Options for emergency control of power grids with high penetration of renewables

Thanh Long Vu, *Member, IEEE*, Konstantin Turitsyn, *Member, IEEE*

***Abstract*— Power systems are experiencing a historical transformation in their paradigm and architecture. As such, the classical methods ensuring secure operation of power grids, which are largely developed several decades ago, need to be reassessed. At the same time, new generations of smart electronic devices provide fast actuation to smart power grids. Also, transmission resources are continuously installed into the system and will be ubiquitously available in the future. These assets provide new opportunities for power grids' operation controls. In this paper, we sketches the ideas of viable alternatives for traditional remedial actions of power grids with high penetration of renewables, in which the renewables are integrated with synchronverters to mimic the dynamics of conventional generators and transmission lines are equipped with FACTS devices. In these novel emergency control schemes, the power electronics resources are exploited to control the inertia/damping of the imitated generators as well as the susceptances of transmission lines in order to quickly stabilize the power system under emergency situations. The proposed controls are designed by solving convex optimization problems, making them scalable to large scale power grids. Also, these emergency control schemes can be implemented with minor investment by exploiting the plentiful transmission facilities ubiquitously available on the existing power grids.**

Keywords: Renewable, emergency control, transient stability

I. INTRODUCTION

Renewable generations, such as wind and solar, are increasingly installed into power grids all over the world in an effort to reduce CO2 emissions from the electricity generation sector. Yet, their natural intermittency introduces high uncertainty that can compromise the grid's stability, while the low inertia of renewable generators challenges the grid's controllability. As a result, the next-generation power grids will be increasingly vulnerable to unfavorable weather conditions and component failures, which can eventually lead to major outages. Therefore, critical/emergency states of power grids will appear frequently, and thus, emergency control, i.e., the action to recover the stability of a power grid when a critical situation is detected, should be paid serious attention.

Although the existing emergency controls of power grids, such as remedial actions, special protection schemes (SPS), and load shedding [1]–[4], make current power grids reasonably stable to disturbances, their drawbacks are twofold. First, some of these emergency actions rely on interrupting electrical service to customers and result in huge economic cost due to power interruptions, e.g. about \$79 billion a year in the US [5]. Second, protective devices are usually only effective for individual elements, but less effective in preventing the whole grid from collapse. Recent major blackouts exhibit the inability of operators to prevent grid from cascading failures [6], regardless of the good performance of individual protective devices. The underlying reason is the lack of coordination among protective devices, which makes them incapable of maintaining the stability of the whole grid. These drawbacks call for system-level, cost-effective solutions to the emergency control of power grids.

On the other hand, new generations of smart electronic devices provide fast actuation to smart power grids. Also, transmission resources are continuously installed into the system and will be ubiquitously available in the future. Motivated by the aforementioned observations, this paper aims to extract more value out of the existing and future fast-acting electronic resources and transmission facilities to quickly stabilize the power grid when it is about to lose synchronism after experiencing contingencies (but the voltage is still well-supported). In particular, we present two options regarding the control of fault-on and post-fault dynamics to stabilize power systems under the fault. In the fault-on emergency control, we will tune the inertia and damping of the renewable generators by using synchronverter technology [7] such that the fault-on dynamics is bounded, by which the fault-cleared state at a fixed clearing time is guaranteed to be inside the stability region of the post-fault equilibrium point. Therefore, when the fault is cleared at the clearing time, the post-fault dynamics will evolve from the fault-cleared state to the equilibrium point. In the post-fault emergency control, the transmission susceptance and power injection will be adjusted by FACTS devices such that the power network/flow is suitably restructured and the post-fault dynamics is driven from a possibly unstable fault-cleared state through a set of equilibrium points to the desired equilibrium points.

Mathematically, these controls are designed by utilizing the Lyapunov function family approach [8], which leads to solving convex optimization problems possibly scalable to large-scale power grids. In the practical side, the proposed emergency controls can be implemented by exploiting the smart electronics devices, such as synchronverters and FACTS devices, which will be widely available on the future power grids, and thus, requires minor investment. In addition, it reduces the need for load shedding which causes serious damage to customers in traditional emergency control.

Thanh Long Vu and Konstantin Turitsyn are with the Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, 02139 USA, e-mail: {longvu, turitsyn}@mit.edu.

978-1-5090-5200-4/16/$31.00 ©2016 IEEE

II. Emergency Control Problem

In normal conditions, a power grid operates at a stable equilibrium point of the pre-fault dynamics. Following some contingencies, the system evolves according to the fault-on dynamics laws and moves away from the pre-fault equilibrium point to a fault-cleared state δ_0. After the fault is cleared, the system evolves according to the post-fault dynamics.

