Robust transient stability assessment of renewable power grids†

Thanh Long Vu, *Member, IEEE,* and Konstantin Turitsyn, *Member, IEEE*

***Abstract*— Large scale renewable generation is increasingly installed into power grids all over the world in an effort to reduce CO2 from electricity sector. Yet, the inherent intermittent nature of renewable generations, such as wind and solar, introduces high uncertainty into system operation and may compromise the grid stability. As such, stability assessment of power grid with high penetration of renewables is an important issue. Due to the renewable generations uncertainty, the transient stability of renewable power grid can be assessed by simulating power systems dynamics with different level of renewable generations, which leads to highly computational cost. In this paper, we present a robust stability certificate that can rigorously guarantee the grids stability with respect to the variation in power injections. Interestingly, quadratic Lyapunov function approach is presented to transient stability assessment, offering real-time construction of stability certificates. The effectiveness of the proposed techniques is numerically illustrated on a number of IEEE test cases.**

I. Introduction

The electric power grids are experiencing a historical paradigm shift with the installment of large-scale inertia-less renewable generations. The inherently intermittent nature of renewable generations makes it difficult to balance the power demand and supply, which is necessary to the reliable operation of existing power grids. Moreover, the uncontrollability of inertia-less renewable generators challenges the operators in maintaining the power systems stability. Therefore, the stability assessment of renewable power systems is an important problem that needs to be carefully addressed when there is high penetration of renewable generations in modern power grids.

Looking in more details, we can see that the operating point of a renewable power system is constantly moving in an unpredictable way because of the intermittent renewable generations, together with changing loads, external disturbances, and real-time clearing of electricity markets. In normal practice, to ensure power system security, the operators have to repeat the security and stability assessment approximately every 15 minutes. This cause enormous computing resources due to the high complexity and large scale of power systems. As such, we aim at a robust transient stability assessment toolbox that can certify stability of power systems in the presence of some uncertainty in power injections.

In particular, this paper will present a robust transient stability certificate that can guarantee the stability of post-fault power systems with respect to a set of unknown equilibrium points. It is worth to note that this stability problem is unusual from the control theory point of view, since most of the existing stability analysis techniques in control theory implicitly assume that the equilibrium point is known exactly. Also, to the best of our knowledge, only few approaches have analyzed the influences of uncertainty onto power systems dynamics, which are based on time-domain simulations [1], [2] and moment computation [3].

Mathematically, the proposed robust stability assessment approach for power systems is based on quadratic Lyapunov functions. Existing approaches to this problem are based on energy function [4] and Lur'e-Postnikov type Lyapunov function [5]–[7], both of which are nonlinear non-quadratic and generally non-convex functions. The convexity of quadratic Lyapunov functions enables the real-time construction of the stability certificate and real-time stability assessment. This is an advancement compared to the energy function based methods, where computing the critical UEP for stability analysis is generally an NP-hard problem.

The paper is structured as follows. In Section II, we present the standard structure-preserving model of power systems, and formulate the robust stability problem of power grids which involves the uncertainty in the equilibrium points due to renewable intermittency. Section III introduces the quadratic Lyapunov functions-based approach for the construction of a robust stability certificate for renewable power systems stability assessment. Finally, Section IV illustrates the effectiveness of this certificate through numerical simulations.

II. System Model and Problem Formulation

A. Power System Model

In this paper, we consider the case when the buses' phasor angles may significantly fluctuate but the buses' voltages are still well-supported. For such situations, we utilize the standard structure-preserving model to describe the post-fault dynamics of generators and loads [8]. Mathematically, the gird is described by an undirected graph $\mathcal{A}(\mathcal{N},\mathcal{E})$, where $\mathcal{N}=\{1,2,\ldots,|\mathcal{N}|\}$ is the set of buses and $\mathcal{E}\subseteq\mathcal{N}\times\mathcal{N}$ is the set of transmission lines connecting those buses. Here, $|A|$ denotes the number of elements in set A. The sets of generator buses and load buses are denoted by $\mathcal{G}$ and $\mathcal{L}$. We assume that the grid is lossless with constant voltage magnitudes $V_k, k\in\mathcal{N}$, and the reactive powers are ignored. Then, the structure-preserving model of the system is given

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by:

$$m_k\ddot{\delta}_k + d_k\dot{\delta}_k + \sum_{j\in\mathcal{N}_k} a_{kj}\sin(\delta_k-\delta_j) = P_{m_k}, k\in\mathcal{G}, \quad (1a)$$

$$d_k\dot{\delta}_k + \sum_{j\in\mathcal{N}_k} a_{kj}\sin(\delta_k-\delta_j) = -P^0_{d_k}, k\in\mathcal{L}, \quad (1b)$$

