Ab-initio calculations of inclusive scattering reactions

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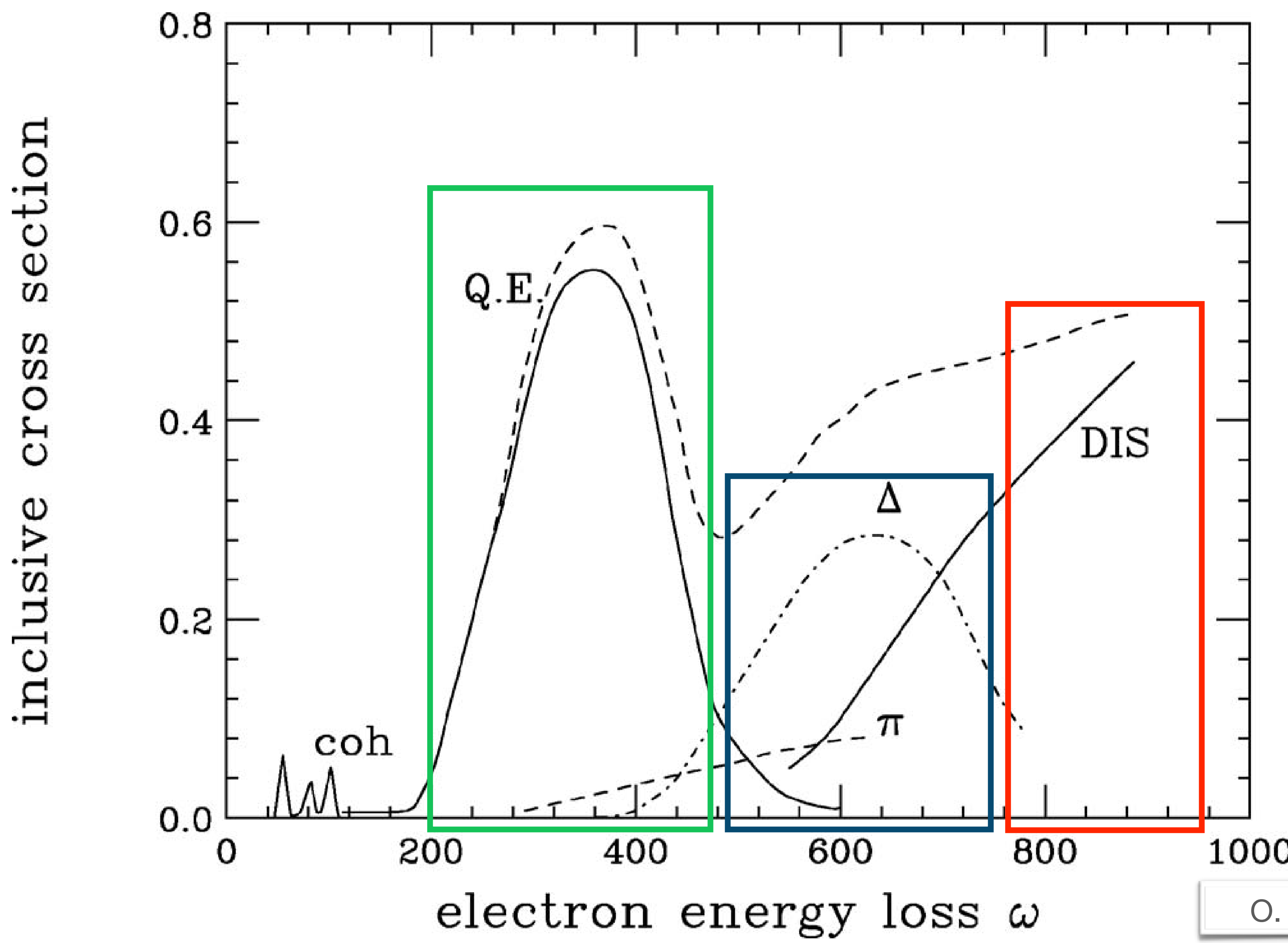
Electron-nucleus scattering

Schematic representation of the inclusive cross section as a function of the energy loss.



- Broad peak due to quasi-elastic electron-nucleon scattering.
- Excitation of the nucleon to distinct resonances (like the Δ) and pion production.
- Deep Inelastic Scattering region, productions of hadrons other than protons and neutrons

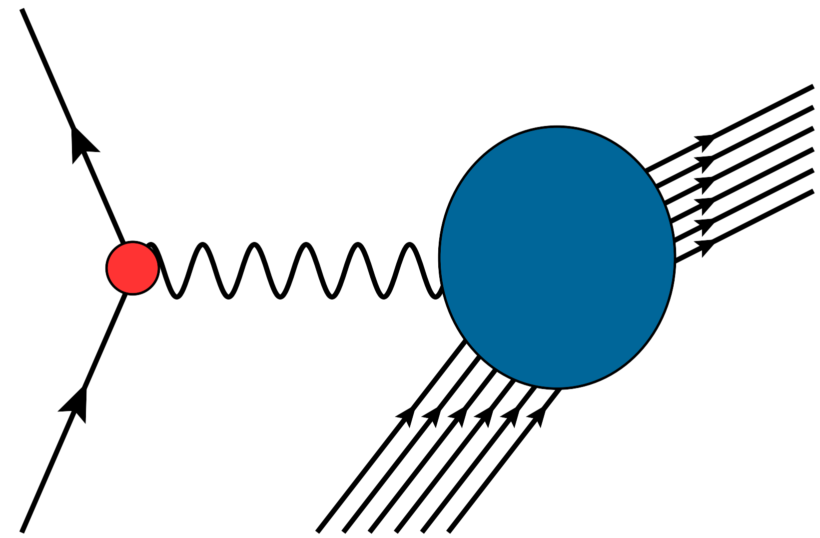
O. Benhar, et al. RMP **80**, 189 (2008)

The different reaction mechanisms can be clearly identified

Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus and the hadronic final state is undetected can be written as

$$\frac{d^2\sigma}{d\Omega_\ell dE_{\ell'}} = L_{\mu\nu} W^{\mu\nu}$$



- The Leptonic tensor is fully specified by the lepton kinematic variables. For instance, in the electron-nucleus scattering case

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k\,k') + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta$$

- The Hadronic tensor contains all the information on target response

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^\nu(q)|0\rangle \delta^{(4)}(p_0 + q - p_f)$$

Non relativistic nuclear many-body theory (NMBT) provides a fully consistent theoretical approach allowing for an accurate description of |0>, independent of momentum transfer.

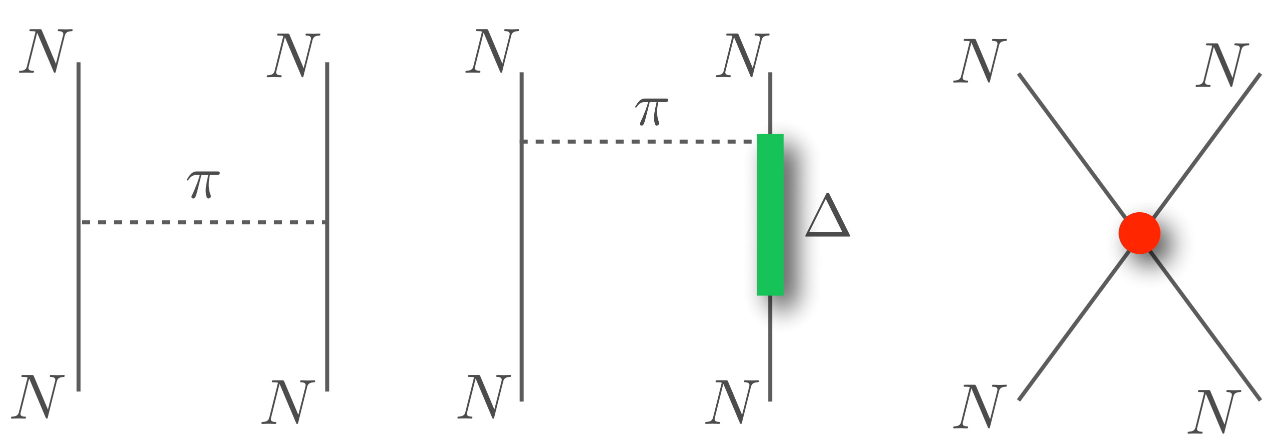
Non relativistic Nuclear Many Body Theory

- Within NMBT the nucleus is described as a collection of A point-like nucleons, the dynamics of which are described by the non relativistic Hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

Argonne v_{18} is a finite, local, configuration-space potential which has been fit to ~4300 np and pp scattering data below 350 MeV of the Nijmegen database, low-energy nn scattering parameters, and deuteron binding energy.

Some of the diagrams included in this potential are



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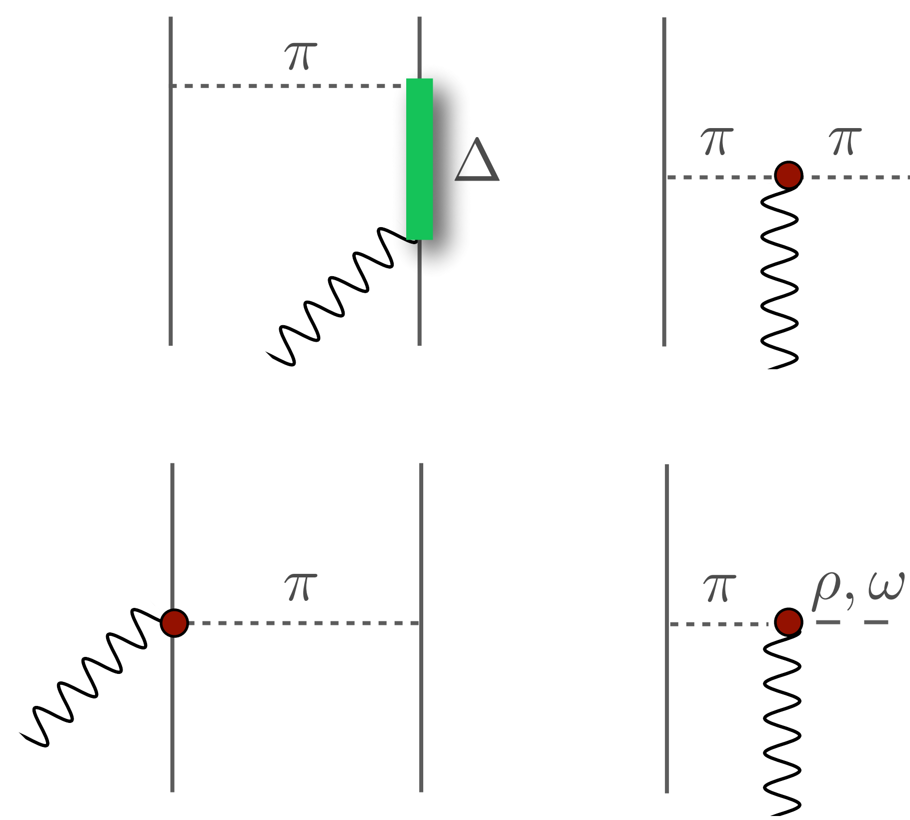
$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

The nuclear electromagnetic current is constrained through the continuity equation

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

• The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions.

