JOINT ANTENNA AND USER SELECTION ALGORITHM FOR UPLINK OF MULTIUSER MIMO SYSTEMS USING SEQUENTIAL MONTE CARLO OPTIMIZATION

Yangyang Zhang†*, Chunlin Ji*‡*, Wasim Q. Malik*†*, Yi Liu*†*, Dominic C. O'Brien*† *and David J. Edwards*†

†Department of Engineering Science, University of Oxford, UK
‡Institute of Statistics and Decision Sciences, Duke University, USA

ABSTRACT

A cross-layer optimization design is developed for the uplink of multiuser multiple-input multiple-output (MIMO) systems, in which the user-based selection scheduling is executed at the medium access control (MAC) layer, while the antenna selection is performed at the physical (PHY) layer. In order to obtain the optimal cross-layer design, a framework based on sequential Monte Carlo (SMC) optimization is presented to jointly consider the user and antenna selection. With the proposed joint user and antenna selection algorithm, the sum capacity of the multiuser MIMO uplink converges to within 99% of that obtained by exhaustive search method, while the complexity is substantial low.

Index Terms— Sequential Monte Carlo (SMC) optimization, multiple-input multiple-output (MIMO), antenna selection, user selection, sum capacity.

1. INTRODUCTION

Future wireless communication systems are expected to provide higher data rates to meet the increasing requirements of a range of services. To this end, a technique known as multiple-input multiple-output (MIMO), has been extensively investigated [1]. MIMO wireless systems have been demonstrated to provide substantially higher link performance than traditional systems without any extra bandwidth by using multiple antenna arrays. In practice, however, the high complexity of MIMO eclipses its contributions for future wireless communication systems. Therefore several promising techniques, such as antenna selection [2] and user selection [3], have been proposed to reduce the hardware complexity while obtaining good system performance.

In this paper, a cross-layer optimization design is proposed based on sequential Monte Carlo (SMC) optimization to jointly consider the user and antenna selection at MAC and PHY layer, respectively, in each time-slot. The proposed algorithm can achieve the near-optimal sum capacity for uplink multiuser MIMO systems with substantially lower complexity.

This work was supported by EPSRC grant GR/T21769/01 and K. C. Wong Scholarship from the University of Oxford.

2. UPLINK MULTIUSER MIMO SYSTEM MODEL

In Fig. 1, we consider the uplink multiuser MIMO system with N_R receive antennas at the base station (BS) and N_T transmit antennas at the k_{th} user. Then the received signal at the base station is represented as [4]

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{v} \tag{1}$$

where $\mathbf{y} \in \mathcal{C}^{N_R \times 1}$ is the received signal vector and $\mathbf{x}_k \in \mathcal{C}^{N_T \times 1}$ is the transmitted signal vector for the k^{th} mobile user with Tr $(\mathbf{Q}_k) \leq \overline{P}_k$ where $\mathbf{Q}_k = \mathbb{E}\{\mathbf{x}_k^{\mathrm{H}} \mathbf{x}_k\}$. Tr $(\cdot)$ stands for the trace operation of its corresponding argument, $\mathbf{Q}_k$ is the signal covariance matrix, $\overline{P}_k$ is the average power constraint for mobile user k, $\mathbb{E}\{\cdot\}$ denotes the statistical expectation and $(\cdot)^{\mathrm{H}}$ represents the Hermitian operation. The vector $\mathbf{v} \in \mathcal{C}^{N_R \times 1}$ is the independently identically distributed (i.i.d.) additive white Gaussian noise vector with distribution $\mathcal{CN}(0, N_0\mathbf{I})$, where N_0 is the average noise power. The channel is described by an $N_R \times N_T$ complex random matrix, denoted by $\mathbf{H}_k$ whose entries, $[\mathbf{H}_k]_{(i,j)}, i = \{1 \dots N_R\}; j = \{1 \dots N_T\}$, represent the channel fading coefficient between the i^{th} receive antenna of BS and the j^{th} transmit antenna of mobile user k. For the uncorrelated channels, the entries of $\mathbf{H}_k$ follow the i.i.d. complex Gaussian distribution $\mathcal{CN}(0, 1)$. Moreover, it is assumed that perfect channel state information (CSI) is available at the receiver only. The optimal sum capacity for the MIMO multiple-access channel is [4]

$$C(\{\mathbf{H}\}_{k=1}^{K}) = \max_{\sum_{k=1}^{K} \mathrm{Tr}(\mathbf{Q}_k) \leq P_s} \log_2 \det(\mathbf{I}_{N_R} + \frac{1}{N_0} \sum_{k=1}^{K} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^{\mathrm{H}}) \tag{2}$$

