TRANSMIT ANTENNA SUBSET SELECTION FOR MIMO-OFDM WIRELESS COMMUNICATION SYSTEMS

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ABSTRACT

Multiple-input multiple-output (MIMO) wireless communication systems allow for high data rates and improved quality of transmission but at the expense of the hardware complexity due to multiple antenna elements and radio frequency (RF) chains. In recent years, antenna selection technique has been proposed to reduced the hardware complexity. In this paper, we present a low complexity transmit antenna subset selection algorithm based on the cross entropy optimization (CEO) method for antenna selection MIMO orthogonal frequency division multiplexing (MIMO-OFDM) systems. The capacity implemented by our algorithm converges to within 99% optimal capacity obtained by exhaustive search (ES). This capacity convergence is independent of the number of selected transmit antennas. Furthermore, this algorithm requires approximately 50% of the computational complexity of the ES method. Moreover, it also leads to the positive effect for the system bit error rate (BER) performance. Compared with a non-selection system, approximately 5 dB signal-to-noise ratio (SNR) gain at the BER of 1×10^{-2} can be achieved by our algorithm.

Index Terms— Antenna selection, channel capacity, cross entropy optimization, frequency selective, MIMO-OFDM.

1. INTRODUCTION

Due to the continuing demand for high data rates and spectral efficiency, MIMO techniques are considered attractive for future wireless communication systems. The introduction of the OFDM technique is expected to offer improved performance in combating adverse frequency selective fading encountered in wideband MIMO wireless systems [1]. However, a major impediment in MIMO based systems is the cost of hardware, because multiple antenna elements and RF chains are deployed at each terminal. Therefore, a promising technique named antenna selection has been proposed to reduce the hardware complexity (save on RF chains) while retaining many diversity benefits [2]. It employs a number of RF chains, each of which is switched to serve multiple antennas [3].

In recent years, there has been increasing interest in applying antenna subset selection to MIMO systems over frequency selective channels [4], [5]. Several algorithms have been developed for selecting the optimal antenna subset. For example, an exhaustive search (ES) method is exploited to maximize the MIMO-OFDM capacity in [4]. It involves searching all possible antenna sets for all OFDM subcarriers, and results in high computational complexity especially for large array systems. This is not attractive for practical systems, even though this selection algorithm offers optimal capacity performance. In this paper, a low complexity antenna subset selection algorithm based on the cross entropy optimization (CEO) method [6] is proposed to maximize the capacity of the MIMO-OFDM system following either the capacity or the norm criteria [5].

2. SYSTEM MODEL

Fig. 1 shows a MIMO-OFDM system with M_T transmit and M_R receive antennas over an L-tap frequency selective channel. Let $\mathbf{h}_l$ be an $M_R \times M_T$ matrix, which denotes the channel response matrix in time domain of the l-th significant delayed path, for $l = \{0, \ldots, L-1\}$. Assume that $\mathbf{h}_l$ is an uncorrelated channel matrix whose entries $h_l(m_r, m_t)$ follow the independently and identically distributed (i.i.d.) complex Gaussian distribution $\mathcal{CN}(0, 1)$. The channel frequency response matrix of the n-th subcarrier for our N-tone MIMO-OFDM system can be described using another $M_R \times M_T$ matrix $\mathbf{H}_n$:

$$\mathbf{H}_n = \sum_{l=0}^{L-1} \mathbf{h}_l e^{-j2\pi nl/N} \tag{1}$$

Therefore, the received signal for the n-th subcarrier at the receiver is:

$$\mathbf{r}_n = \mathbf{H}_n \mathbf{s}_n + \mathbf{v}_n \tag{2}$$

where $\mathbf{s}_n$ is the transmitted data for the n-th subcarrier, and $\mathbf{v}_n \sim \mathcal{CN}(0, \mathbf{I}_{M_R})$ is additive white Gaussian noise satisfying $\varepsilon\{\mathbf{v}_n \mathbf{v}_{n'}^H\} = \mathbf{I}_{M_R}\delta[n - n']$. Here, $\varepsilon\{\cdot\}$ and $\{\cdot\}^H$ stand for the statistical expectation and the Hermitian operation, respectively. We further assume that perfect channel state information (CSI) is available at the receiver but not at the transmitter. Additionally, the total available power is assumed to be allocated uniformly across all space-frequency subcahnnels [7]. So, the mutual information of the N-tone MIMO-OFDM system is:

