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Design of vibratory energy harvesters under stochastic parametric uncertainty: a new optimization philosophy

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Abstract

Vibratory energy harvesters as potential replacements for conventional batteries are not as robust as batteries. Their performance can drastically deteriorate in the presence of uncertainty in their parameters. Parametric uncertainty is inevitable with any physical device mainly due to manufacturing tolerances, defects, and environmental effects such as temperature and humidity. Hence, uncertainty propagation analysis and optimization under uncertainty seem indispensable with any energy harvester design. Here we propose a new modeling philosophy for optimization under uncertainty; optimization for the worst-case scenario (minimum power) rather than for the ensemble expectation of the power. The proposed optimization philosophy is practically very useful when there is a minimum requirement on the harvested power. We formulate the problems of uncertainty propagation and optimization under uncertainty in a generic and architecture-independent fashion, and then apply them to a single-degree-of-freedom linear piezoelectric energy harvester with uncertainty in its different parameters. The simulation results show that there is a significant improvement in the worst-case power of the designed harvester compared to that of a naively optimized (deterministically optimized) harvester. For instance, for a 10% uncertainty in the natural frequency of the harvester (in terms of its standard deviation) this improvement is about 570%.

Keywords: energy harvesting, vibration, optimization, uncertainty, stochastic, piezoelectric

(Some figures may appear in colour only in the online journal)

1. Introduction

The considerable reduction in power consumption of electronics in addition to the scalability issue of conventional batteries have made harvesting energy from ambient vibration, a universal and abundant source of energy, a viable alternative to bulky and costly conventional batteries [1]. To efficiently harvest energy from the excitation source, mechanical and electrical parameters of the harvester should be well optimized and finely tuned. Finding exact or approximate optimal deterministic parameters for electromagnetic [2] and piezoelectric [3–7] energy harvesters has been comprehensively studied in the literature for the linear harvesters. For the nonlinear energy harvesters, researchers have mainly studied the effects of mechanical potential shape [8–12] or the harvesting circuitry [13–15]. All these studies have assumed deterministic system parameters to optimize the harvested power; however, manufacturing tolerances, wear and tear and material degradation, and humidity, temperature and environmental conditions among others result in parametric uncertainty in the system. Uncertainty in the system usually necessitates two types of analysis: uncertainty propagation and sensitivity analysis, and optimization under uncertainty for robust design.

Although researchers have explored the topics of sensitivity analysis and optimization under uncertainty in other fields like controls, finance, and production planning, they have not received much attention in the field of energy harvesting. Ng and Liao [16] studied voltage and charge sensitivity of three unimorph, series triple layer and parallel triple layer cantilever piezoelectric energy harvesters (PEHs) to vertical tip force for

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different values of metal layer thickness and Young's modulus. Ali *et al* [17] studied with the help of Monte Carlo (MC) simulations, the effect of uncertainty in harmonic excitation frequency, mechanical damping and electromechanical coupling on the ensemble expectation of the harvested power of a linear PEH. They optimized deterministic dimensionless time constant and electromechanical coupling coefficient as a function of standard deviation in the excitation frequency.

Godoy and Trindade [18] used MC simulations to study the effect of parametric uncertainties in piezoelectric and dielectric constants of the piezoelectric layer and the inductance of the harvesting circuit on the mean and confidence levels of different output variables of the system. Franco and Varoto [19] studied the geometric and electrical parametric uncertainty in a cantilever PEH. They used MC simulations for sensitivity analysis and used stochastic optimization to optimize the parameters for the ensemble expectation of the harvested power.

There are even fewer studies on uncertainty propagation and optimization under uncertainty when it comes to nonlinear vibratory energy harvesters (VEHs). Mann *et al* [20] studied the uncertainty propagation in the harvested power of a nonlinear electromagnetic energy harvester with respect to the uncertainty in different mechanical and electrical parameters. Using harmonic balance method they derived and used the approximate nonlinear frequency responses for monostable hardening and softening harvesters in addition to bistable harvester for uncertainty propagation analysis. More recently Madankan *et al* [21] studied the uncertainty quantification in a nonlinear mono-stable PEH with the help of conjugate unscented transformation quadrature points. With this technique they approximated the system statistics in terms of power and maximum deflection moments (up to fourth order) which were then used to approximate joint and marginal probability density functions of power and maximum tip deflection using maximum entropy principle.

Most of the studies about uncertainty in the context of energy harvesting discussed above are about uncertainty quantification [16, 18, 20, 21]. Only two of the above-mentioned studies explored optimization under uncertainty [17, 19]. Approaches to optimization under uncertainty have followed a variety of modeling philosophies, including expectation minimization, minimization of deviations from goals, minimization of maximum costs, and optimization over soft constraints [22]. The two optimization studies mentioned here are of the expectation-minimization type (minimization of the negative of the ensemble average of the harvested power).

