

Probing 3N-SRCs Using Inclusive and Exclusive Reactions

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#motivations:

- High Density Fluctuations in Nuclei

Can such fluctuations be probed? - 2N, 3N SRCs

What they tell us about the nuclear forces at short distances

QCD/ Hadron transitions

Partons in the nuclear medium/EMC effects

Nature of the nuclear core

Nature of 3N forces

- Astrophysical Implications

Conceptually: How to probe nuclei at short nucleon separations

- Probe bound nucleons at **large** internal momenta
- Need high energy probes to resolve such nucleons **in** nuclei

#Theory of High Energy eA Scattering:

Emergence of High Energy Dynamics

- Short Range Nucleon Correlations(SRCs) are important feature of Nuclear Dynamics
- Transition from hadronic to quark-gluon degrees of freedom in cold-nuclei “should happen” through SRCs
- Internal momenta relevant to SRCs $p \sim M_N$ - Relativistic
- Such states are probed in high energy processes: high energy approximations can be applied in description of SRC dynamics

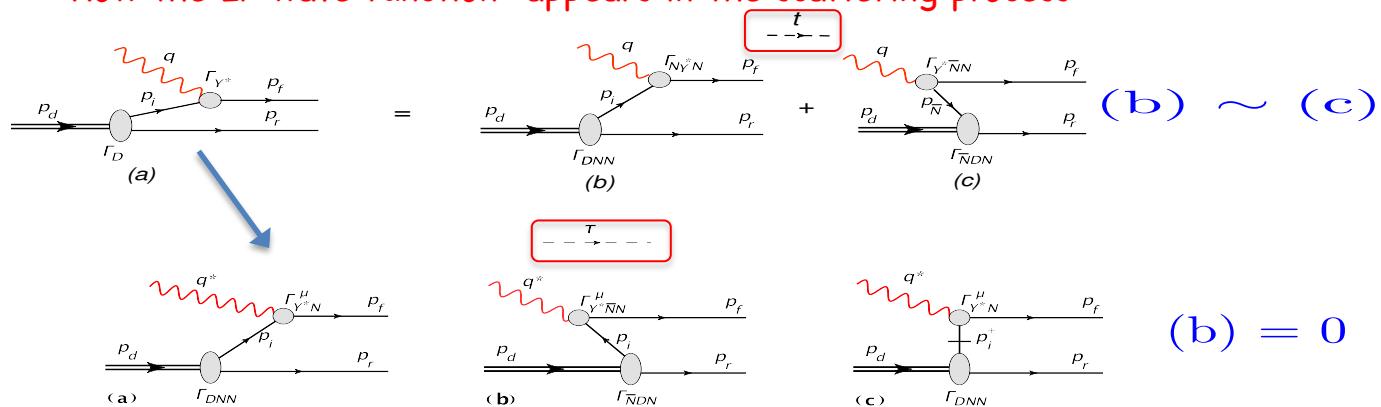
Light-Front wave function of the Nucleus

$$\Psi_{LF}(\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \dots, \mathcal{Z}_n, \tau)$$

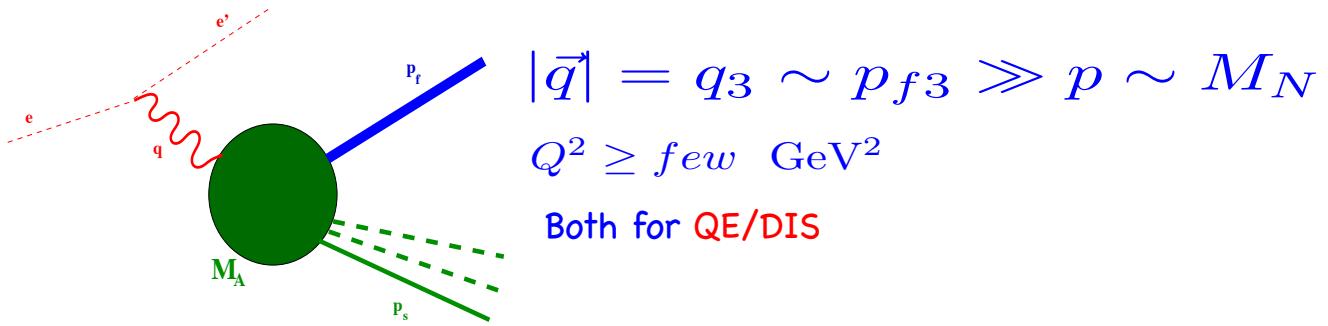
- in the momentum space

$$\Psi_{LF}(\alpha_1, p_{1\perp}; \alpha_2, p_{2\perp}; \alpha_3, p_{3\perp}; \dots, \alpha_n, p_{n\perp}) \quad \alpha_i = \frac{p_{i-}}{p_{A-}/\bar{A}}$$

- How the LF wave function appears in the scattering process



High Energy Approximations:

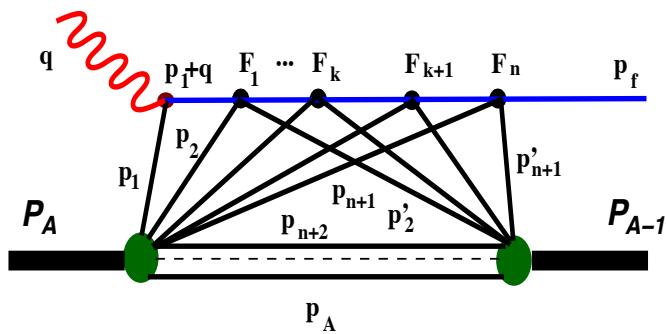
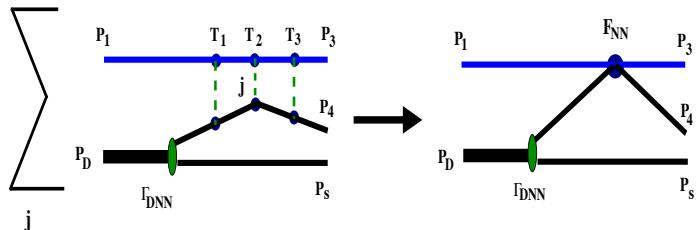
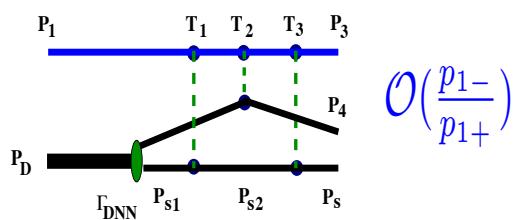


- Emergence of the **small parameter**

$$\frac{q_-}{q_+} = \frac{q_0 - q_3}{q_0 + q_3} \ll 1 \quad \mathcal{O}\left(\frac{q_-}{q_+}\right)$$

$$\frac{p_{f-}}{p_{f+}} = \frac{E_f - p_{f3}}{E_f + p_{f3}} \ll 1 \quad \mathcal{O}\left(\frac{p_{f-}}{p_{f+}}\right)$$

Emergence of “effective” theory

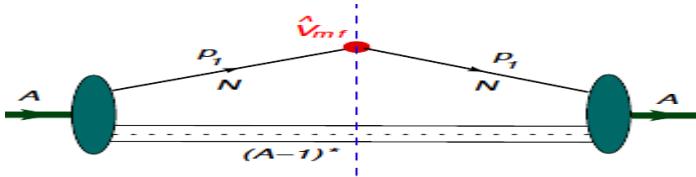


Effective Feynman
Diagrammatic Rules

M.S. IJMS 2001

Wave function?

Spectral Function Calculations



$$S_A^{MF} = -Im \int \chi_A^\dagger \Gamma_{A,N,A-1}^\dagger \frac{p_1 + m}{p_1^2 - m^2 + i\varepsilon} \hat{V}^{MF} \frac{p_1 + m}{p_1^2 - m^2 + i\times\varepsilon} \left[\frac{G_{A-1}(p_{A-1})}{p_{A-1}^2 - M_{A-1}^2 + i\varepsilon} \right]^{on} \Gamma_{A,N,A-1} \chi_A \frac{d^4 p_{A-1}}{i(2\pi)^4}$$

$$\hat{V}^{MF} = ia^\dagger(p_1, s_1) \delta^3(p_1 + p_{A-1}) \delta(E_m - E_\alpha) a(p_1, s_1)$$

$$\psi_{N/A}(p_1, s_1, s_A, E_\alpha) = \frac{\bar{u}(p_1, s_1) \Psi_{A-1}^\dagger(p_{A-1}, s_{A-1}, E_\alpha) \Gamma_{A,N,A-1} \chi_A}{(M_{A-1}^2 - p_{A-1}^2) \sqrt{(2\pi)^3 2E_{A-1}}}$$

$$S_A^{MF}(p_1, E_m) = \sum_{\alpha} \sum_{s_1, s_{A-1}} | \psi_{N/A}(p_1, s_1, s_A, E_\alpha) |^2 \delta(E_m - E_\alpha)$$

1. Short-Range Correlations

for large $k > k_{Fermi}$

$$n_A^{LC}(\alpha, p_t) \approx a_{NN}(A, z) n_{NN}^{LC}(\alpha, p_t)$$

Frankfurt, Strikman Phys.
Rep, 1988
Day, Frankfurt, Strikman,
MS, Phys. Rev. C 1993