where equation (1a) represents the dynamics at generator buses and equation (1b) the dynamics at load buses. In these equations, with $k \in \mathcal{G}$, then $m_k > 0$ is the generator's dimensionless moment of inertia, $d_k > 0$ is the term representing primary frequency controller action on the governor, and P_{m_k} is the input shaft power producing the mechanical torque acting on the rotor of the k^{th} generator. With $k \in \mathcal{L}$, then $d_k > 0$ is the constant frequency coefficient of load and $P^0_{d_k}$ is the nominal load. Here, $a_{kj} = V_k V_j B_{kj}$, where B_{kj} is the (normalized) susceptance of the transmission line $\{k,j\}$ connecting the k^{th} bus and j^{th} bus, $\mathcal{N}_k$ is the set of neighboring buses of the k^{th} bus. Note that, the system described by equation (1) has many stationary points δ^*_k that are characterized, however, by the angle differences $\delta^*_{kj} = \delta^*_k - \delta^*_j$ (for a given P_k) that solve the following system of power flow-like equations:

$$\sum_{j\in\mathcal{N}_k} a_{kj}\sin(\delta^*_{kj}) = P_k, k\in\mathcal{N}, \quad (2)$$

where $P_k = P_{m_k}, k \in \mathcal{G}$, and $P_k = -P^0_{d_k}, k \in \mathcal{L}$.

We assume that there is a solution δ^* of equations (2) satisfying $|\delta^*_{kj}| \leq \gamma < \pi/2, \forall\{k,j\} \in \mathcal{E}$. In most of power systems test cases and practical power systems, this assumption holds true if the power injection satisfy the following condition:

$$\|L^\dagger p\|_{\mathcal{E},\infty} \leq \sin\gamma. \quad (3)$$

Here, $p = [P_1, ..., P_{|\mathcal{N}|}]^T$ is power injection, $\|x\|_{\mathcal{E},\infty} = \max_{\{i,j\}\in\mathcal{E}} |x(i) - x(j)|$, and $L^\dagger$ is the pseudoinverse of the network Laplacian matrix.

It should be noted that, there are many solutions of (2) due to the nonlinear power flows. Accordingly there is may equilibrium points with their own region of attraction. Therefore, it is difficult to analyze the stability region of the stable equilibrium point δ^*.

B. Robust Stability Assessment Problem

In contingency screening for transient stability, we consider three types of dynamics of power systems, namely pre-fault dynamics, fault-on dynamics and post-fault dynamics. In normal conditions, a power grid operates at a stable equilibrium point of the pre-fault dynamics. Following some contingencies, the system evolves according to the fault-on dynamics laws and moves away from the pre-fault equilibrium point to a fault-cleared state δ_0. After the fault is cleared, the system evolves according to the post-fault dynamics. The transient stability assessment problem addresses the question of whether the post-fault dynamics converges from the fault-cleared state to a post-fault stable equilibrium point δ^*_{post}. Figure 1 shows the transient stability of the post-fault dynamics originated from the fault-cleared states to the stable post-fault equilibrium.

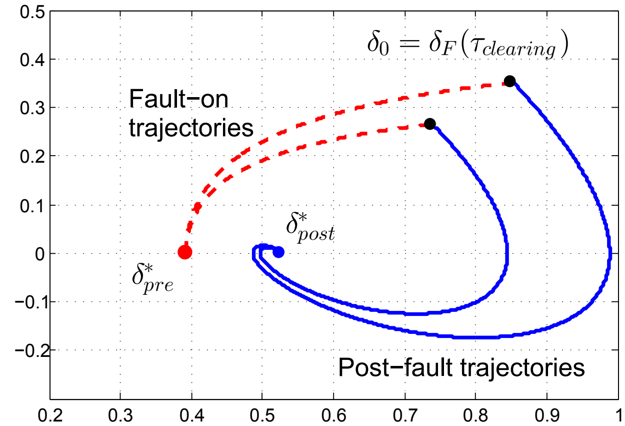

Fig. 1. Convergence of the post-fault dynamics from two different fault-cleared states $\delta_F(\tau_{clearing})$, which are obtained from two different fault-on dynamics at the clearing times $\tau_{clearing}$, to the post-fault equilibrium point δ^*_{post}.

The robust transient stability problem involves situations where there is uncertainty in power injections P_k, the sources of which are intermittent renewable generations and varying power consumptions. Particularly, while the parameters m_k, d_k are fixed and known, the power generations P_{m_k} and load consumption $P^0_{d_k}$ are changing in time. As such, the post-fault equilibrium δ^*_{post} defined by (2) also variates. This raises the need for a robust stability certificate that can certify stability of post-fault dynamics with respect to a set of equilibria. When the power injections P_k change in each transient stability assessment cycle, such a robust stability certificate can be repeatedly utilized in the "offline" certification of system stability, eliminating the need for assessing stability on a regular basis. Formally, we consider the following robust stability problem:

(P) Robust stability w.r.t. a set of unknown equilibria: *Determine if a given fault-cleared state* δ_0 *is stable when the equilibrium point is unknown and belongs to the set* $\Delta(\gamma) = \{delta^* : |\delta^*_{kj}| \leq \gamma < \pi/2, \forall\{k,j\} \in \mathcal{E}\}$.

