

Towards Electronics-based Emergency Control in Power Grids with High Renewable Penetration

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Abstract—Many traditional emergency control schemes in power systems accompany with power interruption, yielding severely economic damages to customers. Aiming at alleviating this remarkable drawback, this paper sketches the ideas of a viable alternative for traditional remedial controls for power grids with high penetration of renewables, in which the renewables are integrated with synchronverters to mimic the dynamics of conventional generators. In this novel emergency control scheme, the power electronics resources are exploited to control the inertia and damping of the imitated generators in order to quickly compensate for the deviations caused by fault and thereby bound the fault-on dynamics and stabilize the power system under emergency situations. The control design is based on solving convex optimization problems tractable for large scale power grids. This emergency control not only saves investments and operating costs for modern and future power systems, but also helps to offer seamless electricity service to customers. Simple numerical simulation will be used to illustrate the concept of this paper.

I. INTRODUCTION

The aging US power grid is approaching its physical limits with the high penetration of intermittent renewables, large volume of power storage and EVs, and ubiquitous presence of massive loads. As a result, the stressed system is especially vulnerable to extreme events. As such, monitoring and maintaining the grid stability are immensely important tasks.

Currently, the grid under emergency situations mainly relies on remedial actions [1]–[3], special protection systems (SPS) [4], [5] and load shedding [6]–[8] to quickly rebalance power and hopefully stabilize the system. However, many of these emergency actions lead to interrupting electricity service to customers. The unexpected load shedding are extremely harmful to customers since it may cause enormously high economic damage. On the other hand, the protective devices are usually only effective for individual elements, but less effective in preventing the grid from collapse, and in many cases may split the grid into islands or lead to cascading failures [9]. The underlying reason is the lack of coordination among protective devices and the difference in their timescales, which together make them incapable to maintain the grid stability at the system level.

These issues on economic efficiency and system stability call for a new generation of emergency controls, which can

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maintain the system stability and reduce the damages to customers. In this paper, we aim to bring the emergency control problem to the attention of the control community, and hence we sketch the ideas of a novel emergency control scheme for renewable-integrated power grids by exploiting the emerging power electronics resources. Remarkably, this emergency control scheme can reduce the needs in power interruptions due to load shedding, and maintain the transient stability of power grids under emergency situations.

In particular, this paper brings in the following novelties:

- We model the power grids with high levels of renewable penetration, in which the renewable generators are integrated with the synchronverters [10]. The synchronverters will control the renewable generators to imitate the dynamics of the conventional generators, which is described by the classical swing equations. Also, the inertia and damping parameters of these generators can be quickly adjusted in a wide range of values.
- Exploiting such electronics resources, we formulate the emergency control problem in this renewable power system, which aims to maintain the transient stability of the system following a line tripping by appropriately tune the inertia and damping of the imitated generators.
- We solve this emergency control by applying our recently introduced quadratic Lyapunov function-based transient stability certificate [11]. In particular, we present sufficient conditions on the inertia and damping of the imitated generator such that when applied to the fault-on dynamics, the fault-cleared state after a fixed clearing time still stays inside the region of attraction of the post-fault equilibrium point. Note-worthily, the sufficient conditions in many case can be formulated as a set of linear matrix inequalities (LMIs), which can be solved quickly by convex optimization enabling the fast response of the remedial actions.

II. NETWORK MODEL AND EMERGENCY CONTROL PROBLEM

A. Network Model

Consider a grid including conventional generators, renewable generators, loads, and transmission lines connecting them. We assume that all the renewables are integrated with synchronverter [10], which will control the renewables to mimic the dynamics of conventional generator, and thus we call both conventional and renewable generators as gen-

erators. In this paper we consider the standard structure-preserving model to describe components and dynamics in the grid [12]. Also, we assume that the voltages in the system are well supported, and we will only care about the phasors dynamics. Mathematically, the grid is described by an undirected graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{1, 2, \dots, |\mathcal{N}|\}$ is the set of buses and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of transmission lines connecting those buses. Here, $|A|$ denotes the number of elements in the set A . The sets of conventional/renewable generator buses and load buses are denoted by $\mathcal{G} = \mathcal{G}_C \cup \mathcal{G}_R$ and \mathcal{L} and labeled as $\{1, \dots, |\mathcal{G}|\}$ and $\{|\mathcal{G}| + 1, \dots, |\mathcal{N}|\}$. We assume that the grid is lossless with constant voltage magnitudes $V_k, k \in \mathcal{N}$, and the reactive powers are ignored.