When the channel state information (CSI) is not available to the transmitter, a reasonable transmission strategy is equal power allocation for each user [4]. Then $\mathbf{Q}_k = \overline{P}_k \mathbf{I}_{N_T} / N_T$ where the uniform power, $\overline{P}_k = P_s / K$, and P_s is the total power.

For the uplink multiuser MIMO systems, a joint antenna and user selection module (JAUSM) is centralized at the base station to generate the selected index to maximize the sum capacity of the system. Moreover, the number of selected mo-

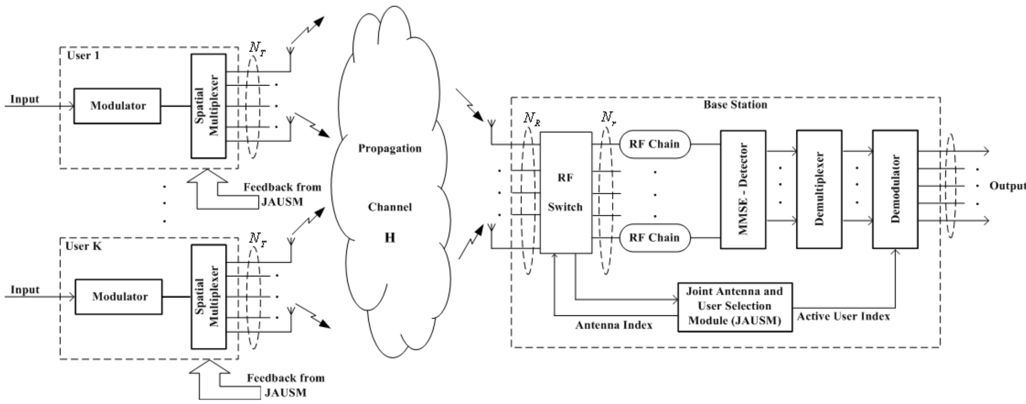

Fig. 1. Block diagram of the uplink multiuser MIMO system.

bile users and the number of selected receive antennas are denoted by K_{sel} and N_r,respectively. We further denote the set of all $\mathcal{Q} = \binom{N_R}{N_r} \times \sum_{j=1}^{K_{sel}} \binom{K}{j}$ subset as $\mathbf{\Omega} = \{\omega_1, \cdots, \omega_{\mathcal{Q}}\}$ and the indicator of the selected subset of antennas and users by a two-element vector index $\boldsymbol{\omega}_k = (\boldsymbol{\omega}_{k,A}; \boldsymbol{\omega}_{k,U})$ where $\boldsymbol{\omega}_k \in \mathbf{\Omega}$. The elements of $\boldsymbol{\omega}_{k,A}$ and $\boldsymbol{\omega}_{k,U}$ indicate whether a certain receive antenna and user is selected or not. According to (2), the sum capacity associated with the selection is described as

$$C(\{\mathbf{H}\}_{\boldsymbol{\omega}_k}) = \max_{\boldsymbol{\omega}_k \in \mathbf{\Omega}} \log_2 \det(\mathbf{I}_{N_R} + \frac{\rho}{N_T} \sum_{k=1}^{K_{sel}} \overline{\mathbf{H}}_k \overline{\mathbf{H}}_k^{\mathrm{H}}) \quad (3)$$

where $\rho = \bar{P}_k / N_0$ is the average signal-to-noise ratio (SNR). The matrix $\overline{\mathbf{H}}_k$ has dimensions $N_r \times N_T$ and its entries define the channel gains from transmit antennas of selected user k to the selected receive antennas.

