Age Optimal Information Gathering and Dissemination on Graphs

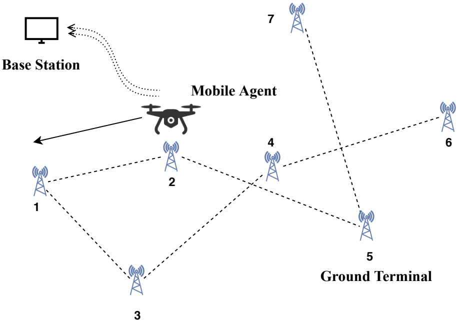
Vishrant Tripathi, Rajat Talak, and Eytan Modiano

Lab for Information & Decision Systems, MIT

May 2, 2019

Introduction

- Dynamic phenomena of interest happening at different locations
- Recorded by a mobile agent moving around between these locations and sent to a base station
- Timely exchange of updates needed to ensure fresh information at the base station



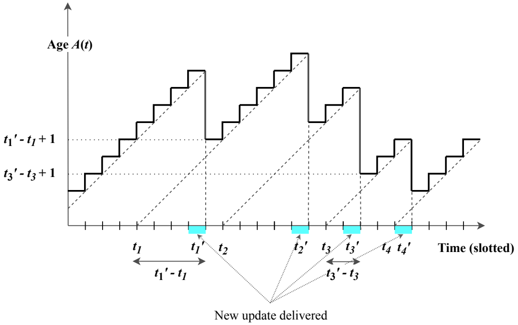
Motivation

- Examples -
  - measuring traffic, temperature and pollution in cities
  - ocean monitoring using underwater autonomous vehicles
  - surveillance using UAVs
  - web crawler collecting information
- Having the freshest available data is essential to system performance
- AoI - new metric that characterizes freshness of information
- Low AoI $\implies$ fresh information $\implies$ better performance

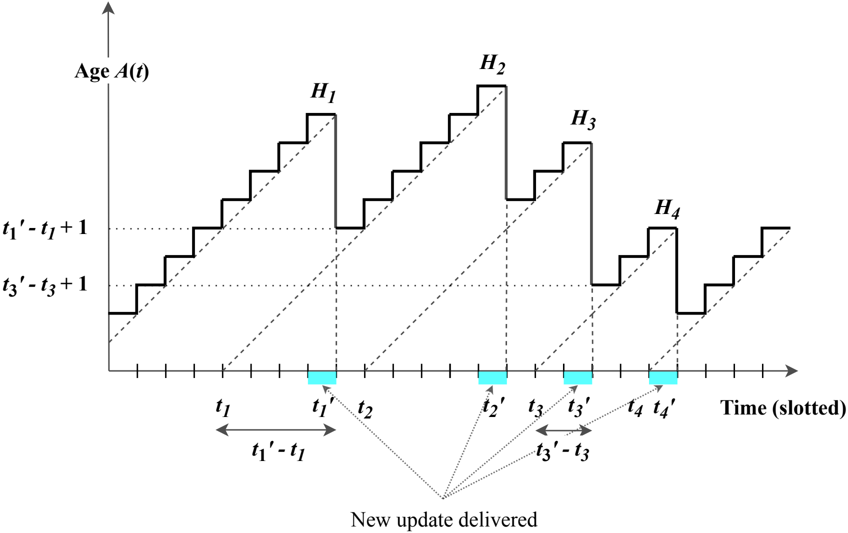
What is Age of Information?

Age of Information $A(t)$ at the destination increases linearly with time, unless it receives a useful update about the source. When an update is received age drops to the time since the update was generated.

$$A(t+1) = \begin{cases} A(t)+1, & \text{if no update at time } t \\ t+1-t_i, & \text{if update } i \text{ is delivered.} \end{cases}$$



What is Age of Information?



Age Metrics

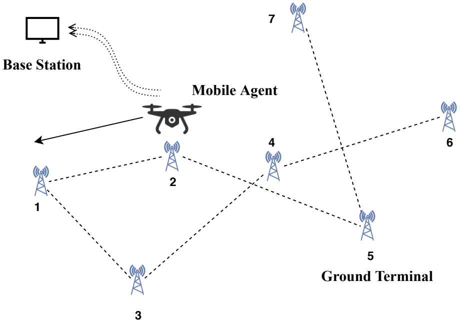
Peak Age - average of peak values of the age process $A(t)$

$$A^{\mathrm{p}} \triangleq \limsup_{T\to\infty} \frac{\sum_{t=1}^{t=T} A(t)\mathbb{1}_{\{\text{update delivered at time } t\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{\text{update delivered at time } t\}}},$$

