non-trivial effects. Example: we saw above that $\mathcal{I}\text{Slave works on Master's birthday}$ is not free of $\mathcal{I}\text{Slave works on Friday}$. So if we let $S$ be the first of these and $\sim \varphi$ be the second (so $\varphi$ says Slave takes Friday off), we should have a case where $S^+ \neq S$. 
Let's try it. Master first says that Slave is to work on her (Master's) birthday. That gives us the desired S. Then Master further commands that Slave is to work on Friday. That gives us $S' = !\neg \varphi(S)$ = the worlds where Slave works Friday and Master's birthday. Then Master permits Slave not to work on Friday. $S^+$ can't be the worlds where Slave works on Master's birthday again because that by hypothesis is tainted with Slave's working on Friday, which he has just been permitted not to do. A better because more Friday-free candidate for $S^+$ is the set of worlds where Slave works on Master's birthday unless Master's birthday falls on a Friday. So permitting Slave to take Friday off in a context where Slave was required only work on Master's birthday has a non-trivial effect on the sphere of permissibility. I take it that an utterance that makes non-trivial changes to what's permissible is fully entitled to its reputation as a communicative and information-giving.

Like remarks apply to asserting that $\varphi$ and then immediately allowing it might be that $\neg \varphi$. Suppose I've asserted that it will rain on my birthday. Allowing it might not rain on Friday has the effect, I'm suggesting, of asserting it will rain on Friday and then taking it back. Once again this does not leave everything as it was. The prediction that remains is that it will rain on my birthday, provided anyway that my birthday doesn't fall on a Friday.

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I have just argued that forbidding and then immediately permitting $\varphi$ can enlarge the sphere of permissibility, and also that asserting $\varphi$ and then immediately allowing that maybe that's wrong can enlarge the sphere of believability. So to the extent changing the relevant sphere suffices for "saying" something, permitting the already permissible (admitting as mabe true what was already maybe true), I have explained how these sorts of permissions and admissions can be informative.

But there are also cases where permitting what I've just forbidden (admitting that a previous assertion might be wrong) leaves the sphere just as it was. An example might be this. Nothing has been said about Bob's location, but I know you want to find him. What is accomplished by saying "He might be in
his office," when no one has suggested otherwise? Likewise what is accomplished by saying climbing after kittens is permitted when tree climbing was not forbidden?

It seems to me these things are not so mysterious, once we distinguish what has been forbidden, in the sense that the command has been given, and what is forbidden, in the sense that it's against the rules but counselor may not have got around to announcing it yet. The campers may know when the first permission is given that nothing has been forbidden, but they have no idea what might or might not be forbidden in the more important sense of being off limits. When they hear tree climbing after kittens is permitted, they learn an upper bound on what is forbidden, namely that it doesn't include tree climbing after kittens. This is not, to my mind, because the counselor has said climbing after kittens is not forbidden; she has the ability to forbid but not, as we're imagining the game, the ability to comment on the extent of the forbidden. What the counselor has done is "shown" climbing after kittens is not forbidden by staging a confrontation with some imagined off-screen forbider, and cancelling what that imagined person has attempted to.

The same is going on, it seems to me, when I say to someone looking for Bob that he might be in his office. The distinction we need this time is between what has been asserted, in the sense that someone has actually said where Bob is, and what is "in evidence," in the sense that it's understood to be so even if no one has got around to announcing it yet. Before I spoke my friend might have been wondering whether an assertion that Bob was not in his office was in the cards. I satisfy his curiosity not by saying that an assertion to that effect is not in the cards; my subject matter is Bob and his office, not assertions about them. I satisfy my friend's curiosity by showing that an assertion to that effect is not in the cards, by staging a confrontation with someone imagined to have made the assertion, and striking his comments from the record.

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The question arises how much help this is with the puzzles noted at the beginning for the personal ignorance and update semantics theories of "might." Let's talk about one of the puzzles for each. The personal ignorance theory has trouble explaining why I can say, "I might vote Libertarian
and I might not," when I know darn well how I am going to vote. The present theory can say that in declaring I might vote Libertarian I am showing my audience by example as it were that no assertion is forthcoming on the topic of how I am going to vote. Update semantics has trouble explaining why allowing Ringo might stay away from the party leaves intact the information that John, Paul, and George will be there. The present theory can say that this is just one more instance of Lewis's problem of proportional cancellation: prior assertions that entail the falsity of what we learned might be true are cancelled, while prior assertions that do not entail the falsity what we learned might be true are left in place.

