

# Semi-Supervised Learning on Data Streams via Temporal Label Propagation

Tal Wagner



Joint work with: Sudipto Guha



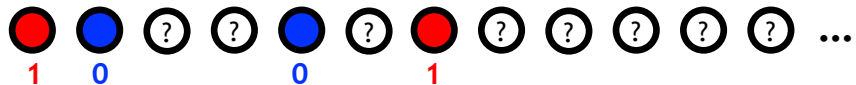
Shiva Kasiviswanathan



Nina Mishra



# Semi-Supervised Learning on a Stream



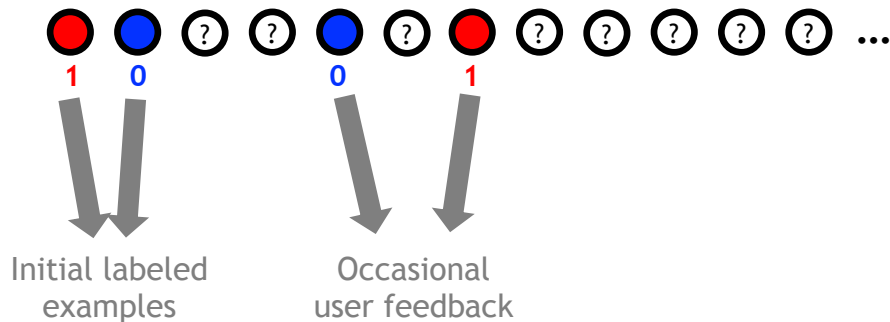
Problem:

- ▶ Data points arrive on a stream
- ▶ Few are labeled
- ▶ Most are unlabeled
- ▶ Task: Label points on-the-fly





# Semi-Supervised Learning on a Stream



## Goals:

- ▶ **Time:** Label points quickly
- ▶ **Space:** Stream too large to fully store
- ▶ Learn from unlabeled data



*Background:*

# Offline Semi-Supervised Learning

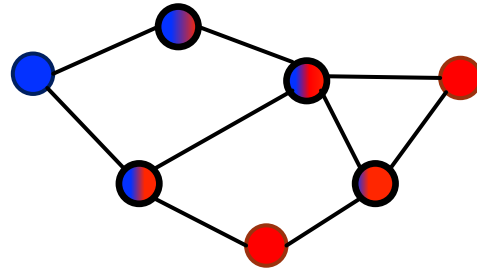
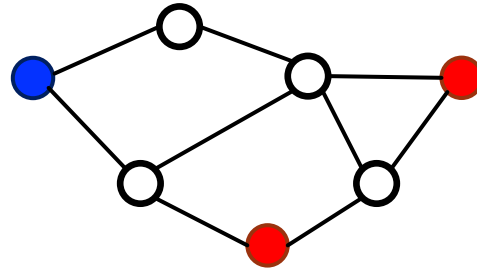
# Offline Semi-Supervised Learning: Label Propagation

[Zhu, Ghahramani, Lafferty ICML'03]

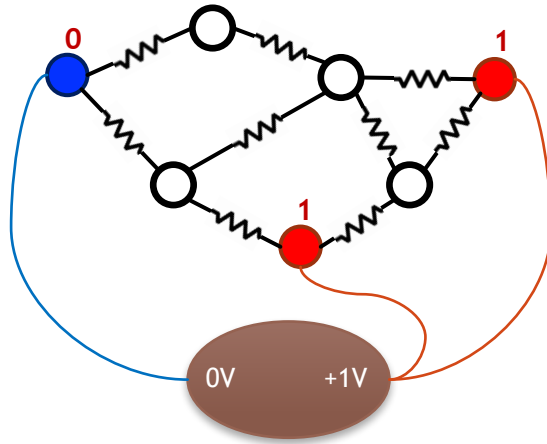


[ICML 10-Year Classic Paper Prize 2013]

