Short Range Correlations and Final State Interactions in Electron-Nucleus Scattering

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2nd Workshop on Quantitative Challenges in SRC and MEC Research
MIT, March 20-23,2019

Nuclear Cross Section at Large Momentum Transfer

⋆ Consider a nuclear transition matrix element

$$\mathcal{M}^{\mu}_{0\mathrm{F}} = \langle \mathrm{F} | J^{\mu}_{A} | 0 \rangle$$

- the target ground state $|0\rangle$ can be described to remarkable accuracy within non relativistic nuclear many-body theory (NMBT)
- at large momentum transfer NMBT is no longer applicable to describe either the final state $|\mathrm{F}\rangle$ or the nuclear current operator J^{μ}_{A}
- however, in this regime the impulse approximation (IA), based on the assumption that the beam particle couples to individual nucleons, is expected to be applicable:

$$J^{\mu}_{A} \to \sum_{i=1}^{A} j^{\mu}_{i}$$

⋆ The IA scheme naturally leads to factorisation of the A-particle final state.

$$|\mathrm{F}\rangle = |n_{\mathrm{A}-1}\rangle \otimes |p\rangle$$

The High-Energy Approximation

- ⋆ At high energy, the state describing the knocked out nucleon can be written in eikonal approximation, and assuming that the spectator particles be frozen, that is, behave as fixed scattering centres

$$H_{\mathrm{A}} = H_{\mathrm{A}-1} + T_1 + \sum_{j=2}^{\mathrm{A}} v_{1j} = H_{\mathrm{A}-1} + T_1 + H_{\mathrm{FSI}}$$

- ⋆ A-particle evolution operator within the frozen-spectators approximation

$$\mathrm{e}^{-iH_A t} = \mathrm{e}^{-i(H_{\mathrm{A}-1} + T_1 + H_{FSI})t} \to \mathrm{e}^{-iH_{\mathrm{A}-1}t}\mathrm{e}^{-i(T_1 + H_{FSI})t}$$

- ⋆ In eikonal approximation

$$\langle x_1' | \mathrm{e}^{-i(T_1 + H_{FSI})t} | x_1 \rangle = \langle x_1' | \mathrm{e}^{-iT_1 t} | x_1 \rangle \, \Omega_p(\boldsymbol{x}_1, t)$$

⋆ Propagation of the knocked out particle driven by

$$\Omega_p(\boldsymbol{x},t) = \frac{1}{\rho_A(\boldsymbol{x})}\langle 0|\frac{1}{A}\sum_{i=1}^{\mathrm{A}}\mathcal{P}_z\prod_{j\neq i}[1-\Gamma_p(\boldsymbol{x}_i+\boldsymbol{v}t-\boldsymbol{x}_j)]\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}_i)|0\rangle$$

$$\Gamma_p(\boldsymbol{x}) = \theta(z)\gamma_p(\boldsymbol{x}_\perp) \quad , \quad \gamma_p(\boldsymbol{x}_\perp) = -\frac{i}{2}\int\frac{d^2k_\perp}{(2\pi)^2}f_p(\boldsymbol{k}_\perp)\mathrm{e}^{i\boldsymbol{x}_\perp\cdot\boldsymbol{x}_\perp}$$

where $f_p(\boldsymbol{k}_\perp)$ is the measured NN scattering amplitude

⋆ Final State

$$\langle x_1,\ldots,x_\mathrm{A}|\mathrm{F}\rangle = \langle x_2,\ldots,x_\mathrm{A}|\mathrm{n}\rangle\otimes\Omega(\boldsymbol{x}_1,t)\frac{1}{\sqrt{V}}\mathrm{e}^{i\mathbf{p}\cdot\boldsymbol{x}_1}$$

⋆ The leading contribution to $\Omega(\boldsymbol{x}_1,t)$ corresponds to the Plane Wave Impulse Approximation (PWIA), in which FSI are neglected

FSI IN INCLUSIVE $e + A \to e' + X$ PROCESSES

Basic Facts

- ⋆ depending on the nature of the interaction, FSI can affect the inclusive cross section, even at large momentum transfer
- ⋆ the inclusive x-section carries information on FSI taking place within a distance $d \sim q^{-1}$ of the primary interaction vertex
- ⋆ at large momentum transfer, the probability of FSI within a distance d is small. However, FSI move strength from the quasi free peak, where the inclusive cross section is large, to the region of low energy loss, $\omega \ll Q^2/2m$, where the x-section is very small. As a consequence, their effects may become appreciable, in fact even dominant.