In the next section, we utilize tools from nonlinear control theory to resolve this problem. Denote the stable post-fault equilibrium point as δ^*. Consider the state vector $x = [x_1, x_2, x_3]^T$, which is composed of the vector of generator's angle deviations from equilibrium $x_1 = [\delta_1 - \delta^*_1, \ldots, \delta_{|\mathcal{G}|} - \delta^*_{|\mathcal{G}|}]^T$, their angular velocities $x_2 = [\dot{\delta}_1, \ldots, \dot{\delta}_{|\mathcal{G}|}]^T$, and vector of load buses' angle deviation from equilibrium $x_3 = [\delta_{|\mathcal{G}|+1} - \delta^*_{|\mathcal{G}|+1}, \ldots, \delta_{|\mathcal{N}|} - \delta^*_{|\mathcal{N}|}]^T$. Let E be the incidence matrix of the graph $\mathcal{G}(\mathcal{N},\mathcal{E})$, so that $E[\delta_1, \ldots, \delta_{|\mathcal{N}|}]^T = [(\delta_k - \delta_j)_{\{k,j\}\in\mathcal{E}}]^T$. Let the matrix C be $E[I_{m\times m}\ O_{m\times n}; O_{(n-m)\times 2m}\ I_{(n-m)\times(n-m)}]$. Then

$$Cx = E[\delta_1 - \delta^*_1, \ldots, \delta_{|\mathcal{N}|} - \delta^*_{|\mathcal{N}|}]^T = [(\delta_{kj} - \delta^*_{kj})_{\{k,j\}\in\mathcal{E}}]^T.$$

Consider the vector of nonlinear interactions F in the simple trigonometric form: $F(Cx) =$

$[(\sin\delta_{kj} - \sin\delta^*_{kj})_{\{k,j\}\in\mathcal{E}}]^T$. Denote the matrices of moment of inertia, frequency controller action on governor, and frequency coefficient of load as $M_1 = \mathrm{diag}(m_1,\dots,m_{|\mathcal{G}|}), D_1 = \mathrm{diag}(d_1,\dots,d_{|\mathcal{G}|})$ and $M = \mathrm{diag}(m_1,\dots,m_{|\mathcal{G}|},d_{|\mathcal{G}|+1},\dots,d_{|\mathcal{N}|})$.

Therefore, the power system (1) is equivalently represented as:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= M_1^{-1}D_1x_2 - S_1M^{-1}E^TSF(Cx)\\ \dot{x}_3 &= -S_2M^{-1}E^TSF(Cx)\end{aligned} \tag{4}$$

where $S = \mathrm{diag}(a_{kj})_{\{k,j\}\in\mathcal{E}}, S_1 = [I_{m\times m} \quad O_{m\times n-m}], S_2 = [O_{n-m\times m} \quad I_{n-m\times n-m}], n = |\mathcal{N}|, m = |\mathcal{G}|$. Accordingly, we have

$$\dot{x} = Ax - BF(Cx), \tag{5}$$

where

$$A = \begin{bmatrix} O_{m\times m} & I_{m\times m} & O_{m\times n-m}\\ O_{m\times m} & -M_1^{-1}D_1 & O_{m\times n-m}\\ O_{n-m\times m} & O_{n-m\times m} & O_{n-m\times n-m}\end{bmatrix},$$

and

$$B = \begin{bmatrix} O_{m\times|\mathcal{E}|}; & S_1M^{-1}E^TS; & S_2M^{-1}E^TS \end{bmatrix}.$$

III. Quadratic Lyapunov Function-based Robust Stability Certificate

In this section, we introduce the robust stability certificate to address the problem **(P)** by utilizing quadratic Lyapunov functions. The construction of these quadratic Lyapunov functions is based on exploiting the strict bounds of the nonlinear vector F in a region surrounding the equilibrium point and solving a linear matrix inequality (LMI).

Firstly, we establish the strict bound of the nonlinearity of the system in regions surrounding the equilibrium point. Indeed, Figure 2 demonstrates a possible bound of the nonlinear interactions $(\sin\delta_{kj} - \sin\delta^*_{kj})$ by the linear functions of angular difference $(\delta_{kj} - \delta^*_{kj})$. We can see from Fig. 2 that for any $\delta_{kj} = \delta_k - \delta_j$ such that $|\delta_{kj}| \le \pi/2$, we have:

$$g_{kj}(\delta_{kj} - \delta^*_{kj})^2 \le (\delta_{kj} - \delta^*_{kj})(\sin\delta_{kj} - \sin\delta^*_{kj}) \le (\delta_{kj} - \delta^*_{kj})^2 \tag{6}$$

where

$$g_{kj} = \min\{\frac{1-\sin\delta^*_{kj}}{\pi/2-\delta^*_{kj}}, \frac{1+\sin\delta^*_{kj}}{\pi/2+\delta^*_{kj}}\} = \frac{1-\sin|\delta^*_{kj}|}{\pi/2-|\delta^*_{kj}|} \tag{7}$$