In this paper, we consider power systems under critical situations when the buses' phasor angles may significantly fluctuate but the buses' voltages are still well-supported. For such situations, we utilize the standard structure-preserving model to describe the post-fault dynamics of generators and loads [9]. Mathematically, the gird is described by an undirected graph $\mathcal{A}(\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \ldots, |\mathcal{N}|\}$ is the set of buses and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of transmission lines connecting those buses. Here, $|A|$ denotes the number of elements in set A. The sets of generator buses and load buses are denoted by $\mathcal{G}$ and $\mathcal{L}$. We assume that the grid is lossless with constant voltage magnitudes $V_k, k \in \mathcal{N}$, and the reactive powers are ignored. Then, the structure-preserving model of the system is given by:

$$m_k \ddot{\delta_k} + d_k \dot{\delta_k} + \sum_{j \in \mathcal{N}_k} a_{kj} \sin(\delta_k - \delta_j) = P_{m_k}, k \in \mathcal{G}, \quad (1a)$$

$$d_k \dot{\delta_k} + \sum_{j \in \mathcal{N}_k} a_{kj} \sin(\delta_k - \delta_j) = - P_{d_k}^0, k \in \mathcal{L}, \quad (1b)$$

where equation (1a) represents the dynamics at generator buses and equation (1b) the dynamics at load buses. In these equations, with $k \in \mathcal{G}$, then $m_k > 0$ is the generator's dimensionless moment of inertia, $d_k > 0$ is the term representing primary frequency controller action on the governor, and P_{m_k} is the input shaft power producing the mechanical torque acting on the rotor of the k^{th} generator. With $k \in \mathcal{L}$, then $d_k > 0$ is the constant frequency coefficient of load and $P_{d_k}^0$ is the nominal load. Here, $a_{kj} = V_k V_j B_{kj}$, where B_{kj} is the (normalized) susceptance of the transmission line $\{k, j\}$ connecting the k^{th} bus and j^{th} bus, $\mathcal{N}_k$ is the set of neighboring buses of the k^{th} bus. Note that, the system described by equation (1) has many stationary points δ_k^* that are characterized, however, by the angle differences $\delta_{kj}^* = \delta_k^* - \delta_j^*$ (for a given P_k) that solve the following system of power flow-like equations:

$$\sum_{j \in \mathcal{N}_k} a_{kj} \sin(\delta_{kj}^*) = P_k, k \in \mathcal{N}, \quad (2)$$

where $P_k = P_{m_k}, k \in \mathcal{G}$, and $P_k = -P_{d_k}^0, k \in \mathcal{L}$.

Our emergency control objective is to adjust the power system dynamics right after the fault happens such that the stability of power system is maintained. Two emergency control options are considered as follows.

A. Fault-on emergency control

Assume that when a line tripping occurs, the system operator can immediately send signals to simultaneously adjust the inertia and damping of the low-inertia generators without any communication and regulation delays. Also, the tuned values of inertia and damping can be kept for at least a time period $[0, \tau_{clearing}]$. Here it should be noted that, since $\tau_{clearing}$ is in the range of hundreds of milliseconds, one of the advantages of the approach presented in this paper is that it requires a very limited amount of power and energy from the additional power sources. Our emergency control problem is how to appropriately tune the inertia and damping of the low-inertia generators (with the help of an additional power sources) to compensate for the disturbance such that after the given clearing time $\tau_{clearing}$, the fault-cleared state is still inside the region of attraction of the post-fault stable equilibrium point δ_{post}^*. If this objective can be obtained, then at the clearing time $\tau_{clearing}$, the fault is cleared, the inertia and damping of the low-inertia generators are brought back to their initial values, and the power system will evolve according to the post-fault dynamics from the fault-cleared state to the stable post-fault equilibrium point.

B. Post-fault emergency control

The critical situations considered in this problem are when the fault-on trajectory is leaving polytope $\Pi/2$ defined by inequalities $|\delta_{kj}| \leq \pi/2, \forall \{k, j\} \in \mathcal{E}$, i.e., the fault-cleared state δ_0 stays outside polytope $\Pi/2$. In normal power systems, protective devices will be activated to disconnect faulted lines/nodes, which will isolate the fault and prevent the post-fault dynamics from instability (this would usually happen at some point beyond a voltage angle difference $\pi/2$).