Inclusive Cross Section

- Factorization yields

$$\frac{d\sigma_{\rm A}}{d\Omega d\omega} = \int d\omega' f_q(\omega-\omega') \left(\frac{d\sigma_{\rm A}}{d\Omega d\omega'}\right)_{\rm PWIA}$$

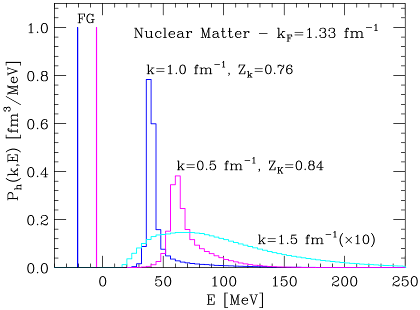
$$\left(\frac{d\sigma_{\rm A}}{d\Omega d\omega'}\right)_{\rm PWIA} = \int d^3k dE P(k,E) \, \frac{d\sigma_{\rm N}}{d\Omega d\omega}$$

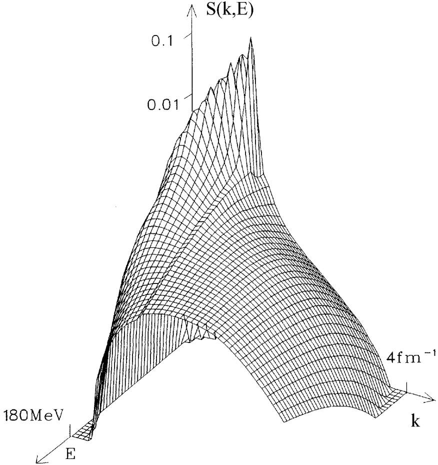
- the spectral function $P(k,E)$ describes the probability of knocking out a nucleon of momentum k from the target leaving the residual system with excitation energy E

$$f_q(\omega) \int \frac{dt}{2\pi} {\rm e}^{i\omega t} \, \bar{\Omega}(t) \quad , \quad \bar{\Omega}(t) = \frac{1}{\rm A} \int d^3x \rho_{\rm A}(\boldsymbol{x}) \, \Omega(\boldsymbol{x},t)$$

- In the absence of FSI, $\bar{\Omega}(t) \to 1$, and $f(\omega) \to \delta(\omega)$

$P(\mathbf{k}, E)$ of Isospin-Symmetric Nuclear Matter

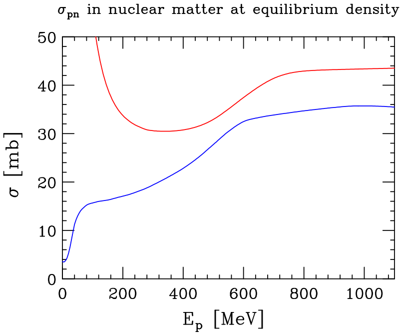




- The spectral functions of medium-heavy nuclei have been extensively studied carrying out systematic measurements of $(e, e'p)$ cross sections

INGREDIENTS OF THE FOLDING FUNCTION

- ⋆ the NN scattering amplitude is written in terms of three parameters, extracted from fits to the data: σ (total x-section), β (slope), and α (ratio between real and imaginary part of)
- ⋆ medium modifications of NN scattering are significant, and must be taken into account



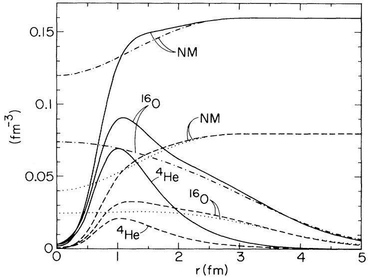
- ⋆ calculations by Vijay Pandharipande and Steven Pieper

⋆ the average of the scattering operator over the position of the spectators is strongly affected by correlation effects

⋆ to see this, consider that the probablity of NN rescattering depends upon the joint probability of finding the the struck particle at position $\boldsymbol{x}_1$ and a spectator at position $\boldsymbol{x}_2$

$$\rho^{(2)}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \rho_{\rm A}(\boldsymbol{x}_1)\rho_{\rm A}(\boldsymbol{x}_2)\, g(\boldsymbol{x}_1, \boldsymbol{x}_2)$$