As the function $(1-\sin t)/(\pi/2-t)$ is decreasing on $[0,\pi/2]$, it holds that

$$g_{kj} \ge \frac{1-\sin\lambda(\delta^*)}{\pi/2-\lambda(\delta^*)} := g > 0 \tag{8}$$

where $\lambda(\delta^*) = \max_{\{k,j\}\in\mathcal{E}}|\delta^*_{kj}| \le \gamma < \pi/2$. Hence, in the polytope $\mathcal{P}$, defined by inequalities $|\delta_{kj}| \le \pi/2$, the nonlinearities F are bounded by:

$$\big(F(Cx) - gCx\big)^T\big(F(Cx) - Cx\big) \le 0, \forall x \in \mathcal{P}. \tag{9}$$

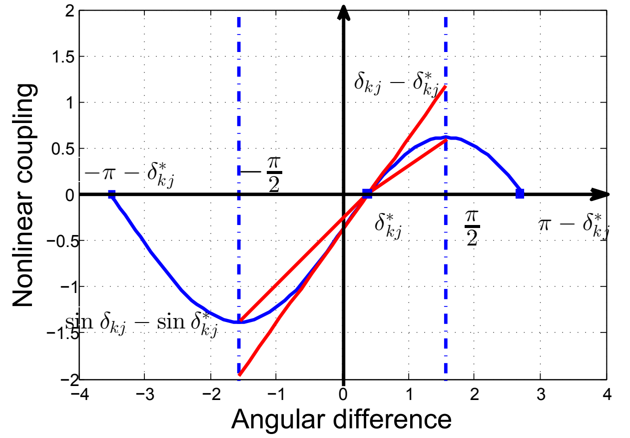

Fig. 2. Strict bounds of nonlinear sinusoidal couplings $(\sin\delta_{kj} - \sin\delta^*_{kj})$ by two linear functions of the angular difference δ_{kj}

Exploiting this strict bound of the nonlinearities F, we can obtain the quadratic Lyapunov function certifying convergence of the structure-preserving model (1) as follows.

Lemma 1: Consider power grids described by the structure-preserving model (1) *and satisfying Assumption 1. Assume that for given matrices A, B, C, there exists a positive definite matrix P of size $(|\mathcal{N}| + |\mathcal{G}|)$ such that*

$$\begin{aligned}&(A - \frac{1}{2}(1+g)BC)^TP + P(A - \frac{1}{2}(1+g)BC)\\ &+ PBB^TP + \frac{(1-g)^2}{4}C^TC \le 0\end{aligned} \tag{10}$$

or equivalently (by Schur complement) satisfying the LMI

$$\begin{bmatrix} \bar{A}^TP + P\bar{A} + \frac{(1-g)^2}{4}C^TC & PB\\ B^TP & -I\end{bmatrix} \le 0 \tag{11}$$

where $\bar{A} = A - \frac{1}{2}(1+g)BC$. Then, along (1)*, the Lyapunov function $V(x(t))$ is decreasing whenever $x(t) \in \mathcal{P}$.*

The proof of this result is in [10] and omitted here. It can be seen that the matrix P obtained by solving the LMI (11) depends on matrices A, B, C and the gain g. Matrices A, B, C do not depend on the parameters P_k in the structure preserving model (1). Hence, we have a common triple of matrices A, B, C for all the equilibrium point δ^* in the set $\Delta(\gamma)$. Also, whenever $\delta^* \in \Delta(\gamma)$, we can replace g in (8) by the lower bound of g as $g = \dfrac{1-\sin\gamma}{\pi/2-\gamma} > 0$. This lower bound also does not depend on the equilibrium point δ^* at all. Then, the matrix P is independent of the set $\Delta(\gamma)$ of stable equilibrium points δ^*. Therefore, Lemma 1 provides us with a common quadratic Lyapunov function for any post-fault dynamics with post-fault equilibrium point $\delta^* \in \Delta(\gamma)$. In the next section, we present the transient stability certificate based on this quadratic Lyapunov function.

A. Transient Stability Certificate

Note that the Lyapunov function $V(x)$ considered in Lemma 1 is decreasing whenever the system trajectory evolves inside polytope $\mathcal{P}$. However, outside polytope $\mathcal{P}$, the Lyapunov function may increase. Here, we construct inside polytope $\mathcal{P}$ an invariant set $\mathcal{R}$ of the post-fault dynamics described by structure-preserving system (1). Then, from any point inside this invariant set $\mathcal{R}$, the post-fault dynamics (1) will only evolve inside $\mathcal{R}$ and eventually converge to the equilibrium point due to the decrease of the Lyapunov function $V(x)$.