• Non relativistic expansion of $\mathbf{J}_{\rm EM}$, in powers $|\mathbf{q}|/m$

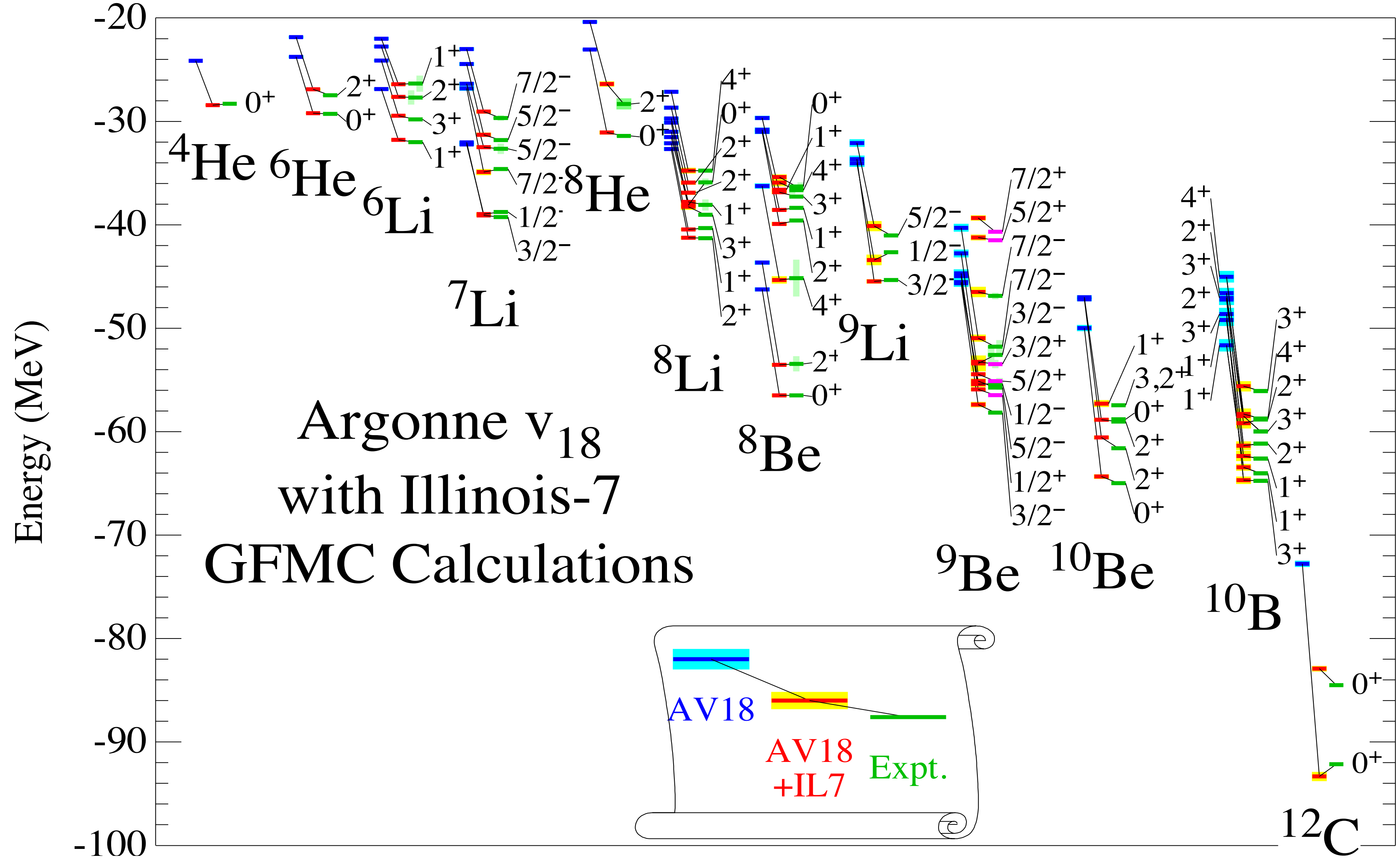


The Green's Function Monte Carlo approach

- Diffusion Monte Carlo methods use an imaginary-time projection technique to enhance the ground-state component of a starting (correlated) trial wave function.

$$\lim_{\tau\to\infty} e^{-(H-E_0)\tau}|\Psi_T\rangle = \lim_{\tau\to\infty}\sum_n c_n\, e^{-(E_n-E_0)\tau}|\Psi_n\rangle = c_0|\Psi_0\rangle$$

- Suitable to solve A ≤ 12 nuclei with ~1% accuracy



J. Carlson et al. RMP 87, 1067 (2015)

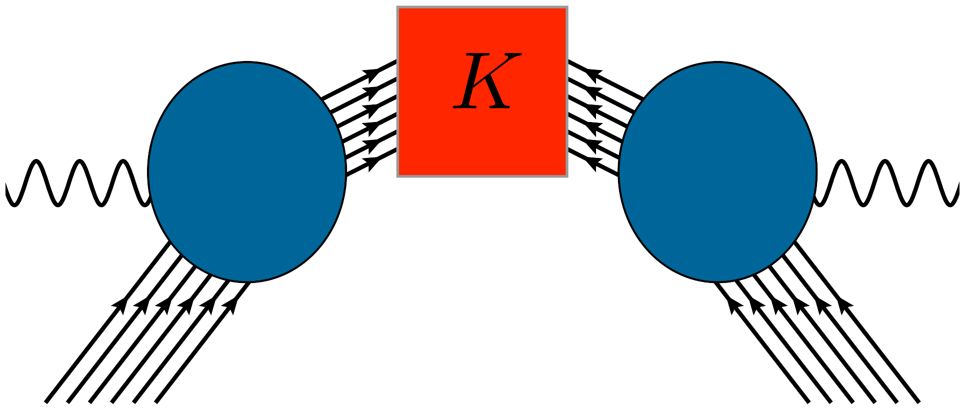
Integral transform techniques

✤ Accurate GFMC calculations of the electroweak responses of ^{4}He and ^{12}C have been recently performed:

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle 0|J^\dagger_\alpha(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

- Valuable information on the energy dependence of the response functions can be inferred from the their integral transforms

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \int d\omega K(\sigma,\omega)R_{\alpha\beta}(\omega,\mathbf{q}) = \langle\psi_0|J^\dagger_\alpha(\mathbf{q})K(\sigma,H-E_0)J_\beta(\mathbf{q})|\psi_0\rangle$$



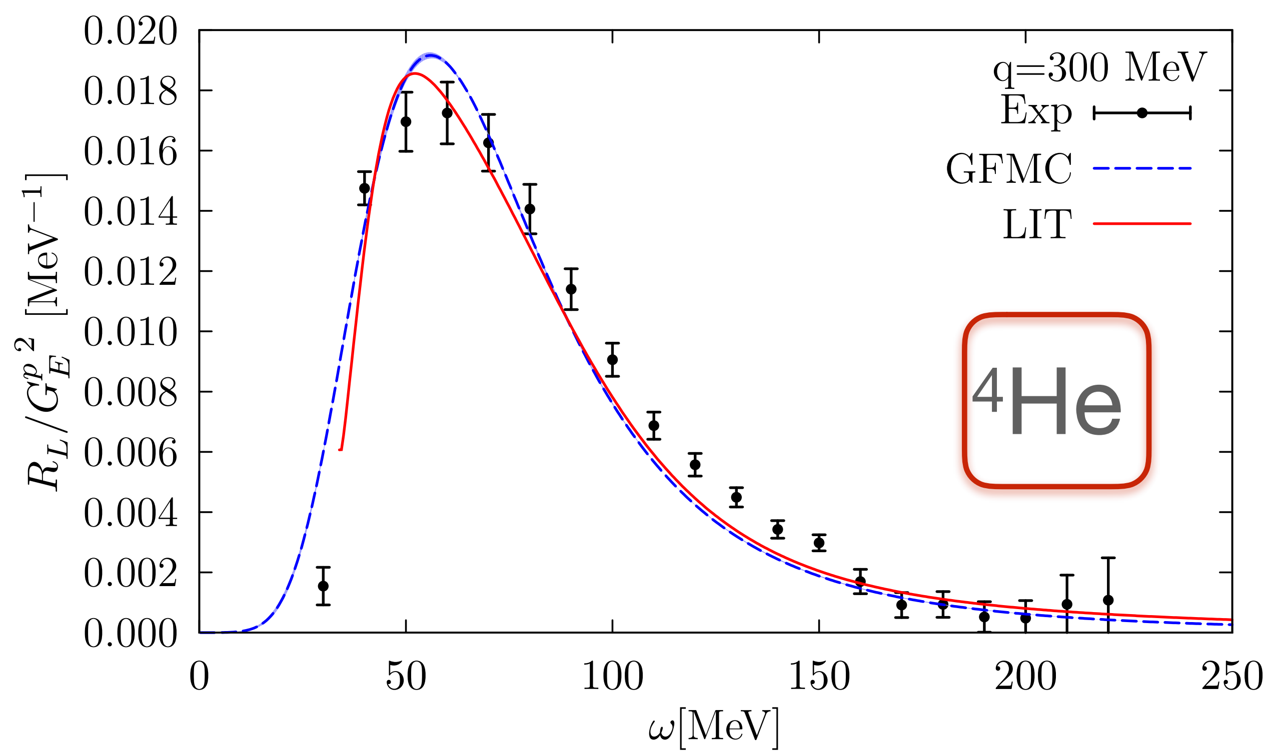
Using the completeness relation for the final states, we are left with a ground-state expectation value

Integral transform techniques

- The Lorentz integral transform (LIT)

$$K(\sigma,\omega) = \frac{1}{(\omega-\sigma_R)^2+\sigma_I^2}$$

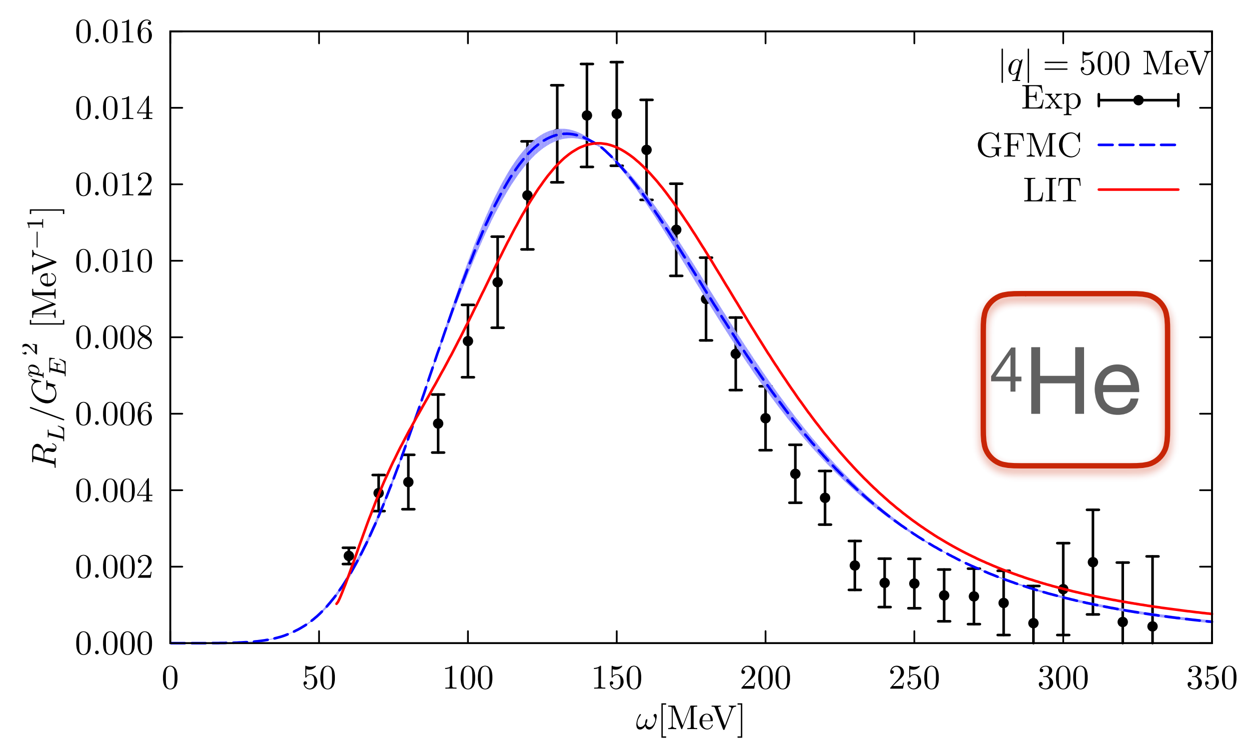
has been successfully exploited in the calculation of nuclear responses:
Using HH: V. D. Efros et al., Phys Lett B 338, 130 (1994)
Using CC: Bacca et al., PRC 76, 014003 (2007), PRL 111, 122502 (2013)