Avoiding disconnecting line/node, our emergency control objective is to make post-fault dynamics become stable by controlling the post-fault dynamics from the fault-cleared state δ_0 to the stable equilibrium point $\delta_{\mathbf{origin}}^*$, which, e.g., may be an optimum point of some optimal power flow (OPF) problem. To achieve this, we consider adjusting the post-fault dynamics through adjusting the susceptance of some selected transmission lines and/or changing power injections. These changes can be implemented by the FACTS devices available on power transmission grids. The rationale of this control is that, by appropriately changing the structure of power systems, we can obtain new post-fault dynamics with a new equilibrium point whose region of attraction contains the fault-cleared state δ_0, and therefore, the new post-fault dynamic is stable. Given a fault-cleared state δ_0 and the stable equilibrium point $\delta_{\mathbf{origin}}^*$, determine the feasible values for susceptances of selected transmission lines and/or feasible power injection such that the post-fault dynamics are driven from the fault-cleared state δ_0 to the original post-fault equilibrium point $\delta_{\mathbf{origin}}^*$.

III. Emergency control solutions

In this section, we report the emergency control solutions recently introduced in [10] and [11]. To this end, we separate the nonlinear couplings and the linear terminal system in (1). For brevity, we denote the stable post-fault equilibrium point for which we want to certify stability as δ^*. Consider the state vector $x = [x_1, x_2, x_3]^T$, which is composed of the vector of generator's angle deviations from equilibrium $x_1 = [\delta_1 - \delta_1^*, \ldots, \delta_{|\mathcal{G}|} - \delta_{|\mathcal{G}|}^*]^T$, their angular velocities $x_2 = [\dot{\delta}_1, \ldots, \dot{\delta}_{|\mathcal{G}|}]^T$, and vector of load buses' angle deviation from equilibrium $x_3 = [\delta_{|\mathcal{G}|+1} - \delta_{|\mathcal{G}|+1}^*, \ldots, \delta_{|\mathcal{N}|} - \delta_{|\mathcal{N}|}^*]^T$. Let E be the incidence matrix of the graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$, so that $E[\delta_1, \ldots, \delta_{|\mathcal{N}|}]^T = [(\delta_k - \delta_j)_{\{k,j\} \in \mathcal{E}}]^T$. Let the matrix C be $E[I_{m\times m}\ O_{m\times n}; O_{(n-m)\times 2m}\ I_{(n-m)\times(n-m)}]$. Then

$$Cx = E[\delta_1 - \delta_1^*, \ldots, \delta_{|\mathcal{N}|} - \delta_{|\mathcal{N}|}^*]^T = [(\delta_{kj} - \delta_{kj}^*)_{\{k,j\} \in \mathcal{E}}]^T.$$

Consider the vector of nonlinear interactions F in the simple trigonometric form: $F(Cx) = [(\sin\delta_{kj} - \sin\delta_{kj}^*)_{\{k,j\}\in\mathcal{E}}]^T$. Denote the matrices of moment of inertia, frequency controller action on governor, and frequency coefficient of load as $M_1 = \operatorname{diag}(m_1, \ldots, m_{|\mathcal{G}|}), D_1 = \operatorname{diag}(d_1, \ldots, d_{|\mathcal{G}|})$ and $M = \operatorname{diag}(m_1, \ldots, m_{|\mathcal{G}|}, d_{|\mathcal{G}|+1}, \ldots, d_{|\mathcal{N}|})$.

Then, the power system (1) can be then expressed as

$$\dot{x} = Ax - BF(Cx), \tag{3}$$

with the matrices A, B given by the following expression:

$$A = \begin{bmatrix} O_{m\times m} & I_{m\times m} & O_{m\times n-m} \\ O_{m\times m} & -M_1^{-1}D_1 & O_{m\times n-m} \\ O_{n-m\times m} & O_{n-m\times m} & O_{n-m\times n-m} \end{bmatrix},$$

$$B = \begin{bmatrix} O_{m\times|\mathcal{E}|}; & S_1 M^{-1} E^T S; & S_2 M^{-1} E^T S \end{bmatrix},$$

and $S = \operatorname{diag}(a_{kj})_{\{k,j\}\in\mathcal{E}}, S_1 = [I_{m\times m} \quad O_{m\times n-m}], S_2 = [O_{n-m\times m} \quad I_{n-m\times n-m}], n = |\mathcal{N}|, m = |\mathcal{G}|$.