Maximizing expected power is an appropriate approach when a large number of harvesters are to be used together (uncertainty in harvester parameters) to power up a device or when one harvester is to be used in an uncertain environment. Imagine 100 harvesters are to be used to power up a device or charge a battery, then maximizing the expected power over parametric uncertainties makes perfect sense as the expected power of the ensemble is a good measure of the total delivered power. Now consider a case where a single harvester powers up a device which requires a minimum power to operate properly. This would be a common setup for self-powered medical implants, wireless sensors and many other applications of energy harvesters. For instance, suppose a hospital decides to purchase medical devices say pacemakers, powered by energy harvesters. In this case it is crucial that for each single device, its harvester delivers a minimum power; otherwise, it will cause serious health-related complexities. In this case, the customer, i.e. the hospital will be interested in a batch of devices with the maximum number of devices fulfilling the minimum power requirement or alternatively, in a batch of devices with the largest minimum power for a given percentage of the total number of devices. It is obvious that the expected power of the batch will be of minimal interest in this case; hence, optimizing the harvesters for the maximum expected power is not practically helpful. This type of demands and problems requires another optimization philosophy: optimization of minimum power (worst-case scenario) and *not* expectation optimization. This paper addresses this type of optimization which is of great importance in the field of energy harvesting and has not yet been addressed.

In this paper we formulate two problems in a generic form: (i) propagation of parametric uncertainty in terms of the worst-case (minimum) power and (ii) optimization of the worst-case power in presence of parametric uncertainty. The later is cast as a min-max optimization. The former analysis provides information about the minimum power delivery of a specific percentage (depending on the confidence-level) of a batch of harvesters. Parametric uncertainties are modelled as Gaussian random variables. Optimization in (ii) is done over deterministic parameters and the mean values of the uncertain parameters which are assumed to be controllable in a mean-value sense to maximize the worst-case performance of the harvesters for a given confidence level and parametric uncertainty. Finally these two problems are applied to a PEH and the results of uncertainty propagation and min-max optimization are presented.

2. Mathematical modeling

A cantilever beam with attached piezoelectric patches is the most common VEH design. Since most of the energy is carried by the lowest excited harmonic of the vibratory structure, the cantilever beam PEH is usually modeled as a single-degree-of-freedom (SDOF) oscillator coupled with an electrical circuit as shown in figure 1. Assuming that the piezoelectric patches are directly connected to a load resistance, and that the harvester is base-excited the governing dynamic equations of the system could be written as [23, 24]

$$\begin{aligned} m\ddot{x} + c\dot{x} + kx + \theta v &= -m\ddot{x}_b \\ C_p\dot{v} + \frac{v}{R} &= \theta\dot{x}, \end{aligned} \tag{1}$$

where m, k, and c are the oscillator's mass, linear stiffness and damping coefficient, respectively. C_p, θ and R are the inherent capacitance of the piezoelectric layer, electromechanical coupling coefficient, and the load resistance, respectively. x, x_b and v are the oscillator's displacement relative to its base, base displacement, and the voltage across the load resistance, respectively.

Average power is the measure of the performance of the harvester. Since the system is linear a closed-form solution for

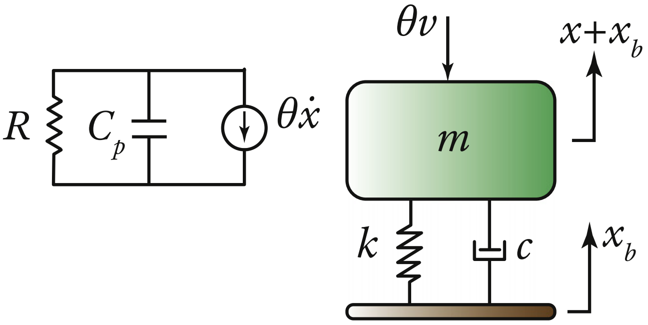

Figure 1. A base-excited PEH modeled as an SDOF oscillator coupled with an electric circuit modeling a load resistance and the inherent capacitance of the piezoelectric layer.

the power could be easily found by applying the Fourier transform to equation (1). Power is conventionally normalized by the square of input acceleration for the harmonic excitation. The normalized peak power[2] could then be written as

$$\left|\frac{P(\omega)}{(X_b\omega^2)^2}\right| = \frac{1}{R}\left|\frac{V}{X_b\omega^2}\right|^2 = \frac{R\theta^2\omega^2}{\left(\left(RC_p\omega_n^2 + 2\zeta\omega_n + \frac{R\theta^2}{m}\right)\omega - RC_p\omega^3\right)^2 + (-\omega_n^2 + (1 + 2\zeta RC_p\omega_n\omega^2)\omega^2)^2}, \qquad (2)$$

where $X_b(\omega)$ and $V(\omega)$ are the Fourier transforms of the base displacement and load voltage, respectively, and ω is the excitation frequency. Also, by convention natural frequency ω_n and damping ratio ζ are introduced which are defined as $\omega_n = \sqrt{k/m}$, and $\zeta = c/2\sqrt{km}$.