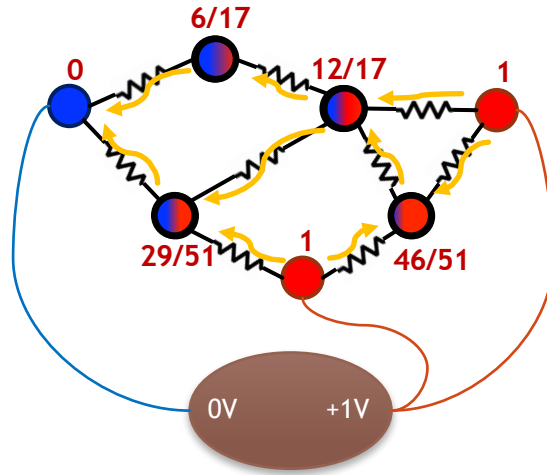
- ▶ Construct graph on data points
- ▶ Propagate labels on graph



# Label Propagation: Electric Intuition

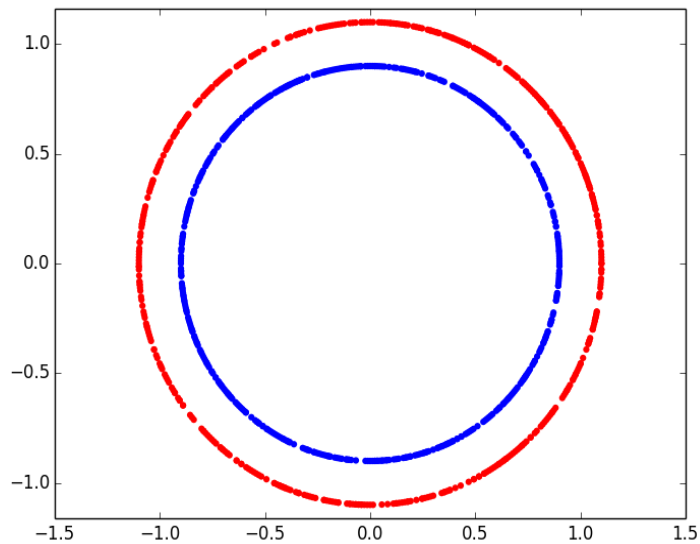
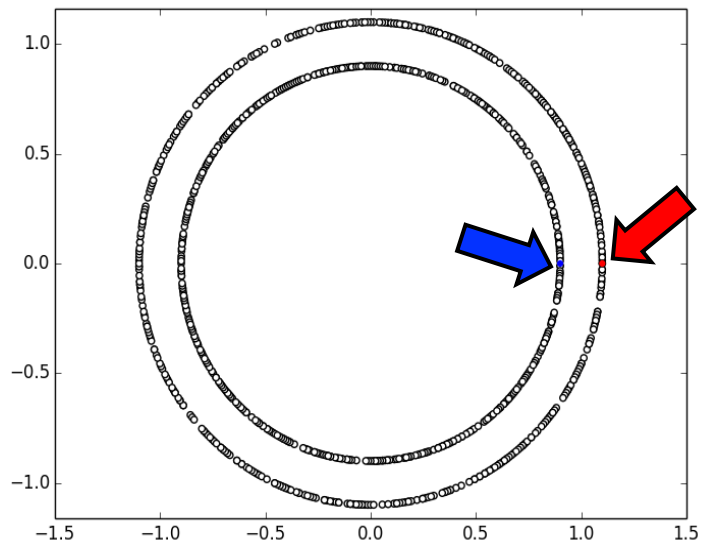


# Label Propagation: Electric Intuition



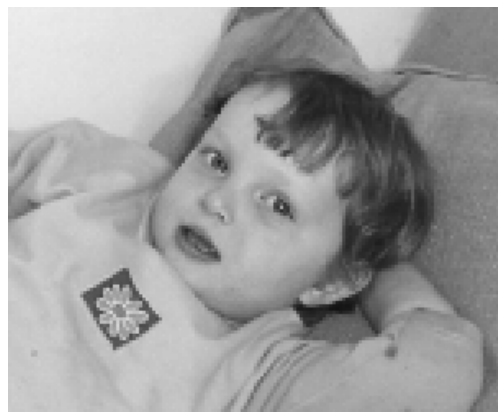
$$f: V \rightarrow [0,1] \text{ computed by } \min_f \sum_{(x,y) \in E} w_{x,y} (f(x) - f(y))^2 \quad \text{s.t.} \quad f(\text{blue}) = 0, f(\text{red}) = 1$$