Indeed, for each edge $\{k, j\}$ connecting the generator buses k and j, we divide the boundary $\partial\mathcal{P}_{kj}$ of $\mathcal{P}$ corresponding to the equality $|\delta_{kj}| = \pi/2$ into two subsets $\partial\mathcal{P}_{kj}^{in}$ and $\partial\mathcal{P}_{kj}^{out}$. The flow-in boundary segment $\partial\mathcal{P}_{kj}^{in}$ is defined by $|\delta_{kj}| = \pi/2$ and $\delta_{kj}\dot{\delta}_{kj} < 0$, while the flow-out boundary segment $\partial\mathcal{P}_{kj}^{out}$ is defined by $|\delta_{kj}| = \pi/2$ and $\delta_{kj}\dot{\delta}_{kj} \geq 0$. Since the derivative of δ_{kj}^2 at every points on $\partial\mathcal{P}_{kj}^{in}$ is negative, the system trajectory of (1) can only go inside $\mathcal{P}$ once it meets $\partial\mathcal{P}_{kj}^{in}$.

Define the following minimum value of the Lyapunov function $V(x)$ over the flow-out boundary $\partial\mathcal{P}^{out}$ as:

$$V_{\min} = \min_{x \in \partial\mathcal{P}^{out}} V(x), \tag{12}$$

where $\partial\mathcal{P}^{out}$ is the flow-out boundary of the polytope $\mathcal{P}$ that is the union of $\partial\mathcal{P}_{kj}^{out}$ over all the transmission lines $\{k, j\} \in \mathcal{E}$ connecting generator buses. Due to the decrease of $V(x)$ inside polytope $\mathcal{P}$, we obtain the following center result regarding transient stability assessment, the proof of which can be found in [10].

Theorem 1: For a post-fault equilibrium point $\delta^* \in \Delta(\gamma)$, from any initial state x_0 staying in set $\mathcal{R}$ defined by

$$\mathcal{R} = \{x \in \mathcal{P} : V(x) < V_{\min}\}, \tag{13}$$

then, the system trajectory of (1) *will only evolve in the set $\mathcal{R}$ and eventually converge to the stable equilibrium point δ^*.*

Theorem 1 allows us to determine if the the fault-cleared state x_0 of the post-fault dynamics will converge to the equilibrium point. If $x_0 \in \mathcal{R}$, i.e. if $x_0 \in \mathcal{P}$ and $V(x_0) < V_{\min}$, then it is guaranteed that the post-fault dynamics is stable. Otherwise, there is no conclusion for the stability or instability of the post-fault dynamics, and the system stability should be assessed by detailed simulations.

Since the Lyapunov function $V(x)$ is convex, it is possible to quickly find the minimum value $V_{\min} = \min_{x \in \partial\mathcal{P}^{out}} V(x)$. Hence, we can perform the transient stability assessment in the real time. This is an remarkable advantage of the proposed approach in comparison to the typically non-convex energy functions and Lur'e-Postnikov type Lyapunov functions.

Note that the proposed transient stability certificate allows us to assess the transient stability of post-fault dynamics whenever the fault-cleared state stays inside polytope $\mathcal{P}$. In practice, polytope $\mathcal{P}$ contains almost all practically interesting configurations. This is due to the fact that outside polytope $\mathcal{P}$ high differences in voltage phasor angles will trigger out protective relay equipment. This situation should be analyzed individually with more realistic models by time-domain simulations.

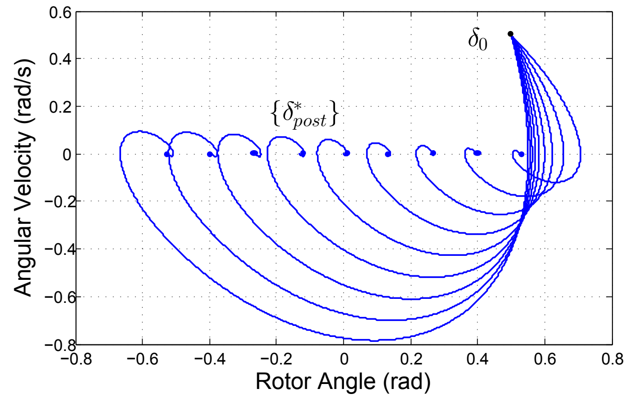

Fig. 3. Robust transient stability of the post-fault dynamics originated from the fault-cleared state $\delta_0 = [0.5 \quad 0.5]^T$ to the set of stable equilibrium points $\Delta(\pi/6) = \{\delta^*_{post} = [\delta^*, 0]^T : -\pi/6 \leq \delta^* \leq \pi/6\}$.

B. Robust Stability w.r.t. Power Injection Variations

This section present a robust stability certificate that can certify transient stability of the post-fault dynamics with respect to power injection variations. In particular, consider the system whose stable equilibrium point belongs to the set $\Delta(\gamma)$. As such, if the varying power injections P_k satisfy the synchronization condition (3), we can apply such stability certificate, while we do not need to know the equilibrium point exactly. This helps us avoid repeating the time and resource-consuming stability assessment on a regular basis, as currently dominating in practice.