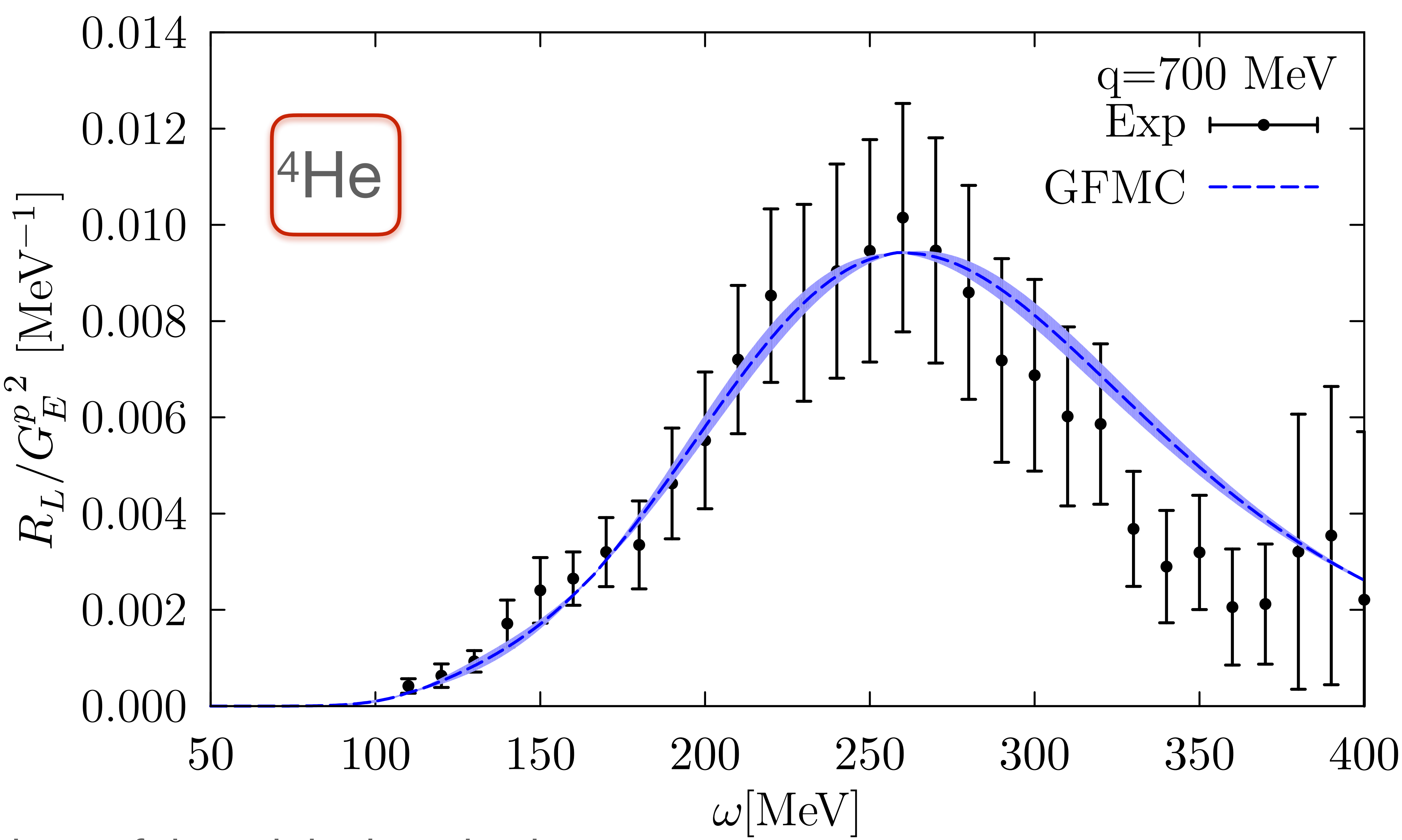
- The Laplace integral transform

$$K(\sigma,\omega) = e^{-\omega\sigma}$$

of the nuclear responses is computed within GFMC and inverted using bayesian techniques: Maximum Entropy
A. Lovato et al, Phys.Rev.Lett. 117 (2016), 082501, Phys.Rev. C97 (2018), 022502



GFMC electromagnetic responses



Limitations of the original method:

★ The quantum mechanical approach (e.g. the kinematics) is non relativistic — relativistic correction up to order q^2/m^2 are included in the currents

★ The computational effort required by the inversion of $E_{\alpha\beta}$ makes the direct calculation of inclusive cross sections unfeasible ⟶ novel algorithm based on first-kind scaling

Relativistic effects in a correlated system

- We extend the applicability of GFMC in the quasielastic region to intermediate momentum transfers by performing the calculation in a reference frame that minimizes nucleon momenta.

- The importance of relativity emerges in the frame dependence of non relativistic calculations at high values of **q**

- In a generic reference frame the longitudinal non relativistic response reads

$$R_L^{fr} = \sum_f \left| \langle \psi_i | \sum_j \rho_j(\mathbf{q}^{fr}, \omega^{fr}) | \psi_f \rangle \right|^2 \delta(E_f^{fr} - E_i^{fr} - \omega^{fr})$$

$$\delta(E_f^{fr} - E_i^{fr} - \omega^{fr}) \approx \delta[e_f^{fr} + (P_f^{fr})^2/(2M_T) - e_i^{fr} - (P_i^{fr})^2/(2M_T) - \omega^{fr}]$$

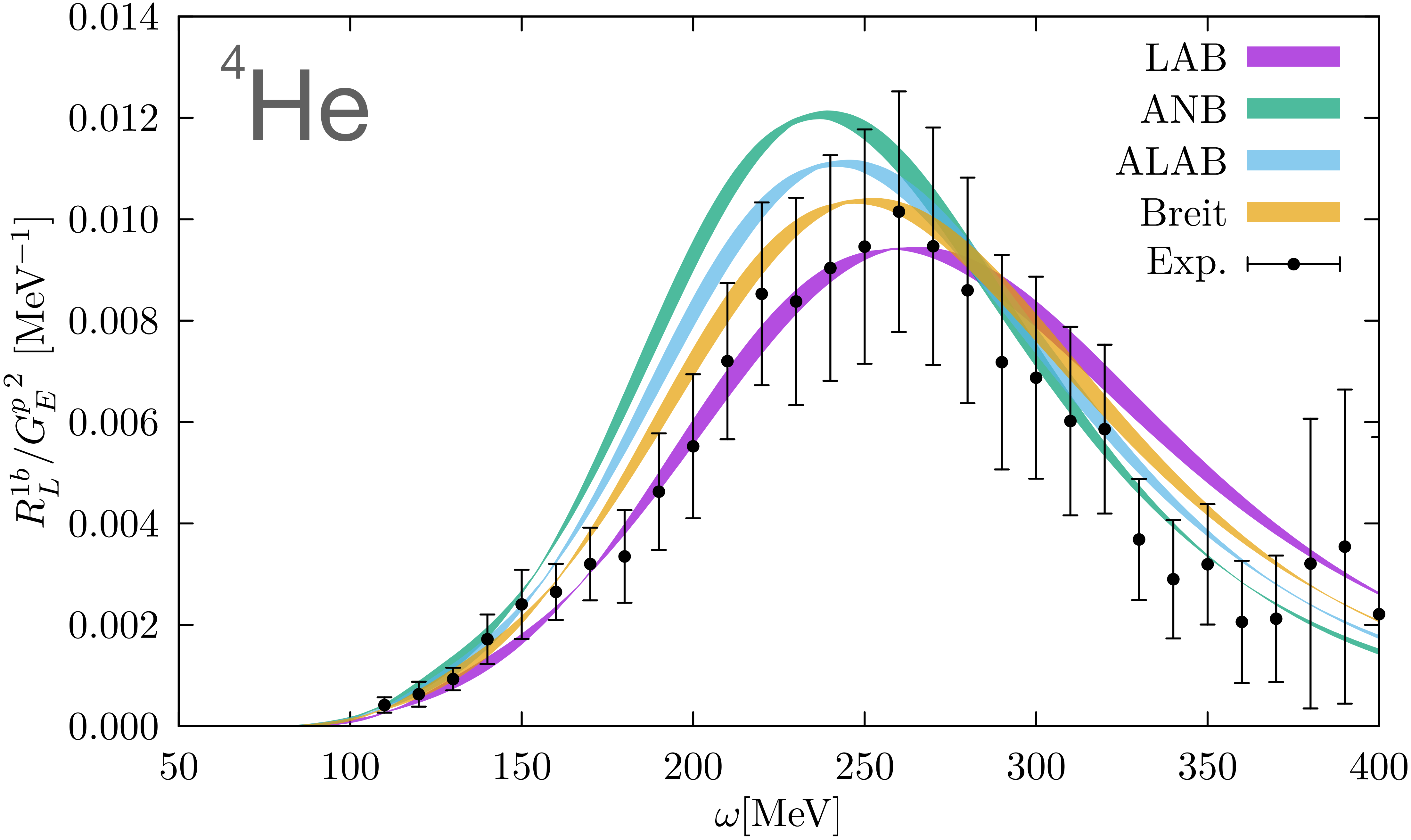
- The response in the LAB frame is given by the Lorentz transformation

$$R_L(\mathbf{q}, \omega) = \frac{\mathbf{q}^2}{(\mathbf{q}^{fr})^2} \frac{E_i^{fr}}{M_0} R_L^{fr}(\mathbf{q}^{\mathbf{fr}}, \omega^{fr})$$

where

$$q^{fr} = \gamma(q - \beta\omega), \ \omega^{fr} = \gamma(\omega - \beta q), \ P_i^{fr} = -\beta\gamma M_0, \ E_i^{fr} = \gamma M_0$$

Relativistic effects in a correlated system

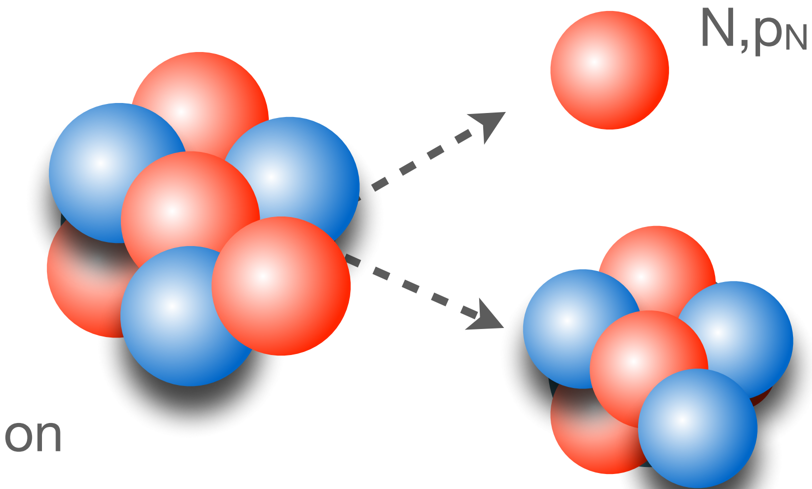


- Longitudinal responses of ^{4}He for |q|=700 MeV in the four different reference frames. The curves show differences in both peak positions and heights.

Relativistic effects in a correlated system

- The frame dependence can be drastically reduced if one assumes a two-body breakup model with relativistic kinematics to determine the input to the non relativistic dynamics calculation

$$p^{fr} = \mu\Big(\frac{p_N^{fr}}{m_N} - \frac{p_X^{fr}}{M_X}\Big)$$
$$P_f^{fr} = p_N^{fr} + p_X^{fr}$$