The simplest approach for solving (3) to obtain the optimal antenna and user subset, $\boldsymbol{\omega}_k^*$, is by an exhaustive search method, namely, finding all possible $\boldsymbol{\omega}_k$ out of $\mathbf{\Omega}$ to obtain the optimal selected user subset $\boldsymbol{\omega}_k^*$ which can yield the maximum $C(\{\mathbf{H}\}_{\boldsymbol{\omega}_k^*})$. However, this method leads to a total of $\binom{N_R}{N_r} \times \sum_{j=1}^{K_{sel}} \binom{K}{j}$ possible combinations and becomes computationally expensive for a multiuser MIMO wireless system with a large number of total users, K and available receive antennas, N_R. In order to reduce the computational cost, we model (3) as the following combinatorial optimization problem:

$$\boldsymbol{\omega}_k^* = \arg \max_{\boldsymbol{\omega}_k \in \mathbf{\Omega}} C(\{\mathbf{H}\}_{\boldsymbol{\omega}_k}) \quad (4)$$

where $\boldsymbol{\omega}_k^*$ denotes the optimal selected antenna and user subset of the objective function, $C(\{\mathbf{H}\}_{\boldsymbol{\omega}_k})$. After transforming (3) into an optimization problem (4), an iterative optimization algorithm can be applied to solve it.

3. SEQUENTIAL MONTE CARLO OPTIMIZATION

We present a novel sequential Monte Carlo (SMC) algorithm for this complicated optimization problem. Sequential Monte Carlo (SMC) is a class of importance sampling and resampling techniques designed to simulate from a sequence of probability distributions, which have become very popular for the last few years to solve sequential Bayesian inference problems in various disciplines [5][7]. It was recently extended to a general framework to deal with static and sequential Bayesian inference, as well as global optimization [6].

To apply SMC for optimization problem, a sequence of artificial intermediate distributions is needed, for example $\pi_n(\cdot) = [\pi(\cdot)]^{\tau_n}$ where $\{\tau_n\}_{n=1}^N$ is such that $0 < \tau_1 < \cdots < \tau_N$ and $1 << \tau_N$ to ensure that $\pi_0(\cdot)$ is easy to sample from and $\pi_N(\cdot)$ is concentrated around the set of global maxima of $\pi(\cdot)$. In this study, the sequence of intermediate distributions can be designed as: $\gamma_n(\boldsymbol{\omega}_k) = \exp\left\{\frac{C(\{\mathbf{H}\}_{\boldsymbol{\omega}_k})}{\tau_n}\right\}$, where $\tau_1 > \cdots > \tau_N > 0$, such that maximizing $C(\{\mathbf{H}\}_{\boldsymbol{\omega}_k})$ is equivalent to maximizing $\pi(\boldsymbol{\omega}_k) = \gamma_n(\boldsymbol{\omega}_k) / \sum_{\boldsymbol{\omega}_k \in \mathbf{\Omega}} \gamma_n(\boldsymbol{\omega}_k)$,

$$\boldsymbol{\omega}^* = \arg \max_{\boldsymbol{\omega}_k \in \mathbf{\Omega}} C(\{\mathbf{H}\}_{\boldsymbol{\omega}_k}) = \arg \max_{\boldsymbol{\omega}_k \in \mathbf{\Omega}} \pi(\boldsymbol{\omega}_k) \quad (5)$$

Given some sequence of distributions, SMC propagates samples forward from one distribution to the next according to a sequence of Markov kernels, K_n, and correcting for the discrepancy between the proposal and the target distribution by importance sampling [6]. Moreover, to ensure that a significant fraction of the particle set have non-negligible weights, the particle representation is resampled using some resampling scheme, whenever the effective sample size (*ESS*) is below a prespecified threshold [7].