$$c = \frac{1}{N}\sum_{n=0}^{N-1}\log[\det(\mathbf{I}_{M_R} + \frac{\rho}{M_T}\mathbf{H}_n\mathbf{H}_n^H)] \tag{3}$$

where N is the total number of OFDM subcarriers, ρ is the SNR per subcarrier. $\det(\cdot)$ and $\mathbf{I}_{M_R}$ denote the determinant operation and the $M_R \times M_R$ identity matrix, respectively. The ergodic capacity of this system is [7]:

$$C = \varepsilon\{c\} \tag{4}$$

In the selection based MIMO-OFDM system, only a subset of transmit antennas M_t ($M_t \leq M_T$) are used at each time slot. We assume that the antenna subset index is sent back to the transmitter from the receiver through an error free and delay free feedback channel. The ergodic capacity associated with antenna selection is modified as:

$$C_{sel}(\boldsymbol{\omega}_q) = \varepsilon\{\frac{1}{N}\sum_{n=0}^{N-1}\log[\det(\mathbf{I}_{M_R} + \frac{\rho}{M_t}[\mathbf{H}_n]_{\boldsymbol{\omega}_q}[\mathbf{H}_n^H]_{\boldsymbol{\omega}_q})]\} \tag{5}$$

where $[\mathbf{H}_n]_{\boldsymbol{\omega}_q} \in \mathcal{C}^{M_R \times M_t}$ denotes the channel frequency response matrix of the n-th subcarrier after selection. Here, $\boldsymbol{\omega}_q$ is the indicator of the selected subset of the transmit antennas and can be defined by

$$\boldsymbol{\omega}_q = \{I_i\}_{i=1}^{M_T}, \quad \{I_i\} \in \{0,1\}; \quad q = 1, 2, \cdots, Q. \tag{6}$$

where i is the index of the columns of $\mathbf{H}_n$ and the indicator function I_i indicates whether the i-th column of $\mathbf{H}_n$ (the i-th transmit antenna) is selected. Q is the number of all possible antenna subsets and can be defined by $Q = \binom{M_T}{M_t}$. Thus (2) can be modified as

$$[\mathbf{r}_n]_{\boldsymbol{\omega}_q} = [\mathbf{H}_n]_{\boldsymbol{\omega}_q}[\mathbf{s}_n]_{\boldsymbol{\omega}_q} + [\mathbf{v}_n]_{\boldsymbol{\omega}_q} \tag{7}$$

where, $[\mathbf{r}_n]_{\boldsymbol{\omega}_q} \in \mathcal{C}^{M_R \times 1}$, $[\mathbf{s}_n]_{\boldsymbol{\omega}_q} \in \mathcal{C}^{M_t \times 1}$ and $[\mathbf{v}_n]_{\boldsymbol{\omega}_q} \in \mathcal{C}^{M_R \times 1}$ denote the received data, transmitted data and the AWGN noise for the n-th subcarrier associated with the selection, respectively.

3. TRANSMIT ANTENNA SUBSET SELECTION ALGORITHM

3.1. Problem Statement

The simplest idea for antenna selection in MIMO-OFDM systems is exhaustive search (ES). It investigates all possible $\binom{M_T}{M_t}$ transmit antenna subsets to estimate the ergodic capacity using (5) so as to obtain the maximum. However, because of singular value decomposition (SVD) computations, this capacity based selection criteria results in high complexity ($\mathcal{O}(NM_R^2M_t)$). Hence, it is usually substituted by a norm-based selection criteria [5]. The norm criteria has low complexity ($\mathcal{O}(NM_RM_t)$). But, even though employing this criteria, the antenna selection with ES algorithm is still not suitable for practical systems due to its high computational complexity. Thus, a low complexity transmit antenna subset selection algorithm is required for a practical implementation.