Conventional generator buses. The dynamics of conventional generators are described by swing equations:

$$m_k \ddot{\delta}_k + d_k \dot{\delta}_k + P_{e_k} - P_{m_k} = 0, k \in \mathcal{G}_C, \quad (1)$$

where, $m_k > 0$ is the dimensionless moment of inertia of the generator, $d_k > 0$ is the term representing primary frequency controller action on the governor. P_{m_k} is the effective dimensionless mechanical torque acting on the rotor and P_{e_k} is the effective dimensionless electrical power output of the k^{th} generator.

Synchronverter-integrated renewable generator buses.

The dynamics of synchronverter-based renewable generators are described by the same set of equations:

$$m_k \ddot{\delta}_k + d_k \dot{\delta}_k + P_{e_k} - P_{m_k} = 0, k \in \mathcal{G}_R, \quad (2)$$

where the inertia m_k and damping d_k are tunable.

Load buses. Let P_{d_k} be the real power drawn by the load at k^{th} bus, $k \in \mathcal{L}$. In general P_{d_k} is a nonlinear function of voltage and frequency. For constant voltages and small frequency variations around the operating point $P_{d_k}^0$, it is reasonable to assume that

$$P_{d_k} = P_{d_k}^0 + d_k \dot{\delta}_k, k \in \mathcal{L}, \quad (3)$$

where $d_k > 0$ is the constant frequency coefficient of load.

AC power flows. The active electrical power P_{e_k} injected from the k^{th} bus into the network, where $k \in \mathcal{N}$, is given by

$$P_{e_k} = \sum_{j \in \mathcal{N}_k} V_k V_j B_{kj} \sin(\delta_k - \delta_j), k \in \mathcal{N} \quad (4)$$

Here, the value V_k represents the voltage magnitude of the k^{th} bus which is assumed to be constant. B_{kj} are the (normalized) susceptance of the transmission line $\{k, j\}$ connecting the k^{th} bus and j^{th} bus. \mathcal{N}_k is the set of neighboring buses of the k^{th} bus. Let $a_{kj} = V_k V_j B_{kj}$.

By power balancing we obtain the structure-preserving model of power systems as:

$$m_k \ddot{\delta}_k + d_k \dot{\delta}_k + \sum_{j \in \mathcal{N}_k} a_{kj} \sin(\delta_k - \delta_j) = P_{m_k}, k \in \mathcal{G}, \quad (5a)$$

$$d_k \dot{\delta}_k + \sum_{j \in \mathcal{N}_k} a_{kj} \sin(\delta_k - \delta_j) = -P_{d_k}^0, k \in \mathcal{L}, \quad (5b)$$

where, the equations (5a) represent the dynamics at generator buses and the equations (5b) the dynamics at load buses.

The system described by equations (5) has many stationary points with at least one stable corresponding to the desired operating point. Mathematically, the state of (5) is presented by $\delta = [\delta_1, \dots, \delta_{|\mathcal{G}|}, \delta_1, \dots, \delta_{|\mathcal{G}|}, \delta_{|\mathcal{G}|+1}, \dots, \delta_{|\mathcal{N}|}]^T$, and the desired operating point is characterized by the buses' angles $\delta^* = [\delta_1^*, \dots, \delta_{|\mathcal{G}|}^*, 0, \dots, 0, \delta_{|\mathcal{G}|+1}^*, \dots, \delta_{|\mathcal{N}|}^*]^T$. This point is not unique since any shift in the buses' angles $[\delta_1^* + c, \dots, \delta_{|\mathcal{G}|}^* + c, 0, \dots, 0, \delta_{|\mathcal{G}|+1}^* + c, \dots, \delta_{|\mathcal{N}|}^* + c]^T$ is also an equilibrium. However, it is unambiguously characterized by the angle differences $\delta_{kj}^* = \delta_k^* - \delta_j^*$ that solve the following system of power-flow like equations:

$$\sum_{j \in \mathcal{N}_k} a_{kj} \sin(\delta_{kj}^*) = P_k, k \in \mathcal{N}, \quad (6)$$

where $P_k = P_{m_k}, k \in \mathcal{G}$, and $P_k = -P_{d_k}^0, k \in \mathcal{L}$.

Assumption 1: There is a solution δ^* of equations (6) such that $|\delta_{kj}^*| \leq \gamma < \pi/2$ for all $\{k, j\} \in \mathcal{E}$.