We also consider the case where excitation is wideband random excitation. In this case we use Parseval's identity which relates the average energy in a signal to its finite Fourier transform as [25, 26]:

$$\overline{P}(t) = \lim_{T\to\infty}\frac{1}{T}\int_0^T \frac{v^2(t)}{R}\mathrm{d}t = \int_0^\infty \frac{S_v(\omega)}{R}\mathrm{d}\omega, \qquad (3)$$

where $S_v(\omega)$ is the power spectral density of the voltage across the load and is related to the input acceleration power spectral density $S_{\ddot{x}_b}(\omega)$ by the relation [27, 28]

$$S_v(\omega) = |H_{\ddot{x}_b}^v(\omega)|^2 S_{\ddot{x}_b}(\omega). \qquad (4)$$

In equation (4) $S_{\ddot{x}_b}(\omega)$ is one-sided power spectral density of input acceleration and $H_{\ddot{x}_b}^v(\omega)$ is the transfer function from input base acceleration $\ddot{x}_b$ to the load voltage v and could be derived based on governing dynamics equations in equation (1) as

$$H_{\ddot{x}_b}^v(\omega) = \frac{R\theta\omega}{\left(\left(RC_p\omega_n^2 + 2\zeta\omega_n + \frac{R\theta^2}{m}\right)\omega - RC_p\omega^3\right) + (-\omega_n^2 + (1 + 2\zeta RC_p\omega_n\omega^2)\omega^2)j}, \qquad (5)$$

where $j = \sqrt{-1}$.

For a deterministic harvester equations (2) and (3) could be used to study the effect of different parameters on the harvested power and to optimize them. Here we assume that some of the parameters are random. This uncertainty in parameters could be a result of manufacturing tolerances or defects, material degradation, or environmental effects such as temperature or humidity. Random parameters ξ_i are modelled as Gaussian variables with mean value of ξ_{mi} and standard deviation of σ_{ξ_i}.

3. Uncertainty propagation and optimization formulation

In this study we investigate the effect of uncertainty on the minimum harvested power, i.e. the worst-case performance, and then optimize the mean uncertain parameters to maximize the minimum power, i.e. optimization for the best worst-case performance. The random parameters are modeled as Gaussian with a mean and a standard deviation. Here we assume the mean value of the parameters (ξ_{mi}) are controllable. Hence we write the ith random parameter as $\xi_i = \xi_{mi} + \delta\xi_i$ where $\delta\xi_i$ is the variation from the mean value. We know that for random variables with Gaussian distribution this variation extends from $-\infty$ to $+\infty$; however, the closer it gets to the tails the smaller gets the probability of the parameter in that range. Therefore, to make the optimization tractable and nontrivial we have to limit the variation $\delta\xi_i$ for a desired confidence level. For example for a 99.7% confidence level, $-3\sigma_{\xi_i} < \delta\xi_i < +3\sigma_{\xi_i}$, and for a 95.5% confidence level we should limit $\delta\xi_i$ as $-2\sigma_{\xi_i} < \delta\xi_i < +2\sigma_{\xi_i}$.

Suppose a manufacturer mass produces a batch of harvesters with parametric uncertainties. It is important for the customer to know that a certain percentage of the harvesters (defining the confidence level), say 95.5% of the harvesters, deliver a minimum required power. To answer this question, the manufacturer should be able to quantify the effect of uncertainties on the worst-case performance (minimum power) for a given confidence level. Moreover, it is clear that the larger the confidence level fulfilling a minimum power requirement or the larger the minimum power for a given

[2] For a harmonic excitation the average power is simply half the peak power; hence, we simply use the peak power as a performance measure.

confidence level is, the better is the quality of that batch. Assuming that the mean value of the uncertain parameters are controllable in the manufacturing process, then the optimization should be carried over the mean values of the random parameters in addition to the deterministic parameters to maximize the worst-case power for a given confidence level or to maximize the confidence level for a given worst-case power. For the optimization in this study we do the former i.e. maximizing the worst-case power for a given confidence level.

As discussed we have two types of problems here for both harmonic and random excitations:

(P1): *Uncertainty propagation:* given the confidence level, find the worst-case (minimum) power as a function of parametric uncertainties (standard deviations σ_{ξ_i}), deterministic parameters, and mean values of uncertain parameters ξ_{mi}:

$$\bar{P}^{\mathrm{wc}}(\xi_{mi}, \sigma_{\xi_i}, \xi_j^{\mathrm{det}}) = \min_{\xi_i}\{\bar{P}(\xi_i, \xi_j^{\mathrm{det}}) : \xi_i \in (\xi_{mi} - \max(\delta\xi_i), \xi_{mi} + \max(\delta\xi_i))\}. \quad (6)$$