- The relative momentum is derived in a relativistic fashion

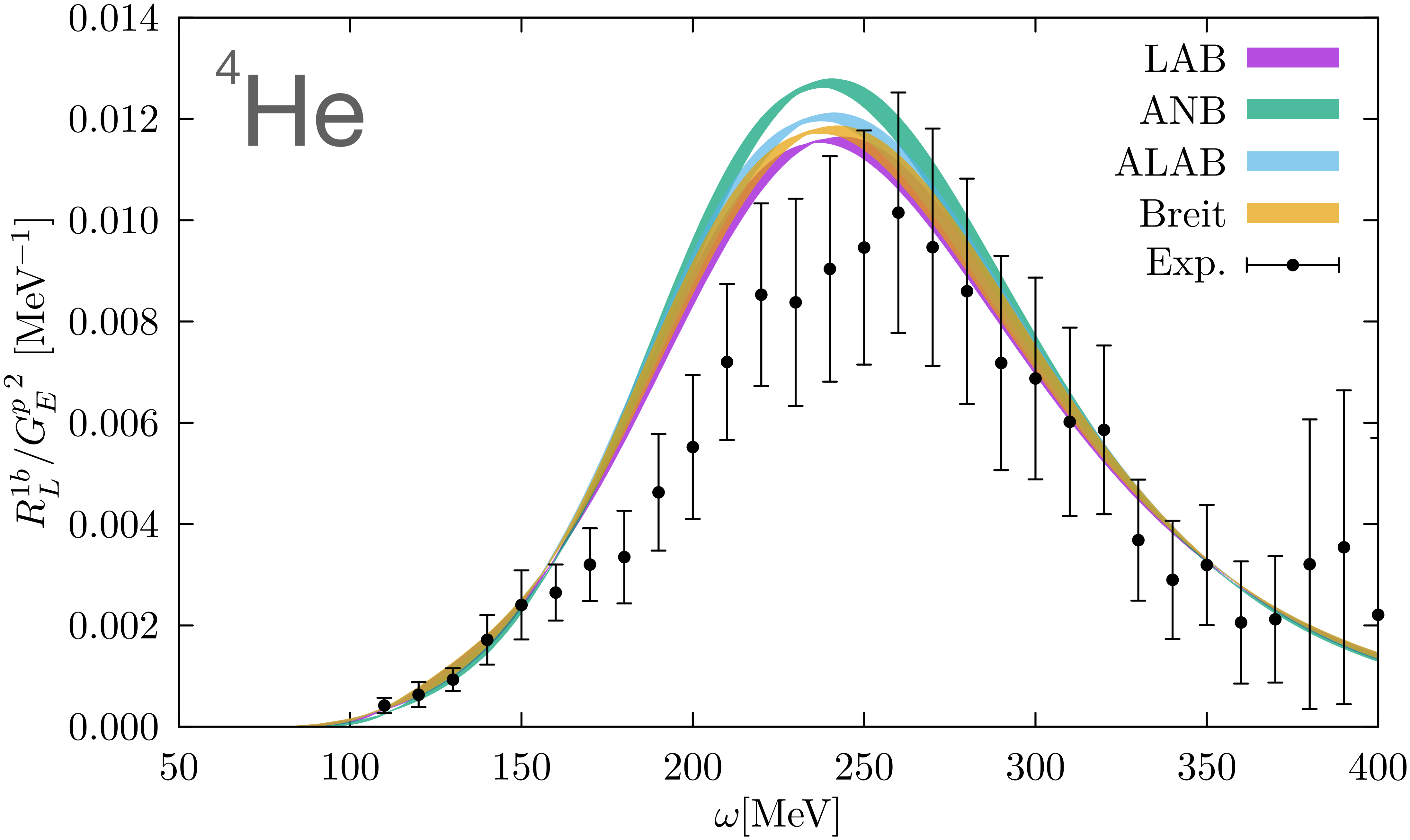
$$E_f^{fr} = \sqrt{m_N^2 + [\mathbf{p}^{fr} + \mu/M_X \mathbf{P}_f^{fr}]^2} + \sqrt{M_X^2 + [\mathbf{p}^{fr} - \mu/m_N \mathbf{P}_f^{fr}]^2}$$
$$\omega^{fr} = E_f^{fr} - E_i^{fr}$$

- And it is used as input in the non relativistic kinetic energy

$$e_f^{fr} = (p^{fr})^2/(2\mu)$$

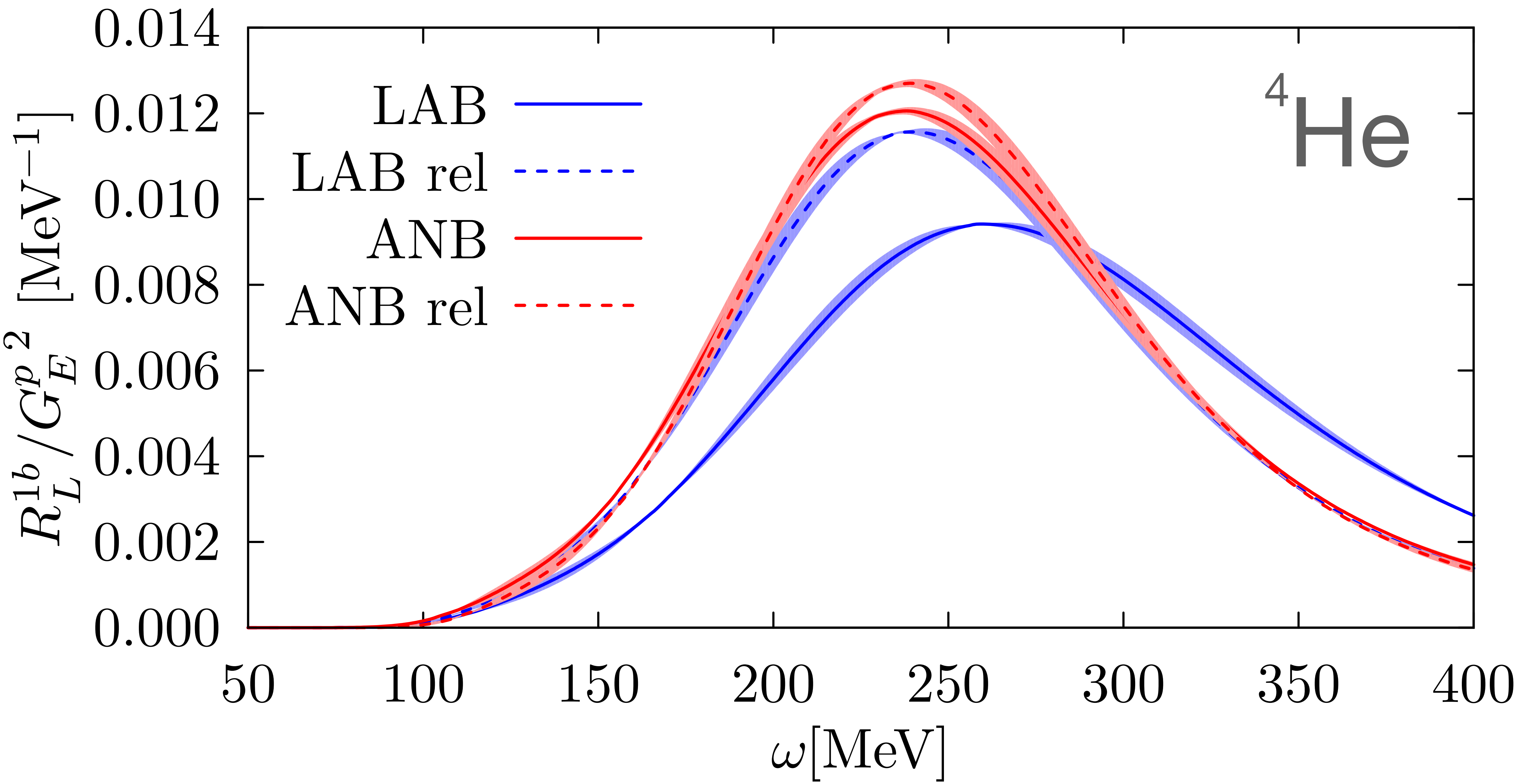
- Analogy with NN potential model where the NN relative scattering momentum p_{12} is determined in a relativistically correct fashion and used $\longrightarrow$ $E_{12}=p_{12}^2/2\mu$

Relativistic effects in a correlated system



- Longitudinal responses of ^{4}He for |q|=700 MeV in the four different reference frames. The different curves are almost identical.

Relativistic effects in a correlated system



• Relativistic effects are much smaller in the ANB frame where the final nucleon momentum is $\propto q/2$, the position of the peak remains almost unchanged

Electron- and neutrino-scattering cross sections

- We start by defining the nuclear response functions, for a given value of **q** and ω

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle 0|J_\alpha^\dagger(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

- Electron case we write the double differential cross section as:

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\mathrm{Mott}}\left[\left(\frac{q^2}{\mathbf{q}^2}\right)^2 R_L + \left(\frac{-q^2}{2\mathbf{q}^2} + tan^2\frac{\theta}{2}\right)R_T\right]$$

where: $R_L = W_{00} \quad , \quad R_T = W_{xx} + W_{yy}$

- Neutrino case:

$$\left(\frac{d\sigma}{dE'd\Omega}\right)_{\nu/\bar{\nu}} = \frac{G^2}{4\pi^2}\frac{k'}{2E_\nu}\left[\hat{L}_{CC}R_{CC} + 2\hat{L}_{CL}R_{CL} + \hat{L}_{LL}R_{LL} + \hat{L}_T R_T \pm 2\hat{L}_{T'}R_{T'}\right] ,$$

- Where the nuclear responses are given by

$$R_{CC} = W^{00} , \quad R_{CL} = -\frac{1}{2}(W^{03} + W^{30}) , \quad R_{LL} = W^{33} , \quad R_T = W^{11} + W^{22} , \quad R_{T'} = -\frac{i}{2}(W^{12} - W^{21})$$

Scaling in the Fermi gas model

- Scaling of the first kind: the nuclear electromagnetic responses divided by an appropriate function describing the single-nucleon physics no longer depend on the two variables ω and **q**, but only upon $\psi(\mathbf{q},\omega)$

Adimensional variables:

$$\lambda = \omega/2m$$
$$\kappa = |\mathbf{q}|/2m$$
$$\tau = \kappa^2 - \lambda^2$$
$$\eta_F = p_F/m$$
$$\xi_F = \sqrt{p_F^2 + m^2}/m - 1$$

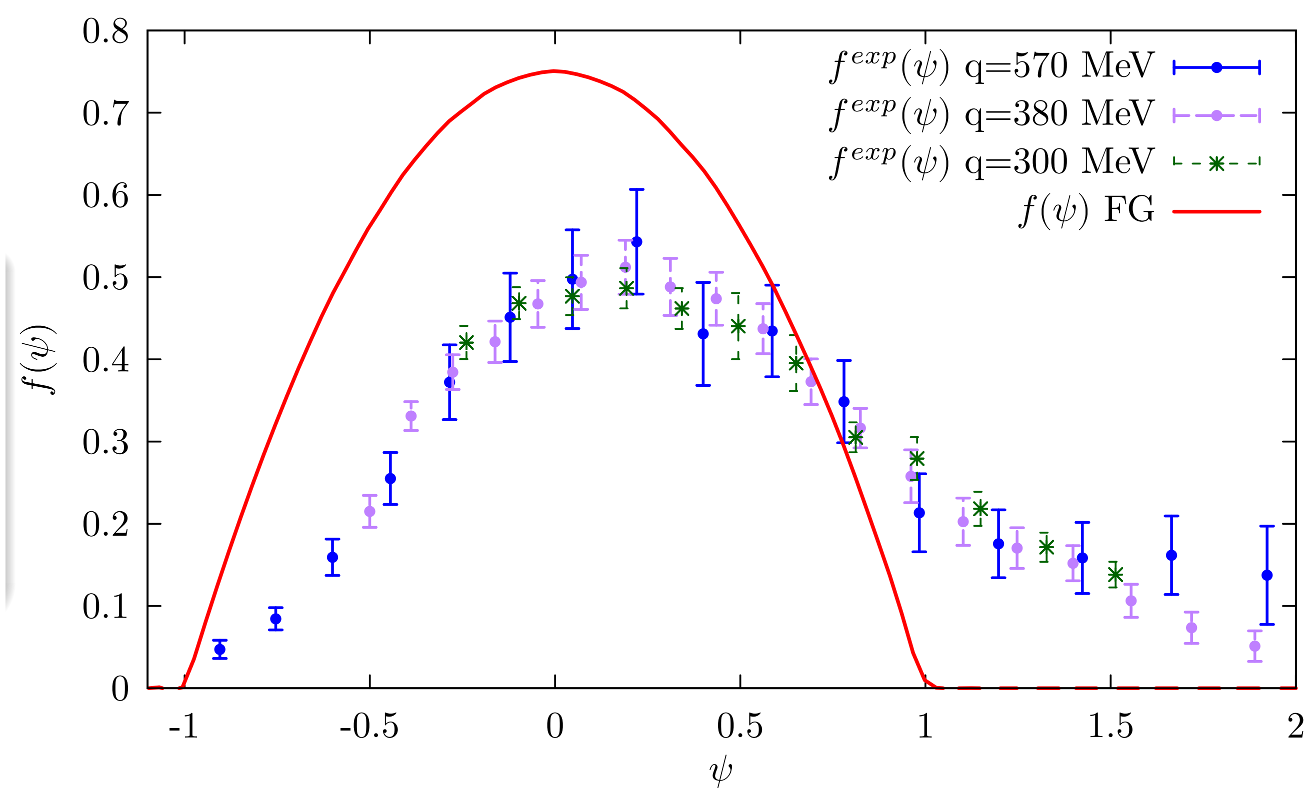
Scaling function:

$$\psi = \frac{1}{\xi_F} \frac{\lambda - \tau}{\sqrt{(1+\lambda)\tau + \kappa\sqrt{\tau(1+\tau)}}}$$

