# 6.097 Pset 3 Solutions

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## 1 Solution to O & W 7.21

1. The Nyquist rate is $2 \times 5000\pi = 10000\pi$. The sampling period must be at most $T_{max} = 2\pi/10000\pi = 2 \times 10^{-4}$s. Since the sampling period is less than this, we <u>can</u> recover $x(t)$ from $x_p(t)$.

2. The sampling period must be at most $T_{max} = 2\pi/(2 \times 15000\pi) = 0.66 \times 10^{-4}$s. Since the sampling period is greater than this, we <u>cannot</u> recover $x(t)$ from $x_p(t)$.

3. $\Im(X(j\omega))$ is not specified. The Nyquist rate is indeterminate. Hence we, in general, <u>cannot</u> recover $x(t)$ from $x_p(t)$.

4. If $x(t)$ is real the Fourier transform is even. Thus, we can conclude that $X(j\omega) = 0$ for all $|\omega| > 5000\pi$. This reduces to part (a).

5. If $x(t)$ is real the Fourier transform is even. Thus, we can conclude that $X(j\omega) = 0$ for all $|\omega| > 15000\pi$. This reduces to part (b).

6. If $X(j\omega) * X(j\omega) = 0$ for all $|\omega| > 15000\pi$, then $X(j\omega) = 0$ for all $|\omega| > 7500\pi$. The sampling period must be at most $T_{max} = 2\pi/(2 \times 7500\pi) = 1.33 \times 10^{-4}$s. Since the sampling period is less than this, we <u>can</u> recover $x(t)$ from $x_p(t)$.

7. $|X(j\omega)| = 0$ for $\omega > 5000\pi$ implies that $X(j\omega) = 0$ for $\omega > 5000\pi$. Since the spectrum for $\omega < 0$ is not specified and $x(t)$ may not be real, in general, we <u>cannot</u> recover $x(t)$ from $x_p(t)$.

## 2 Solution to O & W 7.22

Since $Y(j\omega) = X_1(j\omega)X_2(j\omega)$, $X_1(j\omega)$ is band-limited to $1000\pi$, $X_2(j\omega)$ is band-limited to $2000\pi$, this implies that $Y(j\omega)$ is band-limited to $1000\pi$. By the sampling theorem, for no aliasing, we require

$$\frac{2\pi}{T} > 2(1000\pi) \Leftrightarrow T < \frac{1}{1000}.$$

## 3 Solution to O & W 8.21

1. Now, $w(t) = \cos^2(\omega_c t + \theta_c)x(t)$. To get the result, we simply recall that $\cos^2(A) = 1/2(1 + \cos(2A))$.

2. We require that

$$\omega_M \leq \omega_{co} \leq 2\omega_c - \omega_M.$$

Refer to Fig. 1.

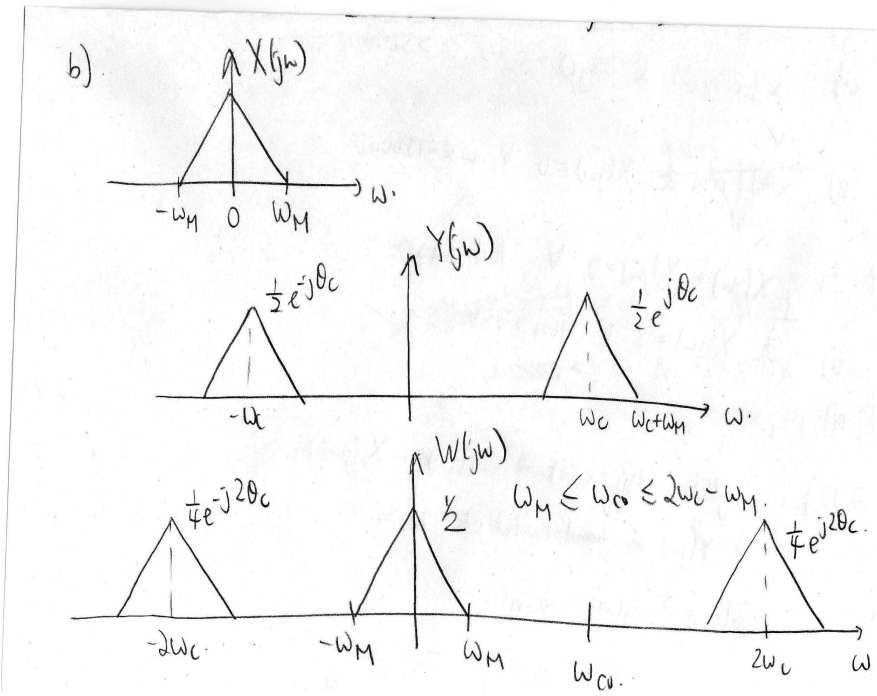


Figure 1: Figure for O & W 8.21