(P2): *Optimization for the worst-case scenario under parametric uncertainty:* given the confidence level, find the optimum mean value of the uncertain parameters ξ_{mi}, and the deterministic parameters ξ_j^{det} to maximize the worst-case (minimum) power:

$$\bar{P}^{\mathrm{wc}}_{\max}(\sigma_{\xi_i}) = \max_{\xi_{mi},\, \xi_j^{\mathrm{det}}} \min_{\xi_i}\{\bar{P}(\xi_i, \xi_j^{\mathrm{det}}) : \xi_i \in (\xi_{mi} - \max(\delta\xi_i), \xi_{mi} + \max(\delta\xi_i))\}, \quad (7)$$

where ξ_j^{det} is the jth deterministic parameter. P2 is also known as min-max optimization problem. To study P1, deterministic parameters and mean values for the uncertain parameters i.e. ξ_{mi} are selected and fixed; then, a search over a grid of $\xi_{mi} - \max(\delta\xi_i) < \xi_i < \xi_{mi} + \max(\delta\xi_i)$ is conducted to find the minimum power. If the grids are fine enough, the grid search algorithm finds the global optimum point in the region of interest. However, if the number of decision variables are relatively large the algorithm slows down. Depending on the confidence level, $\max(\delta\xi_i)$ can adopt different values in terms of the standard deviation σ_{ξ_i}. For instance, for a confidence level of 95.5%, $\max(\delta\xi_i) = 2\sigma_{\xi_i}$. In addition, depending on the number of simultaneous uncertain parameters (decision variables) being studied ($i = 1, 2, \ldots, n$) the search grid will be on a line, surface, or in an n-dimensional hypercube in general. Also, optimum parameters for a deterministic harvester are used as mean values for the uncertain parameters (ξ_{mi}) and the deterministic parameters in P1[3]. This is what we would refer to as *naive* optimization i.e. the optimization of the parameters without considering uncertainties. P1 shows how uncertainty in parameters affects the worst-case power of a naively optimized harvester.

[3] If there is no optimum value a practically reasonable value is selected.

To study P2, the same procedure as described above for P1 is carried out over feasible deterministic parameters ξ_j^{det} and the mean values of the uncertain parameters ξ_{mi} to find the optimum values for the said parameters to maximize the worst-case power. In the next section numerical results are presented and discussed.

4. Numerical results and discussion

We explore the effects of uncertainty in three parameters namely, natural frequency ω_n, load resistance R and electromechanical coupling coefficient θ on the worst-case harvested power for different confidence levels (P1). Then considering these uncertainties, we optimize the deterministic parameters and the mean value of the uncertain parameters to maximize the worst-case power (P2). To be able to visualize the effects we consider two uncertain parameters at a time and optimize over the mean values of those two parameters unless otherwise specified.

Figure 2 shows the normalized worst-case power as a function of normalized uncertainty in natural frequency and load resistance when subjected to harmonic base excitation. Uncertainties are applied to the harvester optimized for deterministic parameters (naive optimization). Worst-case power is normalized by the maximum power of a deterministic harvester and the uncertainties in parameters are normalized by their deterministic optimum values. In all the simulations $m = 0.001$ kg, $\zeta = 0.02$, and $C_p = 100$ nF. Also, $\omega = 70\ \mathrm{rad\ s^{-1}}$ for harmonic excitation. According to figure 2(a), the worst-case power is very sensitive to the natural frequency but not much to the load resistance. Sharp resonance peak and wide peak for the optimum load resistance in linear harvesters explain this sensitivity. Figure 2(b) depicts this dependence on uncertainty in natural frequency for two different uncertainty levels in the load resistance i.e. zero and 20% uncertainty for different confidence levels of 68%, 95.5%, and 99.7%. According to the figure the larger the confidence level the smaller the worst-case power. This is because the larger confidence level simply means the larger deviation in the parameter from its optimum value.

Figure 3 shows dependence of the normalized worst-case power on uncertainties in natural frequency and electromechanical coupling. According to the figure the sensitivity of the worst-case power to the electromechanical coupling coefficient is considerable and larger than that of the load resistance. Next, sensitivity to the same parameters are studied when the harvester is subjected to wide-band random base excitation. The random excitation considered here is stationary and Gaussian with flat power spectral density of $S_{\ddot{x}_b} = 10^{-3}\ \mathrm{g^2\ Hz^{-1}}$ over frequency range of [2, 50] Hz. This profile results in excitation root-mean-square acceleration of 0.22 g and is very similar to the ASTM D4169 standard profile (level 2) for railroad shipment [29].

Figures 4 and 5 show effect of uncertainty in natural frequency, load resistance, and electromechanical coupling coefficient on the worst-case power. According to the figures, worst-case power is not very sensitive to uncertainty in