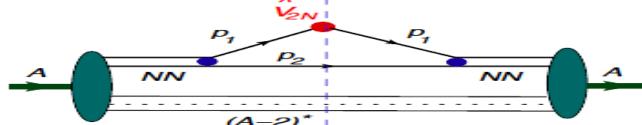
- Experimental observations

Egiyan et al, 2002, 2006
Fomin et al, 2011

$$n_A^{LC}(\alpha, p_t) \approx a_{pn}(A, z) n_d^{LC}(\alpha, p_t)$$

E. Piasetzky, MS, L. Frankfurt,
M. Strikman, J. Watson PRL, 2006

2N SRC model



$$\begin{aligned}
 P_{A,2N}^N(\alpha_1, p_{1,\perp}, \tilde{M}_N^2) = & \sum_{s_2, s_{NN}, s_{A-2}} \int \chi_A^\dagger \Gamma_{A \rightarrow NN, A-2}^\dagger \chi_{A-2}(p_{A-2}, s_{A-2}) \\
 & \times \frac{\chi_{NN}(p_{NN}, s_{NN}) \chi_{NN}^\dagger(p_{NN}, s_{NN})}{p_{NN}^2 - M_{NN}^2} \Gamma_{NN \rightarrow NN}^\dagger \frac{u(p_1, s_1) u(p_2, s_2)}{p_1^2 - M_N^2} \\
 & \times \left[2\alpha_1^2 \delta(\alpha_1 + \alpha_2 + \alpha_{A-2} - A) \delta^2(p_{1,\perp} + p_{2,\perp} + p_{A-2,\perp}) \delta(\tilde{M}_N^2 - \tilde{M}_N^{(2N),2}) \right] \frac{\bar{u}(p_1, s_1) \bar{u}(p_2, s_2)}{p_1^2 - M_N^2} \\
 & \times \Gamma_{NN \rightarrow NN} \frac{\chi_{NN}(p_{NN}, s_{NN}) \chi_{NN}^\dagger(p_{NN}, s_{NN})}{p_{NN}^2 - M_{NN}^2} \chi_{A-2}^\dagger(p_{A-2}, s_{A-2}) \Gamma_{A,NN,A-2} \chi_A \\
 & \times \frac{d\alpha_2}{\alpha_2} \frac{d^2 p_{2,\perp}}{2(2\pi)^3} \frac{d\alpha_{A-2}}{\alpha_{A-2}} \frac{d^2 p_{A-2,\perp}}{2(2\pi)^3}.
 \end{aligned} \tag{1}$$

$$\rho_A(\alpha_N, p_{N,\perp}) = \int P_A(\alpha_N, p_{N,\perp}, \tilde{M}_N^2) \frac{1}{2} d\tilde{M}_N^2$$

O. Artiles & M.S. Phys. Rev. C

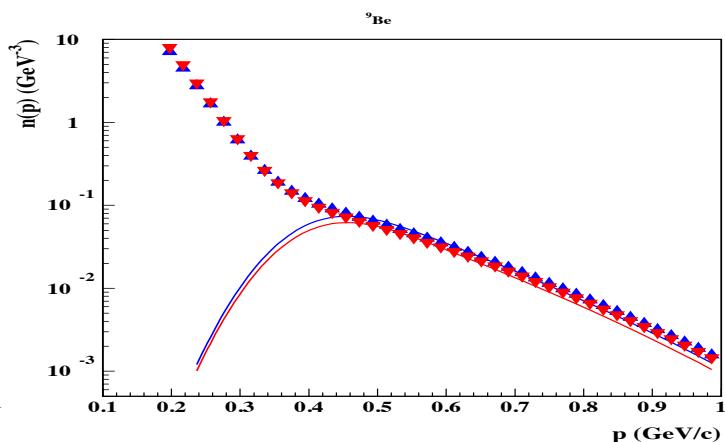
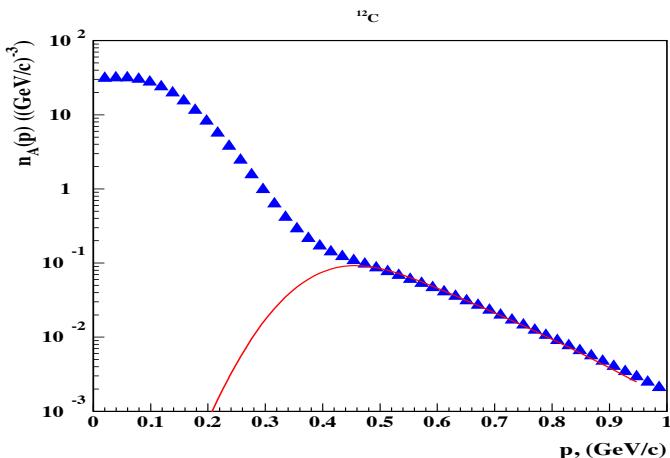
2016

$$\psi_{pn}^{LF}(\alpha, p_\perp) \approx C \psi_d^{LF}(\alpha, p_\perp)$$

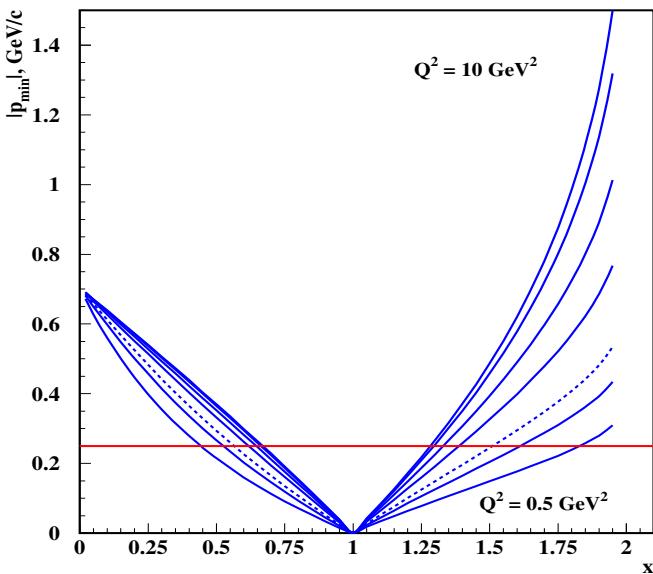
$$\psi_{2N}^{s_{NN}}(\beta_1, k_{1,\perp}, s_1, s_2) = -\frac{1}{\sqrt{2(2\pi)^3}} \frac{\bar{u}(p_1, s_1) \bar{u}(p_2, s_2) \Gamma_{NN \rightarrow NN} \cdot \chi_{NN}(p_{NN}, s_{NN})}{\frac{1}{2}[M_{NN}^2 - 4(M_N^2 + k_1^2)]}$$

$$\psi_{CM}(\alpha_{NN}, k_{NN,\perp}) = -\frac{1}{\sqrt{\frac{A-2}{2}}} \frac{1}{\sqrt{2(2\pi)^3}} \frac{\chi_{NN}^\dagger(p_{NN}, s_{NN}) \chi_{A-2}^\dagger(p_{A-2}, s_{A-2}) \Gamma_{A \rightarrow NN, A-2} \chi_A^{s_A}}{\frac{2}{A}[M_A^2 - s_{NN, A-2}(k_{CM})]}$$

2N SRC model Non Relativistic Approximation

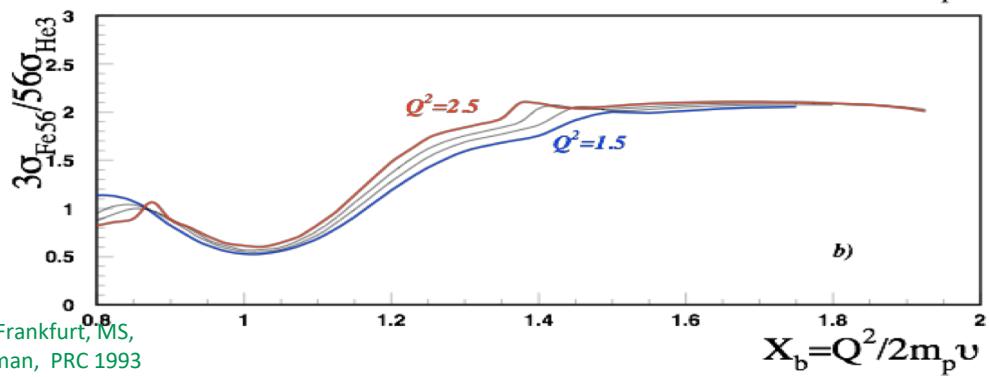
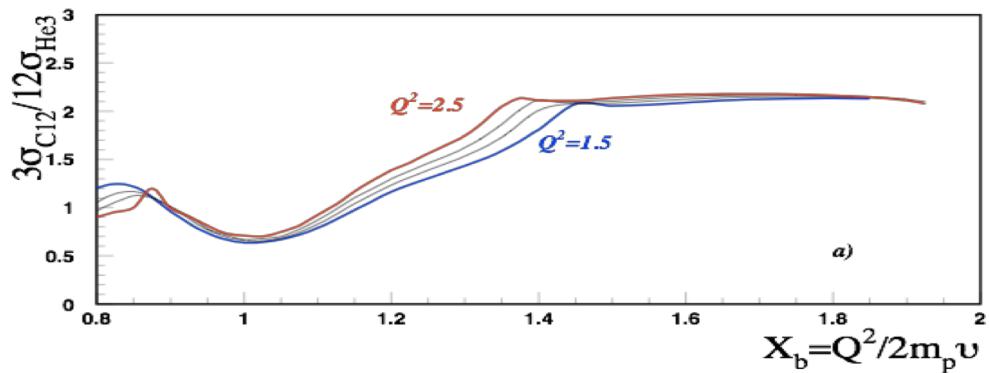


