Effective Interaction from Correlated Wave Functions: Coordinate-Space Renormalisation?

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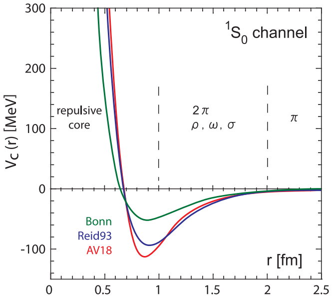
Perturbation Theory with Strongly Repulsive Forces

- ⋆ A prominent feature of the nucleon-nucleon potential is the presence of a strong repulsive core

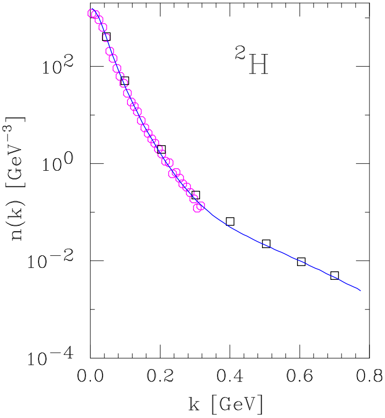
- ⋆ phenomenological and boson-exchange NN potentials in the $^1S_0$ channel
- ⋆ depending on the cutoff, χEFT interactions also feature short-range repulsion

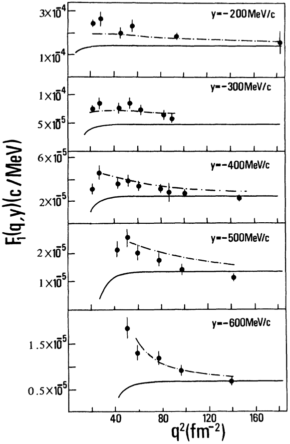


- ⋆ Perturbative calculations of nuclear matter properties can only be performed using softer *effective* interactions, obtained from *renormalisation* of the bare potential

Does it Matter?

- ⋆ Deuteron Momentum Distribution



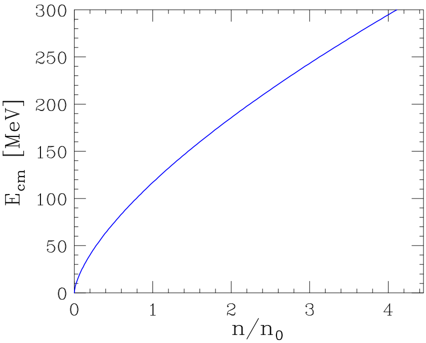


- ⋆ AV18 momentum distribution. Arenhövel analysis of exclusive Saclay data + y-analysis of inclusive SLAC data performed by Ciofi, Pace & Salmè

- ⋆ in strongly degenerate systems, such as neutron star matter, the center-of-mass energy of nucleon-nucleon collisions, $E_{\rm cm}$, is simply related to the particle density, n

- ⋆ Head-on collisions in pure neutron matter at density n

$$E_{\rm cm} = \frac{1}{m}(3\pi^2 n)^{2/3}$$



- ⋆ Potential models used to predict the properties of dense nuclear mater must be capable to describe nucleon-nucleon collisions at energies well beyond pion production threshold

Introducing the Effective Interaction

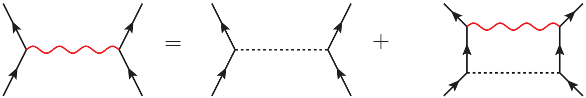
- ⋆ Consider nuclear matter. The eigenstates of H_0 are Fermi gas states $\{|n_{FG}\rangle\}$
- ⋆ Taming the matrix element of the Hamiltonian

$$\langle m_{FG}|H|n_{FG}\rangle \Rightarrow \begin{cases} \langle m_{FG}|H_{\rm eff}|n_{FG}\rangle & (H \Rightarrow H_{\rm eff}) \\ \langle m|H|n\rangle & (\{|n_{FG}\rangle\} \Rightarrow \{|n\rangle\}) \end{cases}$$

  - ▷ Use the *effective* Hamiltonian $H_{\rm eff}$ in standard perturbation theory with Fermi gas basis states, as in the G-matrix approach
  - ▷ Use the *bare* Hamiltonian to do perturbative calculations in the new basis, as in the approach based on Correlated Basis Functions (CBF)
- ⋆ The effective interaction must be designed in such a way as to provide accurate estimates of nuclear matter properties at lowest order of standard perturbation theory

Renormalisation of the Nucleon-Nucleon Potential

- ⋆ In the early days of nuclear matter theory, renormalisation was based on the replacement of the bare interaction, v. with the G-matrix describing nucleon-nucleon scattering in the nuclear medium



$$G = v + v \frac{Q}{H_0 - W} G$$

- ⋆ The G-matrix approach has been extensively employed in conjunction with phenomenological potentials
- ⋆ More recently, soft nucleon-nucleon interactions have been obtained from renormalisation group evolution of potentials derived within χEFT

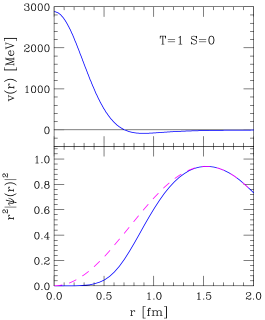
Screening of the Repulsive Core

- ⋆ Renormalisation group evolution essentially amounts to screening the repulsive core of the potential through the action of a momentum-space cutoff, Λ, in momentum space
- ⋆ Screening can also be implemented in *coordinate space*, through a transformation of the basis of eigenstates of the non interacting system

- ⋆ Transformation of the two-nucleons wave function in nuclear matter

$$\phi_{ij}(r) \to \psi_{ij}(r) = f_{ij}(r)\phi_{ij}(r)$$

$$\lim_{r\to\infty} f_{ij}(r) = 1$$



- ⋆ Loosely speaking, the role of the momentum cutoff Λ is played by the correlation range