Average Age - average of the entire age process $A(t)$

$$A^{\mathrm{ave}} \triangleq \limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} A(t).$$

Setup



- Set of ground terminals V
- Mobile agent that collects updates from these locations
- Slotted system - Agent stays at a location and collects an update in every time-slot
- Mobility graph $G(V, E)$ - can only move along edges of this graph

Goal

- For every ground terminal i - age process $A_i(t)$ measures the time since it was last visited by the mobile agent
- Consider 2 metrics - peak age A_i^{p} and average age A_i^{ave} for every terminal
- Using these, we define network peak and average age as -

$$A^{\mathrm{p}} = \sum_{i \in V} w_i A_i^{\mathrm{p}} \quad \text{and} \quad A^{\mathrm{ave}} = \sum_{i \in V} w_i A_i^{\mathrm{ave}},$$

- We want to find trajectories that minimize network peak and average ages A^{p} and A^{ave}

Trajectory Space

- We start by limiting ourselves to random walks on the graph G
- Closed form expressions for network peak and average age can be derived easily
- We will see that this suffices for peak age minimization
- We now formulate the problem as trying to find the random walks that minimize peak and average age on a graph

Age of A Random Walk

Theorem

The network peak and average age for a random walk $\mathbf{P}$ *are given by*

$$A^p(\mathbf{P}) = \sum_{i \in V} \frac{w_i}{\pi_i}, \quad \text{and} \quad A^{ave}(\mathbf{P}) = \sum_{i \in V} \frac{w_i z_{ii}}{\pi_i}, \qquad (1)$$

where π *is the stationary distribution for* $\mathbf{P}$ *and* z_{ii} *are diagonal elements of the matrix* $Z \triangleq (I - \mathbf{P} + \Pi)^{-1}$.

Key Idea - Relate age metrics to moments of return times

Peak Age Minimization

Theorem

Any random walk $\mathbf{P}$ *with the stationary distribution* π^* *that satisfies*

$$\pi_i^* = \frac{\sqrt{w_i}}{\sum_{j \in V} \sqrt{w_j}},$$

achieves minimum network peak age over the space of all trajectories on the graph G.

Similar square root law also observed in P2P networks and AoI scheduling

Peak Age Minimization

This can easily be found in polynomial time using a Metropolis-Hastings random walk

$$\mathbf{P}^{\text{mh}}_{i,j} = \begin{cases} \frac{1}{d_i}\min(1, \frac{\pi^*_j d_i}{\pi^*_i d_j}), & \text{if } i \neq j \text{ and } (i,j) \in E \\ 1 - \sum_{j:j\neq i} \mathbf{P}^{\text{mh}}_{i,j}, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases},$$

where d_i equals the out-degree of node i in the mobility graph G. (can be done in **polynomial time**)

Average Age Minimization

Lemma

In the symmetric case of $w_i = 1, \forall i$, **the average age minimization problem is NP-Hard.**

We show that if we can solve the average age problem in polynomial time, then we can solve the Hamiltonian Cycle problem in polynomial time.

Average Age Minimization

Polynomial Time Heuristic

Theorem

Find the fastest mixing random walk $\mathbf{P}$ *that has a stationary distribution* π^* *on the graph* G*. Then, we have*

$$\frac{A^{ave}(\mathbf{P})}{A^{ave}_{LB}} \leq 8\mathcal{H}^*,$$

where $\mathcal{H}^*$ *is the mixing time of* P *and* A^{ave}_{LB} *is a lower bound on the network average age possible on any graph.*

1

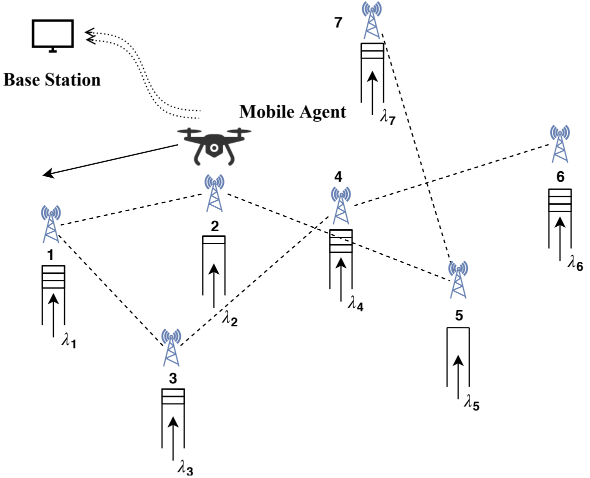
[1] The fastest mixing Markov chain given a fixed stationary distribution can be found in polynomial time by solving a convex program [Boyd, et al. 2004]