# Label Propagation: In Action (I)

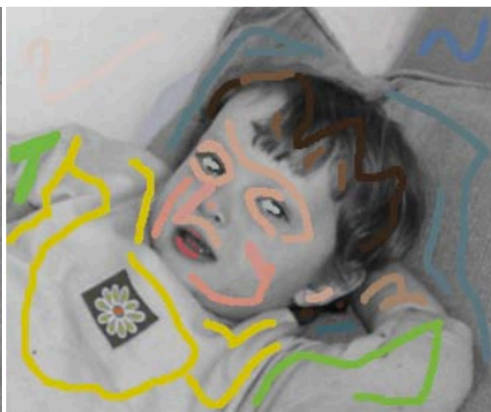


# Label Propagation: In Action (II)

[Levin, Lischinski, Weiss SIGGRAPH'04]



Input



Labeled  
examples



Output

*Handling Data Streams:*

---

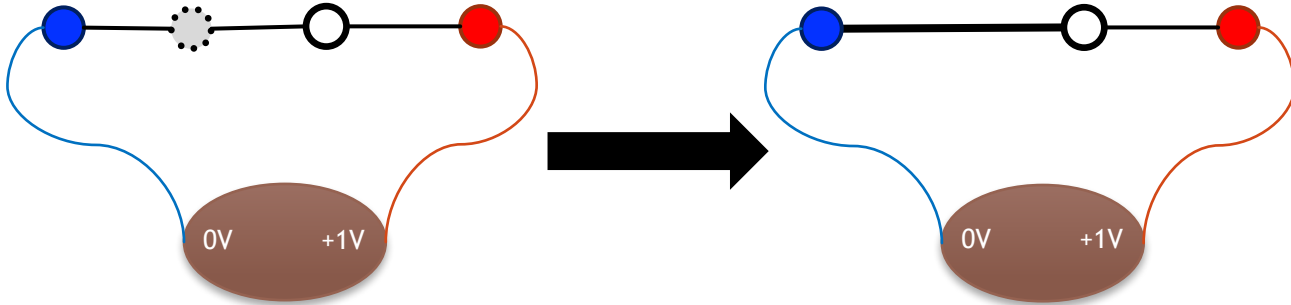
# From Offline to Streaming Label Propagation



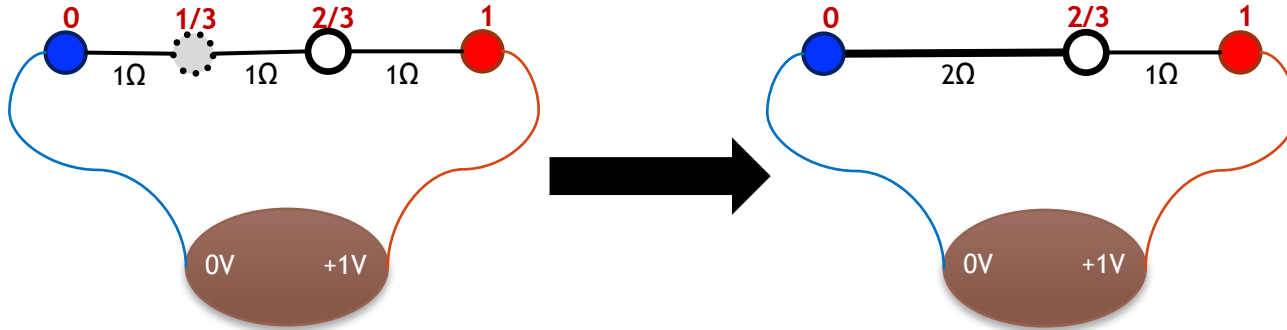
Diagram illustrating the concept of non-terminals and terminals in a sequence. A sequence of circles is shown: red, blue, grey, grey, blue, grey, red, grey, grey, grey, grey, a circle with a question mark, and an ellipsis. Grey circles are labeled *non-terminals* with arrows pointing to them. The circle with the question mark is labeled *terminal* with a red arrow pointing to it.

