Non-coplanar beam orientation optimization for total marrow irradiation using IMRT

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Bone Marrow Transplants (BMTs)

- Method of treatment for blood and bone marrow cancers (leukemia and lymphoma); also for aplastic anemia and sickle cell disease
- To transplant bone marrow, existing bone marrow must be eradicated – current method is total body irradiation (TBI)
- More effective methods of bone marrow elimination – total marrow irradiation (TMI)
Intensity Modulated Radiation Therapy (IMRT)

- Conventional radiotherapy: beam is of homogeneous intensity
- IMRT: beam is broken into numerous beamlets; each beamlet’s intensity is controllable
- Used successfully for other types of cancer (head-and-neck)
Has IMRT been applied to TMI before?

Yes, but

- Not within a mathematical framework
- Previous work studies irradiation from a greater distance
... So what are the problems?

- For a set of beam directions, which beamlet intensities are optimal? And...
- Which beam directions do we use in the first place?
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- Which beam directions do we use in the first place?
Beam Orientation Optimization (BOO)

- Beams are obtained by
  - Rotating the gantry – 10° increments
  - Translating the couch along patient’s vertical axis – 10 cm increments
- Beams are **non-coplanar**
Coplanar vs. non-coplanar

Coplanar
Coplanar vs. non-coplanar
BOO Model Overview

- Set of beam orientations is denoted by $\Theta$
- Quality of set is denoted by function $F(\Theta)$
- Goal is to optimize $F(\Theta)$ over all possible sets of beams
The function $\mathcal{F}$

- Beam set quality can be formulated in many ways
- We use fluence map optimization (FMO) value from Romeijn et al. [2006]
For a set of beam directions, which beamlet intensities are optimal?

- $x_i$ is intensity of beamlet $i$, $i \in B_{\Theta}$
- $z_{js}$ is dose in voxel $j$ in structure $s$, $j \in \{1, \ldots, v_s\}$, $s \in S$
- $z_{js} = \sum_{i \in B_{\Theta}} D_{ijs} x_i$
FMO Model Overview (2)

- The dose in each voxel can be penalized:

\[
F_{js}(z_{js}) = \left[ w_s (T_s - z_{js})^{p_s} + \overline{w}_s (z_{js} - T_s)^{\overline{p}_s} \right]
\]  

(1)

- \(w_s, \overline{w}_s\) are weights for underdosing and overdosing; \(p_s, \overline{p}_s\) are powers for underdosing and overdosing

- Total penalty is \(\sum_{s \in S} \left( \frac{1}{v_s} \sum_{j=1}^{v_s} F_{js}(z_{js}) \right)\)
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Total penalty is:

\[ \sum_{s \in S} \left( \frac{1}{v_s} \sum_{j=1}^{v_s} F_{js}(z_{js}) \right) \]
FMO Model Overview (3)

Model is

\[
\min \sum_{s \in S} \left( \frac{1}{v_s} \sum_{j=1}^{v_s} F_{js}(z_{js}) \right)
\]

subject to

\[
\begin{align*}
z_{js} &= \sum_{i \in B_\Theta} D_{ijs}x_i, \quad \forall j \in \{1, \ldots, v_s\}, \ s \in S \\
x_i &\geq 0, \quad \forall i \in B_\Theta
\end{align*}
\]

FMO model is convex, solvable using projected gradient method
Add/Drop (A/D) Algorithm

- Neighborhood search algorithm
- Previously used in Kumar [2005] and Aleman et al. [2008]
One iteration of A/D

1. Select a beam and a component (gantry angle/\(z\) trans.)
2. Enumerate all the \(\Theta\) in the beam-component neighborhood of current \(\Theta\) (\(\Theta^{(i)}\))
3. Calculate \(\mathcal{F}\) for all of these solutions
4. Set \(\Theta^{(i+1)}\) to the most improving new \(\Theta\) (if any)
One iteration of A/D - graphical example