Bernoulli FCFS Queues

- Updates are generated for every ground terminal i, at rate λ_i (i.i.d. Bernoulli) and get queued in an FCFS queue
- Minimize the network peak age and average age over generation rates $\boldsymbol{\lambda}$ and trajectories $\mathbb{T}$:

$$A_{\mathcal{D}}^{\mathrm{p}*} = \min_{\mathcal{T}\in\mathbb{T},\boldsymbol{\lambda}} \sum_{i\in V} w_i A_i^{\mathrm{p}}, \quad \text{and} \quad A_{\mathcal{D}}^{\mathrm{ave}*} = \min_{\mathcal{T}\in\mathbb{T},\boldsymbol{\lambda}} \sum_{i\in V} w_i A_i^{\mathrm{ave}}$$

Bernoulli FCFS Queues



Separation Principle Policy

Definition

Separation Principle Policy

1. Mobile agent follows the fastest mixing trajectory $\mathbf{P}^*$ obtained earlier
2. Generate updates for the ground terminal i at rate

$$\lambda_i^* = \frac{\pi_i^*}{1 + \sqrt{z_{ii}^* - \pi_i^*}},$$

Performance Bounds

Theorem

The peak and average age of the separation principle policy are bounded by

$$\frac{A^p}{A_{\mathcal{D}}^{p*}} \leq 4\mathcal{H} + 4\sqrt{\mathcal{H}} + 2 \text{ and } \frac{A^{ave}}{A_{\mathcal{D}}^{ave*}} \leq 8\mathcal{H} + 8\sqrt{\mathcal{H}} + 4,$$

where $\mathcal{H}$ is the mixing time of the randomized trajectory $\mathbf{P}^*$.

Key Idea Use FCFS Ber/G/1 queues with vacations to upper bound age processes

Age-Based Policy

Looking beyond random walks

Definition (Age-based trajectory)

In every time slot, the agent moves to the location that has the highest index of $A_i(t)$ among neighbors.

$$m(t+1) = \arg \max_{j:(i,j)\in E} w_j g\left(A_j(t)\right), \tag{2}$$

We set $g(a) = a^2 + a$, similar to the index policy in [Kadota, et al. 2016].

Age-Based Policy

In the **symmetric setting** where all weights are equal, we have

- the age-based policy follows a repeated depth first traversal of the graph G
- can be shown to be **factor-2 average age optimal** regardless of graph size or connectivity
- can be computed offline in polynomial time, since it is equivalent to a DFS traversal

Simulations

We show results for two families of graphs that are commonly used to model wireless sensor networks spread out geographically - random geometric graphs and grid graphs

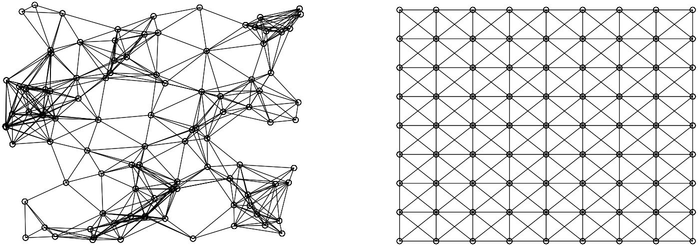


Figure: (a) A random geometric graph with 100 nodes, (b) A grid graph with 81 nodes and diagonal edges.

Numerical Results

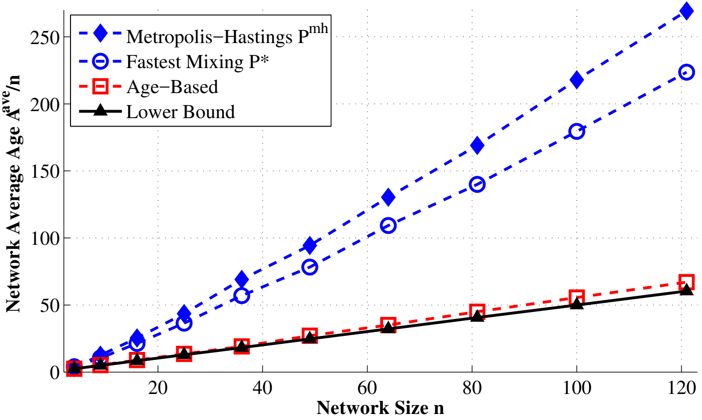


Figure: Information gathering on the grid graph: network average age as a function of network size n