A. Fault-on emergency control solution

In [12], it is showed that if there exists a positive definite matrix P such that

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} + \dfrac{(1-g)^2}{4} C^T C & PB \\ B^T P & -I \end{bmatrix} \leq 0 \tag{4}$$

and

$$V(x_0) < V_{\min} \tag{5}$$

where $\bar{A} = A - \dfrac{1}{2}(1+g)BC$, then, the system trajectory of (1) will converge from the fault-cleared state x_0 to the stable equilibrium point δ^*. Here, $V_{\min}$ is the minimum value of the Lyapunov function $V(x)$ over the flow-out boundary $\partial\mathcal{P}^{out}$ as:

$$V_{\min} = \min_{x\in\partial\mathcal{P}^{out}} V(x), \tag{6}$$

where $\partial\mathcal{P}^{out}$ is the union of $\partial\mathcal{P}_{kj}^{out}$ over all the transmission lines $\{k, j\} \in \mathcal{E}$ connecting generator buses. Applying this result, in [10], we proposed the following fault-on emergency control which guarantee the stability of the post-fault dynamics.

Given the power system under a line tripping and the clearing time $\tau_{clearing}$.

1) Find a positive definite matrix P satisfying the LMI (4).
2) Calculate the minimum value $V_{\min}$ defined as in (6).
3) Let $\mu = \dfrac{\tau_{clearing}}{V_{\min}}$.
4) Find the inertia and damping of the synchronverter-integrated generators such that the following inequality is satisfied:

$$\begin{bmatrix} \bar{A}(m,d)^T P + P\bar{A}(m,d) + \dfrac{(1-g)^2}{4} C^T C & P\bar{B}(m,d) \\ \bar{B}(m,d)^T P & -I \end{bmatrix} \leq 0. \tag{7}$$

5) If there is no such inertia and damping, then repeat from step 1).
6) If such values of inertia and damping exist, then the synchronverters will be used to tune the inertia and damping of the imitated generators and keep these values during the time period $[0, \tau_{clearing}]$. At the clearing time $\tau_{clearing}$, the fault is cleared and the inertia and damping of the imitated generators are tuned back to their initial values.

B. Post-fault emergency control solution

We propose the post-fault emergency control including two steps: (i) changing the power injections to drive the post-fault dynamics from a possibly unstable fault-cleared state to some stable equilibrium point, and (ii) changing the transmission line susceptances to render the post=fault dynamics from the first equilibrium point to the desired equilibrium point. More details can be found in [11].

In particular, we change the power injections in the first step such that the new equilibrium point is as far away from the stability margin as possible. To do that, we utilize the following procedure.

- Minimize the linear function $\|L^\dagger p\|_{\mathcal{E},\infty}$ over the power injection space, where $p = [P_1, ..., P_{|\mathcal{N}|}]^\top$ is the power injection vector, $L^\dagger$ is the pseudoinverse of the network Laplacian matrix, and $\|x\|_{\mathcal{E},\infty} = \max_{\{i,j\}\in\mathcal{E}} |x(i) - x(j)|$. This is a convex optimization problem.
- Calculate the new equilibrium point from the optimum value of the power injection;
- Given the new equilibrium point, utilize the adaptation algorithm proposed in [8] to search for a Lyapunov function in the Lyapunov function family that can certify stability for the fault-cleared state.

In the second step, we utilize discrete changing of the power network such that the post-fault dynamics is driven through a sequence of equilibrium points to the desired equilibrium point. In particular, given the first equilibrium point δ_1^* and the desired equilibrium point $\delta_{\mathbf{origin}}^*$, we determine a sequence of equilibrium point in which δ_i^* is the closest possible equilibrium point to δ_{i-1}^* and the distance between δ_i^* and $\delta_{\mathbf{origin}}^*$ satisfies

$$d_i(\delta_i^*, \delta_{\mathbf{origin}}^*) \leq d_{i-1}(\delta_{i-1}^*, \delta_{\mathbf{origin}}^*) - d, \tag{8}$$

where $d > 0$ is a constant. Here, $d > 0$ is a sufficiently small constant chosen such that the convergence from δ_{i-1}^* to δ_i^* is satisfied for all the equilibrium points, and $d_i(\delta_i^*, \delta)$ is the distance from δ to the equilibrium point δ_i^*, which is defined via $\{B_{kj}^{(i)}\}$, i.e.,

$$d_i(\delta_i^*, \delta) = \sum_{k\in\mathcal{N}} \big(\sum_{j\in\mathcal{N}_k} V_k V_j B_{kj}^{(i)} (\sin \delta_{i_{kj}}^* - \sin \delta_{kj})\big)^2$$
$$= \sum_{k\in\mathcal{N}} \big(P_k - \sum_{j\in\mathcal{N}_k} V_k V_j B_{kj}^{(i)} \sin \delta_{kj}\big)^2.$$

