Optimum Receiver Antenna Selection for Transmit Cyclic Delay Diversity

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Abstract—Cyclic delay diversity(CDD) is an attractive diversity scheme due to its low complexity and compatibility to the existing standard orthogonal frequency division multiplexing systems. This paper presents an novel receiver antenna selection criterion for transmit CDD systems to obtain the maximal diversity gain. The performance of the proposed system is very close to the corresponding orthogonal space-time block coding diversity schemes with optimum receiver antenna selection, but CDD has the advantages of much lower complexity and no data rate reduction. Moreover, in order to make the proposed system simpler for implementation, a low complexity antenna selection algorithm has been presented.

I. INTRODUCTION

Space-time codes have been designed to combat wireless channel fading and improve transmission reliability. This diversity scheme is especially important for orthogonal frequency division multiplexing (OFDM) systems which have poor inherent error performance. One of the well known space-time codes is the orthogonal space-time block coding (OSTBC) scheme [1]. Although the OSTBC scheme obtains full diversity gain, it requires the channel to be stable over a number of successive time slots or subcarriers. Moreover, OSTBC for complex signal constellations decreases the date rate in multiple antenna systems when the transmitter array is larger than two elements [2].

Therefore, another approach, cyclic delay diversity (CDD) has been presented for an arbitrary number of transmitters. It does not suffer data rate reduction or require channel invariance, but offers inferior performance compared to the OSTBC [3]. Furthermore, the compatibility of CDD to existing standards, such as terrestrial digital video broadcasting (DVB-T) as well as wireless local area network (WLAN) standards, with low additional complexity, makes CDD attractive. Because CDD transforms spatial diversity into frequency diversity, more pilot signals are needed for channel state estimation. However, channel state estimation for channels between many transmitters and one receiver (which is mandatory in space-time coding) is not necessary in CDD. It is presented the additional pilot symbol overheads required by CDD and OSTBC are the same [4], [5], [6].

Antenna selection is another diversity scheme which saves radio frequency (RF) chains and therefore significantly cuts down on the system complexity [10]. When channel state information (CSI) is available at the transmitter side, transmit antenna selection outperforms OSTBC (using all the transmit antennas) for a higher coding gain and the same diversity gain [11]. Here we assume CSI is absent at the transmitter, in this case, the previous transmitter diversity schemes can be combined with receiver antenna selection. In this paper, we investigate transmitter cyclic delay diversity and receiver antenna selection (TCDD/RAS) systems.

The rest of this paper is organized as follows. Section II provides the system model. In Section III, an optimum antenna selection criterion is proposed for the TCDD/RAS systems. In Section IV, the simulation results show that this selection criterion outperforms the traditional norm selection and capacity selection criteria which are aimed to maximize the Frobenius norm or the capacity of the selected MIMO channel. Finally, Section V concludes this paper.

II. SYSTEM MODEL AND CHANNEL MODEL

In order to isolate the diversity effect of the CDD transmitter, we assume i.i.d. quasi-static flat Rayleigh fading channels plus additive white Gaussian noise all through this paper. As proved in [7], [8], CDD can retain a substantial diversity gain even in relatively strong frequency-selective channels. Therefore, our method can be further generalized to frequencyselective channels. However, the cyclic delays have to be large enough to be distinguished from the existing resolvable taps in the multipath channel as indicted in [9].

As depicted in Fig.1, the signal bits are modulated after being coded by the forward error control/correction (FEC) block. OFDM is applied as an inverse fast Fourier transform (IFFT) with length of N_s , which is important for the CDD scheme to convert the spatial diversity to frequency diversity. Afterwards, the signal vector is transmitted over N_t separate transmit antennas with cyclic delays δ_n , where *n* denotes the transmitter order. A cyclic prefix is inserted before the signal is sent off to avoid the inter symbol interference (ISI) and inter carrier interference(ICI). Therefore, if the transmit signal vector at one OFDM frame in the frequency domain is