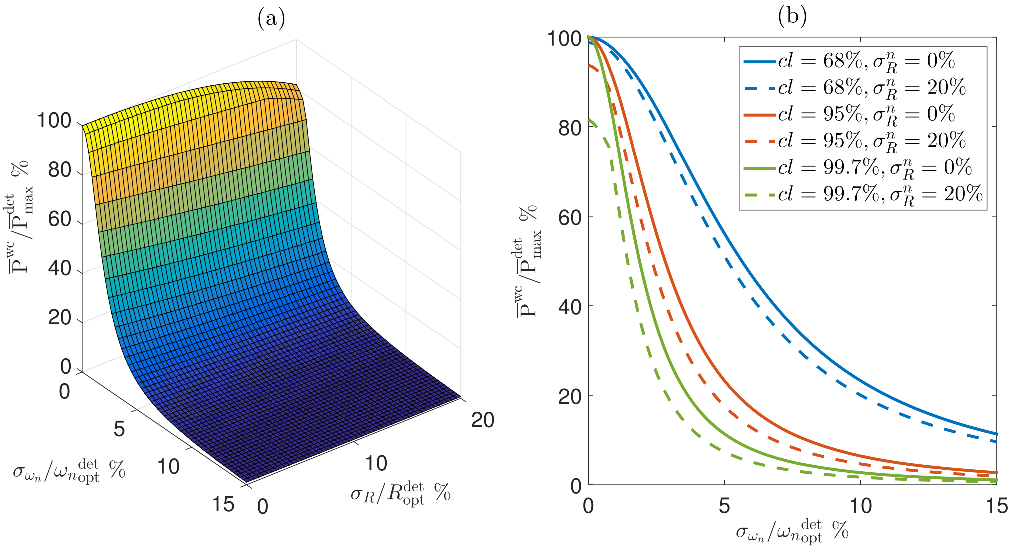

Figure 2. Dependence of normalized worst-case power on normalized uncertainty in natural frequency and load resistance for harmonic excitation: (a) dependence as surface plot for confidence level of 99.7%, (b) dependence on uncertainty in natural frequency for two different normalized uncertainty values in load resistance ($\sigma_R^n = \sigma_R / R_{opt}^{det}\%$), and for three confidence levels of 68%, 95.5%, and 99.7%.

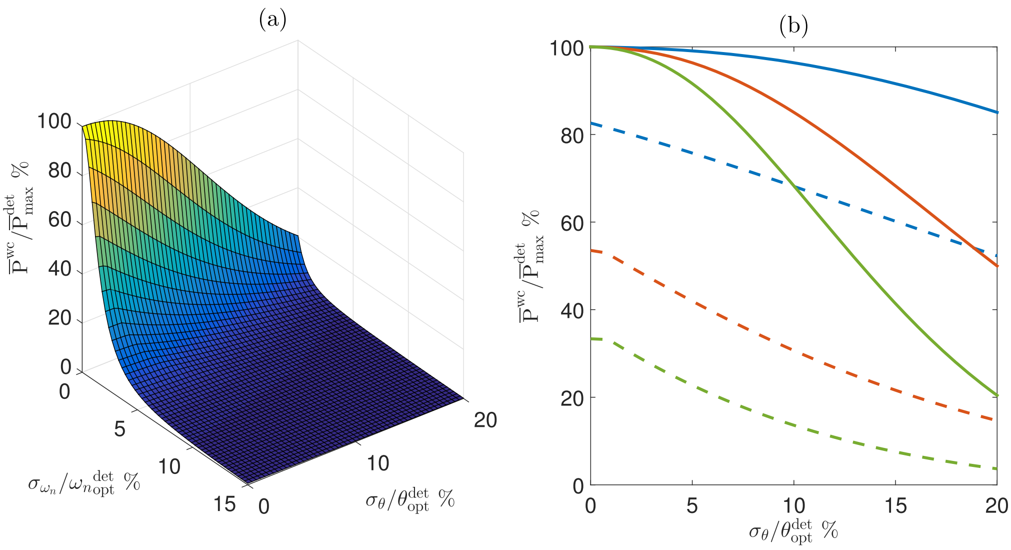

Figure 3. Dependence of normalized worst-case power on normalized uncertainty in natural frequency and electromechanical coupling coefficient for harmonic excitation: (a) dependence as surface plot for confidence level of 99.7%, (b) dependence on uncertainty in electromechanical coupling coefficient for two different normalized uncertainty values in natural frequency of 0% (solid line) and 20% (dashed line), and for three confidence levels of 68% (blue), 95.5% (red), and 99.7% (green).

natural frequency. The reason is two-fold: one is that in general when excitation changes from one harmonic to wide-band, the narrow peak in the harvester power transforms into a wide peak and hence becomes less sensitive to changes in natural frequency. This is because the narrow peak of resonance will be captured over a wider frequency range whereas in the harmonic excitation this peak is captured only at one frequency. Second, the optimum natural frequency is 1 Hz[4] that is a relatively small number; hence an uncertainty of say 15% will change the natural frequency in the worst case (in 3σ sense) by only 0.45 Hz which is not a big enough variation to cause a significant change in the harvested power. Since the worst-case power is not very sensitive to uncertainty in natural frequency for random excitation, we study the effect of uncertainty in load resistance and electromagnetic coupling at zero uncertainty in natural frequency in figures 4(b) and 5 (b). According to the figures, uncertainty in electromechanical coupling coefficient has larger effect on the worst-case power than that of the load resistance.