We note that since $d_i(\delta_i^*, \delta^*)$ is a quadratic function of the susceptance variable $\{B_{kj}^{(i)}\}$, defining δ_i^* can be described by the quadratically constrained quadratic program (QCQP) in the transmission susceptance variables $\{B_{kj}^{(i)}\}$:

$$\min_{\{B_{kj}^{(i)}\}} \; d_i(\delta_i^*, \delta_{i-1}^*) \qquad (9)$$
$$\text{s.t.} \;\; d_i(\delta_i^*, \delta_{\textbf{origin}}^*) \le d_{i-1}(\delta_{i-1}^*, \delta_{\textbf{origin}}^*) - d.$$

Because all the stable equilibrium points are strictly staying inside a polytope with phasor angular differences lees that $pi/2$, we can prove that the functions $d_i(\delta_i^*, \delta_{i-1}^*)$ and $d_i(\delta_i^*, \delta_{\textbf{origin}}^*)$ are strictly convex functions of $\{B_{kj}^{(i)}\}$. Therefore, the above QCQP is a convex optimization problem and can be quickly solved using convex optimization solvers. As such, the proposed procedure to determine equilibrium sequence might be performed for large scale power systems.

IV. Numerical validation for fault-on emergency control: Three-generator system

For illustrating the concept of this paper, we consider the simple yet non-trivial system of three generators, one of which is the renewable generator (generator 1) integrated with the synchronverter. The susceptance of the transmission lines are assumed at fixed values $B_{12} = 0.739$ p.u., $B_{13} = 1.0958$ p.u., and $B_{23} = 1.245$ p.u. Also, the inertia and damping of all the conventional and imitated generator at the normal working condition are $m_k = 2$ p.u., $d_k = 1$ p.u. Assume that the line between generators 1 and 2 is tripped, and then reclosed at the clearing time $\tau_{clearing} = 200ms$, and during the fault-on dynamic stage the time-invariant terminal voltages and mechanical torques given in Tab. II.

Node	V (p.u.)	P (p.u.)
1	1.0566	-0.2464
2	1.0502	0.2086
3	1.0170	0.0378

TABLE I
Voltage and mechanical input

The pre-fault and post-fault equilibrium point is calculated from (2): $\delta^* = [-0.6634 \;\; -0.5046 \;\; -0.5640 \; 0 \; 0 \; 0]^T$. Hence, the equilibrium point stays in the polytope defined by the inequality $|\delta_{kj}| < \pi/10$. As such, we can take $g = (1 - \sin(\pi/10))/(\pi/2 - \pi/10)$. Using CVX in MATLAB to solve the LMI (4), we can obtain the Lyapunov function

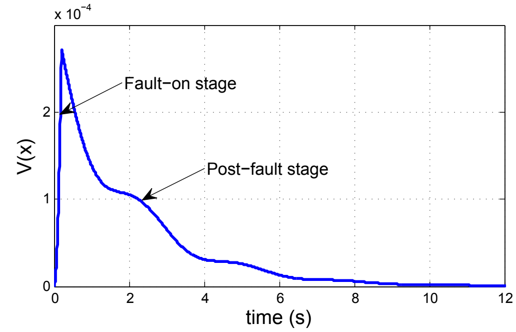

Fig. 1. Variations of the quadratic Lyapunov function $V(x) = x^T Px = (\delta - \delta^*)^T P(\delta - \delta^*)$ during the fault-on and post-fault dynamics.

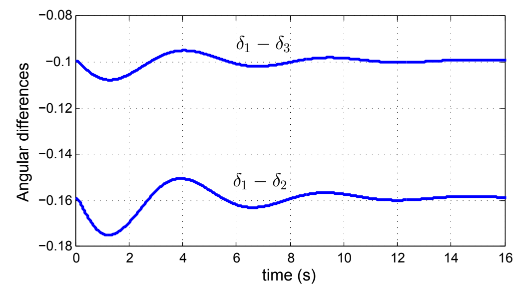

Fig. 2. Variations of the angular differences during the fault-on and post-fault dynamics.

$V(x) = x^T Px$ where the matrix P is

$$\begin{bmatrix} 2.8401 & 1.9098 & 1.9812 & 4.5726 & 4.4349 & 4.4563 \\ 1.9098 & 2.7949 & 2.0263 & 4.4393 & 4.5628 & 4.4578 \\ 1.9812 & 2.0263 & 2.7235 & 4.4502 & 4.4644 & 4.5480 \\ 4.5726 & 4.4393 & 4.4502 & 18.4333 & 17.5302 & 17.6662 \\ 4.4349 & 4.5628 & 4.4644 & 17.5302 & 18.3632 & 17.7364 \\ 4.4563 & 4.4578 & 4.5480 & 17.6662 & 17.7364 & 18.2271 \end{bmatrix}$$