Nuclear Scaling in QE Inclusive $A(e,e')X$ reaction at $x>1$ region



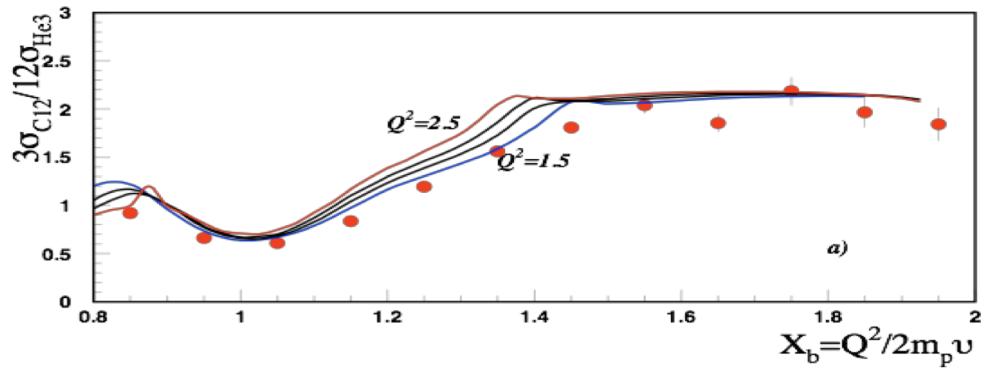
Day, Frankfurt, MS,
Strikman, PRC 1993

$A(e, e')$

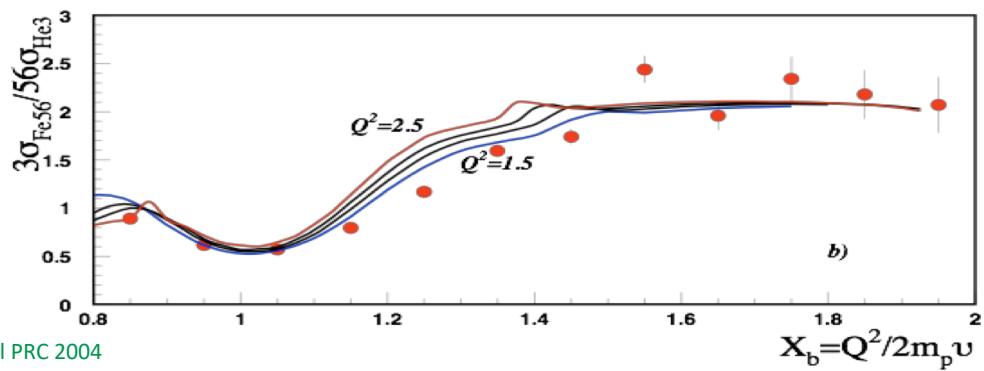


Day, Frankfurt, MS,
Strikman, PRC 1993

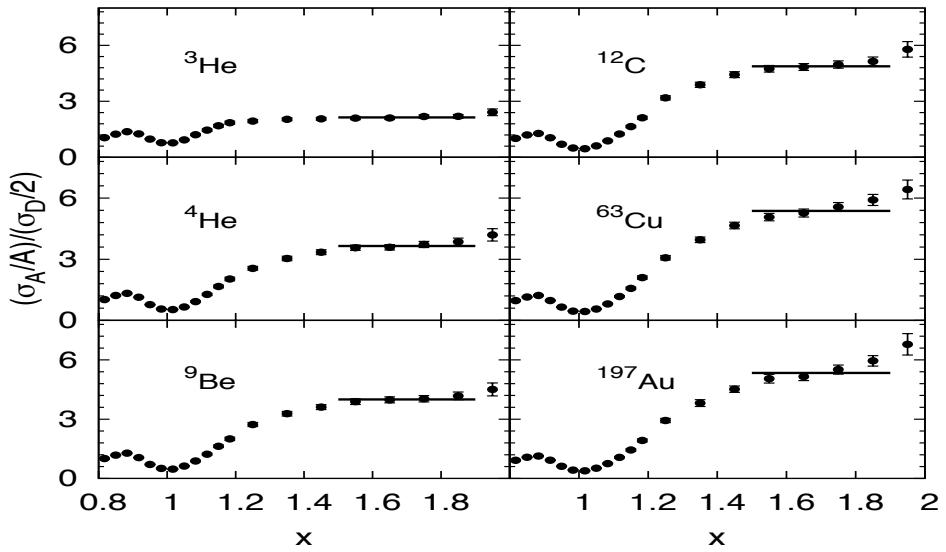
$$A(e,e')$$



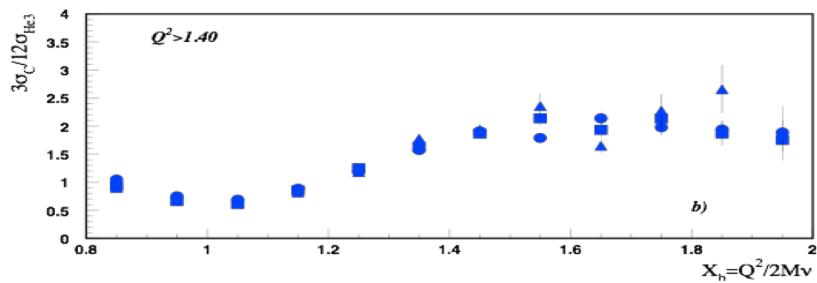
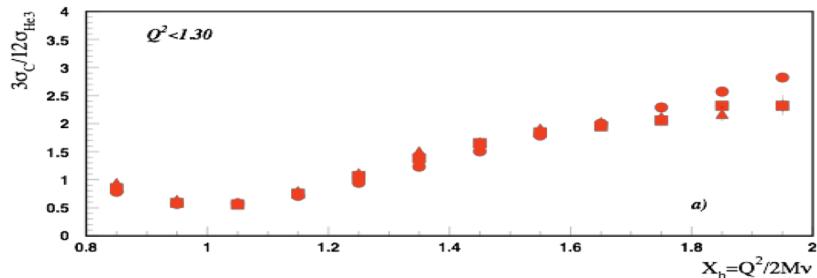
a)



b)



$A(e, e')$



Meaning of the scaling values

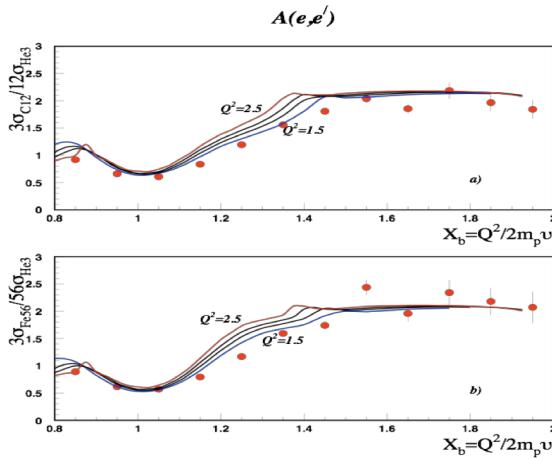
Day, Frankfurt, MS,
Strikman, PRC 1993

Frankfurt, MS, Strikman,
IJMP A 2008

Fomin et al PRL 2011

$$R = \frac{A_2 \sigma [A_1(e, e') X]}{A_1 \sigma [A_2(e, e') X]}$$

For $1 < x < 2$ $R \approx \frac{a_2(A_1)}{a_2(A_2)}$



Egiyan, et al PRL 2006, PRC 2004

a2's as relative probability of 2N SRCs

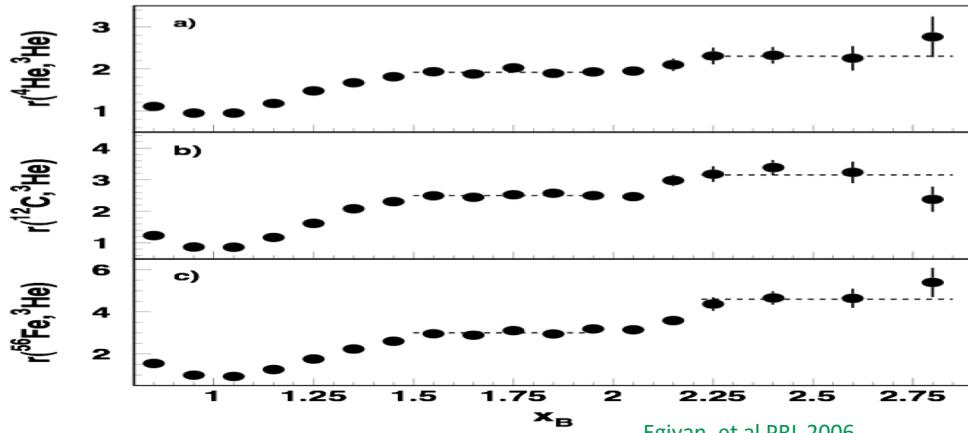
Table 1: The results for $a_2(A, y)$

A	y	This Work	Frankfurt et al	Egiyan et al	Famin et al
^3He	0.33	2.07 ± 0.08	1.7 ± 0.3		2.13 ± 0.04
^4He	0	3.51 ± 0.03	3.3 ± 0.5	3.38 ± 0.2	3.60 ± 0.10
^9Be	0.11	3.92 ± 0.03			3.91 ± 0.12
^{12}C	0	4.19 ± 0.02	5.0 ± 0.5	4.32 ± 0.4	4.75 ± 0.16
^{27}Al	0.037	4.50 ± 0.12	5.3 ± 0.6		
^{56}Fe	0.071	4.95 ± 0.07	5.6 ± 0.9	4.99 ± 0.5	
^{64}Cu	0.094	5.02 ± 0.04			5.21 ± 0.20
^{197}Au	0.198	4.56 ± 0.03	4.8 ± 0.7		5.16 ± 0.22