-

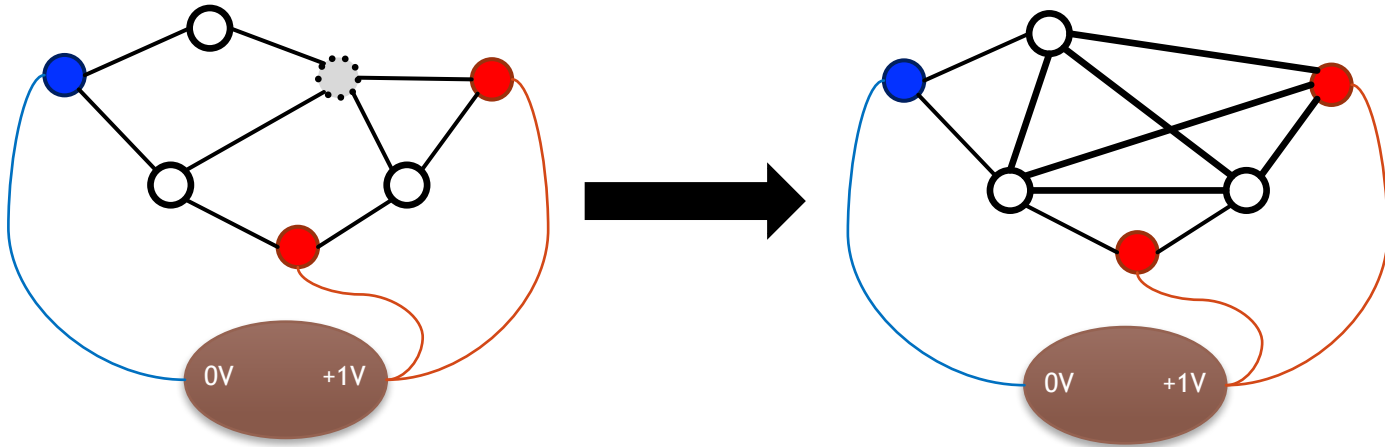
# Graph Reduction



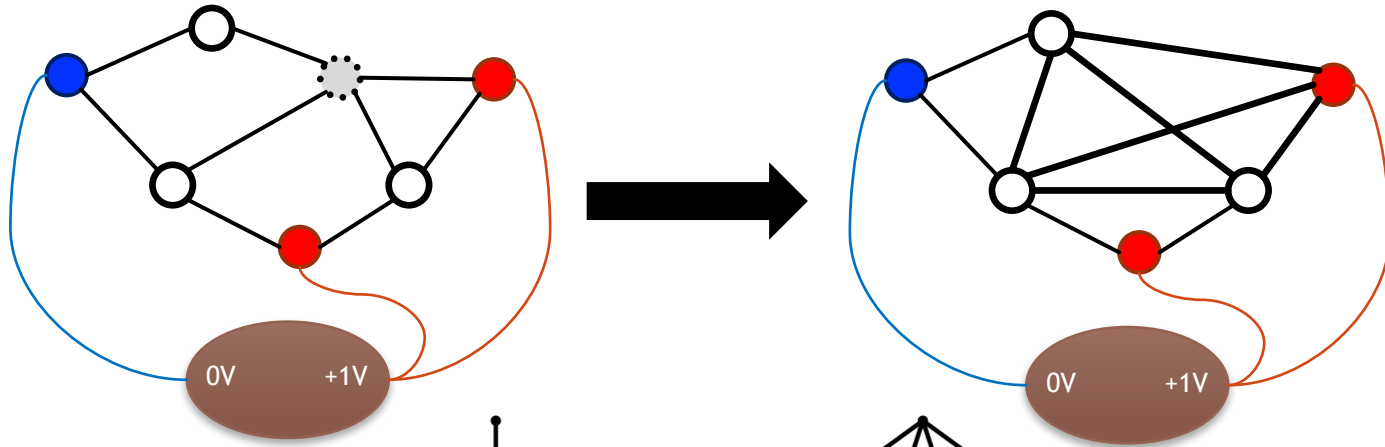
# Graph Reduction



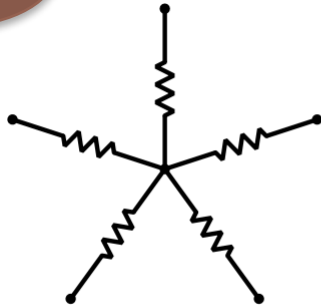
## Graph Reduction: Star-Mesh Transform



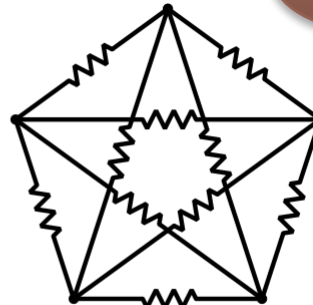
# Graph Reduction: Star-Mesh Transform



“Star-mesh  
transform”  
[Rosen 1924]