It is known that for almost all power systems this assumption holds true if the system satisfies the following synchronization condition:

$$\|L^\dagger p\|_{\mathcal{E}, \infty} \leq \sin \gamma. \quad (7)$$

Here, L^\dagger is the pseudoinverse of the network Laplacian matrix, $p = [P_1, \dots, P_{|\mathcal{N}|}]^T$, and $\|x\|_{\mathcal{E}, \infty} = \max_{\{i, j\} \in \mathcal{E}} |x(i) - x(j)|$. We denote as $\Delta(\gamma)$ the set of equilibrium points δ^* satisfying that $|\delta_{kj}^*| \leq \gamma < \pi/2, \forall \{k, j\} \in \mathcal{E}$. Then, any equilibrium point in this set is a stable operating point.

B. Electronics-based Emergency Control Problem

In normal conditions, a power grid operates at a stable equilibrium point of the pre-fault dynamics. After the initial disturbance (in this paper we consider the disturbance of a line tripping) the system evolves according to the fault-on dynamics laws and moves away from the pre-fault equilibrium point δ_{pre}^* . At the clearing time $\tau_{clearing}$, the fault self-clears, the system is at the fault-cleared state $\delta_0 = \delta_F(\tau_{clearing})$, and the tripped line is reclosed (note that we do not consider the permanent faults). Hence, the system configuration is the same as pre-fault one and the power system experiences the post-fault transient dynamics. The transient stability of the post-fault dynamics is certified if the post-fault dynamics converges from the fault-cleared state to the post-fault stable equilibrium point δ_{post}^* , or more clearly, if the fault-cleared state stays inside the region of attraction of the post-fault stable equilibrium point.

In this paper, we assume that when a fault of line tripping happens, then the system operator can immediately send signals to synchronverters to simultaneously adjust the inertia and damping of the imitated generator without any communication and regulation delays (possible ways to deal with the issue of computation and regulation delays will be discussed in Section IV.C). We also assume that the tuned values of inertia and damping in these imitated generators can be kept in at least a time period $[0, \tau_{clearing}]$. Our emergency control problem is how to appropriately tune the inertia and damping

of the imitated generators to compensate for the disturbance such that after the given clearing time $\tau_{clearing}$, the fault-cleared state is still inside the region of attraction of the post-fault stable equilibrium point δ_{post}^* .

If this objective can be obtained, then at the clearing time $\tau_{clearing}$, the fault self-clears, the inertia and damping of the imitated generators are brought back to their initial values, and the power system will evolve according to the post-fault dynamics from the fault-cleared state to the stable post-fault equilibrium point.

III. QUADRATIC LYAPUNOV FUNCTION-BASED TRANSIENT STABILITY CERTIFICATE

In this section, we recall our recently introduced quadratic Lyapunov function-based transient stability certificate for power systems in [11]. For this end, we separate the nonlinear couplings and the linear terminal system in (5). For brevity, we denote the stable post-fault equilibrium point for which we want to certify stability as δ^* . Consider the state vector $x = [x_1, x_2, x_3]^T$, which is composed of the vector of generator's angle deviations from equilibrium $x_1 = [\delta_1 - \delta_1^*, \dots, \delta_{|\mathcal{G}|} - \delta_{|\mathcal{G}|}^*]^T$, their angular velocities $x_2 = [\dot{\delta}_1, \dots, \dot{\delta}_{|\mathcal{G}|}]^T$, and vector of load buses' angle deviation from equilibrium $x_3 = [\delta_{|\mathcal{G}|+1} - \delta_{|\mathcal{G}|+1}^*, \dots, \delta_{|\mathcal{N}|} - \delta_{|\mathcal{N}|}^*]^T$. Let E be the incidence matrix of the graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$, so that $E[\delta_1, \dots, \delta_{|\mathcal{N}|}]^T = [(\delta_k - \delta_j)_{\{k,j\} \in \mathcal{E}}]^T$. Let the matrix C be $E[I_{m \times m} \ O_{m \times n}; O_{(n-m) \times 2m} \ I_{(n-m) \times (n-m)}]$. Then

$$Cx = E[\delta_1 - \delta_1^*, \dots, \delta_{|\mathcal{N}|} - \delta_{|\mathcal{N}|}^*]^T = [(\delta_{kj} - \delta_{kj}^*)_{\{k,j\} \in \mathcal{E}}]^T.$$