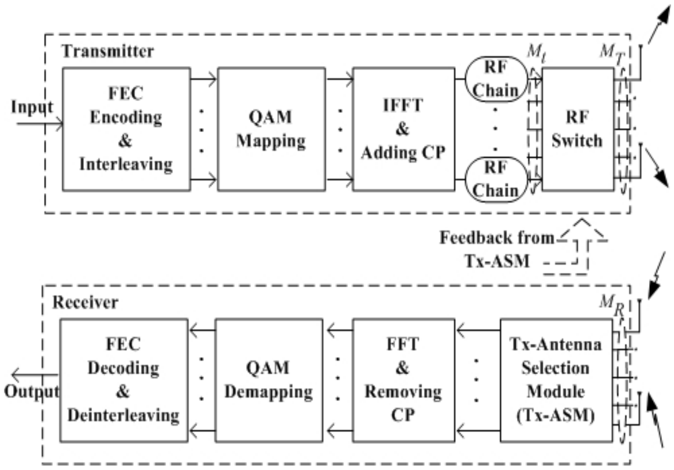

Fig. 1. Block diagram of the transmit antenna selection MIMO-OFDM system.

3.2. Cross Entropy Optimization Method

In this paper, we transform the antenna selection problem into a combinatorial optimization problem. In order to maximize the channel capacity, the cross entropy optimization (CEO) method is presented for an antenna subset selection at the transmitter. The CEO method, a principled adaptive importance sampling, was presented by Rubinstein [8], to estimate the probabilities of rare events in complex stochastic networks. It was extended to solve complicated combinatorial optimization problems, such as the nondeterministic polynomial time (NP) hard problems [9]. The CEO method has been proved to be a global random search procedure in [9] and [10]. In order to employ the CEO method for antenna selection, we will first formulate the antenna selection problem as a combinatorial optimization problem:

$$\boldsymbol{\omega}^* = \arg\max_{\boldsymbol{\omega}_q \in \boldsymbol{\Omega}} C_{sel}(\boldsymbol{\omega}_q) \tag{8}$$

where $\boldsymbol{\omega}^*$ denotes the global optimum of the objective function, $C_{sel}(\boldsymbol{\omega}_q)$, and $\boldsymbol{\Omega}$ is the set of transmit antenna subset selection indicators $\{\boldsymbol{\omega}_1, ..., \boldsymbol{\omega}_Q\}$. The flow of the transmit selection algorithm based on the CEO method is described below:

Transmit antenna subset selection algorithm based on the CEO method [1]

Step 1: Start with an initial value $\mathbf{p}^{(0)} = \{p_i^{(0)}\}_{i=1}^{M_T}$, $\{p_i^{(0)}\} = \frac{1}{2}$. Set the iteration counter $k := 1$;

Step 2: Generate samples $\{\boldsymbol{\omega}_q^{(n)}\}_{n=1}^{N_{CEO}}$ from the density function $f(\cdot, \mathbf{p}^{(k-1)})$ [6], where N_{CEO} is the total number of the samples ;

Step 3: Calculate $\{C_{sel}(\boldsymbol{\omega}_q^{(n,k)})\}_{n=1}^{N_{CEO}}$ and order them from largest to smallest, $C_{sel}^{(1)} \geq \cdots \geq C_{sel}^{(N_{CEO})}$. let $r^{(k)}$ be $(1-\eta)$ sample quantile of the performances: $r^{(k)} = C_{sel}^{(\lceil(1-\eta)N_{CEO}\rceil)}$, where $\lceil\cdot\rceil$ is the ceiling operation and η is the quantile coefficient;

Step 4: Update the parameter $\mathbf{p}^{(k)}$ via

$$p_i^{(k)} = \frac{\sum_{n=1}^{N_{CEO}} I_{\{C_{sel}(\boldsymbol{\omega}_q^{(n,k)}) \geq r^{(k)}\}} I_i(\boldsymbol{\omega}_q^{(n,k)})}{\sum_{n=1}^{N_{CEO}} I_{\{C_{sel}(\boldsymbol{\omega}_q^{(n,k)}) \geq r^{(k)}\}}} \quad (9)$$

Step 5: If stopping criteria is satisfied, then stop; otherwise set $k := k+1$ and go back to step 2. Here, the stopping criteria is the predefined number of iterations.