Denote $\delta = [\delta_1, ..., \delta_{|\mathcal{G}|}, \dot{\delta}_1, ..., \dot{\delta}_{|\mathcal{G}|}, \delta_{|\mathcal{G}|+1}, ..., \delta_{|\mathcal{N}|}]$. The system state x and the fault-cleared state x_0 can be then presented as $x = \delta - \delta^*$ and $x_0 = \delta_0 - \delta^*$. Note that LMI (11) is independent on the equilibrium point δ^*. Employing this observation, we obtain the following robust stability certificate of renewable power systems, the proof of which can be found in [10].

Theorem 2: Consider the post-fault dynamics (1) *with uncertain stable equilibrium point δ^* that satisfies $\delta^* \in \Delta(\gamma)$. Consider a fault-cleared state $\delta_0 \in \mathcal{P}$, i.e., $|\delta_{0_{kj}}| \leq \pi/2, \forall\{k, j\} \in \mathcal{E}$. Suppose that there exists a positive definite matrix P of size* $(|\mathcal{N}| + |\mathcal{G}|)$ *satisfying the LMI* (11) *and*

$$\delta_0^T P \delta_0 < \min_{\delta \in \partial\mathcal{P}^{out}, \delta^* \in \Delta(\gamma)} \left(\delta^T P \delta - 2\delta^{*T} P(\delta - \delta_0)\right) \tag{14}$$

Then, the system (1) *will converge from the fault-cleared state δ_0 to the equilibrium point δ^* for any $\delta^* \in \Delta(\gamma)$.*

Applying Theorem 2, we can assess the transient stability of the system when the power injections P_k variates as follows. First, the synchronization condition (3) is checked. If it is satisfied, the equilibrium point δ^* is in the set $\Delta(\gamma)$. Second, find a positive definite matrix solution P of LMI (11) where $g = (1 - \sin\gamma)/(\pi/2 - \gamma)$. Last, we check inequality (14) for the given fault-cleared state δ_0 staying inside polytope $\mathcal{P}$. If it is satisfied, we conclude that the post-fault dynamics (1) is stable, i.e., it will converge from

δ_0 to the equilibrium point δ^* regardless of the variations in power injections. If not, we should repeat the second step to find other positive definite matrix solution P as well as checking condition (14).

It is worth to note that there may exist many solutions P of the LMI (11). This flexibility can be employed in which we utilize a heuristic algorithm as in [7] to find the best suitable matrix P for certifying stability of power system at a given fault-cleared state δ_0.

In practice, to reduce the conservativeness and computational time in the assessment process, we can off-line compute the common matrix P for any equilibrium point $\delta^* \in \Delta(\gamma)$ and check on-line the condition $V(x_0) < V_{\min}$ with the data (initial state x_0 and power injections P_k) obtained on-line. In some case the initial state can be predicted before hand, and if there exists a positive definite matrix P satisfying the LMI (11) and the inequality (14), then the online assessment is reduced to just checking condition (3) for the power injections P_k.

IV. Numerical Illustrations

To illustrate the effectiveness of the proposed robust stability certificate, we consider two IEEE test case: 2-bus system and 118-bus system. The first one helps us easy to visualize the results, while the second can show the applicability of the proposed approach to large scale power systems.

A. 2-Bus System

For illustration purpose, this section presents the simulation results on the most simple 2-bus power system, described by the single 2-nd order differential equation

$$m\ddot{\delta} + d\dot{\delta} + a \sin \delta - p = 0. \tag{15}$$

For numerical simulations, we choose $m = 0.1$ p.u., $d = 0.15$ p.u., $a = 0.2$ p.u. When the parameters p changes from -0.1 p.u. to 0.1 p.u., the stable equilibrium point δ^* (i.e. $[\delta^* \ 0]^T$) of the system belongs to the set: $\Delta = \{\delta^* : |\delta^*| \leq \arcsin(0.1/0.2) = \pi/6\}$. For the given fault-cleared state $\delta_0 = [0.5 \ \ 0.5]$, using the CVX software we obtain a positive matrix P satisfying the LMI (11) and the condition for robust stability (14) as $P = [0.8228 \ \ 0.1402; 0.1402 \ \ 0.5797]$. The simulations confirm this result. We can see in Fig. 3 that from the fault-cleared state δ_0 the post-fault trajectory always converges to the equilibrium point δ^* for all $\delta^* \in \Delta(\pi/6)$. Figure 4 shows the convergence of the quadratic Lyapunov function to 0.