Consider the vector of nonlinear interactions F in the simple trigonometric form: $F(Cx) = [(\sin \delta_{kj} - \sin \delta_{kj}^*)_{\{k,j\} \in \mathcal{E}}]^T$. Denote the matrices of moment of inertia, frequency controller action on governor, and frequency coefficient of load as $M_1 = \text{diag}(m_1, \dots, m_{|\mathcal{G}|})$, $D_1 = \text{diag}(d_1, \dots, d_{|\mathcal{G}|})$ and $M = \text{diag}(m_1, \dots, m_{|\mathcal{G}|}, d_{|\mathcal{G}|+1}, \dots, d_{|\mathcal{N}|})$. Let $S = \text{diag}(a_{kj})_{\{k,j\} \in \mathcal{E}}$, $S_1 = [I_{m \times m} \ O_{m \times n-m}]$, $S_2 = [O_{n-m \times m} \ I_{n-m \times n-m}]$, $n = |\mathcal{N}|$, $m = |\mathcal{G}|$.

In state space representation, the power system (5) can be then expressed in the following compact form:

$$\dot{x} = Ax - BF(Cx), \quad (8)$$

with the matrices A, B given by the following expression:

$$A = \begin{bmatrix} O_{m \times m} & I_{m \times m} & O_{m \times n-m} \\ O_{m \times m} & -M_1^{-1}D_1 & O_{m \times n-m} \\ O_{n-m \times m} & O_{n-m \times m} & O_{n-m \times n-m} \end{bmatrix},$$

$$\text{and } B = [O_{m \times |\mathcal{E}|}; \ S_1 M^{-1} E^T S; \ S_2 M^{-1} E^T S].$$

The construction of quadratic Lyapunov function is based on the bounding of the nonlinear term F by linear functions of the angular differences. Particularly, we observe that for all values of $\delta_{kj} = \delta_k - \delta_j$ staying inside the polytope \mathcal{P} defined by the inequalities $|\delta_{kj}| \leq \pi/2$, we have:

$$g(\delta_{kj} - \delta_{kj}^*)^2 \leq (\delta_{kj} - \delta_{kj}^*)(\sin \delta_{kj} - \sin \delta_{kj}^*) \leq (\delta_{kj} - \delta_{kj}^*)^2$$

where $g = \min_{\{k,j\} \in \mathcal{E}} (1 - \sin |\delta_{kj}^*|) / (\pi/2 - |\delta_{kj}^*|)$.

For each transmission line $\{k, j\}$ connecting generator buses k and j , define the corresponding flow-in boundary

segment $\partial \mathcal{P}_{kj}^{in}$ of the polytope \mathcal{P} by equations/inequalities $|\delta_{kj}| = \pi/2$ and $\delta_{kj} \dot{\delta}_{kj} < 0$, and the flow-out boundary segment $\partial \mathcal{P}_{kj}^{out}$ by $|\delta_{kj}| = \pi/2$ and $\delta_{kj} \dot{\delta}_{kj} \geq 0$. Consider the quadratic Lyapunov function $V(x) = x^T P x$ and define the following minimum value of the Lyapunov function $V(x)$ over the flow-out boundary $\partial \mathcal{P}^{out}$ as:

$$V_{\min} = \min_{x \in \partial \mathcal{P}^{out}} V(x), \quad (9)$$

where $\partial \mathcal{P}^{out}$ is the union of $\partial \mathcal{P}_{kj}^{out}$ over all the transmission lines $\{k, j\} \in \mathcal{E}$ connecting generator buses. We have the following result, which is a corollary of Theorem 1 in [11]. Hence, the proof is omitted.

Theorem 1: Consider a power system with the post-fault equilibrium point $\delta^ \in \Delta(\gamma)$ and the fault-cleared state x_0 staying in the polytope \mathcal{P} . Assume that there exists a positive definite matrix P such that*

$$\begin{bmatrix} \bar{A}^T P + P \bar{A} + \frac{(1-g)^2}{4} C^T C & P B \\ B^T P & -I \end{bmatrix} \leq 0 \quad (10)$$

and $V(x_0) < V_{\min}$, where $\bar{A} = A - \frac{1}{2}(1+g)BC$. Then, the system trajectory of (5) will converge from the fault-cleared state x_0 to the stable equilibrium point δ^* .

Therefore, a sufficient condition for the transient stability of the post-fault dynamics is the existence of a positive definite matrix P satisfying the LMI (10) and the Lyapunov function at the fault-cleared state is small than the critical value V_{\min} defined as in (9). We will utilize this condition to design the emergency control in the next section.