It was shown that uncertainty in parameters of a naively optimized harvester could drastically decrease its worst-case power. Next we would like to see if optimization of the deterministic parameters and/or mean value of the uncertain

[4] This was the lowest limit for the search for optimum natural frequency; natural frequencies smaller than this result in large vibration displacements.

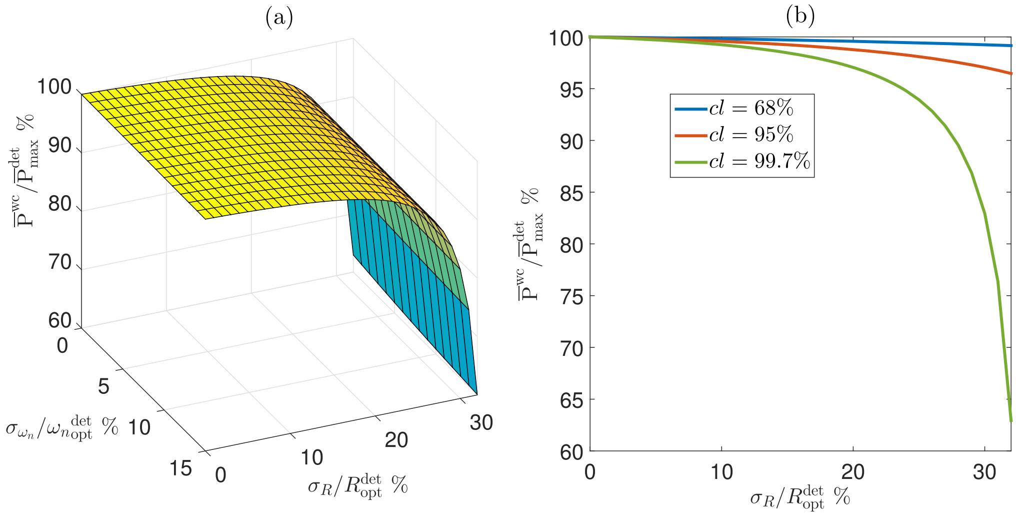

Figure 4. Dependence of normalized worst-case power on normalized uncertainty in natural frequency and load resistance for random excitation: (a) dependence as surface plot for confidence level of 99.7%, (b) dependence on uncertainty in load resistance for zero uncertainty in natural frequency i.e. deterministic ω_n, and for three confidence levels of 68%, 95.5%, and 99.7%.

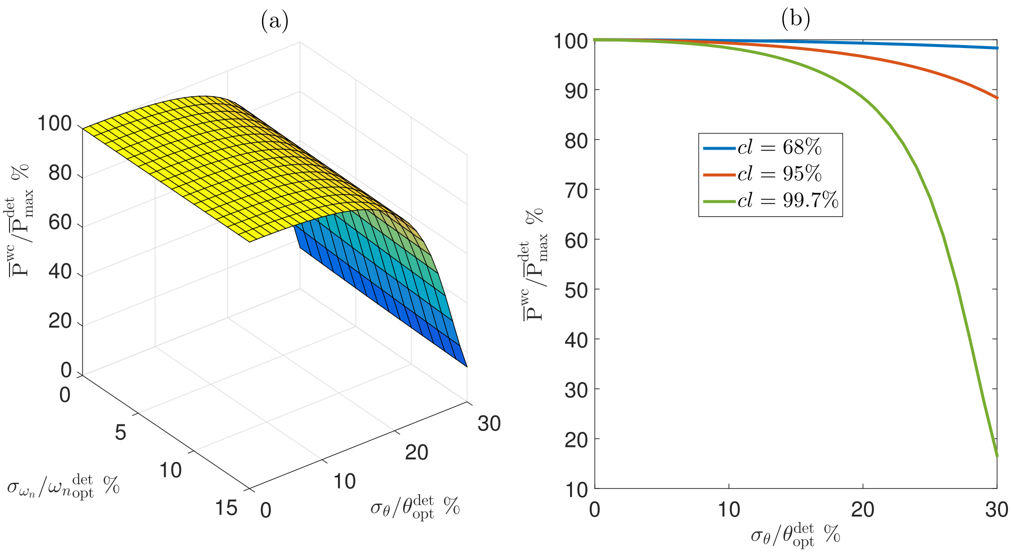

Figure 5. Dependence of normalized worst-case power on normalized uncertainty in natural frequency and electromechanical coupling coefficient for random excitation: (a) dependence as surface plot for confidence level of 99.7%, (b) dependence on uncertainty in electromechanical coupling coefficient for zero uncertainty in natural frequency i.e. deterministic ω_n, and for three confidence levels of 68%, 95.5%, and 99.7%.

parameters with the knowledge of uncertainties in the system will help to decrease the effect of uncertainty on the worst-case power. This could be done by numerically solving the min-max optimization problem in P2.