“star”

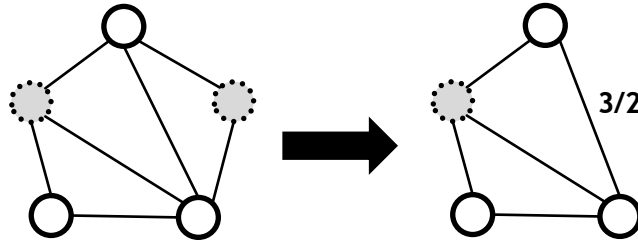


“mesh”

(from Wikipedia)

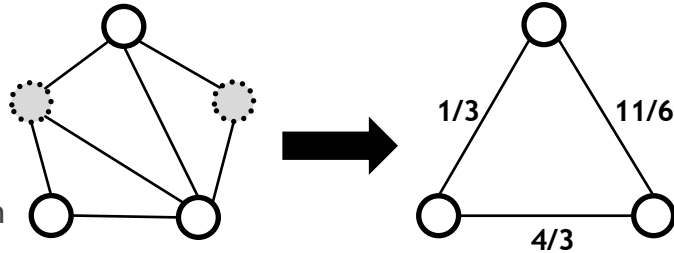
## Graph Reduction: Sequential Star-Mesh Transforms

- ▶ Graph has  $n$  nodes and  $\tau$  terminals
- ▶ Star-mesh the  $n - \tau$  non-terminals one by one



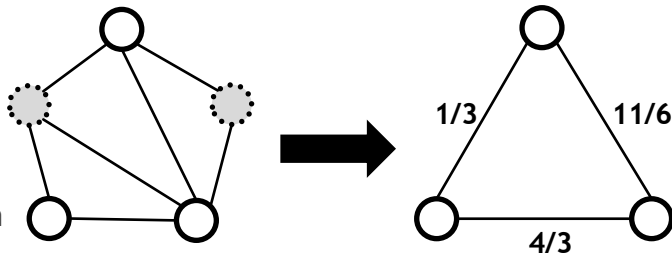
## Graph Reduction: Sequential Star-Mesh Transforms

- ▶ Graph has  $n$  nodes and  $\tau$  terminals
- ▶ Star-mesh the  $n - \tau$  non-terminals one by one
  - ▶ Computes Schur complement of graph Laplacian matrix [Campbell 1922]



# Graph Reduction: Sequential Star-Mesh Transforms

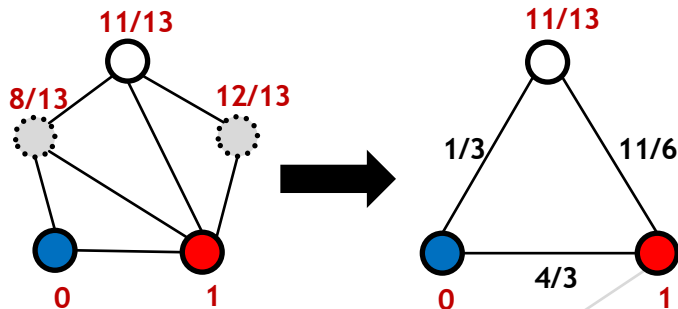
- ▶ Graph has  $n$  nodes and  $\tau$  terminals
- ▶ Star-mesh the  $n - \tau$  non-terminals one by one
  - ▶ Computes Schur complement of graph Laplacian matrix [Campbell 1922]



## ▶ Compression:

From  $\Omega(n \log n)$  to  $O(\tau^2 \log n)$  bits

- ▶ **Theorem:** Label Propagation is preserved on terminals.