$$\mathbf{X}' = [X_1, X_2, \dots, X_{N_s}], \tag{1}$$

the transmit signal matrix after CDD arranging can be denoted



Fig. 1. Transmit CDD and receive antenna selection system model

as

$$\mathcal{X}' = \begin{bmatrix} X_1 & X_2 e^{j\frac{2\pi}{N_s}\delta_1} & \dots & X_{N_s} e^{j\frac{2\pi}{N_s}\delta_1(N_s-1)} \\ X_1 & X_2 e^{j\frac{2\pi}{N_s}\delta_2} & \dots & X_{N_s} e^{j\frac{2\pi}{N_s}\delta_2(N_s-1)} \\ \vdots & \vdots & \ddots & \vdots \\ X_1 & X_2 e^{j\frac{2\pi}{N_s}\delta_{N_t}} & \dots & X_{N_s} e^{j\frac{2\pi}{N_s}\delta_{N_t}(N_s-1)} \end{bmatrix},$$
(2)

where $(\cdot)'$ denotes the transposition operation.

Antenna selection is implemented at the receiver side to choose L_r optimum receive antennas from N_r available receive antennas. Let $\mathbf{H} \in C^{N_t \times N_r \times N_s}$ denotes the original MIMO channel matrix, $\hat{\mathbf{H}} \in C^{N_t \times L_t \times N_s}$ is the channel submatrix selected. Afterwards, maximal ratio combining (MRC) is used to combine the received symbols over the selected antennas. Following this, a standard OFDM demodulator is applied to recover the signal. By virtue of the FFT in the demodulator, CDD converts the multiple input channels into equalized single input channels with increased frequency selectivity. The equalized channel can be denoted as

$$H^{m}_{equ,k}(t) = \sum_{n=1}^{N_{t}} e^{j2\pi \frac{k\delta_{n}}{N_{s}}} H^{(n,m)}_{k}(t),$$
(3)

where $H_k^{(n,m)}(t)$ represents the channel transfer function of the channel between the *n*th transmitter and the *m*th receiver on the *k*th subcarrier in the *t*th OFDM frame, $H_{equ,k}^m(t)$ is the equalized channel transfer function as in a single input channel. The equalized channel matrix in one OFDM frame can be denoted as \mathbf{H}_{equ} , where $\mathbf{H}_{equ} \in \mathcal{C}^{1 \times N_r \times N_s}$.

III. TCDD/RAS SYSTEM ANALYSIS

The received signal at kth subcarrier can be represented as

$$\mathbf{Y}_{k}(t) = \mathcal{X}_{k}(t)\mathbf{H}_{k}(t) + \mathbf{w}_{k}(t)$$

= $X_{k}(t)\mathbf{H}_{equ,k}(t) + \mathbf{w}_{k}(t)$ (4)

where $\mathcal{X}(t)$, $\mathbf{H}_k(t)$, $\mathbf{H}_{equ,k}(t)$ and $\mathbf{w}_k(t)$ denote the transmitted signal matrix \mathcal{X} , the channel coefficient matrix \mathbf{H} , the equalized channel matrix \mathbf{H}_{equ} and the additive white Gaussian noise vector on the *k*th subcarrier in the *t*th OFDM frame. In this way, the MIMO channels can be treated as single-input and multiple-output (SIMO) channels.

In this paper, we apply antenna selection based on the exact channel knowledge (ECK) as discussed in [14]. The best

antenna subset is selected with maximum post processing SNR and therefore the lowest BER.

A. The Norm Selection Criterion

The norm selection was proposed for receiver antenna selection combined with the transmitter OSTBC schemes (TOSTBC/RAS). [9], [15], [16], [17], [18], [19] Let $\hat{\mathbf{H}}$ denote the channel matrix of the selected antenna subset and $\|\cdot\|_F$ denotes the Frobenius norm operation, so the antenna selection principle can be represented as

$$\boldsymbol{\omega}_{Norm}^* = \arg \max_{L_r \leqslant N_r} \left\{ \|\widehat{\mathbf{H}}\|_F^2 \right\}$$
(5)

where ω_{Norm}^* denotes the optimal selection indices of $\hat{\mathbf{H}}$ from \mathbf{H} with the norm selection criterion.