Optimization procedure formulated in P2 is applied to the harvester under harmonic and random excitation. Figures 6 and 7 illustrate how optimization under parametric uncertainty improves worst-case power compared to the naively optimized system i.e. the system optimized for deterministic parameters. Figure 6 shows the normalized maximum worst-case power as a function of normalized uncertainty in natural frequency and load resistance. Optimization is done over mean values of the said uncertain parameters. For comparison, worst-case power of the naively optimized harvester is also plotted. Figure 7 shows the optimized worst-case power as a function of natural frequency and electromechanical coupling over mean values of which the optimization is applied. As could be seen in figures 6 and 7, optimization under uncertainty greatly improves the worst-case power over the naively optimized harvester for harmonic excitation.

Optimization P2 is next applied to the harvester under random excitation. Since it was shown the harvester in this case is quite insensitive to the natural frequency, only load resistance and electromechanical coupling are considered uncertain and random. Figure 8(a) shows the results of optimization over

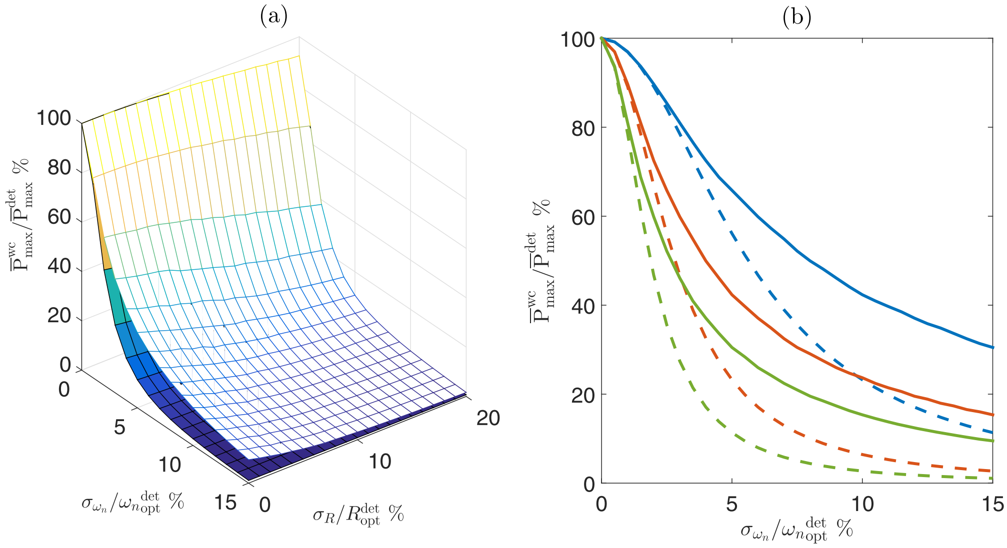

Figure 6. Maximized worst-case power as a function of uncertainty in natural frequency and load resistance for harmonic excitation. (a) the maximum worst-case power (wireframe mesh) compared to the worst-case power of the naively optimized harvester (solid surface) for confidence level of 99.7% (b) maximum worst-case power (solid line) as a function of uncertainty in the natural frequency (no uncertainty in load resistance) compared to the naively optimized harvester (dashed line) for confidence levels of 68% (blue), 95.5% (red), and 99.7% (green).

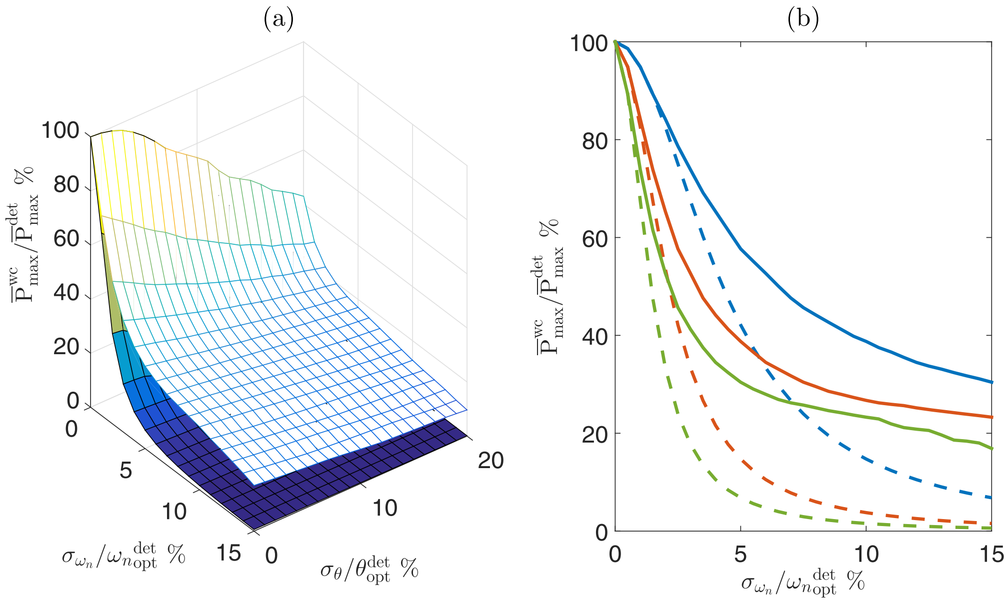

Figure 7. Maximized worst-case power as a function of uncertainty in natural frequency and electromechanical coupling coefficient for harmonic excitation. (a) the maximum worst-case power (wireframe mesh) compared to the worst-case power of the naively optimized harvester (solid surface) for confidence level of 99.7% (b) maximum worst-case power (solid line) as a function of uncertainty in the natural frequency (no uncertainty in electromechanical coupling coefficient) compared to the naively optimized harvester (dashed line) for confidence levels of 68% (blue), 95.5% (red), and 99.7% (green).