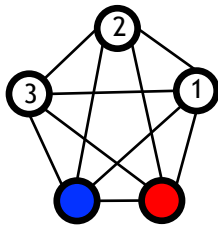
*Our Algorithm:*

# Temporal Label Propagation (TLP)

# Temporal Label Propagation

► Pick small integer  $\tau$

► Store  $\tau$  recent unlabeled points

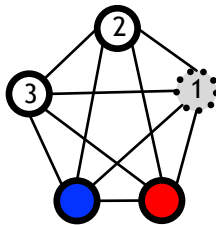


# Temporal Label Propagation

- ▶ Pick small integer  $\tau$
- ▶ Store  $\tau$  recent unlabeled points



- ▶ On point arrival:
  - ▶ **Star-mesh** out oldest point

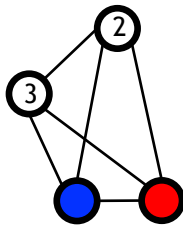


# Temporal Label Propagation

- ▶ Pick small integer  $\tau$
- ▶ Store  $\tau$  recent unlabeled points



- ▶ On point arrival:
  - ▶ **Star-mesh** out oldest point

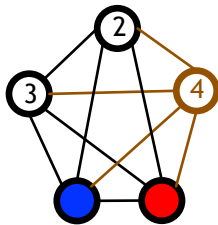


# Temporal Label Propagation

- ▶ Pick small integer  $\tau$
- ▶ Store  $\tau$  recent unlabeled points



- ▶ On point arrival:
  - ▶ **Star-mesh** out oldest point
  - ▶ **Insert** new point

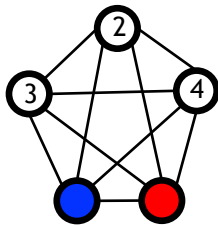


# Temporal Label Propagation

- ▶ Pick small integer  $\tau$
- ▶ Store  $\tau$  recent unlabeled points



- ▶ On point arrival:
  - ▶ **Star-mesh** out oldest point
  - ▶ **Insert** new point
  - ▶ **Propagate** labels

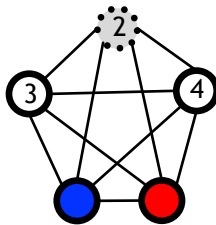


# Temporal Label Propagation

- ▶ Pick small integer  $\tau$
- ▶ Store  $\tau$  recent unlabeled points



- ▶ On point arrival:
  - ▶ **Star-mesh** out oldest point
  - ▶ Insert new point
  - ▶ **Propagate** labels

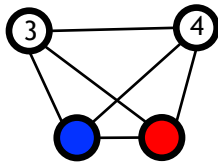


# Temporal Label Propagation

- ▶ Pick small integer  $\tau$
- ▶ Store  $\tau$  recent unlabeled points



- ▶ On point arrival:
  - ▶ **Star-mesh** out oldest point
  - ▶ Insert new point
  - ▶ Propagate labels



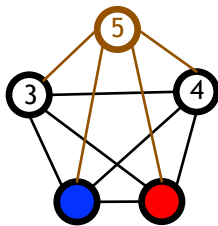


# Temporal Label Propagation

- ▶ Pick small integer  $\tau$
- ▶ Store  $\tau$  recent unlabeled points



- ▶ On point arrival:
  - ▶ **Star-mesh** out oldest point
  - ▶ **Insert** new point
  - ▶ **Propagate** labels

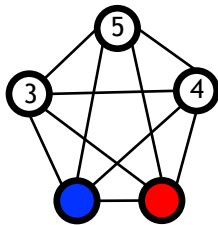


# Temporal Label Propagation

- ▶ Pick small integer  $\tau$
- ▶ Store  $\tau$  recent unlabeled points



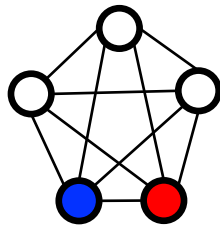
- ▶ On point arrival:
  - ▶ **Star-mesh** out oldest point
  - ▶ **Insert** new point
  - ▶ **Propagate** labels



# Temporal Label Propagation

Guarantees for time step  $n$ :