B. Optimized Selection Criteria for the TCDD/RAS Systems

TCDD/RAS system means the system with transmitter CDD and receiver antenna selection. After selecting L_r antennas from N_r at the receiver, MRC is applied to combine the signal streams over the selected antennas. It has been proven in [13] that the post-processing SNR after MRC is just the sum of the SNRs from all receive antennas. Therefore, in the TCDD/RAS system, the post-processing SNR at the *k*th subcarrier can be denoted as

$$\gamma_{k,TCDD/RAS} = \gamma_0 \sum_{m=1}^{L_r} |H_{equ,k}^m|^2.$$
 (6)

As denoted in Section II, because we assume the channels are flat-fading, the channel transfer function $H_k^{(n,m)}$ can be simplified as $H^{(n,m)}$. Hence the average post-processing SNR over the whole spectrum will be

which is the same as in the TOSTBC/RAS system, where $\mathbb{E}_{k}\{\cdot\}$ denotes the expectation operation over all of the subcarrier orders k, $(\cdot)^*$ denotes conjugation operation. However, in the TCDD/RAS system, as indicated in (6), the post-processing SNRs on different subfrequences are not equal. The correlations between the channels from different transmit antennas have an effect on $|H_{equ,k}^m|^2$ and thereafter make $\gamma_{k,TCDD/RAS}$ frequency selective. Consequently, the BER performance is degraded by this correlation. Therefore, the norm selection criterion for TOSTBC/RAS systems cannot be applied to TCDD/RAS systems directly.

As we know, $\gamma_{k,TCDD/RAS}$ has an lower bound $\gamma_0 \min_k \left\{ \sum_{m=1}^{L_r} |H_{equ,k}^m|^2 \right\}$. The performance of the selected transmit $\stackrel{m}{CDD}$ and receive MRC system will be improved as the smallest possible post-processing SNR increases, since the BER is mostly determined by relatively small SNRs where most errors occur. However, the impairment of SNR on a small number of subcarriers can be combated by channel coding. Therefore in this paper, the cyclic delay selection criterion in [12] is applied. In this case, the sum of equalized channel parameters $H^m_{equ,k}$ only has N_t distinct states. When $N_t \ll N_s$, the channel parameter at each state will have a significant influence on the BER performance. Hence, if we select a receiver subset to have the maximum value of the quantity $\min_{k} \left\{ \sum_{m=1}^{L_{r}} |H_{equ,k}^{m}|^{2} \right\}, \text{ the possible lowest } \gamma_{k,TCDD/RAS} \text{ will}$ be maximized, and consequently, the error probability will be reduced. This maximum minimum post-processing SNR (MMP-SNR) criteria can be represented as

$$\boldsymbol{\omega}_{MMP-SNR}^{*} = \arg \max_{L_r \leqslant N_r} \left\{ \min_{k} \left\{ \sum_{m=1}^{L_r} |H_{equ,k}^m|^2 \right\} \right\} \quad (8)$$

where $\omega_{MMP-SNR}^*$ is the optimal index of selected receivers from the N_r receive antennas with the MMP-SNR selection criterion.

C. The Capacity Selection Criterion

A capacity selection criterion should also be introduced here for comparison. It is designed to select the antenna subset with the maximum possible channel capacity [?]. The selection principle can be denoted as

$$\boldsymbol{\omega}_{Cap}^{*} = \arg \max_{L_r \leqslant N_r} \left\{ \frac{1}{N_s} \sum_{k=1}^{N_s} \log[\det(\mathbf{I}_{N_r} + \frac{\gamma_0}{N_t} \widehat{\mathbf{H}} \widehat{\mathbf{H}}^{\mathsf{H}})] \right\}$$
(9)

where det(·) stands for the determinant operation, $(\cdot)^H$ denotes the Hermitian transpose operation and \mathbf{I}_{N_r} is a $N_r \times N_r$ identity matrix.

