6.097 Review of Signals and Systems

6.097 Problem Set 2: Compiled from various sources.

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1. **DT Convolution**

   Compute the DT convolution $y[n] = x_1[n] * x_2[n]$ of the following pairs of signals.

   (a) $x_1[n] = \alpha^n u[n]$, $x_2[n] = \beta^n u[n]$, where $\alpha \neq \beta$.
   (You may find the following formula useful: For $r \neq 1$, $\sum_{k=m}^{n} r^k = \frac{r^m - r^{n+1}}{1-r}$.)

   (b) $x_1[n] = u[n] - u[n-5]$, $x_2[n] = (0.25)2^n u[-1-n]$.

   (c) $x_1[n] = [-1, -2, -3, -4, 4, 3, 2, 1]$, $x_2[n] = [1, 2, 2, 1]$. ($x_1[n]$ is a sequence of length 8 with non-zero values for $0 \leq n \leq 7$, and $x_2[n]$ is a sequence of length 4 with non-zero values for $0 \leq n \leq 3$.)

2. **DT Fourier Series**

   (a) Use the multiplication property (repeated below) to find the DTFS coefficients of the following.
   Multiplication Property: Let

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk(2\pi/N)n}$$

   and

$$y[n] = \sum_{k=0}^{N-1} b_k e^{jk(2\pi/N)n}$$

   be periodic signals. Given the signal

$$x[n]y[n] = \sum_{k=0}^{N-1} c_k e^{jk(2\pi/N)n},$$

   then

$$c_k = \sum_{l=\langle N \rangle} a_l b_{k-l} = \sum_{l=\langle N \rangle} a_{k-l} b_l$$

   i. $x[n] \cos(\frac{6\pi n}{N})$

   ii. $x[n] \sum_{r=-\infty}^{\infty} \delta[n - rN]$

   iii. $x[n] \left(\sum_{r=-\infty}^{\infty} \delta[n - \frac{rN}{3}]\right)$. Assume N is divisible by 3. (*Optional*)

   (b) Find the Fourier series representation for the signal $x[n]y[n]$, where

$$x[n] = \cos(\pi n/3)$$

   and

$$y[n] = \begin{cases} 1, & |n| \leq 3 \\ 0, & 4 \leq |n| \leq 6 \end{cases}$$

   is periodic with period 12.

3. **DT Fourier Transform**

Let $\tilde{f}_N(t)$ be the N-term approximation of a periodic signal $f(t)$:

$$\tilde{f}_N(t) = \sum_{n=-N}^{n=N} c_n e^{j\omega_0 nt}$$

where the c_n are the Fourier coefficients of $f(t)$. Let $\{\tilde{c}_n\}$ be the Fourier coefficients of $\tilde{f}_N(t)$,

$$\{\tilde{c}_n\} = \mathcal{F}\{\tilde{f}_N(t)\}.$$

(a) Find a sequence $\{d_n\}$ such that $\{\tilde{c}_n\} = \{d_n c_n\}$.

(b) Find $g_N(t)$ such that $\tilde{f}_N(t) = f(t) * g_N(t)$. Is $g_N(t)$ periodic?

4. **DT Fourier Transform**

Consider a system consisting of the cascade of two LTI systems with frequency responses

$$H_1(e^{j\omega}) = \frac{2 - e^{-j\omega}}{1 + \frac{1}{2}e^{-j\omega}}$$

and

$$H_2(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega} + \frac{1}{4}e^{-2j\omega}}$$

(a) Find the difference equation describing the overall system.

(b) Determine the impulse response of the overall system. (*Optional, Tedious*)

5. **The z-Transform**

A causal LTI system has impulse response $h[n]$, for which the z-transform is

$$H(z) = \frac{1 + z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{4}z^{-1}\right)}.$$

(a) What is the region of convergence of $H(z)$?

(b) Is the system stable? Explain.

(c) Find the z-transform $X(z)$ of an input $x[n]$ that will produce the output

$$y[n] = -\frac{1}{3}\left(-\frac{1}{4}\right)^n u[n] - \frac{4}{3}(2)^n u[-n-1].$$

(d) Find the impulse response $h[n]$ of the system.

(e) Find the LCCDE relating the input and output.