Towards Three Nucleon Short Range Correlations

Looking for the Plateau in Inclusive Cross Section Ratios x>2

For $1 < x < 2$ $R \approx \frac{a_2(A_1)}{a_2(A_2)}$ For $2 < x < 3$ $R \approx \frac{a_3(A_1)}{a_3(A_2)}$

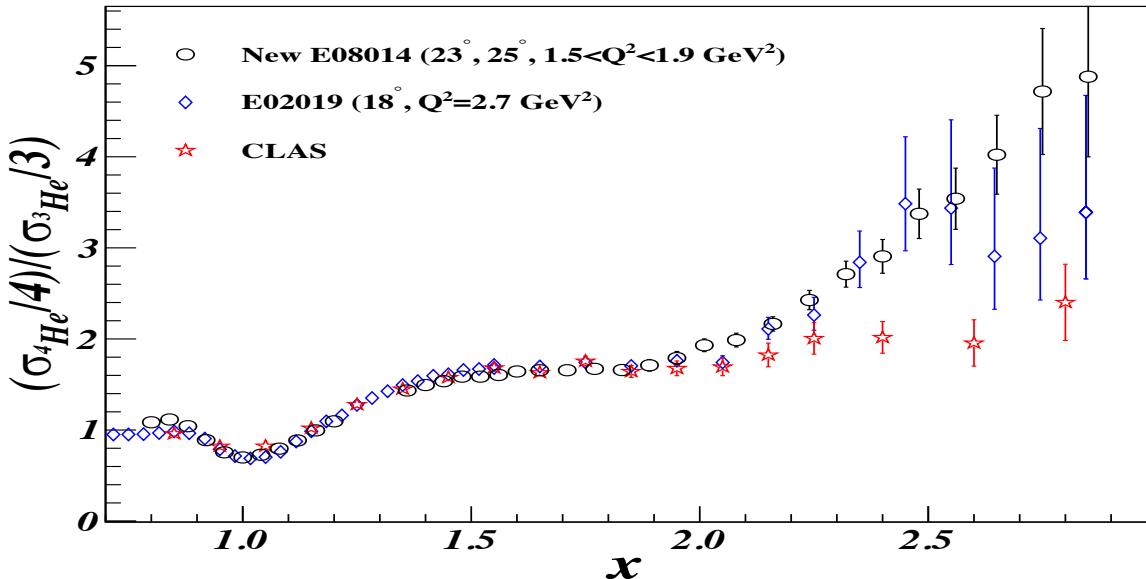


Egiyan, et al PRL 2006

3N SRCs

Z. Ye, et al, 2017

Looking for the Plateau in Inclusive Cross Section Ratios $\times 2$



Egiyan, et al PRL 2006

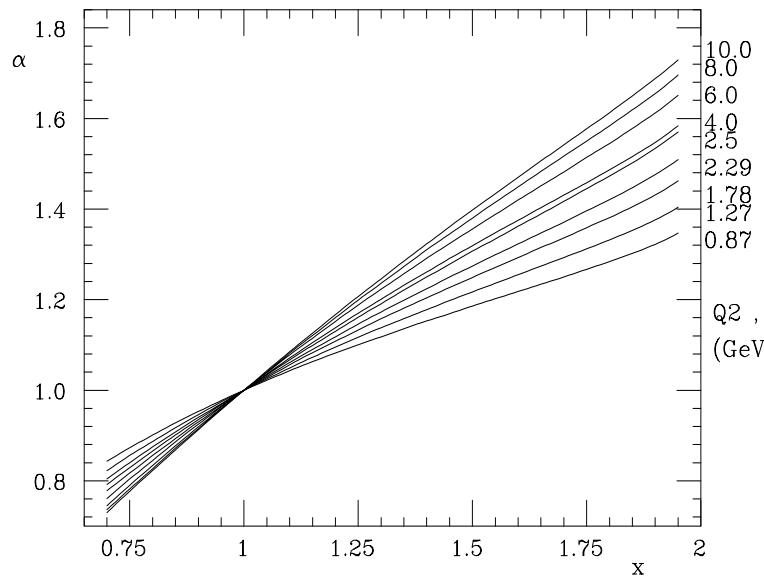
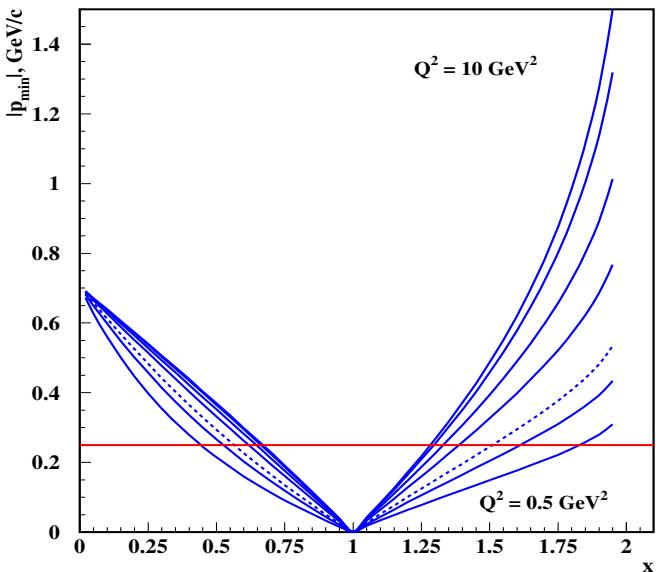
Egiyan, et al PRC 2004

2N SRCs:

Proper Variables of 2N SRC are

- the Light Front Momentum Fraction: $\alpha = \frac{p_N^+}{p_{NN}^+}$
- transverse momentum: p_\perp

Nuclear Scaling in QE Inclusive $A(e,e')X$ reaction at $x>1$ region



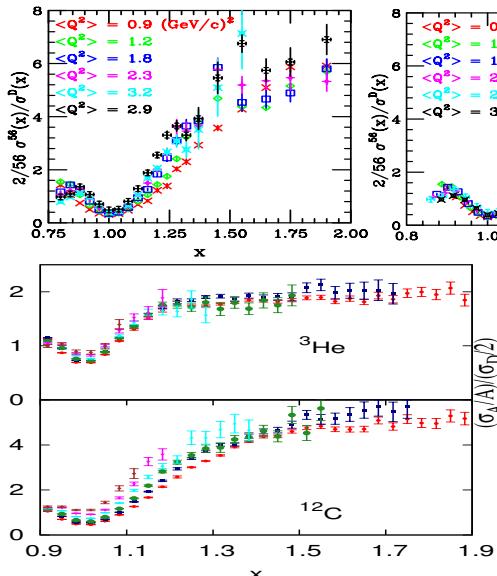
Day, Frankfurt, MS,
Strikman, PRC 1993

Back to inclusive $A(e,e')X$ scattering

$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$

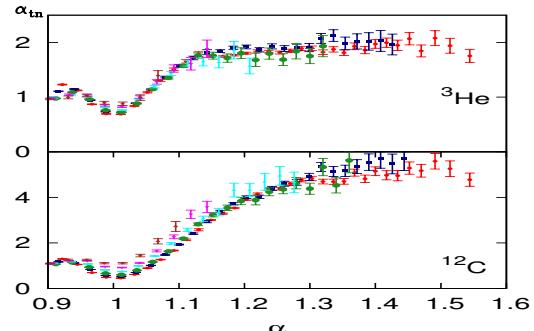
$$1.3 \leq \alpha_{2N} \leq 1.5$$

$$\alpha_{2N} = 2 - \frac{q_- + 2m_N}{2m_N} \left(1 + \frac{\sqrt{W_{2N}^2 - 4m_N^2}}{W_{2N}} \right)$$



J.Arrington, D.Higinbotham
G.Rosner, M.S. Prog. PNP 2012

$$\begin{array}{c|c} \alpha & Q^2 \rightarrow \infty \rightarrow x \\ \alpha & x \rightarrow 1 \rightarrow 1 \end{array}$$