Current solution – $\Theta^{(i)}$, $\mathcal{F}(\Theta^{(i)}) = 700$
One iteration of A/D - graphical example

Select a beam and a component
One iteration of A/D - graphical example

Enumerate solutions in the neighborhood of $\Theta^{(i)}$
One iteration of A/D - graphical example

\[ \Theta_1, \mathcal{F}(\Theta_1) = 490 \]
One iteration of A/D - graphical example

\[ \Theta_2, \mathcal{F}(\Theta_2) = 520 \]
One iteration of A/D - graphical example

\[ \Theta_3, \ F(\Theta_3) = 780 \]
One iteration of A/D - graphical example

\[ \Theta_4, \mathcal{F}(\Theta_4) = 770 \]
One iteration of A/D - graphical example

\[ \Theta_1 \text{ is most improving solution } \rightarrow \text{ set } \Theta^{(i+1)} = \Theta_1 \]
Types of A/D

- Most basic is sequential – (1, G), (1, z), (2, G), (2, z) ...
- Other types: probabilistic, dynamic $\delta$
Probabilistic A/D

- In each iteration, beam and component pair are randomly selected
- Selection probabilities calculated using the average of the recent improvements of each pair, relative to overall average

$$\bar{p}(b, d) = \Pr(B = b, C = d) = \frac{1}{k|D|} + \frac{\alpha}{k|D|} \left( \frac{\Delta_{bdr} - \bar{\Delta}_m}{\bar{\Delta}_m} \right) \quad (3)$$

- Sensitivity of probabilities to recent improvement can be controlled using an additional parameter $\alpha$
Dynamic $\delta$

- In basic A/D, neighborhood size ($\delta$) is constant
- In dynamic $\delta$ A/D, $\delta$ changes to reflect how much solution moved previously
- Large moves $\rightarrow$ large neighborhood sizes
- Small moves $\rightarrow$ small neighborhood sizes

\[
\bar{\delta}_G = c_{zG} ||\bar{\Theta} - \Theta^{(i)}|| \\
\bar{\delta}_z = c_{Gz} ||\bar{\Theta} - \Theta^{(i)}||
\]

(4)

(5)
Results

- Executed on Beowulf CentOS cluster with 32 nodes
- Each node has 8 2GHz processors and 8GB of memory
- In each A/D iteration, each FMO calculation assigned to a different processor
Numerical Results

10 executions, 12 hours, $\delta_G = 20^\circ$, $\delta_z = 20$ cm, random starting point

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<td>SC</td>
<td>11370.4</td>
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<td>Pr, $\alpha = 0$</td>
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<td>Pr, $\alpha = 0.75$</td>
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<td>Pr, $\alpha = 1$</td>
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<td>D$\delta$, $c_{ZG} = 1$</td>
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<td>30.1</td>
<td>36.97%</td>
</tr>
</tbody>
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Treatment Plan Criteria

Criteria used:

- 95% of bone marrow receives more than 12Gy
- At most 20% of bone marrow receives more than 20Gy
- 0% of bone marrow receives more than 25Gy
- Majority of OAR volume receives less than 8Gy
General Results

- Generally all criteria were met
- Hardest to spare well: spinal cord and lungs
DVHs – Sequential Cycling A/D, 30 beams

Non-coplanar beam orientation optimization for TMI
DVHs – Probabilistic A/D ($\alpha = 0.75$), 30 beams
DVHs – Dynamic $\delta$ A/D, 30 beams

![Graph showing DVHs for various organs](image-url)
Conclusions

- IMRT with non-coplanar beams allows for more effective treatment than TBI
- Add/Drop can obtain such plans in a clinically realistic timeframe and environment
The Future

- Other variants of A/D
- Other algorithms
- Clinical realizability
  - Incorporation of patient and organ motion
  - Design of arc therapy plans
Thank you for listening

Questions?