D. Receive Antenna Selection Algorithm with Cross Entropy Optimization Method

In order to execute these selection criteria efficiently, we transform the antenna selection problem into a combinatorial optimization problem, which can be solved by the cross entropy optimization (CEO) method. The CEO algorithm has been proved to be a global random search procedure in [20] and [21]. It was firstly presented by Rubinstein as a principled adaptive importance sampling to estimate the probabilities of rare events in the complex stochastic networks [22]. It has also been adopted to solve complicated combinatorial optimization problems, such as the nondeterministic polynomial time (NP) problems [20]. In order to apply the CEO method to the antenna selection schemes, we have to formulate the antenna selection problem as a combinatorial optimization problem:

$$\boldsymbol{\omega}^* = \arg \max_{\boldsymbol{\omega}_q \in \boldsymbol{\Omega}} S(\boldsymbol{\omega}_q), \tag{10}$$

where ω^* denotes the indicator corresponding to the global optimum of the objective function $S(\omega_q)$, which is used for evaluating the potential solutions, ω_q , and chosen according to the specific selection criteria, such as the norm and the capacity selection criteria. Ω is the set of receive antenna subset selection indicators $\{\omega_1, \dots, \omega_Q\}$. Herein, ω_q is defined as

$$\boldsymbol{\omega}_q = \{I_m\}_{m=1}^{N_r}, \quad I_m \in \{0, 1\}; \quad q = 1, 2, \cdots, Q, \quad (11)$$

where *m* is the receive antenna order and I_m indicates whether the *m*th receive antenna is selected or not. For example, if the first, fourth, fifth and eighth receive antenna are selected out of eight receive antennas, then ω_q will be equal to $\{1, 0, 0, 1, 1, 0, 0, 1\}$. *Q* is the number of all possible antenna subsets and is equal to $\binom{N_r}{L_r}$, where $\binom{x}{y}$ denotes the binomial coefficient, $\frac{x!}{y!(x-y)!}$. The flow of the receive antenna selection algorithm based on the CEO method is described as follows ¹:

Step 1: Start with an initial value $\mathbf{p}^{(0)} = \{p_m^{(0)}\}_{m=1}^{N_r}, p_m^{(0)} = \frac{1}{2}^2$. Set the iteration counter t := 1;

¹Due to space restriction, the detailed description about the antenna selection algorithm with the CEO method will not be presented in this paper but the readers can refer to [23].

²The algorithm converges without the constraint of starting point, but for simplicity we set $p_m^{(0)} = \frac{1}{2}$.

- **Step 2:** Generate samples $\{\omega_q^{(i)}\}_{i=1}^{N_{CEO}}$ from the density function $f(\cdot, \mathbf{p}^{(t-1)})$, where N_{CEO} is the total number of the samples;
- Step 3: Calculate the performance functions $\{S(\omega_q^{(i,t)})\}_{i=1}^{N_{CEO}}$ and order them from largest to smallest, $S^{(1)} \geq \cdots \geq S^{(N_{CEO})}$. let $r^{(t)}$ be (1ρ) sample quantile of the performances: $r^{(t)} = S^{(\lceil (1-\rho)N_{CEO}\rceil)}$, where $\lceil \cdot \rceil$ is the ceiling operation.
- **Step 4:** Update the parameter $\mathbf{p}^{(t)}$ via

$$p_m^{(t)} = \frac{\sum_{i=1}^{N_{CEO}} I_{\{S(\boldsymbol{\omega}_q^{(i,t)}) \ge r^{(t)}\}} I_m(\boldsymbol{\omega}_q^{(i,t)})}{\sum_{i=1}^{N_{CEO}} I_{\{S(\boldsymbol{\omega}_q^{(i,t)}) \ge r^{(t)}\}}} \quad (12)$$

Step 5: If stopping criterion is satisfied, then stop; otherwise set t := t+1 and go back to step 2. Here, the stopping criterion is the predefined number of iterations.

IV. SIMULATION RESULTS

In this section, Monte Carlo simulation results of TCDD/RAS system based on the MMP-SNR selection criterion are presented. Performances of the TCDD/RAS system and the TOSTBC/RAS system with the norm selection criterion (5) and the capacity selection criterion are also provided in Fig.2 for comparison. Firstly we consider the performance comparison of TCDD/RAS system with three different antenna selection rules, and the TOSTBC/RAS system with three different antenna selection criteria as well. Here we set $N_t = 4$, $L_r = 4$ and $N_r = 8$. The OSTBC code in [2] is adopted to form the 4 transmitter OSTBC scheme. The signals are mapped to 16QAM constellation and coded by a half rate convolutional code with a constraint length 3 and a free distance 5.