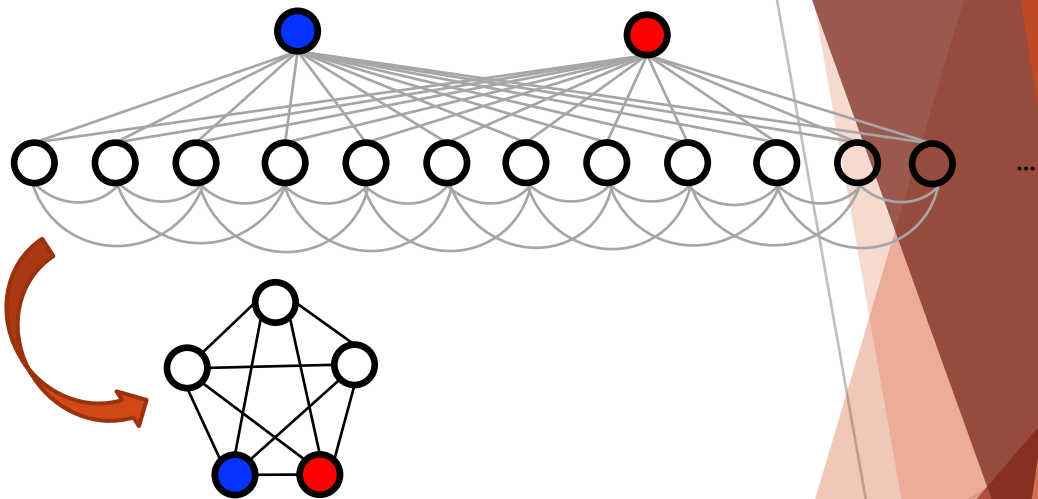
- ▶ Processing time:  $\tilde{O}(\tau^3)$
- ▶ Storage space:  $\tilde{O}(\tau^2 \log n)$



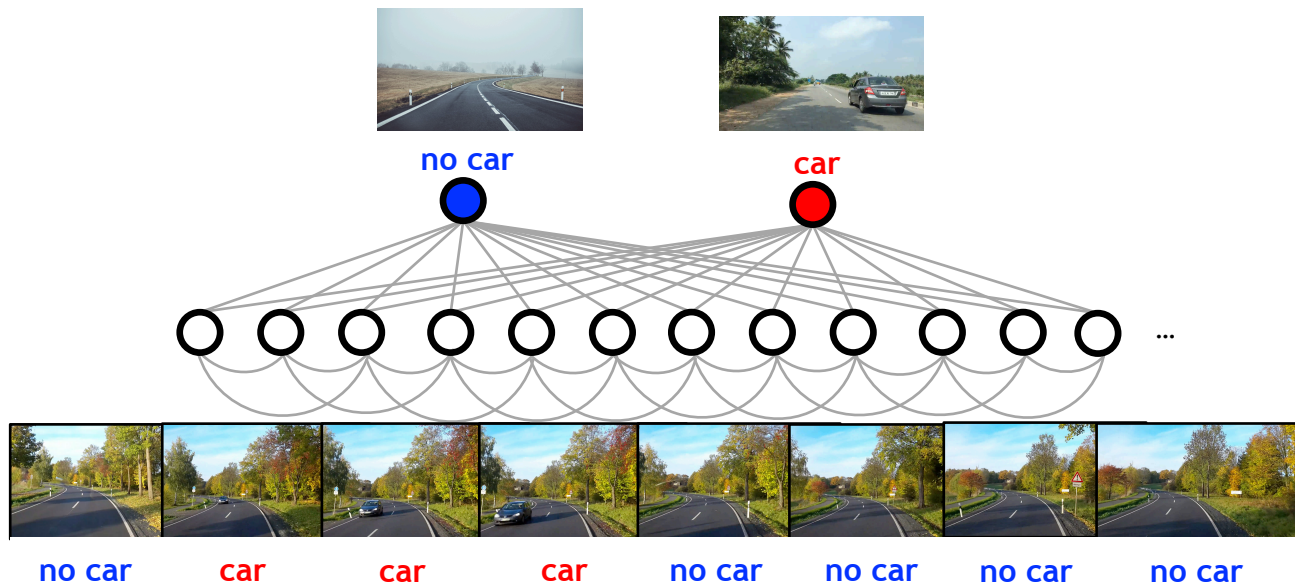
# Temporal Label Propagation

Guarantees for time step  $n$ :