Numerical Results

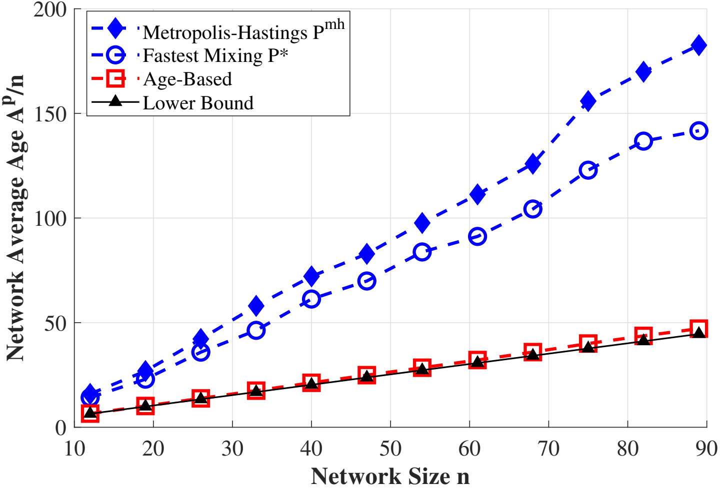


Figure: Information gathering problem in $\mathcal{G}(n, 2/\sqrt{n})$: network average age as a function of network size n

Conclusions

- Peak Age minimization on a graph is easy - can be done in polynomial time
- Average Age minimization is hard - we provide a heuristic
- The performance of the heuristic is better for families of fast mixing graphs
- Looking beyond random walks, we also show that a **greedy age-based trajectory is factor-2 optimal** in a symmetric setting
- Our results extend to Bernoulli queued updates instead of fresh updates

Thank You

Questions?

Supplementary Slides

Extensions

- Look at trajectories other than random walks, extend the age-based approach for the general weighted case
- Consider the case of multiple mobile agents simultaneously gathering or disseminating information
- Add distance weights to the edges of the mobility graph

Peak Age Minimization

Note that if the frequency of visits to a ground terminal i is f_i, then its peak age is given by -

$$A_i^{\mathrm{p}} = \frac{1}{f_i}$$

Thus, for a random walk with a stationary distribution $\vec{\pi}$, the weighted sum of peak ages is given by -

$$A^{\mathrm{p}} = \sum_{i \in V} \frac{w_i}{\pi_i}$$

Average Age Minimization

Let return times to terminal i be $H_{1,i}, H_{2,i}, \ldots$ Then, we have

$$A_i^{\text{ave}} = \lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} A_i(t)\right] = \frac{\mathbb{E}[H_{1,i}^2 + H_{1,i}]}{2\mathbb{E}[H_{1,i}]}.$$

For irreducible Markov chains, we know the following results hold

$$\mathbb{E}[H_{1,i}] = \frac{1}{\pi_i}, \forall i \in V \text{ and}$$

$$\mathbb{E}[H_{1,i}^2] = \frac{-1}{\pi_i} + \frac{2z_{ii}}{\pi_i^2},$$

Average Age Minimization

Finally, we have

$$A_i^{\text{ave}} = \frac{z_{ii}}{\pi_i}.$$

Now, define the quantity $\mathcal{Z} \triangleq \max_i \sum_j |z_{ij} - \pi_j|$.

We get the following upper bound

$$\sum_{i \in V} \frac{w_i z_{ii}}{\pi_i} \leq \sum_{i \in V} \left(\frac{w_i \mathcal{Z}}{\pi_i} + w_i \right) \tag{3}$$

Using $\mathcal{Z} \leq 4\mathcal{H}$ (from [Ailon, et al. 2006]), we get the required result.

Average Age Minimization

Theorem

The fastest mixing randomized trajectory can be found by solving the following convex optimization problem:

$$\begin{aligned}
\underset{\mathbf{P}}{\text{Minimize}} \quad & \mu(\mathbf{P}) = ||\mathbf{P} - \Pi^*||_2, \\
\text{subject to} \quad & P_{i,j} \geq 0, \ \forall (i,j), \\
& \mathbf{P1} = \mathbf{1}, \\
& \pi^* \mathbf{P} = \pi^*, \quad \Pi^*_{i,j} = \pi^*_i \ \forall \ i,j \in V, \\
& P_{i,j} = 0, \forall (i,j) \notin E.
\end{aligned} \tag{4}$$

Here $||A||_2$ *denotes the spectral norm of matrix* A *and* $\pi^*_i = \frac{\sqrt{w_i}}{\sum_{j \in V} \sqrt{w_j}}, \ \forall i \in V.$