IV. SYNCHRONVERTER-BASED EMERGENCY CONTROL DESIGN

A. Control design

In this section, we solve the problem of emergency control described in Section II-B, in which we maintain the power systems transient stability when a fault causes tripping of a line $\{u, v\}$. In particular, we will tune the inertia and damping of synchronverter-integrated generators such that at the fixed clearing time $\tau_{clearing}$, the fault-cleared state x_0 is still inside the region of attraction of the post-fault equilibrium point δ^* . Assume that the tuned inertia and damping can be kept during the time period $[0, \tau_{clearing}]$. Applying Theorem 1, our objective is that: given a positive definite matrix P satisfying the LMI (10), we will find the inertia and damping of the imitated generators such that the fault-cleared state x_0 satisfies $V(x_0) < V_{\min}$.

Indeed, in the grid with well-supported voltages, the fault-on dynamics with the tuned inertia and damping is described by

$$\begin{aligned} \dot{x}_F &= A(m, d)x_F - B(m, d)F(Cx_F) \\ &\quad + B(m, d)D_{\{u,v\}} \sin \delta_{F_{uv}}, \end{aligned} \quad (11)$$

where $D_{\{u,v\}}$ is the vector to extract the $\{u, v\}$ element from the vector of nonlinear interactions F , while $A(m, d)$ and $B(m, d)$ are the new system matrices A, B obtained after the

inertia and damping are tuned. Note that the system matrix C is invariant to the changes of inertia and damping.

We have the following center result of this paper:

Theorem 2: Assume that there exist a positive definite matrix P of size $(|\mathcal{N}| + |\mathcal{G}|)$ satisfying the LMI (10). Let $\mu = \frac{\tau_{clearing}}{V_{min}}$ where V_{min} is defined as in (9). If there exist inertia and damping of the imitated generators such that

$$\begin{aligned} \bar{A}^T P + P \bar{A} + \frac{(1-g)^2}{4} C^T C + P B(m, d) B(m, d)^T P \\ + \mu P B(m, d) D_{\{u, v\}} D_{\{u, v\}}^T B(m, d)^T P \leq 0, \end{aligned} \quad (12)$$

where $\bar{A}(m, d) = A(m, d) - \frac{1}{2}(1+g)B(m, d)C$, then, the fault-cleared state $x_0 = x_F(\tau_{clearing})$ resulted from the fault-on dynamics (11) is still inside the region of attraction of the post-fault equilibrium point δ^* , and the post-fault dynamics following the tripping and reclosing of the line $\{u, v\}$ will return to the original stable operating condition.

Proof: See Appendix VII.

Remark 1: The inequality (12) can be rewritten as

$$\bar{A}(m, d)^T P + P \bar{A}(m, d) + \frac{(1-g)^2}{4} C^T C + P \bar{B} \bar{B}^T P \leq 0,$$

where $\bar{B} = [B(m, d) \quad \sqrt{\mu} B(m, d) D_{\{u, v\}}]$. By Schur complement, this inequality is equivalent with

$$\begin{bmatrix} \bar{A}(m, d)^T P + P \bar{A}(m, d) + \frac{(1-g)^2}{4} C^T C & P \bar{B} \\ \bar{B}^T P & -I \end{bmatrix} \leq 0. \quad (13)$$

When the inertia of synchronverter-integrated generators are fixed, the damping of these generators enters linearly in the system matrices $A(m, d)$, $B(m, d)$ and hence, the inequality (13) is an LMI of these variables. Also, when the damping is fixed, we obtain an LMI with variable as the inverse of inertia. In both these cases, we can quickly solve (13) by convex optimizations. Therefore, the inequality (13) can be solved in polynomial time by a heuristic algorithm, in which we fix the inertia or damping and then use the convex optimizations to solve the corresponding LMI to obtain the optimum value of the remaining variable.

B. Procedure for emergency control

Given the power system under a line tripping and the clearing time $\tau_{clearing}$, we have the following procedure to tune the inertia and damping in the synchronverter-based emergency control:

- 1) Find a matrix $P > 0$ satisfying the LMI (10).
- 2) Calculate the minimum value V_{min} defined as in (9).
- 3) Let $\mu = \frac{\tau_{clearing}}{V_{min}}$.
- 4) Find the inertia and damping of the synchronverter-integrated generators such that the inequality (13) is satisfied. One approach to quickly come up with the solution is to use the heuristic algorithm described in the previous section.
- 5) If there is no such inertia and damping, then repeat from step 1).

- 6) If such values of inertia and damping exist, then the synchronverters will be used to tune the inertia and damping of the imitated generators and keep these values during the time period $[0, \tau_{clearing}]$. At the clearing time $\tau_{clearing}$, the fault self-clears and the inertia and damping of the imitated generators are tuned back to their initial values.

