Irreversible Relaxation

Here we explore how coupling to a continuum of states leads to irrevesible relaxation out of an initial state.
We will calculate the time-evolution of amplitude in an initially prepared state for a finite number of continuum states. The number of continuum states can be adjusted and we see how the occupation of the initial states evolves from purely oscillatory to exponentially damped as the number of states is increased from 1 to 1000. Explore how the number and ditribution of randomly chosen continnum states influences the time-dependence.

Set some variables: Coupling to continuum states: $V := 1$ $\quad maxstates := 10001$

Define a time grid: $j := 0 .. 500$ $\quad \tau_j := \frac{j}{20}$

k=state index variable: $k := 0 .. maxstates$
k=0 is intial state

Rabi frequency: $\Omega(\Delta) := \sqrt{V^2 + \Delta^2}$

$E_k := 0$

Choose the continuum state energies:

(1) Select the range of energies to work with: $\sigma := 10 \cdot V$

(2) Select a random set of energies within 2 σ: $E := \mathrm{runif}(maxstates, -\sigma, \sigma)$

(3) Choose the number of states to include in calculation: $states := 40$

Plot spectrum of continuum density of states:

$$\omega_j := \frac{(j - 250)}{250} \cdot 2 \cdot \sigma$$

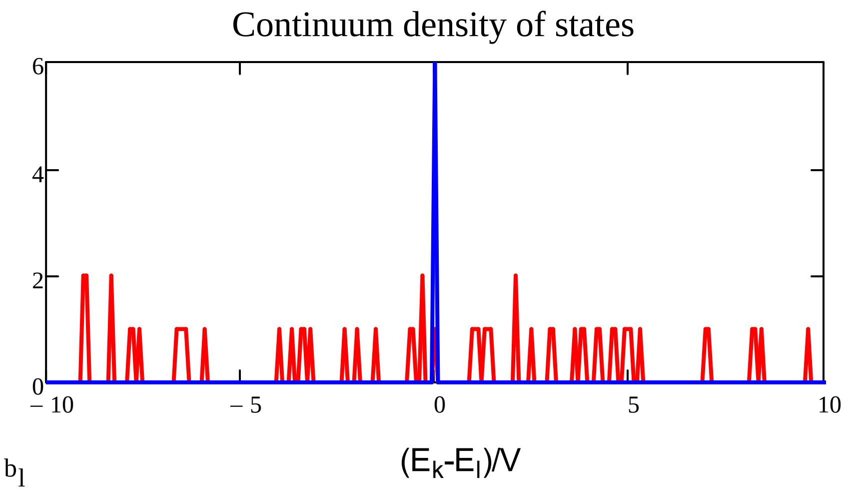
$$Spectrum_j := \sum_{z=1}^{states} \delta\left(\mathrm{round}\left(E_z \cdot 10\right), \mathrm{round}\left(\omega_j \cdot 10\right)\right)$$

$E_0 := 0$

Amplitude in initial and continuum state. An asumption is made here to simplify the calculation. The amplitude in the initial state is split uniformly into equal contributions which exchange only with one continuum state. This preserves more memory of the initial state phase than a full integration of the differential equations for this problem.

$$b_k(\tau, z) := \frac{1}{states} \cdot \frac{-i \cdot V}{\Omega\left(\frac{E_z - E_0}{2}\right)} \cdot e^{-i \cdot \frac{\left(E_z - E_0\right) \cdot \tau}{2}} \cdot \sin\left(\Omega\left(\frac{E_z - E_0}{2}\right) \cdot \tau\right)$$

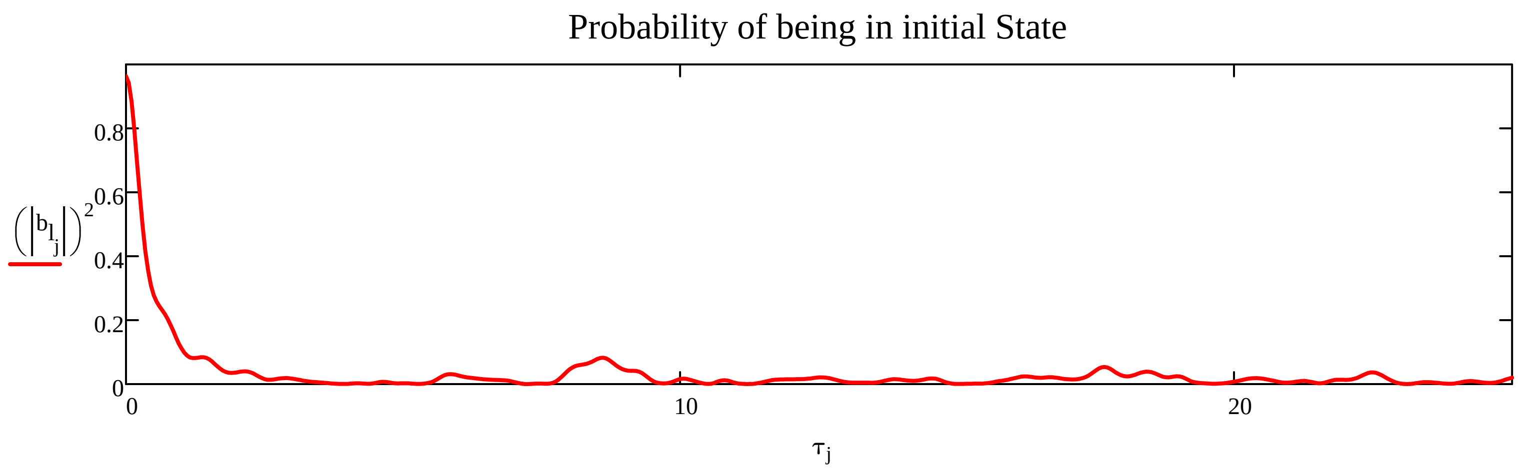
$$b_{init}(\tau, z) := \frac{1}{states} \cdot \frac{i \cdot V}{\Omega\left(\frac{E_z - E_0}{2}\right)} \cdot e^{i \cdot \frac{\left(E_z - E_0\right) \cdot \tau}{2}} \cdot \cos\left(\Omega\left(\frac{E_z - E_0}{2}\right) \cdot \tau\right)$$



Intitial state: sum over loss to each band in continuum:

$$b_{l_j} := \sum_{z=1}^{states} b_{init}\left(\tau_j, z\right)$$

our assumptions wont conserve population, so we'll fix the absolute numbers with it with: $b_l := \frac{b_l}{\max\left(b_l\right)}$



Now let's compare the results to what you get from the Golden rule rate:

Density of states:

Average density of states: $\rho := \frac{states}{2 \cdot \sigma}$ $\rho = 2$

It's more accurate to use delta function! $\rho := \text{if}\left(\text{Spectrum}_{251} = 0, 1, \text{Spectrum}_{251}\right)$ $\rho = 2$

...or input your own number $\rho := 1$

Golden rule rate: $\rho = 1$ $w_{kl} := 2 \cdot V^2 \cdot \rho$ $\Delta E := \sum_{z=1}^{states} \frac{V^2}{\left(E_z - E_0\right)}$ $b_{FGR_j} := \exp\left(-i \cdot \Delta E \cdot \tau_j - \frac{w_{kl}}{2} \cdot \tau_j\right)$