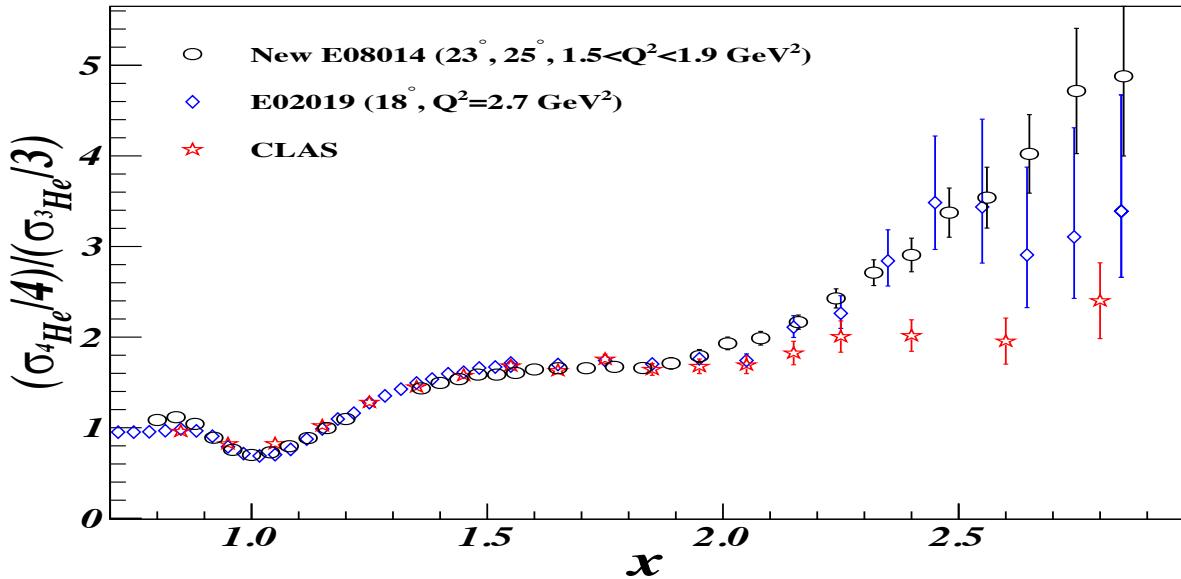


N.Fomin, D.Higinbotham
M.S., P.Sovignon ARNPS, 2017

Three Nucleon Short Range Correlations

Z. Ye, et al, 2017

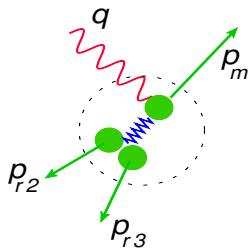
Looking for the Plateau in Inclusive Cross Section Ratios $x > 2$



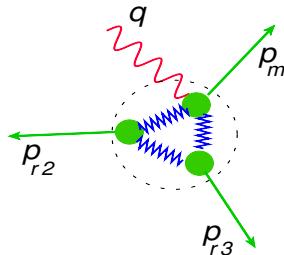
3N SRCs:

Proper Variables of 3N SRC are

- the Light Front Momentum Fraction: $\alpha = \frac{p_N^+}{p_{3N}^+}$
- transverse momentum: p_\perp

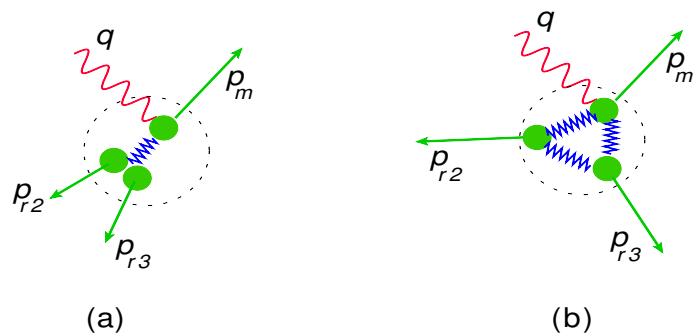
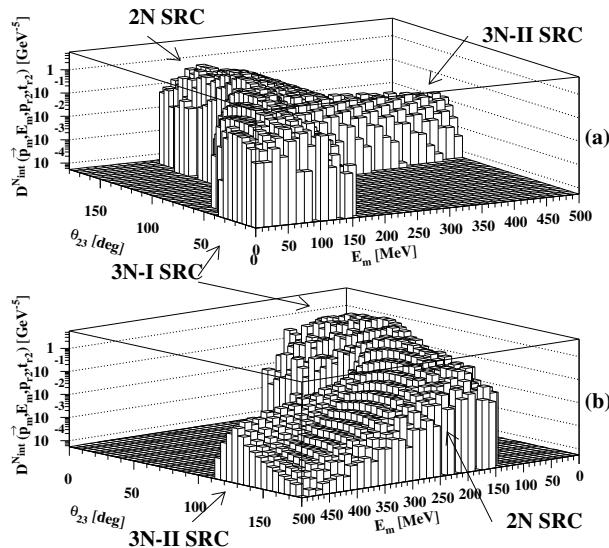


(a)



(b)

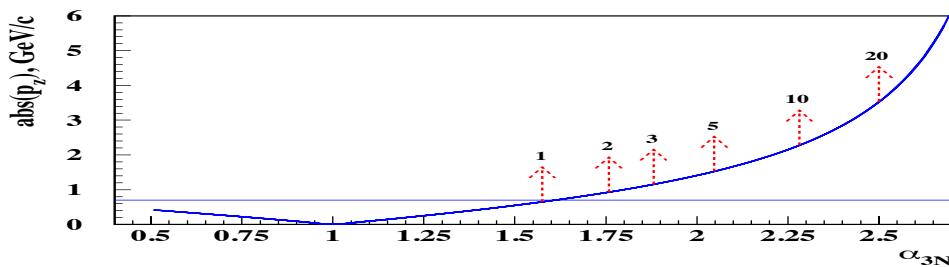
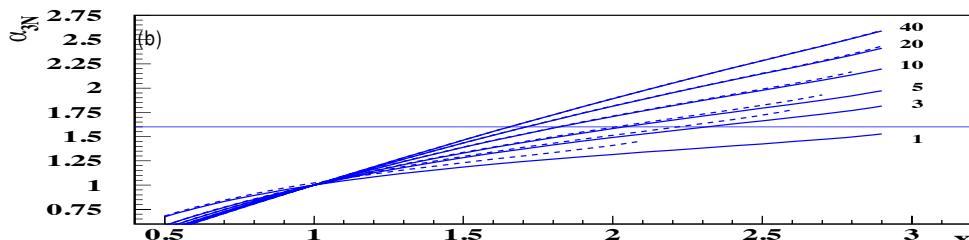
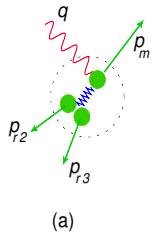
3N SRCs:



3N SRC model $\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha)$ where $\rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$

$$1.6 \leq \alpha_{3N} < 3$$

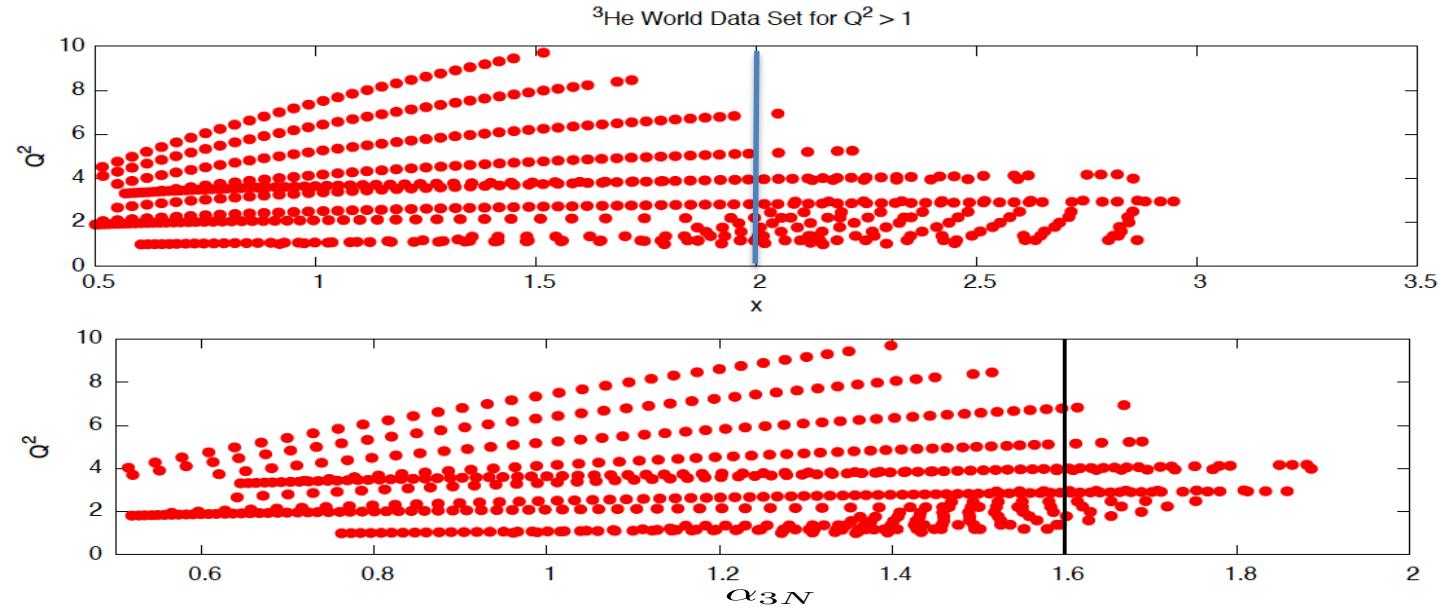
$$q + 3m_N = p_f + p_s$$



3N SRC model $\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha)$ where $\rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$