From Fig.2, it can be seen that the MMP-SNR selection criterion yields the best BER performance among the three antenna selection criteria for TCDD/RAS system, where E_b/N_0 denotes the transmit energy per bit to noise power spectral density ratio. The norm criterion for TCDD/RAS system performs worst. Although the capacity criterion obtains the same diversity gain (which is indicated by the slope of the curve) as the MMP-SNR criterion, the latter obtains a larger coding gain. Meanwhile, the TCDD/RAS system using the MMP-SNR criterion offers almost the same diversity gain as the TOSTBC-RAS system and the coding gain degrades by about 1dB. As explained in Section III, the sacrifice of diversity gain is intended to simplify the antenna selection criterion.

Since for the four transmitter system, the OSTBC code suffers from a data rate loss to a rate of 3/4, we can puncture the convolutional code from a rate of 1/2 to a rate of 2/3 to retain the full data rate as in the CDD system, which has been suggested in [9]. The free distance of the punctured convolutional code is 3. The performance curve of this punctured STBC system is inferior to our TCDD/RAS system when E_b/N_0 is smaller than 7dB as depicted in Figure.3. It is predictable that with a more powerful channel coding scheme, the spatial diversity provided by CDD can be exploited more efficiently, and the performance gap between TCDD/RAS system and TOSTBC/RAS system can be further reduced. But the performance is already commendable even with our simple channel coding scheme.



Fig. 2. Performance comparison of TCDD/RAS systems with three antenna selection criteria and TOSTBC/RAS system with the norm selection criterion, $N_t = 4$, $L_r = 4$, $N_r = 8$



Fig. 3. Performance comparison between exhaustive search and CEO. $N_t = 4$, $L_r = 4$, Nr = 8

Table.I presents the computational complexity comparison between antenna selection schemes based on the CEO and the exhaustive search (ES) algorithms. The complexity is measured by the number of function evaluations $\mathcal{O}(CEO)$ and $\mathcal{O}(ES)$. In this table, t and N_{CEO} are parameters of the CEO algorithm, standing for the numbers of algorithm iterations and samples, respectively. ϑ (equals to $\frac{S_{CEO} - S_{optim}}{S_{optim}}$, S_{CEO} and S_{optim} denote the BER obtained by the CEO algorithm and the ES method) is the performance difference ratio of the bit error rate (BER) produced by the CEO algorithm. From Table.I, we find that the CEO algorithm only requires approximately 50% of the computational complexity as the exhaustive search strategy. From the performance difference ratio, ϑ , it can be observed that the performance difference in terms of BER

TABLE I COMPLEXITY COMPARISONS BETWEEN THE CEO ALGORITHM AND ES METHOD WITH $N_t = 4$

(N_r, L_r)	N_{CEO}	t	$\mathcal{O}(CEO)$	$\mathcal{O}(ES)$	θ
(8,2)	5	3	15	28	$\leq 1\%$
(8,4)	10	3	30	70	$\leq 1\%$
(8,6)	5	3	15	28	$\leq 1\%$

between the the CEO algorithm and the ES method is less than 1%. Fig.3 shows the performance comparison between the CEO and the ES algorithms. From it, we can find that the BER performance obtained by the CEO algorithm is is very close to the optimum performance obtained by the ES method for a wide range of SNR values.



Fig. 4. Performance comparison between TCDD/RAS systems with the receive antenna array size L_r varying. $N_t=4, Nr=8$

Fig.4 shows the performance of TCDD/RAS system with different numbers of selected antennas L_r . Here the simulated system is similar as that of Fig.2, except that L_r varies between 1 and 8. It is seen that when L_r increases, the diversity gain of TCDD/RAS system does not change but the coding gain is enhanced. This feature implies that the diversity order is retained through our antenna selection scheme as if all the receive antennas are used. The result is similar to that described in [14], [17] for OSTBC systems.

V. CONCLUSIONS

From a combined consideration of choosing the cyclic delay and receiver antenna subset, an optimum MMP-SNR antenna selection criterion is derived for the proposed TCDD/RAS system. With this rule, a TCDD/RAS system can achieve nearly full diversity gain with a small coding gain reduction compared to the TOSTBC/RAS system. The diversity gain is retained by the antenna selection scheme as if all the receive antennas are used. However, implementation complexity is much lower and no data rate loss occurs as compared to OSTBC system.

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