B. 118 Bus System

Our test system in this section is the modified IEEE 118-bus test case [11], of which 54 are generator buses and the other 64 are load buses as showed in Fig. 5. The data is taken directly from the test files [11], otherwise specified. The damping and inertia are not given in the test files and thus are randomly selected in the following ranges: $m_i \in [2, 4], \forall i \in \mathcal{G}$, and $d_i \in [1, 2], \forall i \in \mathcal{N}$. The grid originally contains 186 transmission lines. We

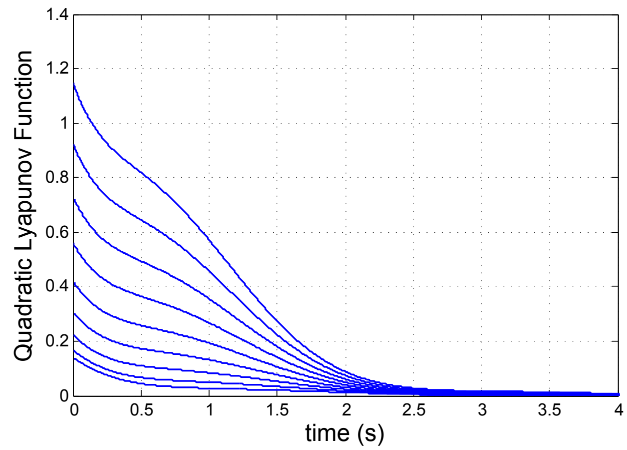

Fig. 4. Convergence of the quadratic Lyapunov function $V(x) = x^T P x = (\delta - \delta^*)^T P(\delta - \delta^*)$ from the initial value to 0 when the equilibrium point δ^* varies in the set $\Delta(\pi/6) = \{\delta^*_{post} = [\delta^*, 0]^T : -\pi/6 \leq \delta^* \leq \pi/6\}$.

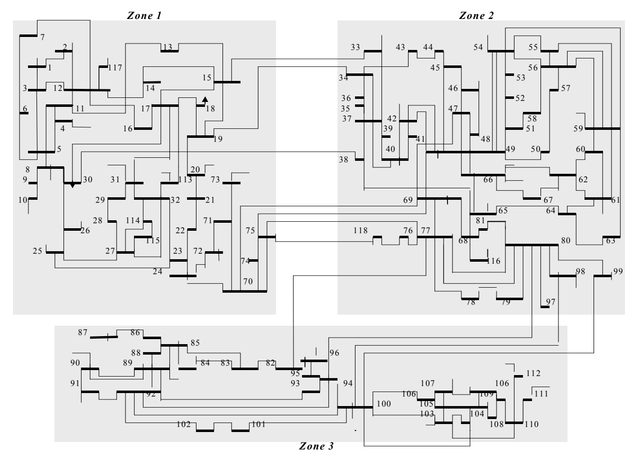

Fig. 5. IEEE 118-bus test case

eliminate 9 lines whose susceptance is zero, and combine 7 lines $\{42, 49\}, \{49, 54\}, \{56, 59\}, \{49, 66\}, \{77, 80\}, \{89, 90\}$, and $\{89, 92\}$, each of which contains double transmission lines as in the test files [11]. Hence, the grid is reduced to 170 transmission lines connecting 118 buses. We renumber the generator buses as $1 - 54$ and load buses as $55 - 118$.

We assume that there are varying generations (possibly due to renewable) at 16 buses $1 - 16$ (i.e. 30% generator buses are varying). The system is initially at the equilibrium point given in [11], but the variations in the renewable generations make the operating condition to change. We want to assess if the system will transiently evolve from the initial state to the new equilibrium points. To make our proposed robust stability assessment framework valid, we assume that the renewable generators have the similar dynamics with the conventional generator but with the varying power output. This happens when we equip renewable generators with synchronverter [12], which will control the dynamics of renewables to mimic the dynamics of conventional generators. Using the CVX software with Mosek solver, we can

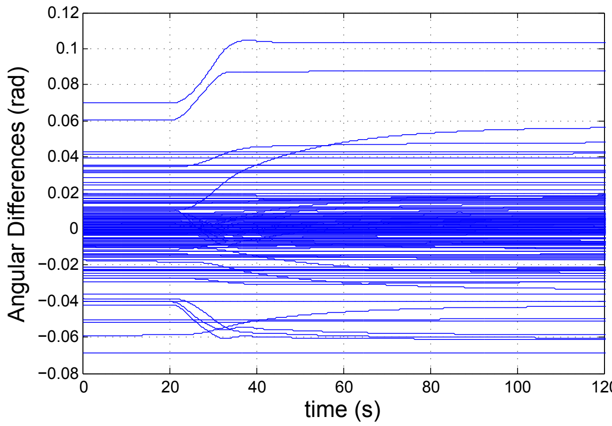

Fig. 6. Transition of the 118-bus system from the old equilibrium to the new equilibrium when the renewable generations increase 50% in the period $[20s, 30s]$