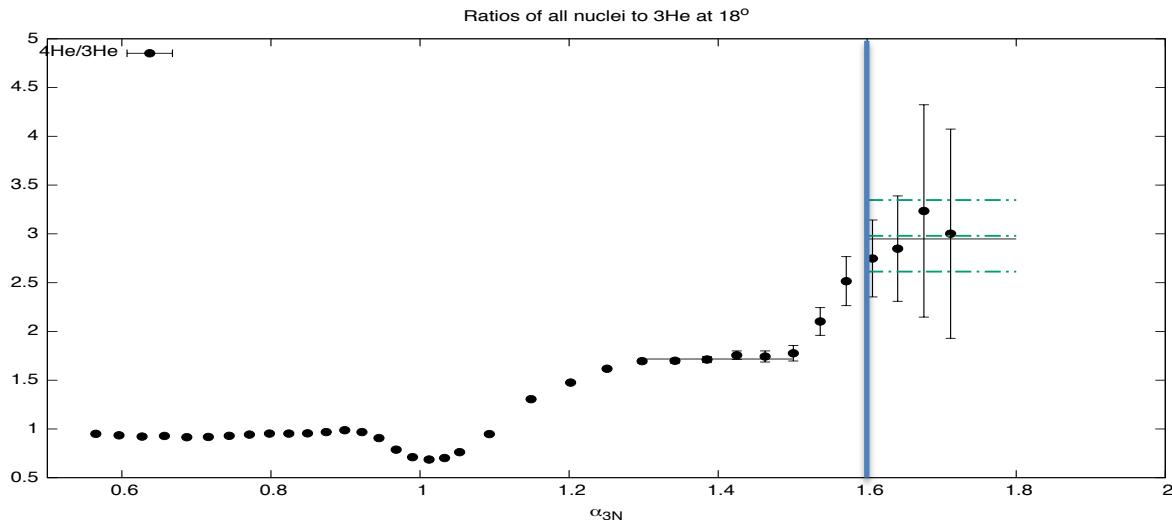
$1.6 \leq \alpha_{3N} < 3$

Donal Day, M.S., Frankfurt, Strikman 2018



3N SRCs

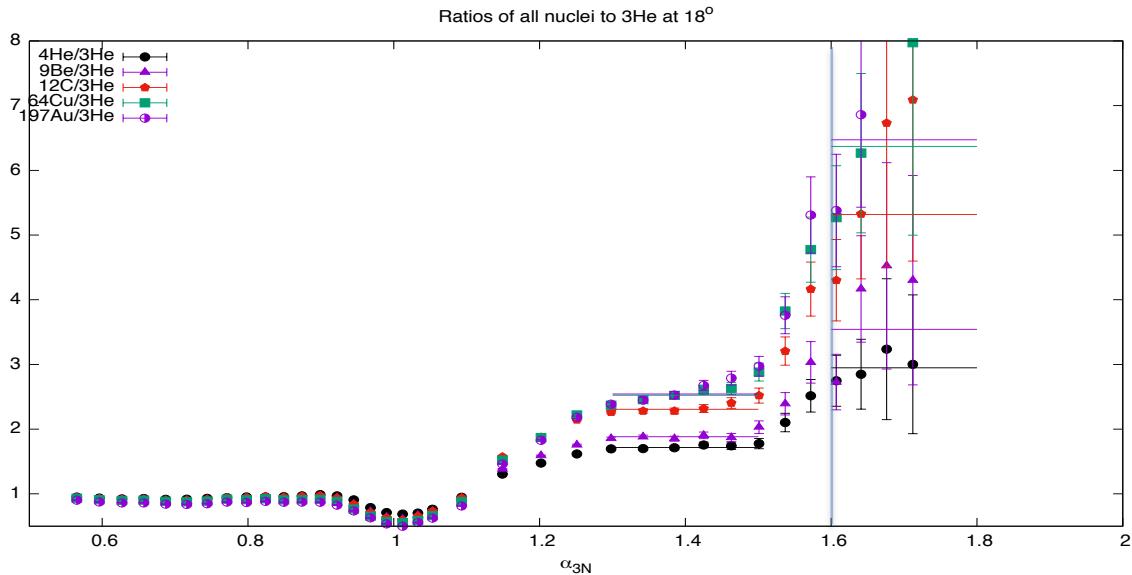
$$\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha) \quad \text{where} \quad \rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$$
$$1.6 \leq \alpha_{3N} < 3$$



JLab - E02019 - Data

3N SRC model $\sigma_{eA} = \sum \sigma_{eN} \cdot \rho_A(\alpha)$ where $\rho_A(\alpha) = \int \rho_A(\alpha, p_\perp) d^2 p_\perp$

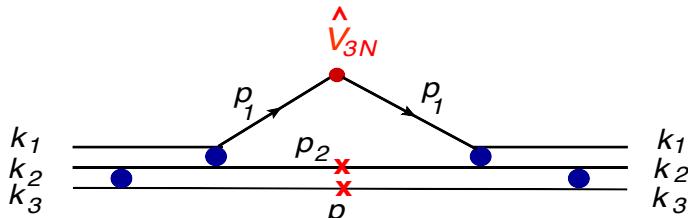
$$1.6 \leq \alpha_{3N} < 3$$



JLab - E02019 - Data

3N SRC: Light-Cone Momentum Fraction Distribution

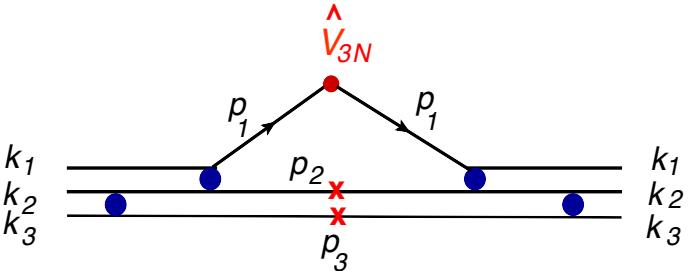
A.Freese, M.S., M.Strikman, Eur. Phys. J 2015
 O. Artiles M.S. Phys. Rev. C 2016



$$\begin{aligned}
 P_{A,3N}^N(\alpha_1, p_{1\perp}, s_1, \tilde{M}_N) = & \sum_{s_2, s_3, s_{2'}, \bar{s}_{2'}} \int \bar{u}(k_1)\bar{u}(k_2)\bar{u}(k_3)\Gamma_{NN \rightarrow NN}^\dagger \frac{u(p_{2'}, \bar{s}_{2'})\bar{u}(p_{2'}, \bar{s}_{2'})}{p_{2'}^2 - M_N^2} \\
 & \times \Gamma_{NN \rightarrow NN}^\dagger \frac{u(p_1, s_1)}{p_1^2 - M_N^2} u(p_2, s_2) \left[2\alpha_1^2 \delta(\alpha_1 + \alpha_2 + \alpha_3 - 3) \delta^2(p_{1\perp} + p_{2\perp} + p_{3\perp}) \delta(\tilde{M}_N^2 - M_N^{3N,2}) \right] \\
 & \times \bar{u}(p_2, s_2) \frac{\bar{u}(p_1, s_1)}{p_1^2 - M_N^2} \Gamma_{NN \rightarrow NN} \frac{u(p_{2'}, s_{2'})\bar{u}(p_{2'}, s_{2'})}{p_{2'}^2 - M_N^2} u(p_3, s_3) \bar{u}(p_3, s_3) \Gamma_{NN \rightarrow NN}^\dagger u(k_1) u(k_2) u(k_3) \\
 & \times \frac{d\alpha_2}{\alpha_2} \frac{d^2 p_{2\perp}}{2(2\pi)^3} \frac{d\alpha_3}{\alpha_3} \frac{d^2 p_{3\perp}}{2(2\pi)^3}, \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 P_{A,3N}^N(\alpha_1, p_{1\perp}, \tilde{M}_N) = & \int \frac{3 - \alpha_3}{2(2 - \alpha_3)^2} \rho_{NN}(\beta_3, p_{3\perp}) \rho_{NN}(\beta_1, \tilde{k}_{1\perp}) 2\delta(\alpha_1 + \alpha_2 + \alpha_3 - 3) \\
 & \delta^2(p_{1\perp} + p_{2\perp} + p_{3\perp}) \delta(\tilde{M}_N^2 - M_N^{3N,2}) d\alpha_2 d^2 p_{2\perp} d\alpha_3 d^2 p_{3\perp}, \tag{1}
 \end{aligned}$$