C. Discussions on computation and regulation delays

Computation and regulation delays may make the proposed emergency control scheme in this paper not yet ready for industrial deployment. To overcome this obstacle, we propose the following off-line computational tasks:

- 1) For each line tripping $\{u, v\}$, we off-line check if there exists a positive definite matrix P satisfying the LMI (10) and the inequality (13) where $A(m, d) = A$, $B(m, d) = B$. If such matrix exists, then the line tripping is safe without any emergency control. The LMI (10) is only dependent on the system matrices A, B, C , and thus is checkable before hand. The inequality (13) is also dependent on V_{min} and thus on the equilibrium point. However, we observe that the equilibrium point in practice usually stays inside a small region. As such, we can obtain some lower bound for V_{min} for all of these equilibrium points. By this way, we can check the inequality (13) for a wide range of post-fault equilibrium points. We believe that by this way we can certify that most of the line trippings in practice are safe.
- 2) For the remaining unsafe line trippings, we calculate before hand if there exist optimum values of inertia and damping of the imitated generators such that the inequality (13) is satisfied. Again, we need to use some bound for V_{min} as above. If these values exist, then we can apply the emergency control described in the previous section right after the fault happens.
- 3) For some unsafe line tripping that there are no inertia and damping satisfying the inequality (13), we may apply traditional emergency control schemes such as load shedding to quickly stabilize the system.

Another way is to allow for a time period of $[0, \tau_{delay}]$ to compensate for the time of computation and regulation. In this period, the fault-on trajectory evolves according to fault-on dynamics (11) where $A(m, d) = A$, $B(m, d) = B$. Again, by finding the positive definite matrix P satisfying both the LMI (10) and the inequality (13) where $A(m, d) = A$, $B(m, d) = B$, we can bound the fault-on dynamics during the delayed period $[0, \tau_{delay}]$. This bound will help us to design the inertia and damping in the period $[\tau_{delay}, \tau_{clearing}]$ such that the fault-cleared state at the clearing time $\tau_{clearing}$ is still inside the region of attraction of the post-fault equilibrium point.

D. Other way to design the emergency control

This section presents another way to determine the optimum value of the inertia and damping of the imitated generators such that the fault-cleared state satisfies $V(x_0) <$

V_{\min} where $V(x) = x^T P x$ and V_{\min} is defined as in (9). The reason is that there are possible situations where we fail to find the inertia and damping by the procedure described in Section IV-B because we require a common Lyapunov function P for both fault-on dynamics and post-fault dynamics. Our observation is that it is easier to find different Lyapunov functions for different dynamics. Therefore, we propose the following procedure to find inertia and damping:

- 1) Find a matrix $P > 0$ satisfying the LMI (10).
- 2) Calculate the minimum value V_{\min} defined as in (9).
- 3) Let $\mu = \frac{\tau_{clearing}}{V_{\min}}$.
- 4) Vary the inertia and damping of the synchronverter-integrated generators, and for each fixed value of inertia and damping, find a matrix $\tilde{P} \geq P$ such that

$$\begin{bmatrix} \bar{A}(m, d)^T \tilde{P} + \tilde{P} \bar{A}(m, d) + \frac{(1-g)^2}{4} C^T C & \tilde{P} \bar{B} \\ \bar{B}^T \tilde{P} & -I \end{bmatrix} \leq 0 \quad (14)$$

- 5) If there is no such matrix \tilde{P} , then repeat from step 1) or step 4) with new values of inertia and damping.
- 6) If such values of inertia and damping exist, then the synchronverters will be used to tune the inertia and damping of the imitated generators and keep these values during the time period $[0, \tau_{clearing}]$. At the clearing time $\tau_{clearing}$, the fault self-clears and the inertia and damping of the imitated generators are tuned back to their initial values.

Since the matrix \tilde{P} satisfies the LMI (14), similar to Theorem 2 we can prove that $x_0^T \tilde{P} x_0 < V_{\min}$. This inequality, together with that $\tilde{P} \geq P$, leads to $x_0^T P x_0 \leq x_0^T \tilde{P} x_0 < V_{\min}$. Applying Theorem 1, we conclude that the fault-cleared state stays inside the region of attraction and therefore the post-fault dynamics is stable.