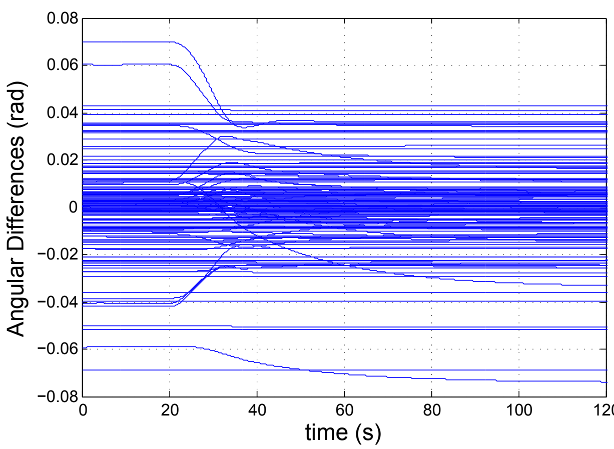

Fig. 7. Transition of the 118-bus system from the old equilibrium to the new equilibrium when the renewable generations decrease 50% in the period $[20s, 30s]$

see that there exists positive definite matrix P satisfying the LMI (11) and the inequality (14) with $\gamma = \pi/12$. As such, the grid will transiently evolve from the initial state to any new equilibrium point in the set $\Delta(\pi/12)$. To demonstrate this result by simulation, we assume that in the time period $[20s, 30s]$, the power outputs of the renewable generators increase 50%. Since the synchronization condition $\|L^\dagger p\|_{\mathcal{E},\infty} = 0.1039 < \sin(\pi/12)$ holds true, we can conclude that the new equilibrium point, obtained when the renewable generations increased 50%, will stay in the set $\Delta(\pi/12)$. From Fig. 6, we can see that the grid transits from the old equilibrium point to the new equilibrium point when the renewable power outputs increase. Similarly, if in the time period $[20s, 30s]$ the power outputs of the renewable generators decrease 50%, then we can check that $\|L^\dagger p\|_{\mathcal{E},\infty} = 0.0762 < \sin(\pi/12)$. Therefore by the robust stability certificate, we conclude that the grid evolves from the old equilibrium point to the new equilibrium point, as confirmed in Fig. 7.

V. Conclusions

In this paper, we formulated the robust transient stability problem for nonlinear power grids. This problem arises due to the uncertainty in power injections, and hence, involving the stability assessment of a power grid at a given fault-cleared state when post-fault equilibrium point is unknown. To solve this problem, we introduced a robust stability certificate that can guarantee the stability of a fault-cleared state with respect to a set of equilibrium points. The proposed robust stability certificate can help system operators screen contingencies in multiple power injection profiles, without relying on computationally wasteful real-time simulations. Interestingly, the proposed robust stability certificate is constructed by utilizing convex quadratic Lyapunov functions, which could allow us to perform the stability assessment in the real time. Multiple IEEE test cases were tested to demonstrate the effectiveness of the proposed stability certificate to large-scale power systems.

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References

[1] I. A. Hiskens and J. Alseddiqui, "Sensitivity, approximation, and uncertainty in power system dynamic simulation," *Power Systems, IEEE Transactions on*, vol. 21, no. 4, pp. 1808–1820, 2006.

[2] Z. Y. Dong, J. H. Zhao, and D. J. Hill, "Numerical simulation for stochastic transient stability assessment," *Power Systems, IEEE Transactions on*, vol. 27, no. 4, pp. 1741–1749, 2012.

[3] S. V. Dhople, Y. C. Chen, L. DeVille, and A. D. Domínguez-García, "Analysis of power system dynamics subject to stochastic power injections," *Circuits and Systems I: Regular Papers, IEEE Transactions on*, vol. 60, no. 12, pp. 3341–3353, 2013.

[4] H.-D. Chiang, *Direct Methods for Stability Analysis of Electric Power Systems*, ser. Theoretical Foundation, BCU Methodologies, and Applications. Hoboken, NJ, USA: John Wiley & Sons, Mar. 2011.

[5] D. J. Hill and C. N. Chong, "Lyapunov functions of Lur'e-Postnikov form for structure preserving models of power systems," *Automatica*, vol. 25, no. 3, pp. 453–460, May 1989.

[6] R. Davy and I. A. Hiskens, "Lyapunov functions for multi-machine power systems with dynamic loads," *Circuits and Systems I: Fundamental Theory and Applications, IEEE Transactions on*, vol. 44, no. 9, pp. 796–812, 1997.

[7] T. L. Vu and K. Turitsyn, "Lyapunov functions family approach to transient stability assessment," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1269–1277, March 2016.

[8] A. R. Bergen and D. J. Hill, "A structure preserving model for power system stability analysis," *Power Apparatus and Systems, IEEE Transactions on*, no. 1, pp. 25–35, 1981.

[9] F. Dorfler, M. Chertkov, and F. Bullo, "Synchronization in complex oscillator networks and smart grids," *Proceedings of the National Academy of Sciences*, vol. 110, no. 6, pp. 2005–2010, 2013.

[10] T. L. Vu and K. Turitsyn, "A framework for robust assessment of power grid stability and resiliency," *IEEE Transactions on Automatic Control*, vol. PP, no. 99, pp. 1–1, 2016.

[11] https://www.ee.washington.edu/research/pstca/pf118/pg_tca118bus.htm.

[12] Qing-Chang Zhong and Weiss, G., "Synchronverters: Inverters That Mimic Synchronous Generators," *Industrial Electronics, IEEE Transactions on*, vol. 58, no. 4, pp. 1259–1267, April 2011.