The coupled two-level system: Comparing exact and 1st order perturbation theory

Given a fixed value of the coupling:

$$V := 1$$

For a value of the energy splitting $\Delta = (E_1 - E_2)/2$, the Rabi Frequency is

$$\Omega(\Delta) := \sqrt{\Delta^2 + V^2}$$

If the system is prepared in one state, and we ask "what is the time-dependent amplitude in the other?", the exact treatment of the probability is

$$P_{exact}(t,\Delta) := \left(\frac{V^2}{\Omega(\Delta)^2}\right)\cdot\sin\left(\Omega(\Delta)\cdot\frac{t}{2}\right)^2$$

The approximation to this solution from first-order perturbation theory is

$$P_{1storder}(t,\Delta) := \left(\frac{V^2}{\Delta^2}\right)\cdot\sin\left(\Delta\cdot\frac{t}{2}\right)^2$$

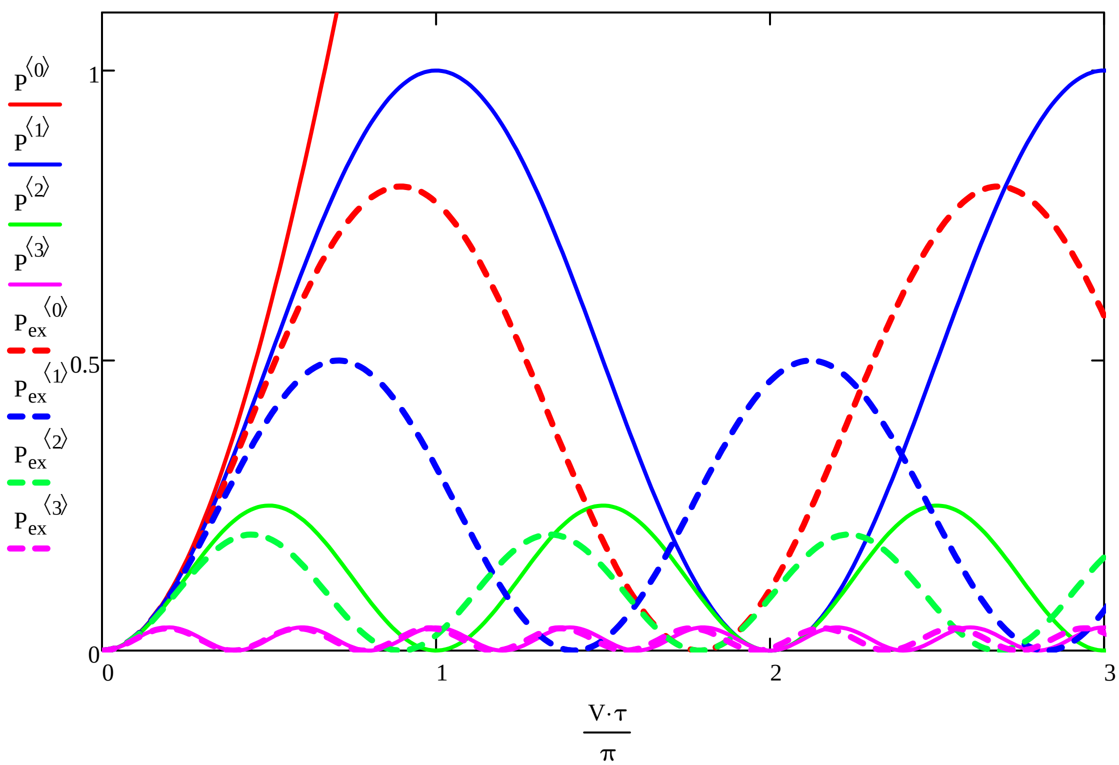
Now let's compare the two forms of the solution for the time dependence of the pobability, using different values of the energy splitting Δ. We will set Δ in units of V

$$i := 0..300 \qquad j := 0..3$$

First the time-domain:

$$\tau_i := \frac{i}{30} \qquad \Delta := \begin{pmatrix} 0.5\cdot V \\ 1\cdot V \\ 2\cdot V \\ 5\cdot V \end{pmatrix} \qquad P_{i,j} := P_{1storder}(\tau_i,\Delta_j) \qquad P_{ex_{i,j}} := P_{exact}(\tau_i,\Delta_j)$$

The exact is plotted as dashes, while the perturbation theory result is solid. Cne starts to reach quantitative agreement for the time-dependent behavior for P_{max} <10% or Δ > 4V.



Now lets look at the dependence of P on energy splitting Δ: for different values of the time delay.

$$\Delta_i := \frac{i - 150.1111}{10}$$

Set time points:

$$T := \begin{pmatrix} 0.2 \\ 0.4 \\ 0.6 \\ 1 \end{pmatrix} \qquad T := \frac{T \cdot \pi}{V}$$

$$S_{i,j} := P_{1storder}\left(T_j, \Delta_i\right) \qquad S_{ex_{i,j}} := P_{exact}\left(T_j, \Delta_i\right)$$

The exact is plotted as dashes, while the perturbation theory result is solid.