The choice of transition kernels is critical in SMC [6]. Here K_n is set as a Markov chain transition kernel in the adaptive Metropolized independence sampler, which has already shown a good perform in User Selection [3]. Unlike the sampling strategy in [3], under the framework of SMC, it is no need to concern about the ergodicity when using adaptive sampling scheme and the convergence of the algorithm is guaranteed under mild conditions [6]. Another virtue is easy to implement SMC in a parallel computing fashion if needed. The adaptive sequential Monte Carlo (SMC) optimization is applied to the joint antenna and user selection as following:

Step 1: At time $n = 1$, sample N particles $[\boldsymbol{\omega}_k]_{n=1}^{(i)} \sim q(\boldsymbol{\omega}; \boldsymbol{p}_0)$ ($i = 1, ..., N$) with an initial value $\boldsymbol{p}_0$, and compute $W_1^{(i)} \propto \frac{\gamma([\boldsymbol{\omega}_k]_{n=1}^{(i)})}{q([\boldsymbol{\omega}_k]_{n=1}^{(i)}; \boldsymbol{p}_0)}$. If $ESS \equiv \left[\sum_{i=1}^{N} (W_n^{(i)})^2\right]^{-1} < N/2$, resample the particle representation $\left\{W_1^{(i)}, [\boldsymbol{\omega}_k]_{n=1}^{(i)}\right\}$

At time $n \geq 2$, iterate steps 2, 3 and 4

Step 2: Sample $[\boldsymbol{\omega}_k]_n^{(i)} \sim K([\boldsymbol{\omega}_k]_{n-1}^{(i)}, \cdot)$: run an adaptive Metro-polized independence sampler using the proposal $q(\boldsymbol{\omega}; \boldsymbol{p}_{n-1})$ [3]; compute the important weights

$$W_n^{(i)} \propto W_{n-1}^{(i)} \frac{\gamma_n\left([\boldsymbol{\omega}_k]_{n-1}^{(i)}\right)}{\gamma_{n-1}\left([\boldsymbol{\omega}_k]_{n-1}^{(i)}\right)} \tag{6}$$

Step 3: Resampling: If $ESS < N/2$, resample the particle representation $\left\{W_n^{(i)}, [\boldsymbol{\omega}_k]_n^{(i)}\right\}$

Step 4: Update the parameter $\boldsymbol{p}_n$ of the adaptive proposal via

$$\boldsymbol{p}_n = \boldsymbol{p}_{n-1} + r_n \left(\frac{\sum_{i=1}^{N} W_n^{(i)}([\boldsymbol{\omega}_k]_n^{(i)} - \boldsymbol{p}_{n-1})}{\sum_{i=1}^{N} W_n^{(i)}}\right) \tag{7}$$

where each p_j in $\boldsymbol{p} = [p_1, \ldots, p_K]^T$, represents the probability of the j^{th} user to be chosen and r_n is a sequence of decreasing step-sizes.

4. SIMULATION RESULTS

Fig. 2 shows the sum capacity averaged over $1,000$ channel realizations versus SNR for $N_T = 2$, $N_R = 8$, $N_r = 4$ and $K_{sel} = 2$. It is shown that the performance obtained by proposed joint antenna and user selection (JAUS) algorithm and the exhaustive search method are nearly same . In fact, the simulation results indicate that the performance difference between these two selection schemes is within 1%. Moreover, in Fig. 3, we find that the sum capacity increases as the number of users (K) and receive antennas (N_R) increase when the number of selected antenna (N_r) and users (K_{sel}) are constant. The reason for this gain is that the systems experience the multiuser diversity provided by the spatially distributed multiuser structure and antenna selection diversity provided by the various receive antennas. Finally, the complexity order comparison between the exhaustive search method and SMC optimization algorithm has been depicted in Fig. 4. It can be seen that the complexity gap is increasing with the increase of K and N_R for a fixed number of K_{sel} and N_r, which indicates the efficiency of our proposed algorithm.