V. NUMERICAL VALIDATION

For illustrating the concept of this paper, we consider the simple yet non-trivial system of three generators, one of which is the renewable generator (generator 1) integrated with the synchronverter. The susceptance of the transmission lines are assumed at fixed values $B_{12} = 0.739$ p.u., $B_{13} = 1.0958$ p.u., and $B_{23} = 1.245$ p.u. Also, the inertia and damping of all the conventional and imitated generator at the normal working condition are $m_k = 2$ p.u., $d_k = 1$ p.u. Assume that the line between generators 1 and 2 is tripped, and then reclosed at the clearing time $\tau_{clearing} = 200ms$, and during the fault-on dynamic stage the time-invariant terminal voltages and mechanical torques given by: $V = [1.0566 \ 1.0502 \ 1.0170]^T$, $P = [-0.2464 \ 0.2086 \ 0.0378]^T$. The pre-fault and post-fault equilibrium point is calculated from (6): $\delta^* = [-0.6634 \ -0.5046 \ -0.5640 \ 0 \ 0 \ 0]^T$. Hence, the equilibrium point stays in the polytope defined by the inequality $|\delta_{kj}| < \pi/10$. As such, we can take $g = (1 - \sin(\pi/10))/(\pi/2 - \pi/10)$. Using CVX in MATLAB to solve the LMI (10), we can obtain the Lyapunov function

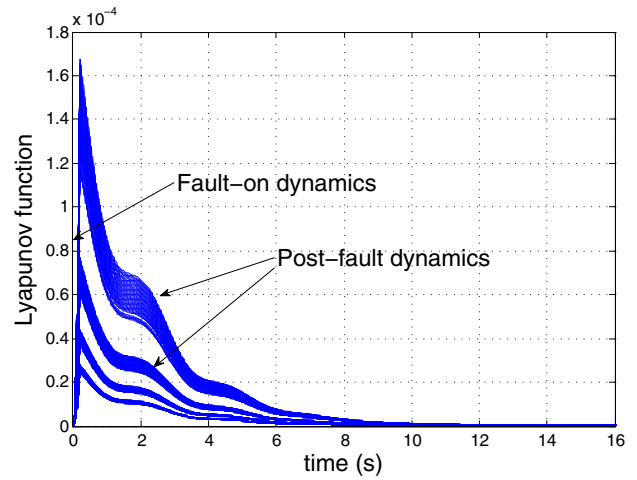


Fig. 1. Variations of the quadratic Lyapunov function $V(x) = x^T P x = (\delta - \delta^*)^T P (\delta - \delta^*)$ during the post-fault and fault-on dynamics with different values of inertia and damping.

$V(x) = x^T P x$, where P is

$$\begin{bmatrix} 2.8401 & 1.9098 & 1.9812 & 4.5726 & 4.4349 & 4.4563 \\ 1.9098 & 2.7949 & 2.0263 & 4.4393 & 4.5628 & 4.4578 \\ 1.9812 & 2.0263 & 2.7235 & 4.4502 & 4.4644 & 4.5480 \\ 4.5726 & 4.4393 & 4.4502 & 18.4333 & 17.5302 & 17.6662 \\ 4.4349 & 4.5628 & 4.4644 & 17.5302 & 18.3632 & 17.7364 \\ 4.4563 & 4.4578 & 4.5480 & 17.6662 & 17.7364 & 18.2271 \end{bmatrix} \quad (15)$$

The minimum value: V_{\min} is $V_{\min} = 0.8139$ and $\mu = \tau_{clearing}/V_{\min} = 0.2457$.

Here, we illustrate the effectiveness of the procedure described in Section IV.D. We use the Lyapunov function $V(x) = x^T P x$ for the post-fault dynamics, and then solve the LMI (14) to find the Lyapunov function $x^T \tilde{P} x$ for the fault-on dynamics for each varied value of new inertia and damping. We can see that there are many values of inertia and damping such that the LMI (14) has positive definite solution. This means that there are many values of inertia and damping to compensate for the dynamics deviation caused by the faulted line and thereby stabilize the power systems. We draw in Fig. 1 the dynamics of Lyapunov function $V(x) = x^T P x$ during the fault-on and post-fault dynamics, where the fault-on dynamics is controlled by different feasible values of inertia and damping. This confirms that there are many suitable values of inertia and damping for the emergency control.