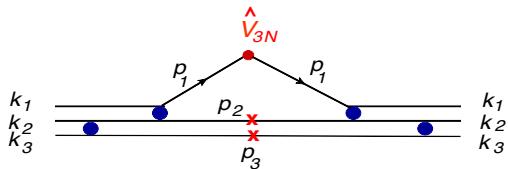
3N SRC: Light-Cone Momentum Fraction Distribution



$$\begin{aligned}
 \rho_{3N}(\alpha_1) = & \int \frac{1}{4} \left[\frac{3 - \alpha_3}{(2 - \alpha_3)^3} \boxed{\rho_{pn}(\alpha_3, p_{3\perp}) \rho_{pn}\left(\frac{2\alpha_2}{3 - \alpha_3}, p_{2\perp} + \frac{\alpha_1}{3 - \alpha_3} p_{3\perp}\right)} + \right. \\
 & \left. \frac{3 - \alpha_2}{(2 - \alpha_2)^3} \boxed{\rho_{pn}(\alpha_2, p_{2\perp}) \rho_{pn}\left(\frac{2\alpha_3}{3 - \alpha_2}, p_{3\perp} + \frac{\alpha_1}{3 - \alpha_2} p_{2\perp}\right)} \right] \delta\left(\sum_{i=1}^3 \alpha_i - 3\right) \\
 & d\alpha_2 d^2 p_{2\perp} d\alpha_3 d^2 p_{3\perp}, \tag{1}
 \end{aligned}$$

$$\rho_{pn}(\alpha, p_\perp) \approx a_2(A) \rho_d(\alpha, p_\perp)$$

3N SRC: Light-Cone Momentum Fraction Distribution



$$-\rho_{3N} \sim a_2(A, z)^2$$

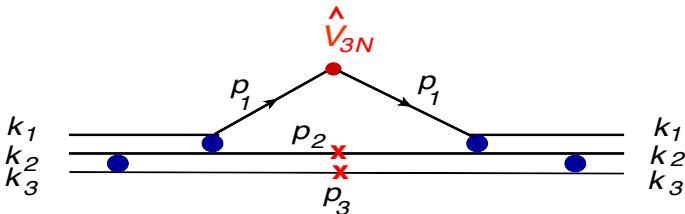
- For $A(e, e') X$ reactions: $\sigma_{eA} = \sum_N \sigma_{eN} \rho_{3N}(\alpha_{3N})$

- Defining: $R_3(A, Z) = \frac{3\sigma_{eA}}{A\sigma_{e^3He}} \mid_{\alpha_{3N} \geq \alpha_{3N}^0}$

- We predict: $R_3(A, Z) = \frac{9}{8} \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3} \left(\frac{a_2(A, Z)}{a_2(^3He)} \right)^2 = \frac{9}{8} \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3} R_2^2(A, Z),$

- Where: $R_2(A, Z) = \frac{3\sigma_{eA}}{A\sigma_{e^3He}} \mid_{1.3 \leq \alpha_{3N} \leq 1.5}$ where: $\alpha_{3N} \approx \alpha_{2N}$

3N SRC: Light-Cone Momentum Fraction Distribution



$$-\rho_{3N} \sim a_2(A, z)^2$$

- ppp and nnn strongly suppressed compared with ppn or pnn
- pp/nn recoil state is suppressed compared with pn

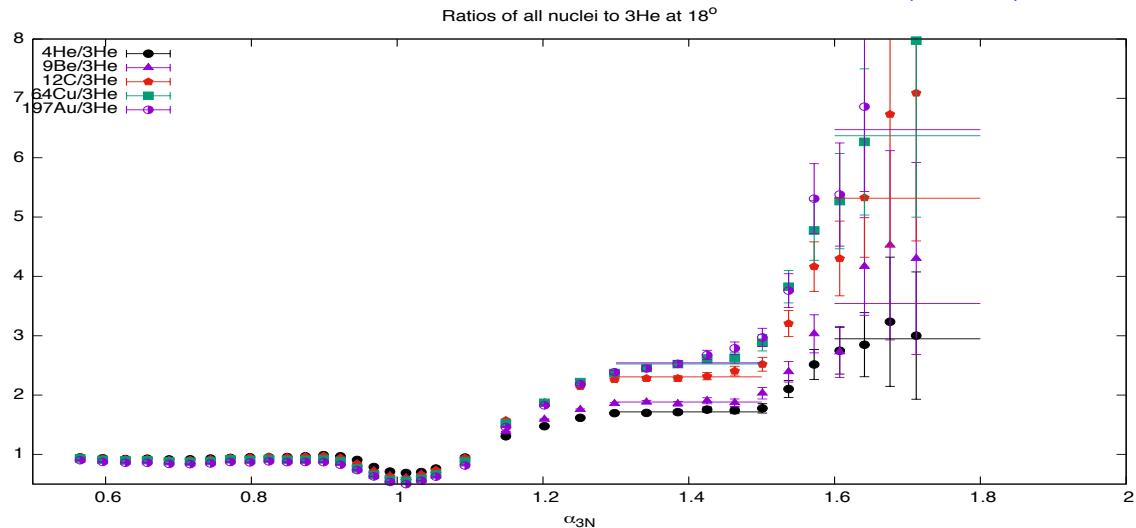
$$\begin{aligned} R_3(A, Z) &= \frac{9}{8} \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3} \left(\frac{a_2(A, Z)}{a_2({}^3He)} \right)^2 = \\ &\frac{9}{8} \frac{(\sigma_{ep} + \sigma_{en})/2}{(2\sigma_{ep} + \sigma_{en})/3} R_2^2(A, Z), \end{aligned}$$

3N SRC model

$$R_2 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.3 \leq \alpha_{3N} \leq 1.5 \quad 1.6 \leq \alpha_{3N} < 3$$

$$R_3 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.6 \leq \alpha_{3N} \leq 1.8$$

$$R_3(A, Z) \approx R_2(A, Z)^2$$

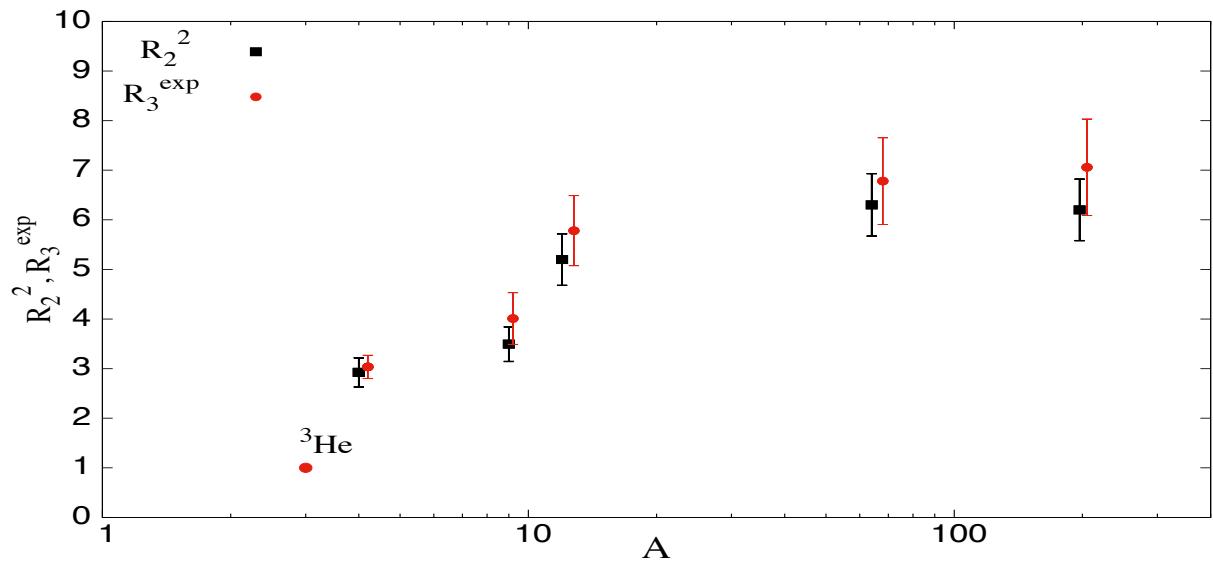


3N SRC model

$$R_2 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.3 \leq \alpha_{3N} \leq 1.5 \quad 1.6 \leq \alpha_{3N} < 3$$

$$R_3 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.6 \leq \alpha_{3N} \leq 1.8 \quad R_3(A) = R_2(A)^2$$

D.Day, L.Frankfurt,M.S, M.Strikman
ArXiv 2018



3N SRC model

Defining: $a_3(A, Z) = \frac{3}{A} \frac{\sigma_{eA}}{(\sigma_{e^3He} + \sigma_{e^3H})/2}$