Then the minimum value $V_{\min}$ is $V_{\min} = 0.8139$ and thus $\mu = \tau_{clearing}/V_{\min} = 0.2457$. Again, using the CVX to solve the LMI (7) where the inertia of the imitated generator is fixed, we obtain the optimum damping $d_1 = 1$ p.u. Figure 1 shows that the Lyapunov function $V(x) = x^T Px = (\delta - \delta^*)^T P(\delta - \delta^*)$ evolves from 0 to some value during the fault-on dynamics and then converge to 0 during the post-fault dynamics. This means that the post-fault power system is transiently stable under the effect of the proposed emergency control. Similarly, Fig. 2 shows that the angular differences of the generators (i.e. $\delta_1 - \delta_2$ and $\delta_1 - \delta_3$) converge to the constant values during the post-fault dynamics, confirming the stability of the system.

V. Numerical validation for post-fault emergency control: Three-generator system

For illustrating the concept of this paper, we consider the simple yet non-trivial system of three generators in the Kron-reduction model. The susceptance of the transmission lines are: $B_{12} = 0.739$ p.u., $B_{13} = 1.0958$ p.u., and $B_{23} = 1.245$ p.u. The inertia and damping of all the generators are: $m_k = 2$ p.u., $d_k = 1$ p.u. Assume that the line between generators 1 and 2 is tripped, and when the fault is cleared, this line is re-closed. After the fault is cleared, the bus voltages V_k and mechanical inputs P_{m_k} of the post-fault dynamics are given in Tab. II.

Node	V (p.u.)	P (p.u.)
1	1.0566	0.7536
2	1.0502	1.2086
3	1.0170	-1.9622

TABLE II
VOLTAGE AND MECHANICAL INPUT

The post-fault equilibrium point is calculated from (2): $\delta^*_{\mathbf{origin}} = [0.1973\ 0.3365\ -0.6308\ 0\ 0\ 0]^T, \dot{\delta}^*_{\mathbf{origin}} = 0.$ However, the fault-cleared state, with angles $\delta_0 = [1.624\ -0.023\ 0.041]^\top$ and generators angular velocity $[-0.016\ -0.021\ 0.014]^\top$, stays outside polytope $\Pi/2$. By our adaptation algorithm, we do not find a suitable Lyapunov function certifying the convergence of this fault-cleared state to the original equilibrium point $\delta^*_{\mathbf{origin}}$, so this fault-cleared state may be unstable. We will design emergency control actions to bring the post-fault dynamics from the possibly unstable fault-cleared state to the equilibrium point $\delta^*_{\mathbf{origin}}$. All the convex optimization problems associated in the design will be solved by CVX software.

A. Designing the first equilibrium point

Assume that the two generators 2-3 are dispatchable, which will be used to design the first equilibrium point. With the original power injection, $\|L^\dagger p\|_{\mathcal{E},\infty} = 0.9673$, which is very near 1, meaning that the original equilibrium point is near the stability margin. Using CVX software to minimize $\|L^\dagger p\|_{\mathcal{E},\infty}$, we obtain the new power injections at generators 2-3 as follows: $P_2 = 0, P_3 = -0.7536$. Accordingly, the minimum value of $\|L^\dagger p\|_{\mathcal{E},\infty} = 0.2501$, and the new equilibrium point is $\delta_1^* = [-0.2984\ -0.5838\ -0.7583]^\top, \dot{\delta}_1^* = 0$.

Next, we apply the fault-dependent stability certificate in Section III. With the new equilibrium point δ_1^*, the fault-cleared state δ_0 stays inside polytope $\mathcal{Q}$. Using the adaptation algorithm presented in [8], after some steps we find that there is a Lyapunov function in this family such that $V(x_0) < V_{\min}$. As such, when we turn on the new power injections, the post-fault dynamics are stable and the post-fault trajectory will converge from the fault-cleared state to the new equilibrium point δ_1^*. After that, we switch power injections back to the original values.

B. Designing the other equilibrium points by changing transmission impedances

Using the adaptation algorithm, we do not find a suitable Lyapunov function certifying that $\delta_1^* \in \mathbf{SR}_{\mathbf{origin}}$. As such, the new equilibrium point δ_1^* may stay outside the stability region of the original equilibrium point $\delta^*_{\mathbf{origin}}$. We design the impedance adjustment controllers to render the post-fault dynamics from the new equilibrium point back to the original equilibrium point.