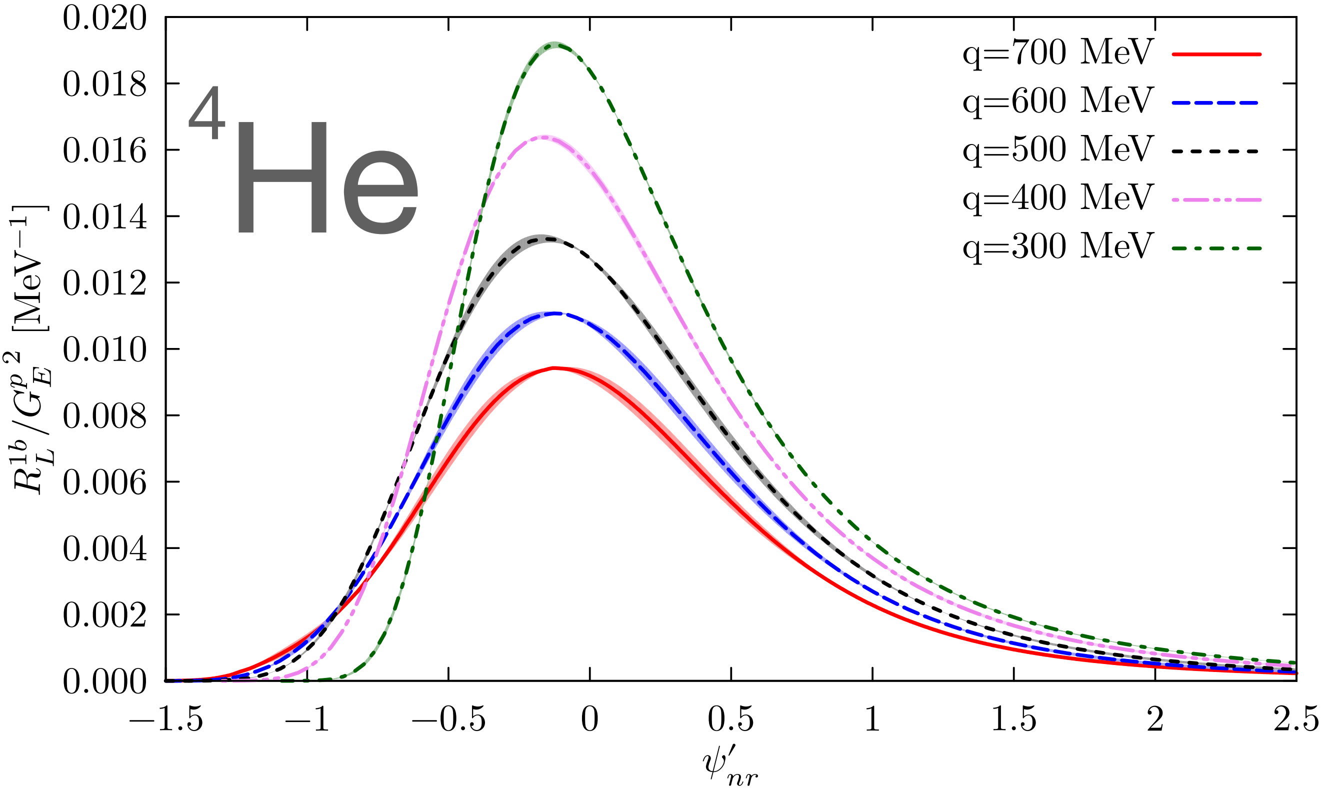
In the Fermi Gas the L and T responses have the same functional form :

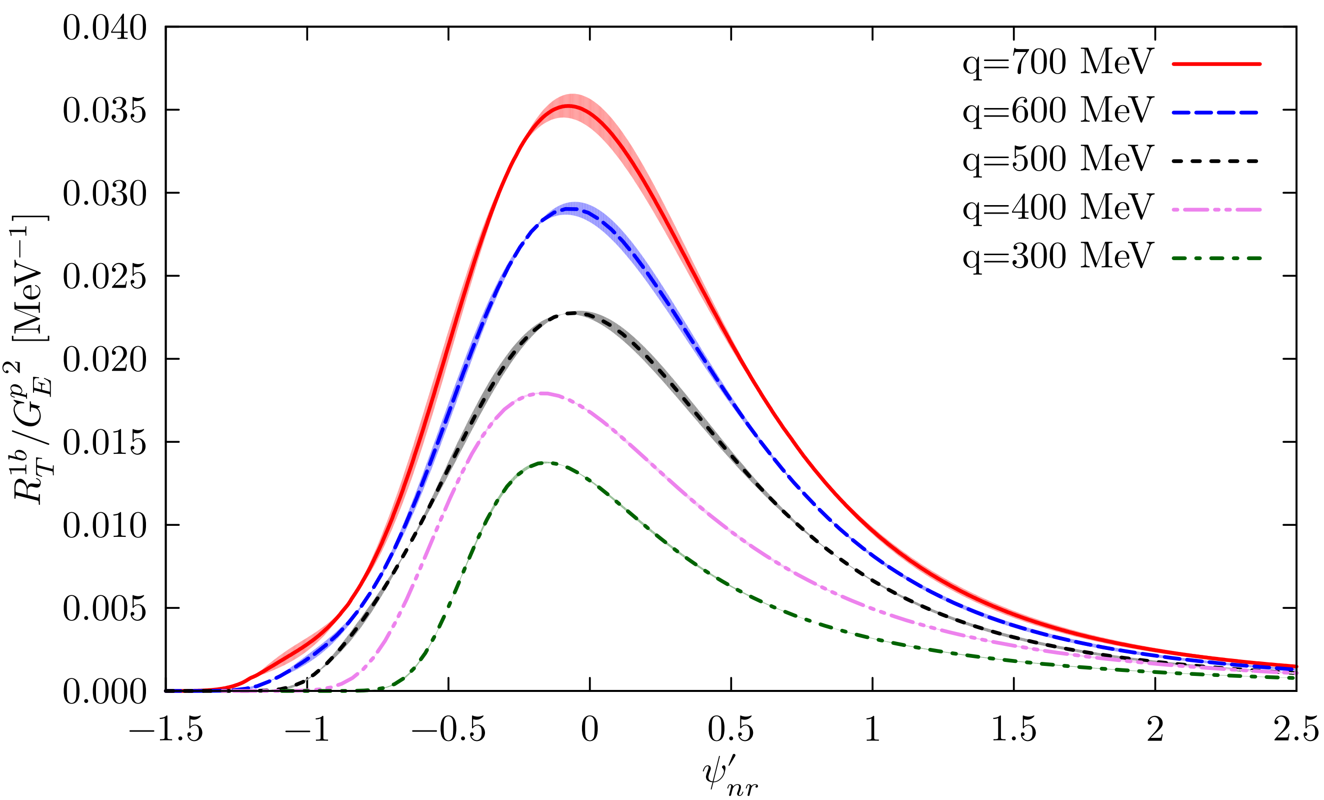
$$R_{L,T} = (1-\psi^2)\theta(1-\psi^2) \times G_{L,T}$$

- In the Fermi Gas picture only statistical correlations are accounted for



Scaling as a tool to interpolate the responses





- In order to obtain the GFMC inclusive electron-nucleus cross sections we developed a novel interpolation algorithm based on the scaling of the nuclear responses.

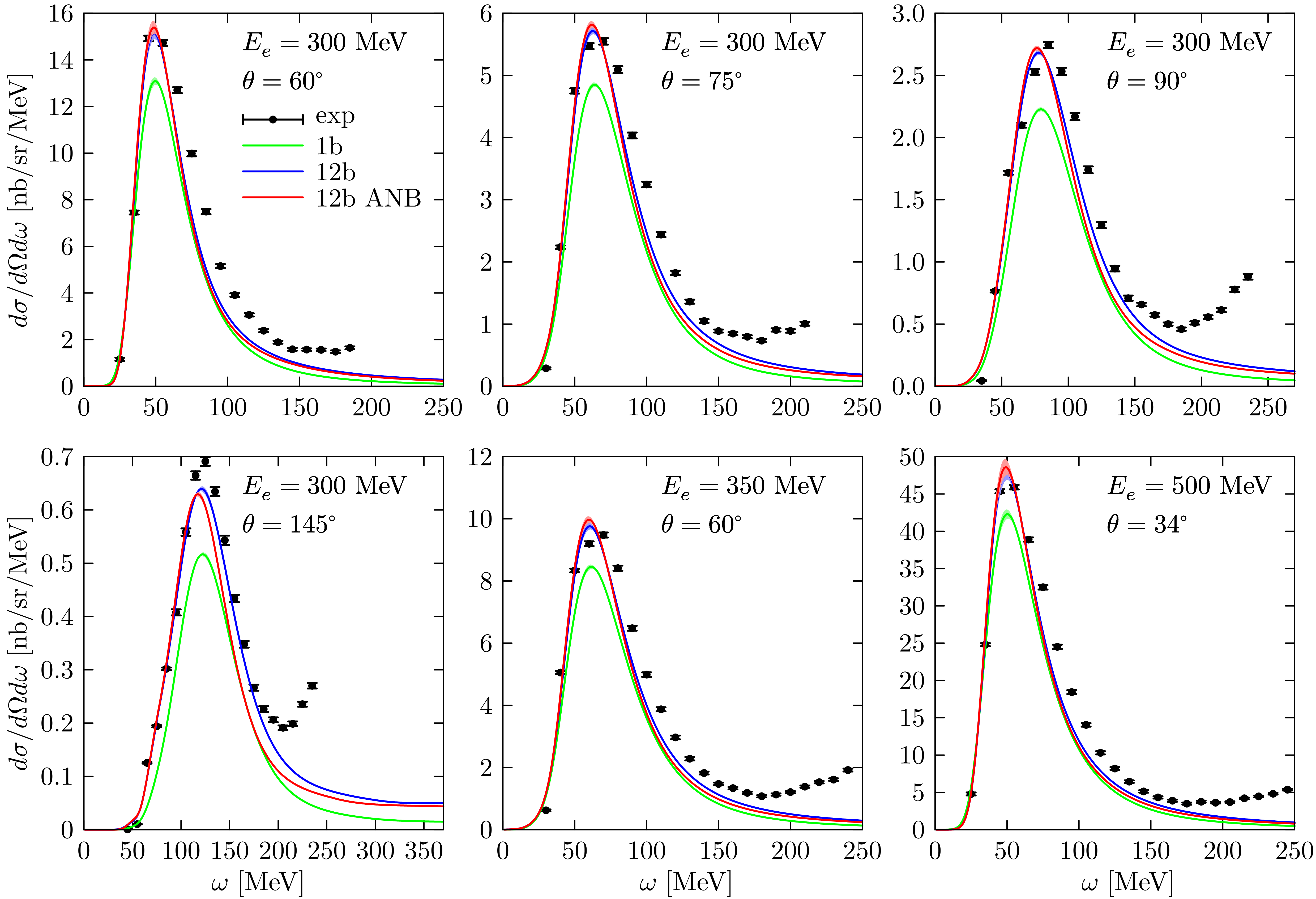
- For a fixed value of E_e and θ_e

$$Q^2 = 4E_e(E_e - \omega)\sin^2\frac{\theta_e}{2}\ ,\ |\mathbf{q}| = \sqrt{Q^2 + \omega^2}$$