natural frequency and mean value of load resistance where only the load resistance is uncertain and figure 8(b) shows the results where the only uncertain parameter is the electromechanical coupling coefficient and optimization is carried over natural frequency and the mean value of the electromechanical coupling coefficient. As seen in both sub-figures there is a considerable increase in the worst-case power when uncertainties are taken into account in the parametric optimization.

5. Conclusions

In this study, we proposed a new modeling philosophy for optimization of energy harvesters under parametric uncertainty. Instead of optimizing for ensemble expectation of average harvested power, we optimize for the worst-case (minimum) power based on some confidence level over the deterministic parameters and mean values of the random

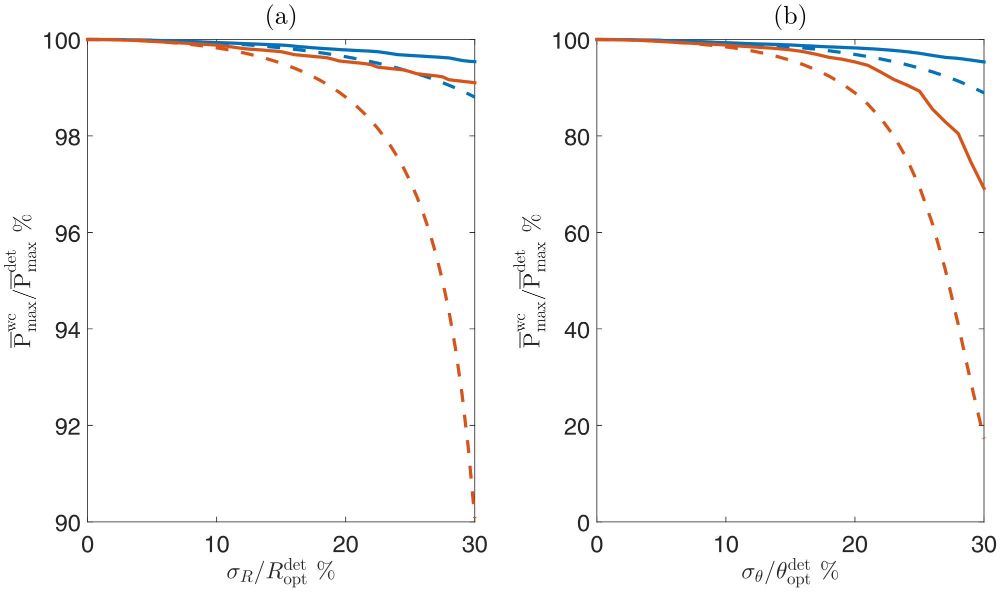

Figure 8. Maximum worst-case power (solid line) compared to the naively optimized harvester (dashed line) for confidence levels of 95% (blue) and 99.7% (red) as a function of uncertainty in (a) load resistance and (b) electromechanical coupling coefficient for random excitation.

parameters. The proposed optimization philosophy is practically very useful when there is a minimum requirement on the harvested power such as those in medical implants and wireless sensors. We also introduced a different notion of uncertainty propagation i.e. propagation in the worst-case power instead of the ensemble expected power. Based on this new modeling philosophy, we presented a very generic and architecture-independent formulation for uncertainty propagation (P1) and optimization under uncertainty (P2).

Next, we applied analysis methodologies P1 and P2 to a simple model of a piezoelectric energy harvester. We have considered parametric uncertainty in natural frequency, load resistance, and electromechanical coupling coefficient of the harvester. Also, both harmonic and wide-band excitation were considered. Direct application of P1 showed that for harmonically excited PEH, the worst-case power of the harvester is highly sensitive to its natural frequency and then to its electromechanical coupling but not very sensitive to the load resistance. However, when the PEH is excited by the wide-band excitation, the worst-case power is not very sensitive to the natural frequency of the harvester but is sensitive to its load resistance and electromechanical coupling.

For the harmonic excitation, the optimization P2 was done over mean values of the natural frequency and load resistance or natural frequency and electromechanical coupling. For the random excitation, since the worst-case power was not sensitive to uncertainty in natural frequency, the optimization was done over the deterministic natural frequency and the mean value of the load resistance or electromechanical coupling coefficient. It was shown that for both harmonic and random excitation, the optimized system taking into account the parametric uncertainties is much more robust to parametric uncertainties in terms of its worst-case power compared to the naively optimized (deterministically optimized) harvester.

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