Assume that the impedances of transmission lines $\{1,2\}$ and $\{2,3\}$ can be adjusted, by FACTS devices integrated

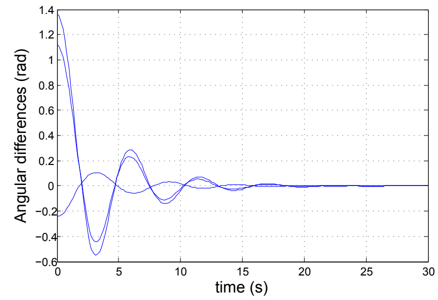

Fig. 3. Effect of power injection control: Convergence of buses angles from the fault-cleared state to the first equilibrium point δ_1^* in the post-fault dynamics

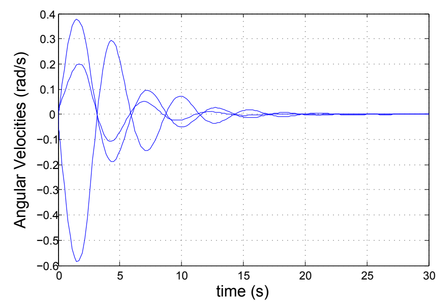

Fig. 4. Effect of power injection control: Convergence of generators frequencies to the base value in post-fault dynamics.

with these lines, in the ranges: $0.6 \leq B_{12} \leq 1p.u., 1 \leq B_{23} \leq 3p.u.$ The distance from the first equilibrium point to the original equilibrium point is calculated as $d_1(\delta_1^*, \delta^*_{\mathbf{origin}}) = 2.915$. Let $d = d_1(\delta_1^*, \delta^*_{\mathbf{origin}})/2 + 0.1 = 1.5607$, and solve the following convex QCQP with variable $B_{12}^{(2)}$ and $B_{23}^{(2)}$:

$$\min_{\{B_{kj}^{(2)}\}} d_2(\delta_2^*, \delta_1^*) \tag{10}$$
$$\text{s.t. } d_2(\delta_2^*, \delta^*_{\mathbf{origin}}) \leq d_1(\delta_1^*, \delta^*_{\mathbf{origin}}) - d = 1.3607.$$

Solving this convex QCQP problem, we obtain the new susceptances at transmission lines $\{1,2\}$ and $\{2,3\}$ as $B_{12}^{(2)} = 0.6p.u.$ and $B_{23}^{(2)} = 2.1949p.u.$, with which the distance from the second equilibrium point to the first equilibrium point and the original equilibrium point are given by $d_2(\delta_2^*, \delta_1^*) = 3.2050$ and $d_2(\delta_2^*, \delta^*_{\mathbf{origin}}) = 1.3607$. Using the adaptation algorithm, we can check that $\delta_1^* \in \mathbf{SR}_2$ and $\delta_2^* \in \mathbf{SR}_{\mathbf{origin}}$.

C. Control performance

We subsequently perform the following control actions:

- Changing the power injections of generators 2-3 to $P_2 = 0, P_3 = -0.7536$. The system trajectories will converge from the fault-cleared state to the first equilibrium point δ_1^*, as shown in Figs. 3-4.
- Switch the power injections to the original values and change the susceptances of transmission lines lines $\{1,2\}$ and $\{2,3\}$ to $B_{12}^{(2)} = 0.6p.u.$ and $B_{23}^{(2)} = 2.1949p.u.$ The system trajectories will converge from the first equilibrium point to the second equilibrium point, as shown in Figs. 5-6.

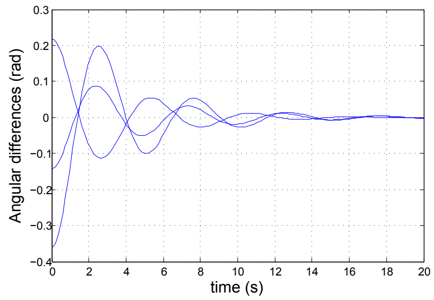

Fig. 5. Effect of susceptance control: Convergence of buses angles from δ_1^* to the second equilibrium point in post-fault dynamics.

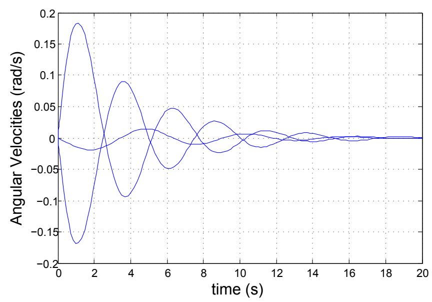

Fig. 6. Effect of susceptance control: Frequency dynamics of the generators to the base value in post-fault dynamics.