4. SIMULATION RESULTS

Fig. 2 shows the ergodic capacity of a 64-tone MIMO-OFDM system over 10,000 OFDM symbol periods with M_t ($M_t = 4$) transmit antennas and M_R ($M_R = 4$) receive antennas. Here, the M_t antennas are selected from a M_T ($M_T = 8$) transmit antenna array. This figure indicates that the ergodic capacity obtained by the CEO selection algorithm is nearly the same as the optimal result achieved by exhaustive search (ES) for a wide range of SNR. Moreover, the result obtained by the norm criteria based selection algorithm is near optimal in low SNR region but degrades as the SNR increases. Compared with either the random selection or the non-selection ($M_t = M_T = 4$) ergodic capacity performance, the norm criteria based algorithm still offers a superior performance even though the SNR is high. The cumulative distribution function (CDF) curves of the corresponding capacities are plotted in Fig. 3. It denotes that the ergodic capacity obtained by the CEO algorithm coincides with the ES method for various outage rates. In addition, Fig. 4 shows that the obtained capacity coinciding with the ES result is not be influenced by the number of selected transmit antennas.

Table 1 addresses that the complexity comparisons in terms of the number of function evaluations among the capacity criteria based CEO and ES selection algorithms. In this table, N_{CEO} and k are the required parameters for the CEO algorithm. $\vartheta = C_{CEO}/C_{optimal}$, is the convergence ratio of the ergodic capacity. C_{CEO} and $C_{optimal}$ denote the ergodic capacity obtained by the CEO algorithm and the ES method, respectively. From Table 1, we find that the CEO algorithm requires lower computation compared with the exhaustive search strategy. It requires approximately 50% of the computational complexity of the ES method. Besides, the convergence ratio, ϑ, indicates that the capacity obtained by the CEO algorithm converges to within 99% of the optimal result implemented by the ES algorithm. In order to validate the system BER performance with our proposed antenna selection algorithm, a 16-QAM modulated MIMO-OFDM system with a minimum mean square error (MMSE) receiver is considered in this paper. The BER performance is shown in Fig. 5. In contrast to the random and the non-selection systems, the selection diversity gain obtained by either the capacity or the norm criteria based CEO selection algorithm improves the system BER. The improvement is more significant when the capacity criteria is adopted. Compared with the non-selection BER performance, approximately 5 dB SNR gain at the BER of 1×10^{-2} is obtained by the capacity criteria based CEO selection algorithm.

Table 1. Complexity comparisons between the CEO algorithm and ES method with $M_T = 8$, $M_R = 4$.

(M_T, M_t)	N_{CEO}	k	$\mathcal{O}(CEO)$	$\mathcal{O}(ES)$	ϑ
$(8,2)$	5	3	15	28	$\geq 99\%$
$(8,4)$	10	3	30	70	$\geq 99\%$
$(8,6)$	5	3	15	28	$\geq 99\%$

5. CONCLUSIONS

In this paper, we have presented a low complexity transmit antenna subset selection algorithm based on the cross entropy optimization (CEO) method for antenna selection MIMO - OFDM wireless communication systems. The proposed selection algorithm obtains near-optimal capacity while holding very fast convergence. It guarantees that the obtained capacity converges to within 99% optimal capacity achieved by exhaustive search. The capacity convergence is not affected by the number of selected transmit antennas. In addition, approximately 50% of the computational complexity of the ES method is required by our algorithm. We have also confirmed that the proposed CEO antenna selection algorithm offers a reduction in the system bit error rate. Compared with the non-selection BER performance, approximately 5 dB SNR gain at the BER of 1×10^{-2} can be achieved by our algorithm. From all results, we conclude that the CEO based transmit antenna subset selection algorithm is promising for the practical antenna selection MIMO-OFDM wireless communication systems.