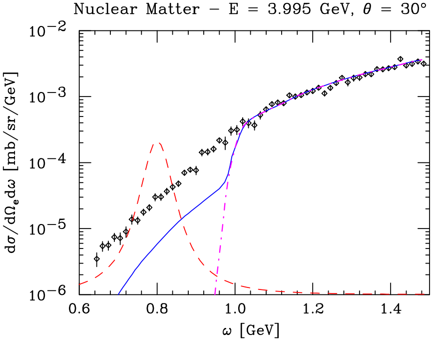
⋆ calculations performed by the Urbana-Argonne-JLab group



⋆ FSI are strongly suppressed at $|\boldsymbol{x}_1, \boldsymbol{x}_2| \lesssim 1$ fm

How it Works

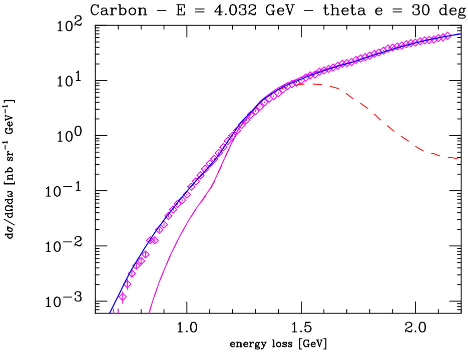
- the real part of the NN scattering amplitude shifts the position of the peak of the folding function, whereas the imaginary part determines its width



- SLAC data extrapolated to isospin-symmetric matter by Donal Day and Ingo Sick
- folding function shown in linear scale, and multiplied by a factor 10^{-3}

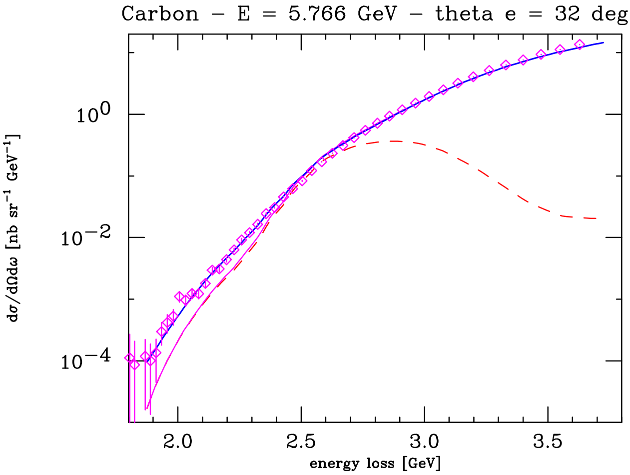
Comparison to Carbon Data

- JLab E89-008 data, $Q^2_{\rm qe} \approx 2.78~{\rm GeV}^2$



- Recall: no adjustable parameters involved!

⋆ JLab E02-019 data, $Q^2_{\rm qe} \approx 6.12\ \mathrm{GeV}^2$



Summary & Outlook

- ⋆ No matter whether you see them as noise obscuring correlation effects or signal carrying relevant information, FSI in the response of interacting fermion systems are significant, and must be accounted for at quantitative level

- ⋆ A consistent framework for the description of FSI can be built based a combination of well established results of nuclear many-body theory and data. While improvements and extensions are certainly needed, applications of the formalism to inclusive processes at large momentum transfer appear to be quite promising

- ⋆ Existing applications to exclusive processes are limited to the two-body breakup channel and light nuclei. Generalisation to more complex final states and heavier targets may require the development of Monte Carlo generators based on the same assumptions and dynamical model

Backup slides

Nucleon-Nucleon (NN) Correlations

- ⋆ NN correlations lead to the excitation of nucleon pairs to continuum states outside the Fermi sea, thus reducing the occupation probability of shell-model states by as much as 20%
- ⋆ Correlation effects can be unambiguously identified in the two-point Green's function exploiting the Källén-Lehmann representaion

$$G_h(\mathbf{k}, E) = \frac{Z_h}{E - e_h - i\tau_h^{-1}} + G_h^B(\mathbf{k}, E)$$