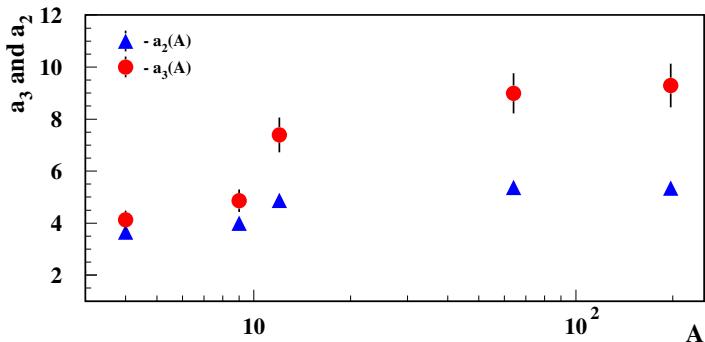
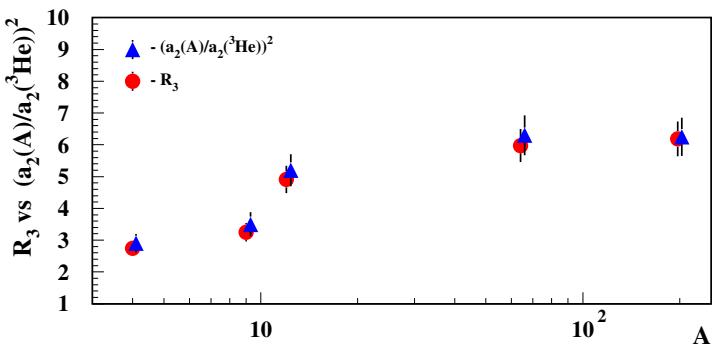
One relates: $a_3(A, Z) = \frac{(2\sigma_{ep} + \sigma_{en})/3}{(\sigma_{ep} + \sigma_{en})/2} R_3(A, Z)$

A	a_2	R_2	R_2^{exp}	R_2^2	R_3^{exp}	a_3
3	2.14 ± 0.04	NA	NA	NA	NA	1
4	3.66 ± 0.07	1.71 ± 0.026	1.722 ± 0.013	2.924 ± 0.29	3.034 ± 0.23	4.55 ± 0.35
9	4.00 ± 0.08	1.84 ± 0.027	1.878 ± 0.018	3.38 ± 0.38	4.01 ± 0.52	6.0 ± 0.78
12	4.88 ± 0.10	2.28 ± 0.027	2.301 ± 0.021	5.2 ± 0.5	5.78 ± 0.71	8.7 ± 1.1
27	5.30 ± 0.60	NA	NA	NA	NA	NA
56	4.75 ± 0.29	NA	NA	NA	NA	NA
64	5.37 ± 0.11	2.51 ± 0.027	2.502 ± 0.024	6.3 ± 0.63	6.780 ± 0.875	10.2 ± 1.3
197	5.34 ± 0.11	2.46 ± 0.028	2.532 ± 0.026	6.05 ± 0.6	7.059 ± 0.970	10.6 ± 1.5

3N SRC model

$$R_2 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.3 \leq \alpha_{3N} \leq 1.5 \quad 1.6 \leq \alpha_{3N} < 3$$

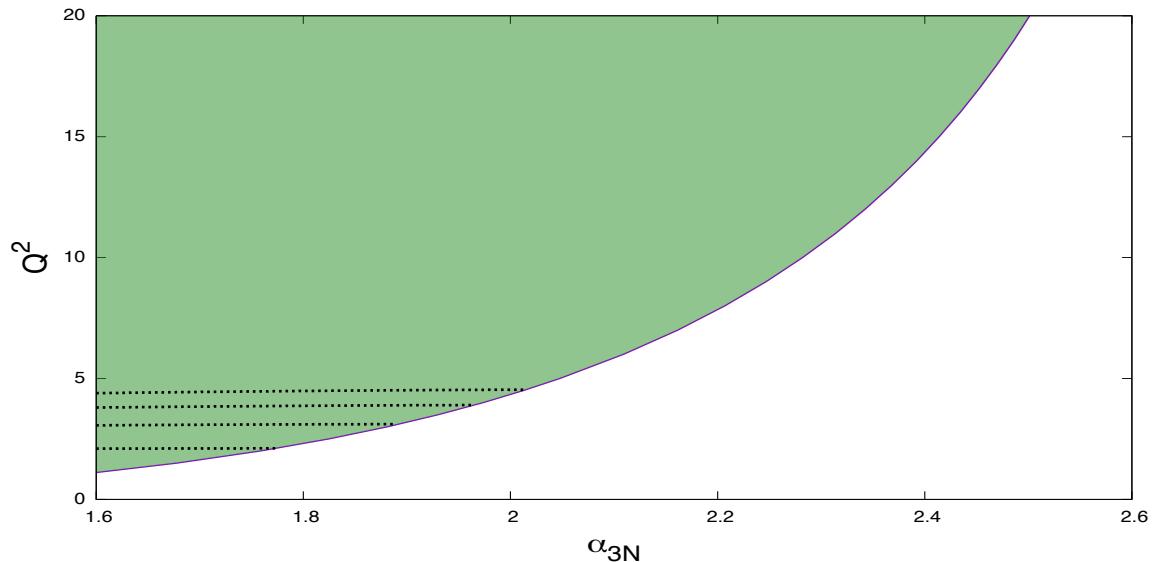
$$R_3 = \frac{3\sigma_{eA}(\alpha_{3N})}{A\sigma_{3A}(\alpha_{3N})} \quad 1.6 \leq \alpha_{3N} \leq 1.8 \quad R_3(A) = R_2(A)^2$$



3N SRC Summary & Outlook

- Proper variable for studies of 2N and 3N SRS are Light-Cone momentum fractions: α_{2N} , α_{3N}
- It seems we observed first signatures of 3N SRCs in the form of the “scaling”
- Existing data in agreement with the prediction of: $R_3(A, Z) \approx R_2(A, Z)^2$
- Unambiguous verification will require larger Q2 data to cover larger α_{3N} region
- Reaching $Q^2 > 5 \text{ GeV}^2$ will allow to reach: $\alpha_{3N} > 2$

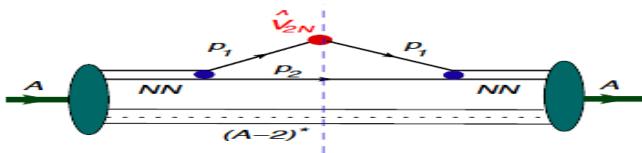
3N SRC Outlook



3N SRCs in Exclusive Processes: $A(e,e'pp)X$

Genuine pp-2NSRC vs pnp 3NSRC

Considering $A(e, e' pp)X$ process: looking for apparent pp short range correlations

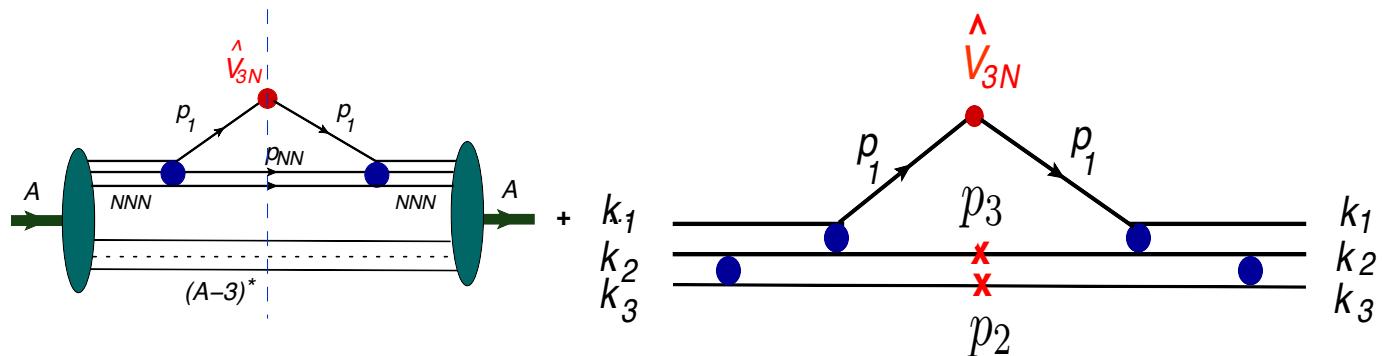


$$D_{A,2N}(\alpha_1, p_{1\perp}, \alpha_2, p_{2\perp}) = \rho_{pp}(\beta_1, k_{1\perp}) \rho_{CM}(\alpha_{cm}, p_{cm,\perp}) \frac{\alpha_2}{\alpha_{cm}}$$

$$\beta_1 = \frac{2\alpha_1}{\alpha_{cm}} \quad \beta_2 = \frac{2\alpha_2}{\alpha_{cm}}$$

$$k_{1\perp} = p_{1\perp} - \frac{\beta_1}{2} p_{cm,\perp}$$

Considering $A(e,e'pp)X$ process: looking for apparent pp short range correlations



$$D_{A,3N}(\alpha_1, p_{1\perp}, \alpha_2, p_{2\perp}) = \frac{3-\alpha_2}{2(2-\alpha_3)^2} \rho_{pn}(\beta'_3, k'_{3\perp}) \rho_{pn}(\beta_1, k_{1\perp}) \alpha_3$$

$$\alpha_2 = 3 - \alpha_1 - \alpha_3$$

$$\beta_1 = \frac{2\alpha_1}{3-\alpha_2}$$

$$k_{1\perp} = p_{1\perp} + \frac{\beta_1}{2} p_{2\perp}$$

$$\beta'_3 = 2 - \beta_2$$

$$\beta_2 \approx \alpha_2$$

$$k'_3 \approx -p_{2\perp}$$

$$k_1 \\ k_2 \\ k_3$$

Decay Function