[1] A detailed description of antenna selection by the CEO method can be found in [6]

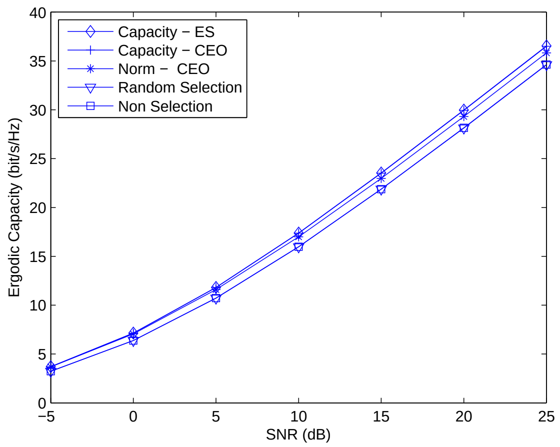

Fig. 2. Ergodic capacity versus SNR with $M_t = 4$, $M_T = 8$, $M_R = 4$ and $L = 3$.

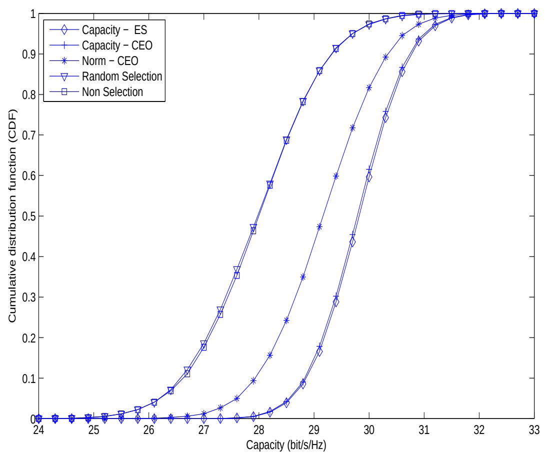

Fig. 3. CDF versus ergodic capacity with $M_t = 4$, $M_T = 8$, $M_R = 4$ and $L = 3$.

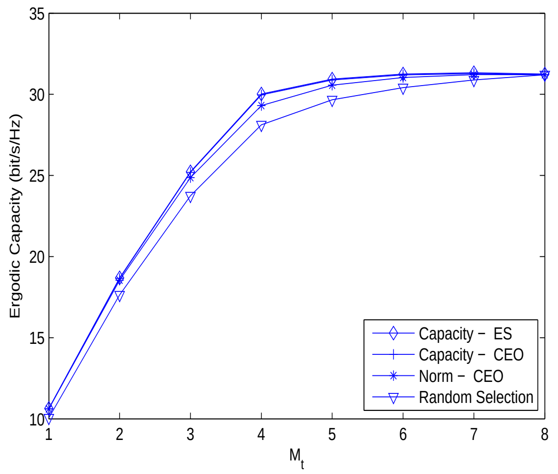

Fig. 4. Ergodic capacity versus selected transmit antennas with $M_T = 8$, $M_R = 4$, $L = 3$ and SNR = 20 dB.

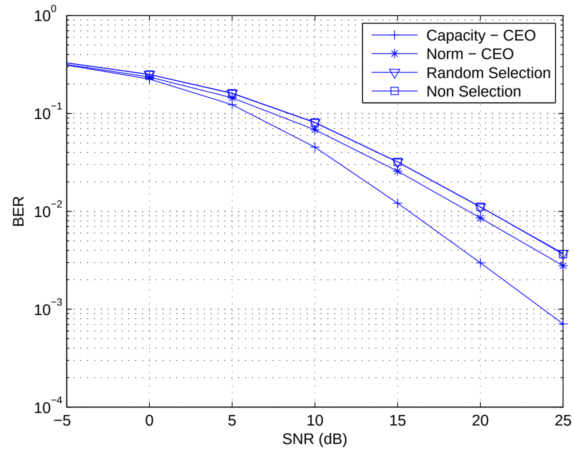

Fig. 5. System BER versus SNR with $M_t = 4$, $M_T = 8$, $M_R = 4$ and $L = 3$. (MMSE)

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