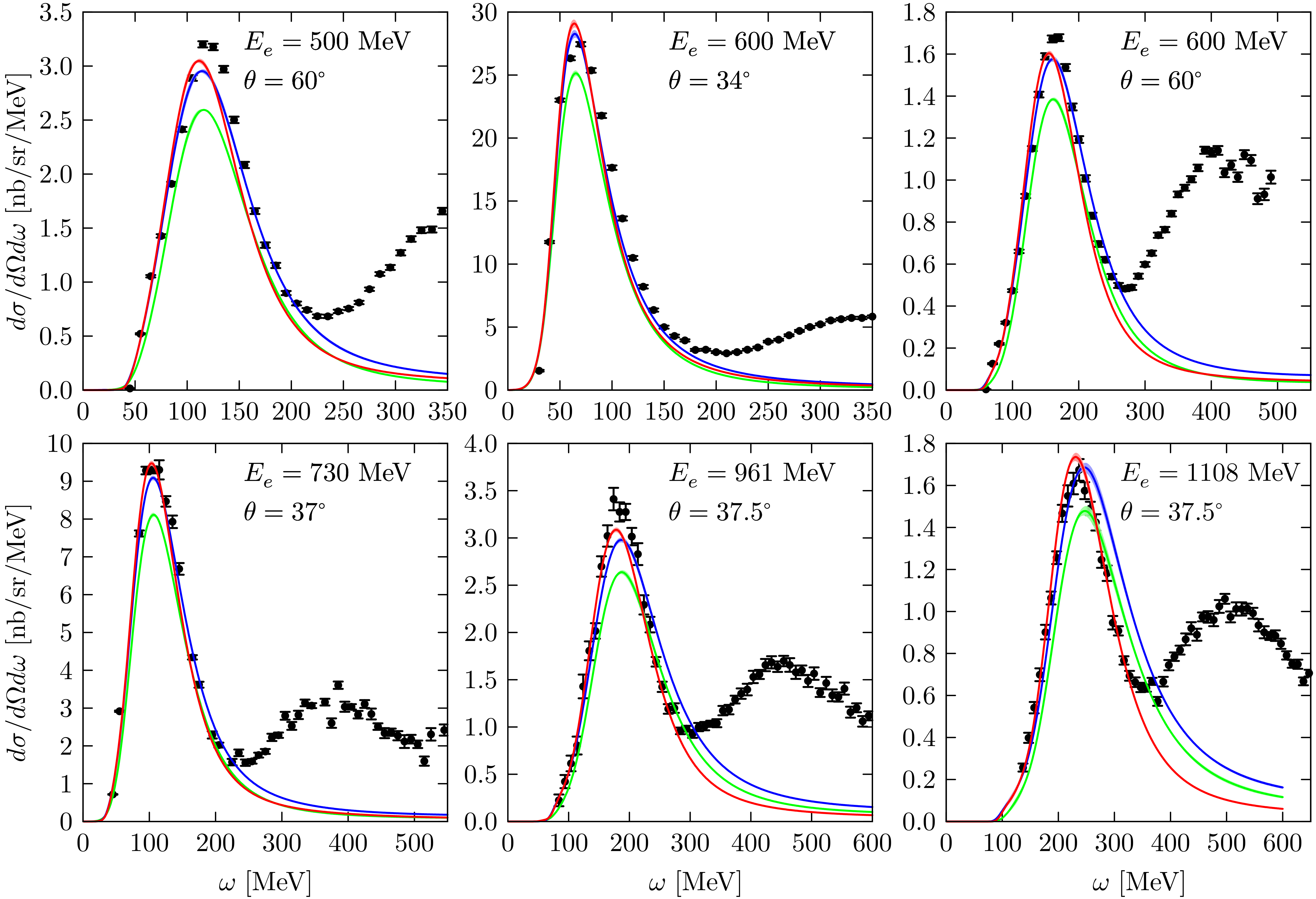
- We first compute ψ'_{nr} then the set of $R_{L,T}(\psi'_{nr}, \mathbf{q})$ is interpolated in $|\mathbf{q}|$.

- For a given value of ψ'_{nr} the curves corresponding to different values of $|\mathbf{q}|$ are almost perfectly aligned and monotonic functions of $|\mathbf{q}|$. Using the concept of scaling, largely improves the accuracy of the interpolation procedure and reduces the computational cost

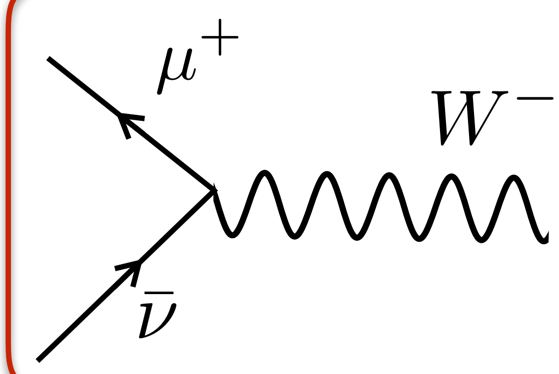
Scaling as a tool to interpolate the responses



Scaling as a tool to interpolate the responses

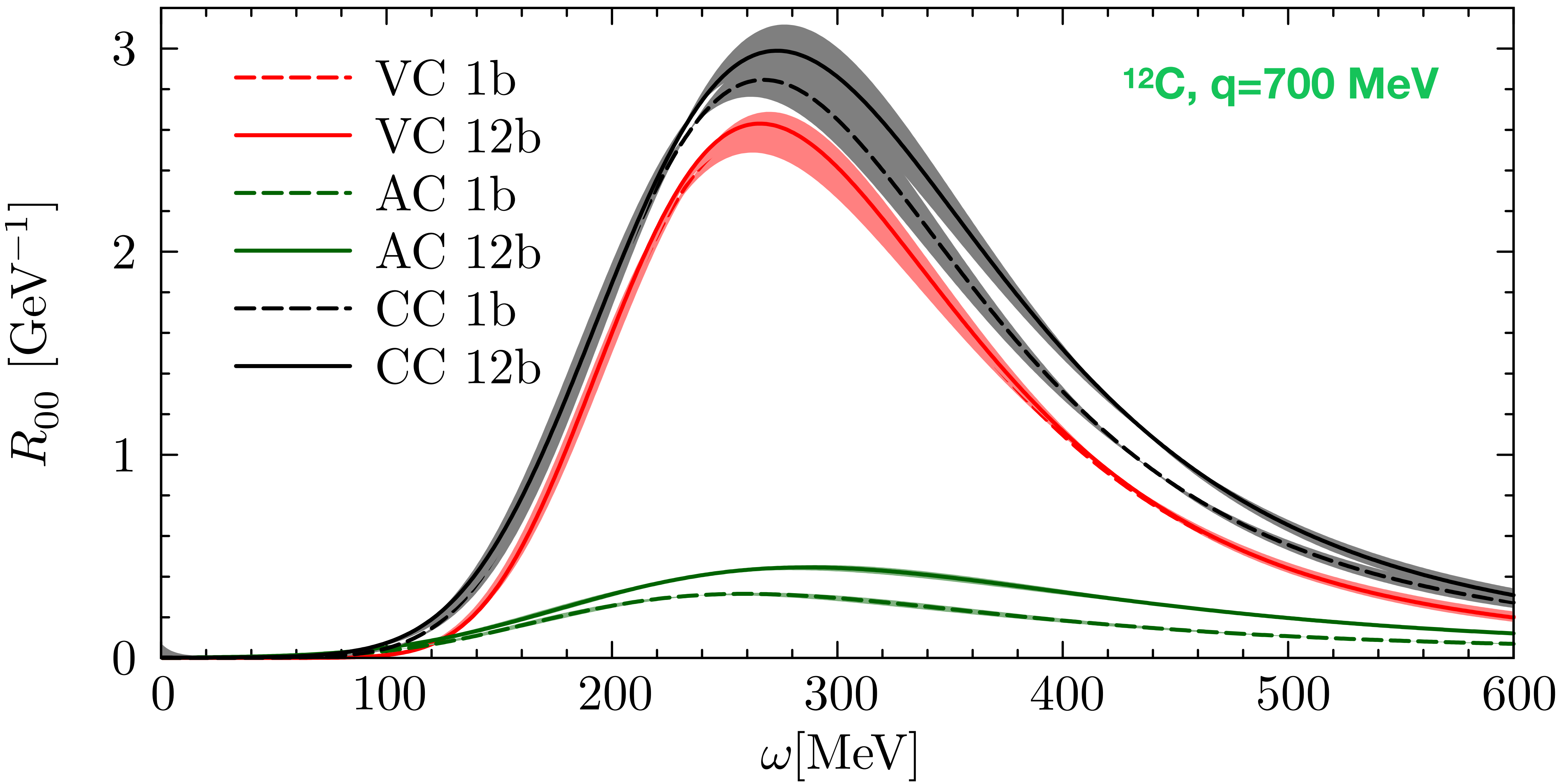


^{12}C charge-current response



- We computed the charged-current response function of ^{12}C

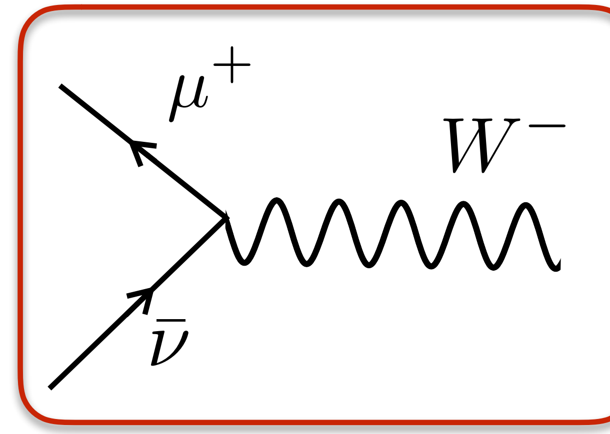
- Two-body currents have little effect in the vector term, but enhance the axial contribution at energy larger than quasi-elastic kinematics



A. Lovato et al. in preparation

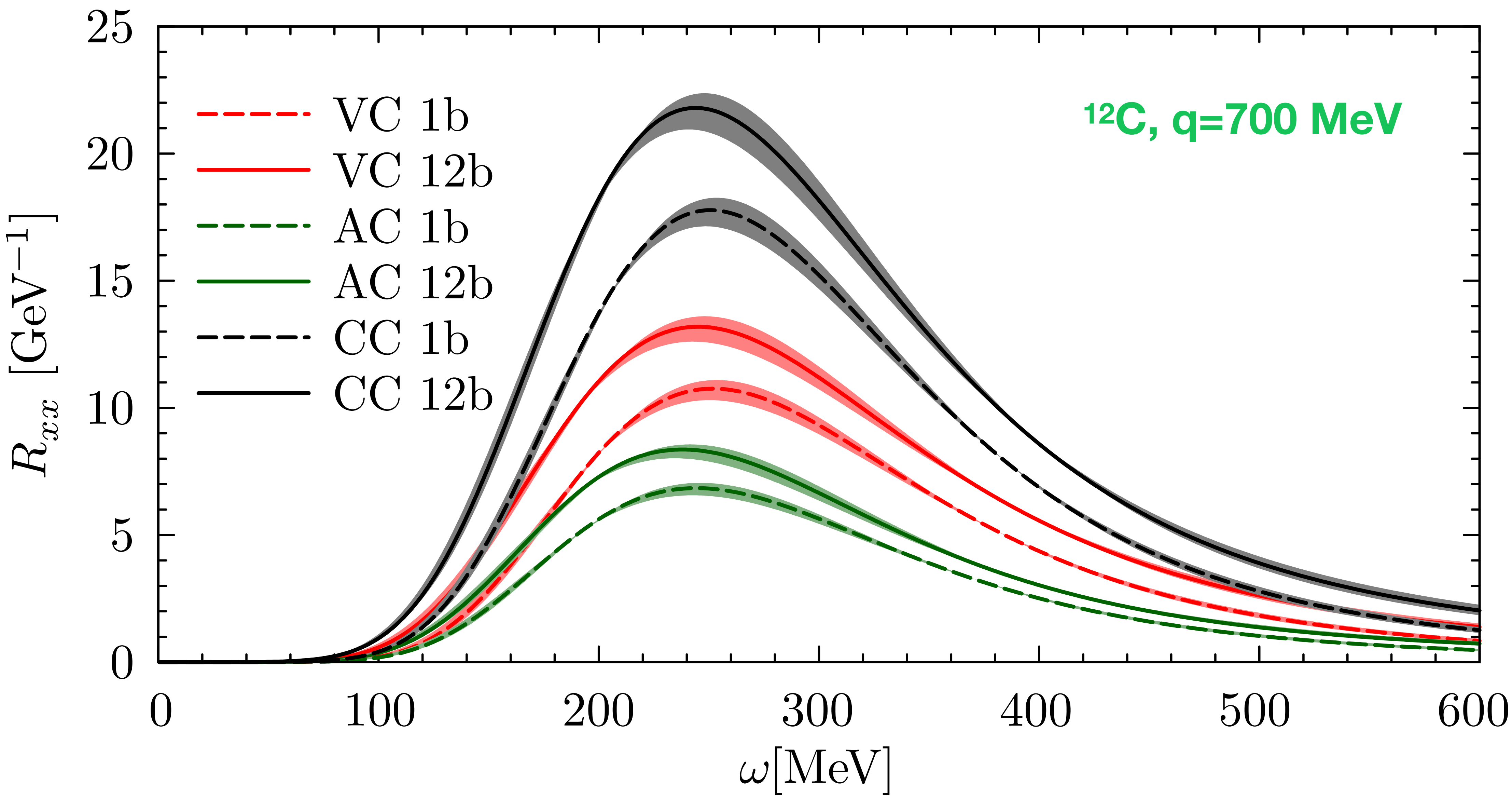
Preliminary

^{12}C charge-current response



- We computed the charged-current response function of ^{12}C

- Two-body currents have a sizable effect in the transverse response, both in the vector and in the axial contributions