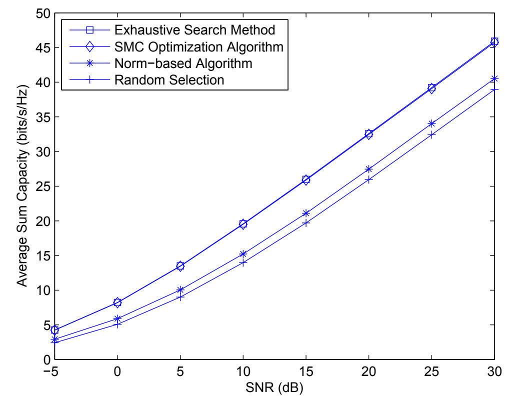

Fig. 2. Average sum capacity versus SNR at $K_{sel} = 2$ and $K = 32$ with $N_r = 4$, $N_R = 8$ and $N_T = 2$.

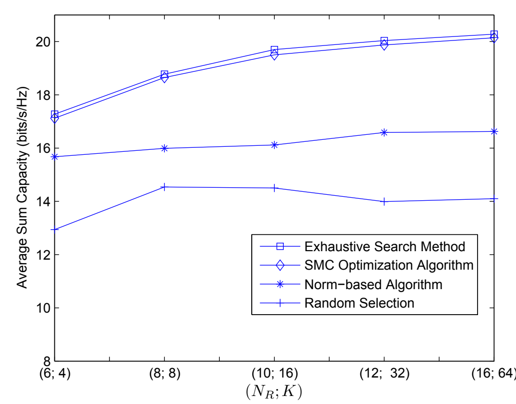

Fig. 3. Average sum capacity versus $(N_R; K)$ at $N_r = 4$ and $K_{sel} = 2$ with $N_T = 2$ and SNR = 10dB.

5. CONCLUSION

This paper presented a cross-layer optimization design for the uplink MIMO systems. In order to obtain the maximal system sum capacity, a joint user and antenna selection algorithm based on sequential Monte Carlo (SMC) optimization is presented. With the proposed algorithm, we can achieve results within 99% of the optimal capacity obtained by exhaustive search method with a substantial low complexity. Moreover, because of the nature parallel mechanism of SMC optimization, it can be easy to implement on the parallel computing facilities to further speed up computation. Considering the low complexity and parallelity, the SMC optimization makes the proposed cross-layer optimization design easier to implement for practical multiuser MIMO systems.

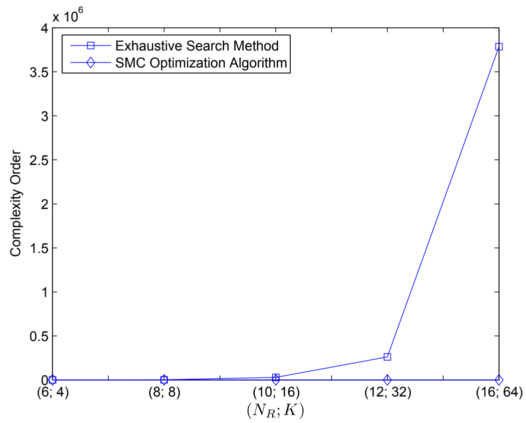

Fig. 4. Complexity order versus $(N_R; K)$ at $N_r = 4$ and $K_{sel} = 2$ with $N_T = 2$ and SNR = 10dB.

6. REFERENCES

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecom.*, vol. 10, pp. 585-595, Nov. 1999.

[2] S. Sanayei and A. Nosratinia, "Antenna selection in MIMO systems," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 68-73, Oct. 2004.

[3] Y. Zhang, C. Ji, Y. Liu, W. Malik, D. O' Brien and D. Edwards, "A Low Complexity User Scheduling Algorithm for Uplink Multiuser MIMO Systems," submitted to *IEEE Trans. on Wireless Communication*.

[4] D. Tse and P. Viswanath, "Fundamentals of Wireless Communication," Cambridge University Press, 2005.

[5] A. Doucet, J. De Freitas and N. Gordon (eds.), "Sequential Monte Carlo Methods in Practice," New York: Springer, 2001.

[6] P. Del Moral, A. Doucet and A. Jasra, "Sequential Monte Carlo samplers," *J. Roy. Statist. Soc. B*, 68, pp. 411-436, 2006.

[7] J. Liu, Monte Carlo Strategies in Scientific Computing," New York: Springer, 2001.