VI. CONCLUSIONS AND PATH FORWARD

This paper was dedicated to bring into the attention of the control community the problem of emergency control in power systems. In particular, we discussed a novel emergency control for power grids with high penetration of renewables by exploiting the emerging power electronics resources. For this end, we modeled the structure preserving model of such systems where the renewables are integrated with the synchronverters which will control the renewable

generator to mimic the dynamics of the conventional generators but with tunable damping and inertia. On top of this model, we formulated the emergency control to maintain the transient stability of post-fault dynamics following a given line tripping by intelligently adjusting the damping and inertia of the imitated generators. Applying our recently introduced quadratic Lyapunov function-based transient stability certificate, we showed that this problem can be solved through a number of convex optimizations in the form of linear matrix inequalities. The numerical simulations showed that this emergency control is effective to recover transient stability after critical line trippings.

We also discussed possible ways to make this novel emergency scheme ready for industrial employment. Particularly, we sketched ways to address the issues of computation and regulations delays, either by offline scanning contingencies and calculating the emergency actions before hand, or by allowing specific delayed time for computation.

Future works would demonstrate the proposed emergency control scheme on large IEEE prototype and large dynamic realistic power systems with renewable generation at various locations and with different levels of renewable penetration. Critical faults when the fault-cleared state is driven out of the $\pi/2$ polytope will be addressed by utilizing the Lyapunov functions family approach [13], [14]. Also, we would investigate the potential values of transmission facilities, which are ubiquitously equipped in the existing power grids such as FACTS devices, to intelligently control the transmission network as an alternative remedial action to emergency situations.

VII. APPENDIX: PROOF OF THEOREM 2

We have the derivative of $V(x)$ along the fault-on trajectory (11) as follows:

$$\dot{V}(x_F) = W(x_F) - S^T S + 2x_F^T P B(m, d) D_{uv} \sin \delta_{F_{uv}} + x_F^T [A(m, d)^T P + P A(m, d) - C^T K_1^T K_2 C + R^T R] x_F,$$

where $W(x_F) = (F - g C x_F)^T (F - C x_F)$, $R = B(m, d)^T P - \frac{1}{2}(1 + g)C$, and $S = F + (B(m, d)^T P - \frac{1}{2}(1 + g)C)x_F$. On the other hand, $2x_F^T P B(m, d) D_{\{u,v\}} \sin \delta_{F_{uv}} \leq \frac{1}{\mu} \sin^2 \delta_{F_{uv}} + \mu x_F^T P B(m, d) D_{\{u,v\}} D_{\{u,v\}}^T B(m, d)^T P x_F$. Therefore,

$$\dot{V}(x_F) \leq W(x_F) - S^T S + x_F^T \tilde{Q} x_F + \frac{1}{\mu} \sin^2 \delta_{F_{uv}} \quad (16)$$

where $\tilde{Q} = A(m, d)^T P + P A(m, d) - C^T K_1^T K_2 C + R^T R + \mu P B(m, d) D_{\{u,v\}} D_{\{u,v\}}^T B(m, d)^T P$. Note that $W(x_F) \leq 0, \forall x_F \in \mathcal{P}$, and from (12), we have $\tilde{Q} \leq 0$. Hence,

$$\dot{V}(x_F) \leq \frac{1}{\mu} \sin^2 \delta_{F_{uv}} \leq \frac{1}{\mu}, \forall x_F \in \mathcal{P}. \quad (17)$$

Assume that $x_F(\tau_{clearing})$ is not in the set \mathcal{R} . Note that the boundary of \mathcal{R} is constituted of the segments on flow-in boundary $\partial \mathcal{P}^{in}$ and the segments on the sublevel sets of the Lyapunov function. It is easy to see that the flow-in boundary

$\partial \mathcal{P}^{in}$ prevents the fault-on dynamics (11) from escaping \mathcal{R} . Therefore, the fault-on trajectory can only escape \mathcal{R} through the segments which belong to sublevel set of the Lyapunov function $V(x)$. Denote τ be the first time at which the fault-on trajectory meets one of the boundary segments which belong to sublevel set of the Lyapunov function $V(x)$. Hence $x_F(t) \in \mathcal{R}, \forall 0 \leq t \leq \tau$. From (17) and $\mathcal{R} \subset \mathcal{P}$, we have

$$V(x_F(\tau)) - V(x_F(0)) = \int_0^\tau \dot{V}(x_F(t)) dt \leq \frac{\tau}{\mu} < \frac{\tau_{clearing}}{\mu} = V_{\min} \quad (18)$$

Note that $x_F(0)$ is the pre-fault equilibrium point, and thus equals to post-fault equilibrium point. Hence $V(x_F(0)) = 0$. By definition, we have $V(x_F(\tau)) = V_{\min}$, which is a contradiction with (18). \square

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