A. Lovato et al. in preparation

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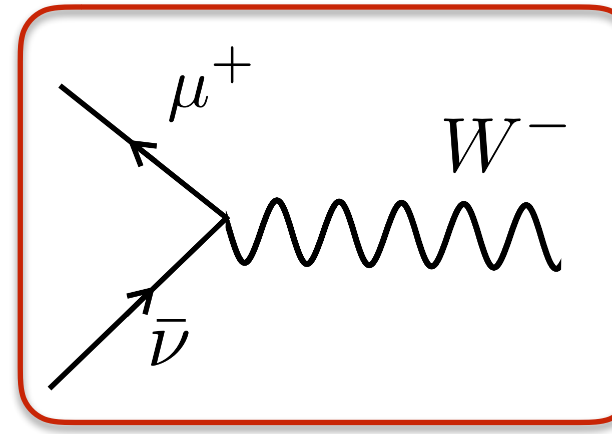
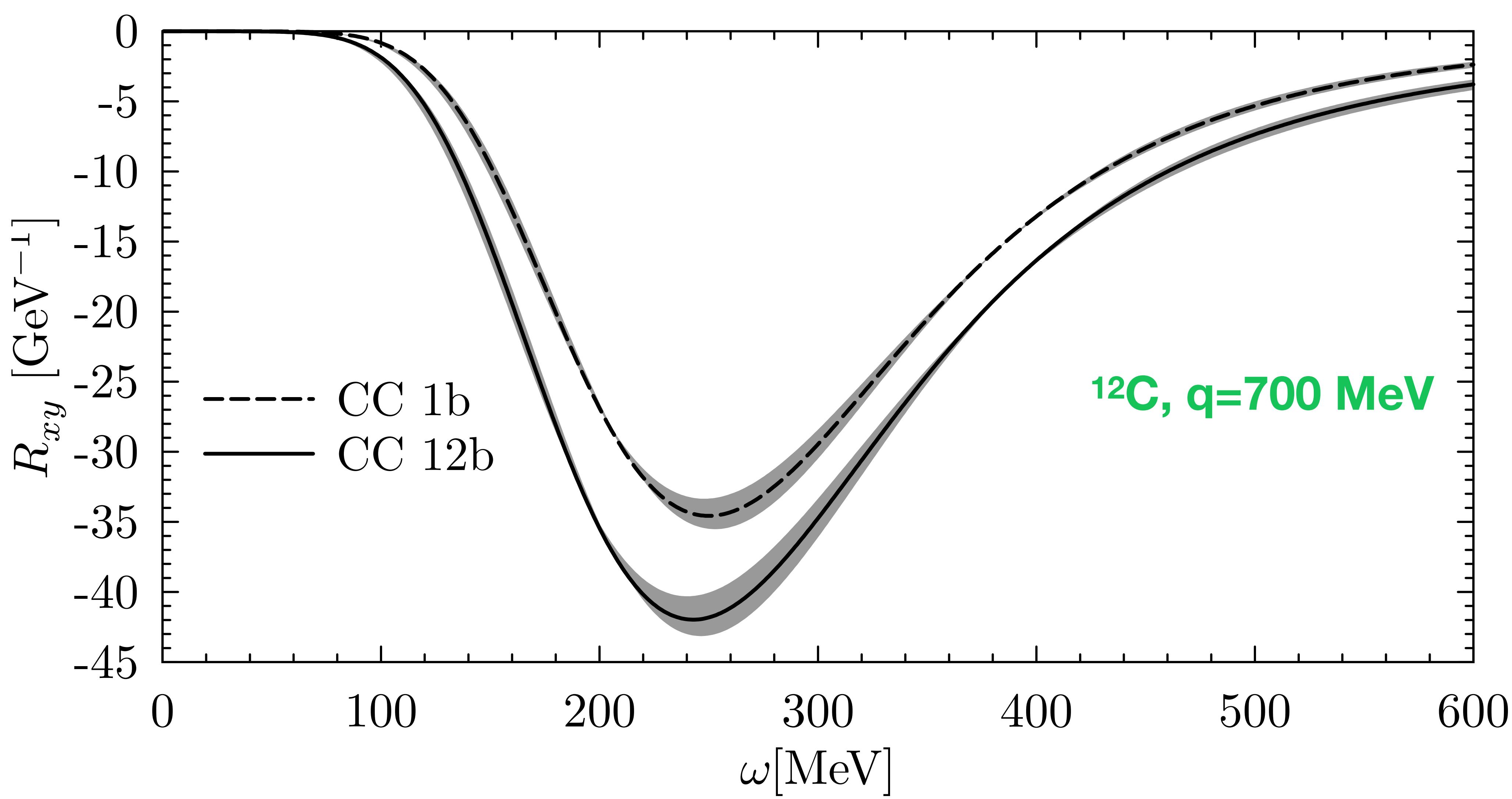
^{12}C charge-current response



- We computed the charged-current response function of ^{12}C

- Two-body currents have a sizable effect in the interference between the axial and vector current contributions, important to asses neutrino/antineutrino event rates



A. Lovato et al. in preparation

Preliminary

Summary

- Relativistic effects in the kinematics can be accounted for choosing a reference frame that minimizes the nucleon momentum + two-fragment model

- Using the concept of scaling of the electromagnetic responses we were able to efficiently interpolate the response functions and obtain cross sections

- Neutrino physics is entering a new precision era; realistic models of nuclear dynamics are fundamental for an accurate analysis of neutrino oscillation data

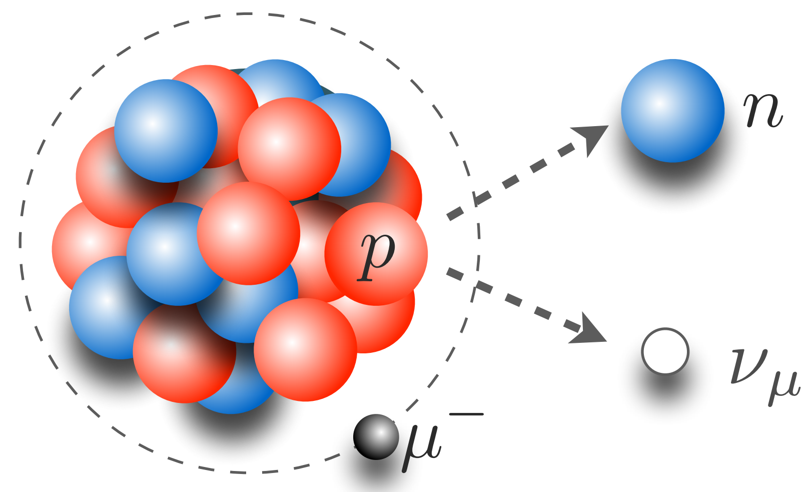
- Neutral and Charge current response functions have been obtained within GFMC. Generalize what has been already done for the electromagnetic case compare with the MiniBooNE data: two-body current contribution is needed to explain the excess.

- Implement chiral currents obtained from the chiral potential developed by M.Piarulli in the electroweak response functions

GFMC results for muon capture in ^{4}He

- Negative muons can be captured by the nucleus in a weak-interaction process resulting in the change of one of the protons into a neutron and a neutrino emission: inverse process of charge current neutrino scattering

The muon rest mass is converted in energy shared by the emitted neutrino and recoiling final nucleus



A calculation of the total inclusive rate requires requires knowledge of both the low-lying discrete states and higher-energy continuum spectrum of the final nucleus; it is given in terms of five response functions

$$\frac{d\Gamma}{dE_\nu} = \frac{G_V^2}{2\pi} |\psi(0)|^2 E_\nu^2 \left[R_{00}(E_\nu) + R_{zz}(E_\nu) + R_{0z}(E_\nu) + R_{xx}(E_\nu) - R_{xy}(E_\nu)\right]$$

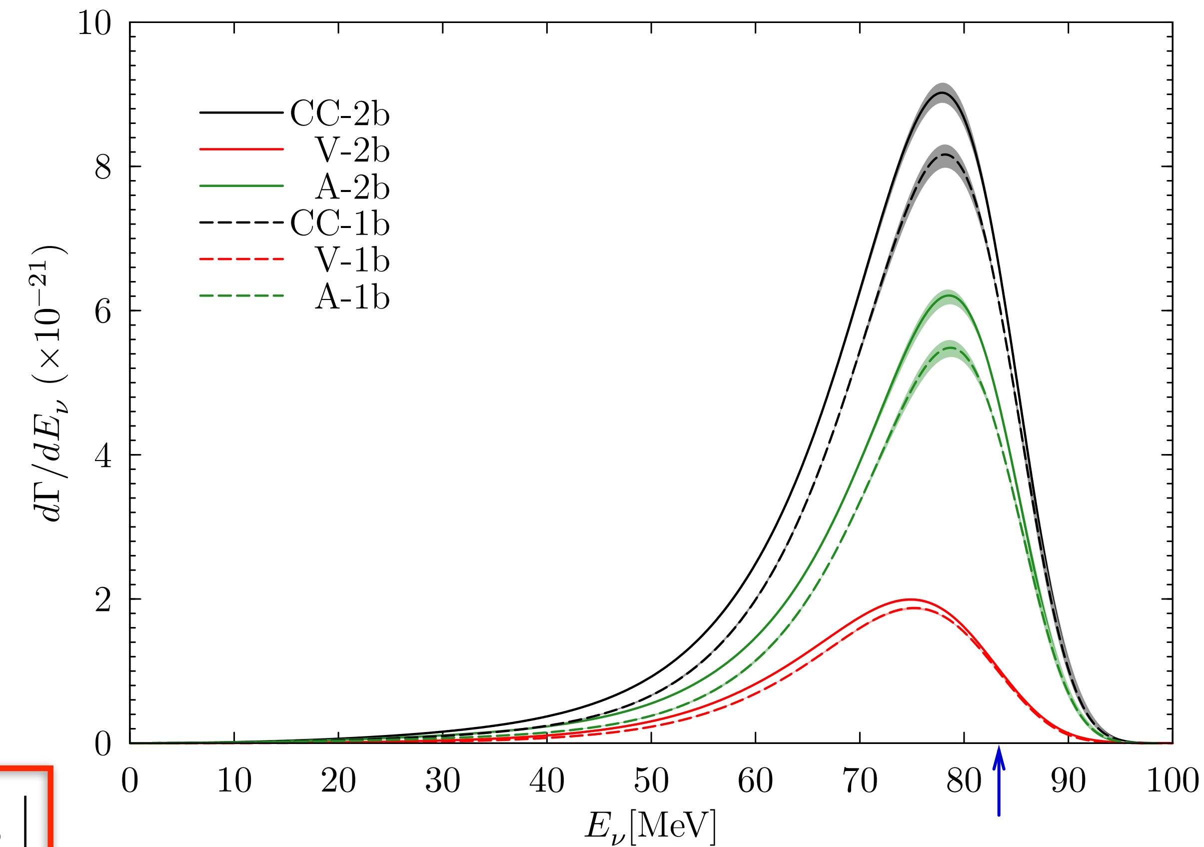
Atomic wave function of the muon approximated as $\longrightarrow$ $\psi(x) \simeq \psi(0) = (Z\alpha\mu)^3/\pi$

Muon capture in ^{4}He

AL, N. Rocco, R. Schiavilla (in prep.)

The differential capture rate can be computed interpolating the response functions at

$$\begin{cases} \omega = m_\mu + m_n - m_p - E_\nu \\ |\mathbf{q}| = E_\nu \end{cases}$$



Integrating the differential capture rate, we get the following total rates

	V-1b	V-2b	A-1b	A-2b	CC-1b	CC-2b
$\Gamma(s^{-1})$	65 ± 1	73 ± 1	171 ± 6	200 ± 6	265 ± 9	306 ± 9

- Two-body currents increase the capture rate by about 15%.