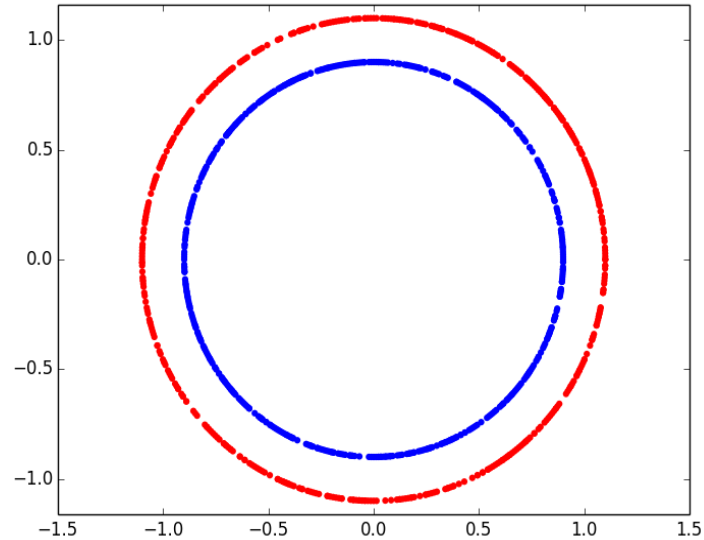
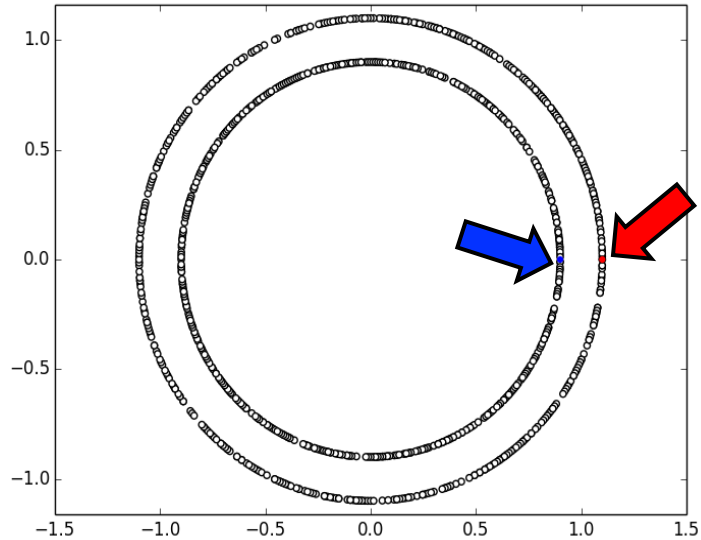
- ▶ Processing time:  $\tilde{O}(\tau^3)$
- ▶ Storage space:  $\tilde{O}(\tau^2 \log n)$
- ▶ Labels propagate on a graph containing the entire stream so far



# Temporal Vicinity Graph

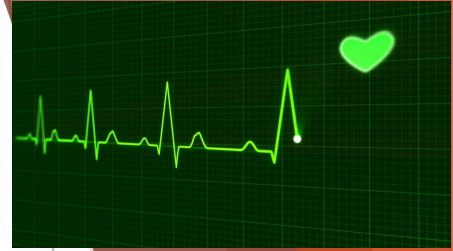


# Demo: Two Rings in Angular Order



Real Data:

# Classifying Irregular Heartbeats on EKG



Class 1  
("APC")

Class 2  
("VPC")

EKG signal:

Our results:

False Class 1

True Class 1

True Class 2

False Class 2

Labeled  
examples  
(2 per  
class)

94.9%  
accuracy

- Data from [physionet.org](https://physionet.org)
- Class 1: Atrial premature contraction (APC)
- Class 2: Ventricular premature contraction (VPC)
- Normal heartbeats are ignored
- Overall accuracy: **94.9%** with  $\tau = 5$  and 2 labeled examples per class

# Conclusion

- ▶ **Principled** and **practical** approach to streaming SSL
  - ▶ Rich, robust, well-studied mathematical framework
  - ▶ Simple to implement
- ▶ **Future directions:**
  - ▶ More applications of **compression** for SSL
    - ▶ Distributed networks, memory-limited devices (edge, GPU, ...)
  - ▶ **Extensions and variants** of Label Propagation on streams
    - ▶ Support regularization, noisy labels, interpretability, ...
  - ▶ Trade **approximation / randomization** for even better performance
    - ▶ Our algorithm is **exact** and **deterministic**

*Thank you*  
*Questions?*