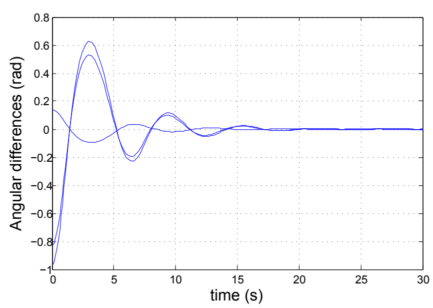

Fig. 7. Automatic convergence of buses angles from the second equilibrium point to $\delta^*_{\mathbf{original}}$ in post-fault dynamics when the susceptances are recovered to original value.

- Switch the susceptances of transmission lines $\{1, 2\}$ and $\{2, 3\}$ to the original values. The system trajectories will converge from the second equilibrium point to the original equilibrium point $\delta^*_{\mathbf{original}}$, as shown in Figs. 7-8.

VI. Conclusions

In this paper, we presented the options for emergency control of power systems with high penetration of renewables. In particular, the adjustment of fault-on dynamics and post-fault dynamics is utilized to maintain the transient stability of power systems under and after the faults. In addition, we reported the solutions to these problems by employing the fast response of power electronics devices such as synchronverters and FACTS devices to tune either inertia/damping or the transmission susceptance/power injection. It should be noted that such emergency control solutions eliminate the

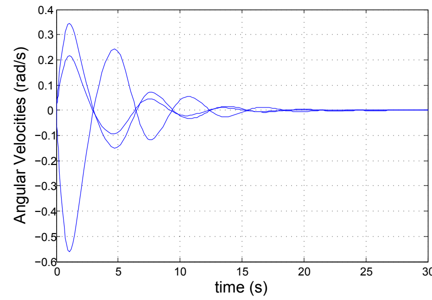

Fig. 8. Automatic convergence of the generators' frequency to the base value in post-fault dynamics when the susceptances are recovered to original value.

need for load shedding which cause enormous economic impact to customers. As an illustration for the principle of these emergency control solutions, we numerically demonstrate the performance of such remedial actions on a three-generator power system. Extensive illustration on large scale power system needs to be carried out before such controls can be considered in practice.

References

[1] V. Vittal, *Emergency Control and Special Protection Systems In Large Electric Power Systems*. Boston, MA: Birkhäuser Boston, 2003, pp. 293–314.

[2] W. Fu, S. Zhao, J. McCalley, V. Vittal, and N. Abi-Samra, "Risk assessment for special protection systems," *Power Systems, IEEE Transactions on*, vol. 17, no. 1, pp. 63–72, Feb. 2002.

[3] S. Koch, M. D. Galus, S. Chatzivasileiadis, and G. Andersson, "Emergency control concepts for future power systems," {*IFAC*} *Proceedings Volumes*, vol. 44, no. 1, pp. 6121 – 6129, 2011, 18th {IFAC} World Congress. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1474667016445854

[4] J. Machowski, J. Bialek, and J. Bumby, *Power System Dynamics: Stability and Control*. John Wiley & Sons, 2011.

[5] E. O. Lawrence, K. H. Lacommare, J. H. Eto, K. H. Lacommare, and J. H. Eto, "Cost of power interruptions to electricity consumers in the United States (U.S.)," 2005.

[6] "Final report on the August 14, 2003 blackout in the United States and Canada: Causes and recommendations," *http://energy.gov/sites/prod/files/oeprod/DocumentsandMedia/BlackoutFinal-Web.pdf*.

[7] Q. C. Zhong and G. Weiss, "Synchronverters: Inverters that mimic synchronous generators," *Industrial Electronics, IEEE Transactions on*, vol. 58, no. 4, pp. 1259–1267, 2011.

[8] T. L. Vu and K. Turitsyn, "Lyapunov Functions family approach to transient stability assessment," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1269–1277, Mar. 2016.

[9] A. R. Bergen and D. J. Hill, "A structure preserving model for power system stability analysis," *Power Apparatus and Systems, IEEE Transactions on*, no. 1, pp. 25–35, 1981.

[10] T. L. Vu, S. Chatzivasileiadis, and K. Turitsyn, "Towards electronics-based emergency control in power grids with high renewable penetration," in *2016 American Control Conference (ACC)*, July 2016, pp. 6773–6778.

[11] T. L. Vu, S. Chatzivasileiadis, H.-D. Chiang, and K. Turitsyn, "Structural emergency control paradigm," *arXiv preprint arXiv:1607.08183*, 2016.

[12] T. L. Vu and K. Turitsyn, "A framework for robust assessment of power grid stability and resiliency," *IEEE Transactions on Automatic Control*, vol. PP, no. 99, pp. 1–1, 2016.