- The predicted value is consistent with the lower range of available experimental determinations

	Exp [47]	Exp [48]	Exp [49]	Th [50]	Th [51]
$\Gamma(s^{-1})$	336 ± 75	375^{+30}_{-300}	364 ± 46	345 ± 110	278

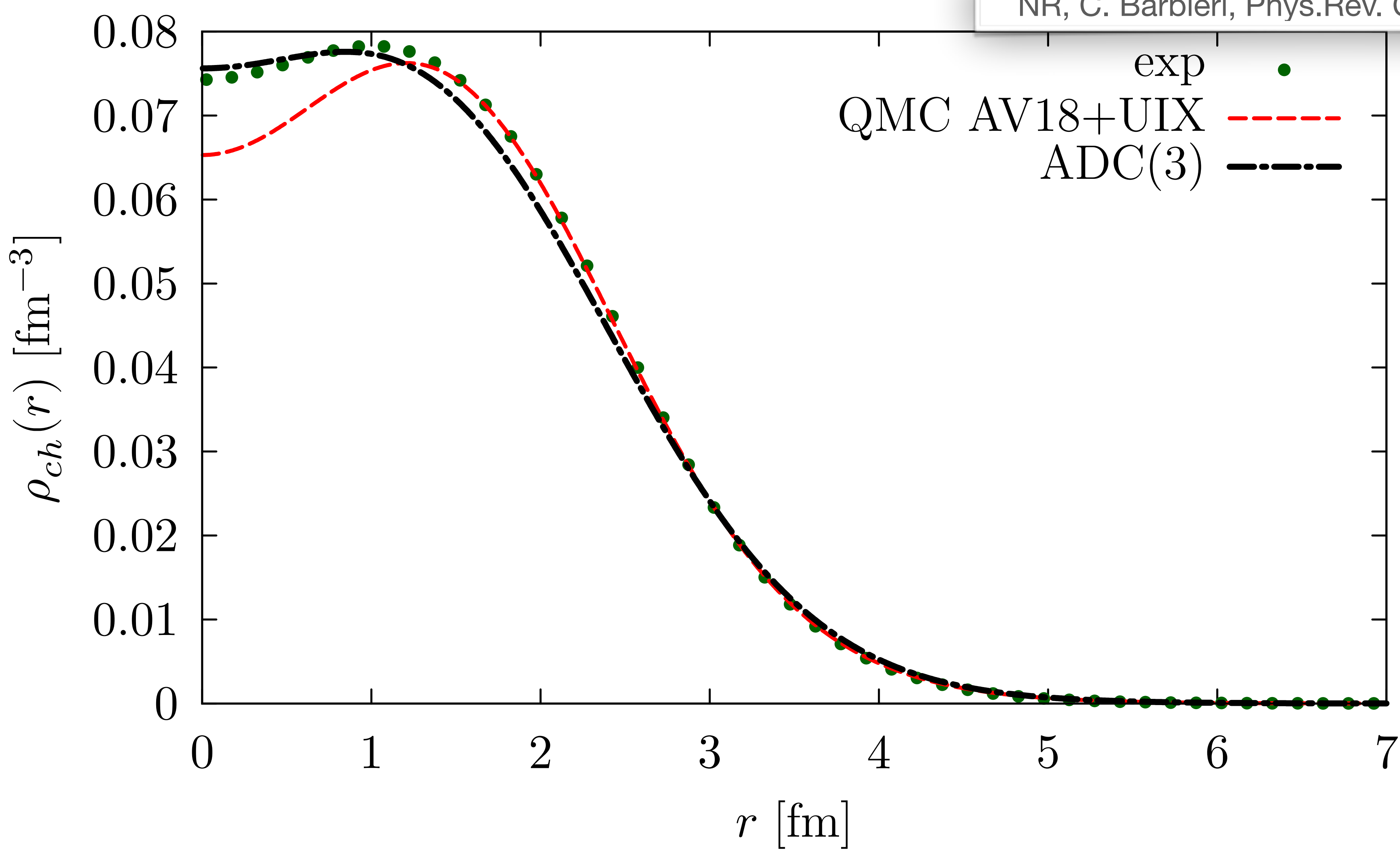
Back up slides

Benchmark the nuclear model: ^{16}O charge density distribution

- The nuclear charge density distribution is the Fourier transform of the charge elastic form factor:

$$\rho_{ch}(r') = \int \frac{d^3q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}'} \; F_L(\mathbf{q})$$

NR, C. Barbieri, Phys.Rev. C98 (2018) 025501



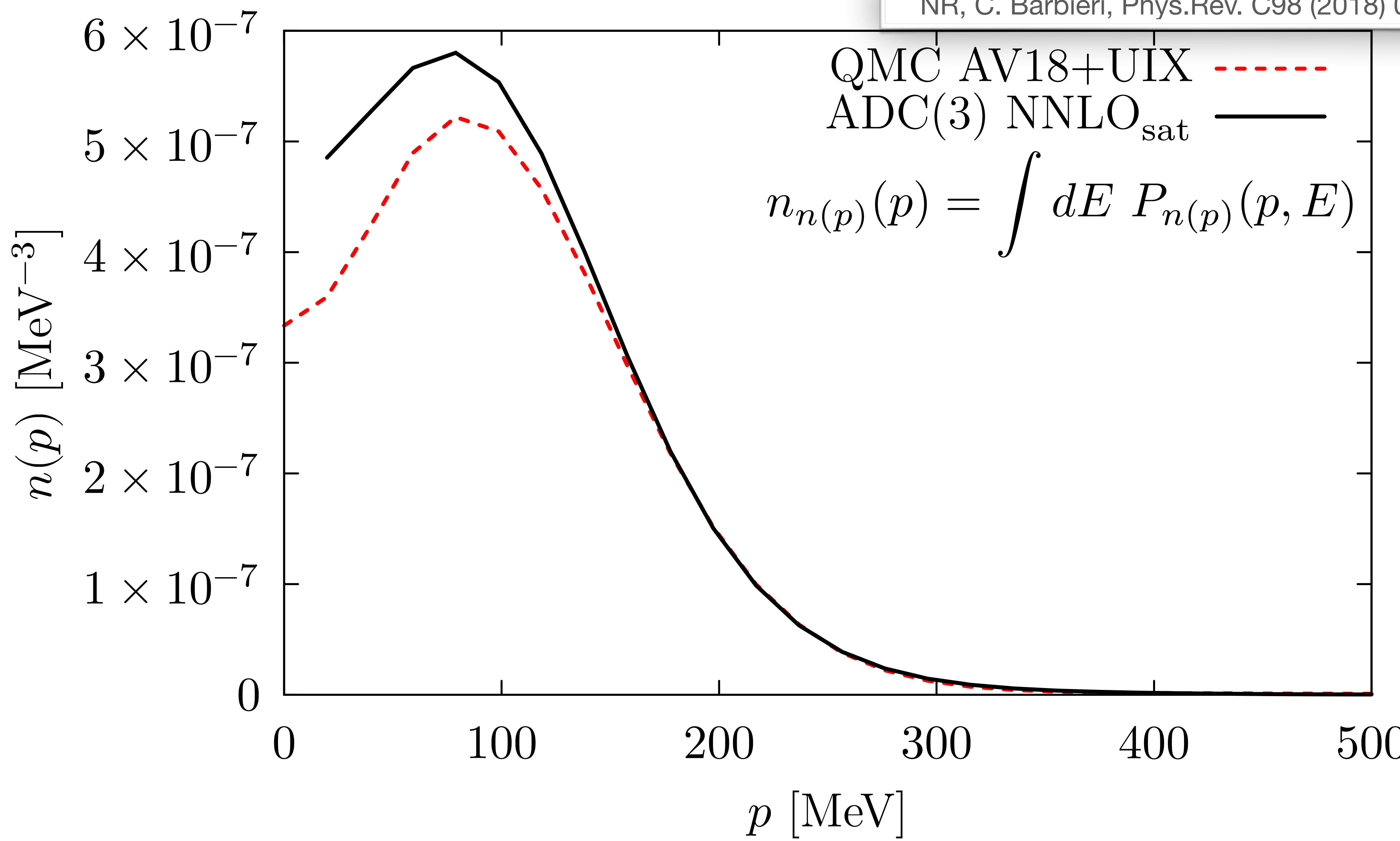
- Nice agreement between the SCGF and QMC calculations

- SCGF results agree with experiments (corroborates the goodness of NNLO$_{sat}$)

Benchmark the nuclear model: ^{16}O momentum distribution

- Single particle momentum distribution of ^{16}O

NR, C. Barbieri, Phys.Rev. C98 (2018) 025501

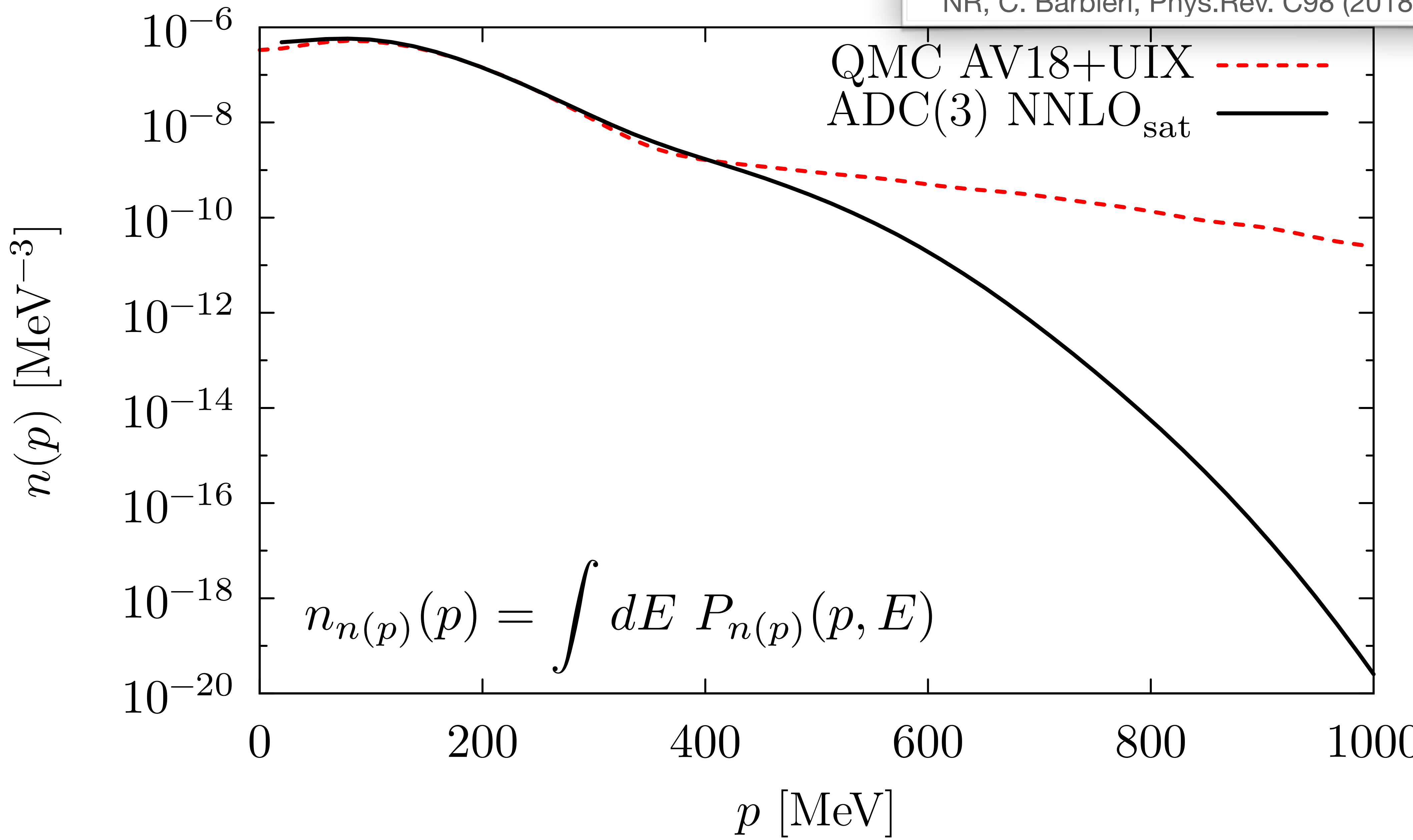


- The momentum distribution reflects the fact that NNLO$_{sat}$ is softer the AV18+UIX.

Benchmark the nuclear model: ^{16}O momentum distribution

- Single particle momentum distribution of ^{16}O, log scale

NR, C. Barbieri, Phys.Rev. C98 (2018) 025501



- The momentum distribution reflects the fact that NNLO$_{\text{sat}}$ is softer than AV18+UIX.