A word finally about the puzzle of conjunctive disjunctions. Suppose you are hungry and I tell you: You may have a piece of cake or a piece of pie. You reach for the pie and I snatch it away. What gave you the idea that was a permissible disjunct? Permission to have cake or pie doesn't mean permission to have pie, not any more than asserting there's cake or pie is in part asserting there's pie. It's really only a piece of cake you can have, and you have cake OR pie only insofar as that's a consequence of its being OK to have cake. Or suppose you are looking for Bob and I say: he could be in his office or at the beach. When you decide to look for him at the beach I say, what are you doing? That Bob might be at place one or place two hardly entails he might be at place two!

Except of course intuitively these entailments are there. Permission to have cake or pie does seem ordinarily to involve permission to have the one and permission to have the other. That Bob might be here or there does seem ordinarily to mean that he might be here and he might be there.

Does the cancellation theory predict this? I claim it does. Suppose the sphere of permissibility is S, and I give you permission to A ∨ B. Assume that both A and B are forbidden in S, that is, no S-world is an A- or B-world, since if not the theory has us change S to S + !(A ∨ B) before the new permission is processed. The result we want (you are permitted to A and you are permitted to B) now follows in a few steps.

1. S + jϕ is ~ϕ-free (by URP)
2. S + j(AvB) is ~(AvB)-free (1)  -- from here on, R is short for S + j(AvB)
3. Where R is false, that is for reasons compatible with the truth of ~(AvB) (2, F)
4. R is nowhere false because A is true, or because B is true (3)
5. R does not entail ~A, and it does not entail ~B (4)
6. R contains A-worlds and B-worlds. (5)
7. So i(AvB) conveys permission to A and permission to B. (6)

Proof of FACT

(p3) says that the right expansion should bring in at least one ϕ-world. It follows that any package of commands all of whose members are consistent with ϕ is inadmissible. For such a package fails to enlarge the sphere of permissibility, as it has to be enlarged to make room for ϕ-worlds. (p2) says that the right expansion should bring in only ϕ-worlds. It follows that any package of commands none of whose members is consistent with ϕ is inadmissible. For that kind of package expands the sphere of possibility to include every world, including ¬ϕ-worlds.

So, any admissible package of commands <ψi> has members consistent with ϕ and members inconsistent with ϕ. Let's use χ for the conjunction of all ψi's individually inconsistent with ϕ, and ψ for the conjunction all ψi's individually consistent with ϕ. In this notation,

(a) S = the set of (χ ∧ ψ)-worlds
(b) S+ = the set of ψ-worlds
(c) S+ - S = the set of (ψ ∧ ¬χ)-worlds

Our reasoning above has established that <ψi> is admissible only if

∀ all worlds in S+ - S are ϕ-worlds (from (p2))
∃ some worlds in S+ - S are ϕ-worlds (from (p3))

It follows from ∀ that for all S+-worlds w, χ holds in w iff ϕ does not hold in w. Proof: The "only if" direction is trivial since each of χ's conjuncts is by definition inconsistent with ϕ. For the "if" direction, suppose contrapositively that χ does not hold in w. w cannot be an S-world
because S-worlds have to satisfy all the \( \psi \)'s. But then \( w \) is in \( S^+ - S \). And according to \( \forall \), every world in that set satisfies \( \varphi \). If \( \chi \) agrees with \( \neg \varphi \) on \( S^+ \)-worlds, then (since \( |\psi| \) is the set of \( S^+ \)-worlds), we can infer from (a) that

(d) \( S = \) the set of \( (\neg \varphi \land \psi) \)-worlds

It follows from \( \exists \) that \( \psi \) is consistent with \( \varphi \) (we already knew its conjuncts were). Proof: Suppose not. Then \( S^+ = \) the set of \( \psi \)-worlds does not contain any \( \varphi \)-worlds. But \( \exists \) implies that \( S^+ \) does contain \( \varphi \)-worlds, since \( S^+ - S \) contains them. This lets us infer from (d) that

FACT \( S = |\neg \varphi| \cap |\psi| \) for some \( \psi \) consistent with \( \varphi \).

QED