- ⋆ e_h Z_h and τ_h^{-1} are the position, strength and width of the peaks appearing in the missing energy spectra
- ⋆ the correlation contribution $G_h^B(\mathbf{k}, E)$ exhibits a smooth energy-dependence
- ⋆ In the absence of correlations

$$Z_k \to 1 \quad , \quad G_h^B(\mathbf{k}, E) \to 0$$

Nuclear Spectral Function

- The analytic structure of the two-point Green's function is reflected by the spectral function

$$P(\mathbf{k}, E) = \sum_{h \in F} Z_h |M_h(\mathbf{k})|^2 F_h(E - e_h) + P_B(\mathbf{k}, E)$$

- Compare to the IPM
  - Momentum dependence

$$|M_h(\mathbf{k})|^2 = |\langle h | a_{\mathbf{k}} | 0 \rangle|^2 \to |\phi_h(\mathbf{k})|^2$$

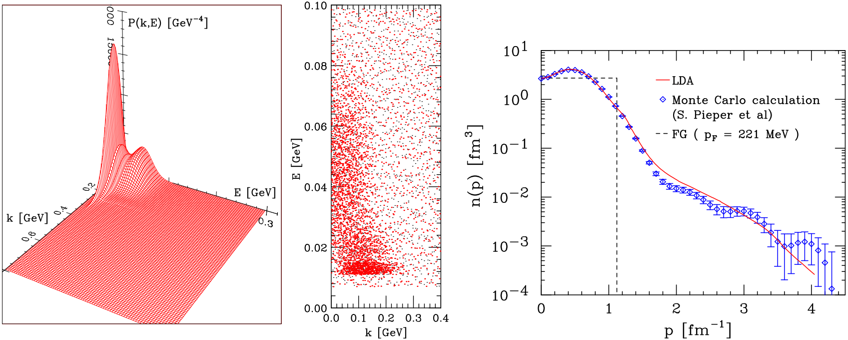
  - Energy distribution

$$F_h(E - e_h) = \frac{1}{\pi} \frac{\tau_h}{(E - e_h)^2 + (\tau_h^{-1})^2} \to \delta(E - e_h)$$

- The spectral function yields the probability of removing a particle with momentum $\mathbf{k}$ from the target ground state leaving the residual system with excitation energy E

SPECTRAL FUNCTION OF ^{16}O

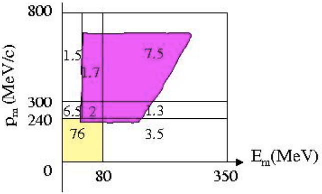
- The spectral function of medium-mass nuclei has obtained combining $(e, e'p)$ data and results of accurate nuclear matter calculations within the Local Density Approximation (LDA)

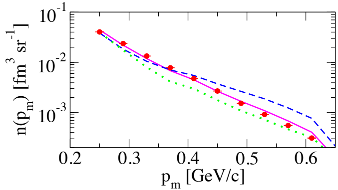


- shell model states account for $\sim 80\%$ of the strenght
- the remaining $\sim 20\%$, arising from NN correlations, is located at high momentum and large removal energy ($\mathbf{k} \gg k_F, E \gg \epsilon$)

Measured Correlation Strength

- the correlation strength in the 2p2h sector has been measured by the JLAB E97-006 Collaboration using a carbon target
- strong energy-momentum correlation: $E \sim E_{thr} + \frac{A-2}{A-1}\,\frac{\mathbf{k}^2}{2m}$

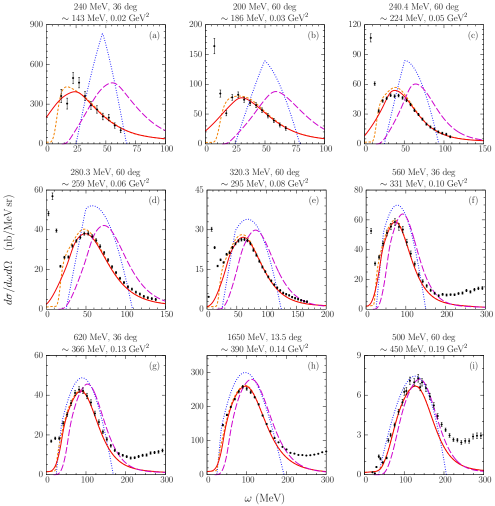




- Measured correlation strength 0.61 ± 0.06, to be compared with the theoretical predictions of *ab initio* approaches 0.64 (CBF) and 0.56 (G-Matrix)

FSI at Low Momentum Transfer

Carbon Quasi Elastic Cross Section: IA+FSI



⋆ A. Ankowski et al, PRD 91 033005, (2015)