THE CBF EFFECTIVE INTERACTION

- The Correlated Basis Function (CBF) formalism is based on the transformation from Fermi gas (FG) states to correlated states

$$|n_{FG}\rangle \to |n\rangle = F|n_{FG}\rangle \quad , \quad F = \mathcal{S}\prod_{j>i} f_{ij}$$

- The definition of the CBF effective interaction follows from the requirement (note: H include both the two- and three-nucleon potentials)

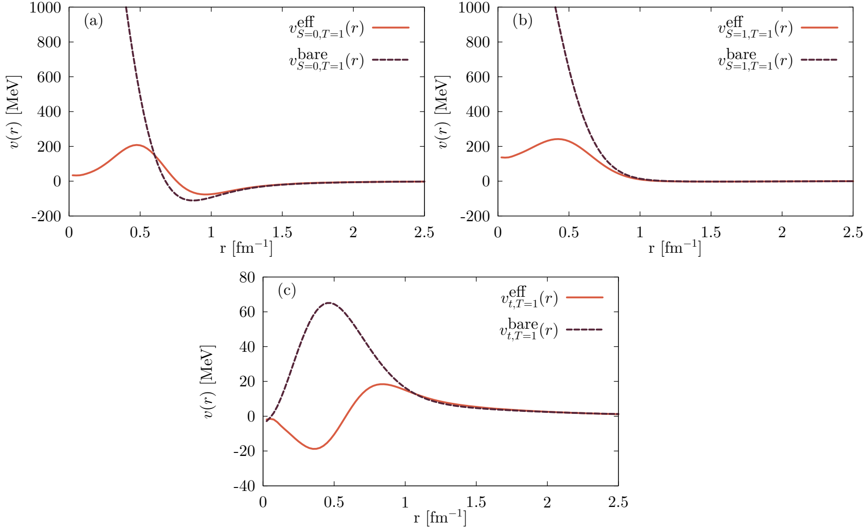
$$\langle H \rangle = \langle 0|H|0\rangle = \frac{3}{5}\frac{k_F^2}{2m} + \langle 0_{FG}|V_{\rm eff}|0_{FG}\rangle$$

implying

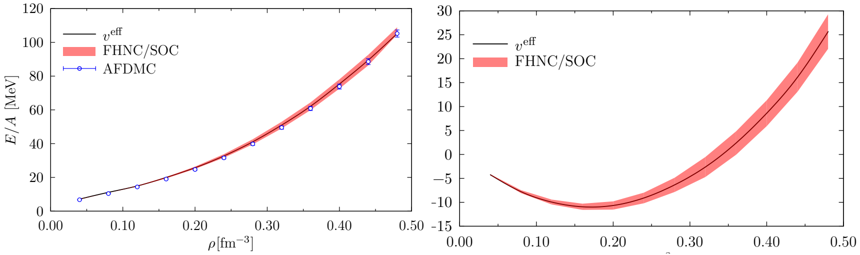
$$H_{\rm eff} = H_0 + V_{\rm eff} = F^\dagger H F$$

- For any given density, the operator F is determined in such a way as to reproduce the value of $\langle H \rangle$ obtained from Quantum Monte Carlo or Variational FHNC/SOC calculations at third order of the cluster expansion

⋆ CBF effective interaction in the $T = 1$ channel at nuclear matter equilibrium density, obtained from the Argonne $v_6' + UIX$ nuclear Hamiltonian (A. Lovato and OB)



- ⋆ Density dependence of the ground state energy per nucleon of unpolarized pure neutron matter (PNM) and isopspin-symmetric nuclear matter (SNM) obtained from the Argonne $v_6' + UIX$ nuclear Hamiltonian



- ⋆ Note that the $v_6' + UIX$ Hamiltonian, while yielding saturation at $\rho \approx \rho_0 = 0.16\ \mathrm{fm}^{-3}$, underestimates the equilibrium energy of SNM by ~ 5 MeV, corresponding to a $\sim 15\%$ underestimate of the interaction energy

Nuclear Matter Energy and Single-Particle Spectrum

- The ground state energy per baryon can be computed at first order in the effective interaction—that is, in Hartree–Fock approximation—for fixed baryon density and arbitrary proton fraction and polarizartions

$$\frac{E}{N_B} = \sum_{\mathbf{k}\lambda} \frac{\mathbf{k}^2}{2m} n_\lambda(\mathbf{k}) + \frac{1}{2} \sum_{\mathbf{k}\lambda\ ,\mathbf{k}'\lambda'} \langle \mathbf{k}\lambda\ \mathbf{k}'\lambda' | v^{\rm eff} | \mathbf{k}\lambda\ \mathbf{k}'\lambda' \rangle_A \ n_\lambda(\mathbf{k}) n_{\lambda'}(\mathbf{k}')$$

where $\lambda = 1, 2, 3, 4$ corresponds to $p \uparrow$, $p \downarrow$, $n \uparrow$, $n \downarrow$, and

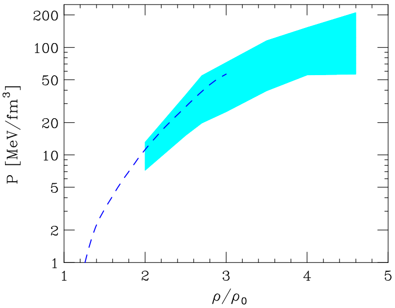
$$n_\lambda(\mathbf{k}) = \theta(k_{F_\lambda} - |\mathbf{k}|) \quad , \quad k_{F_\lambda} = (3\pi^2 \rho_\lambda)^{1/3}$$

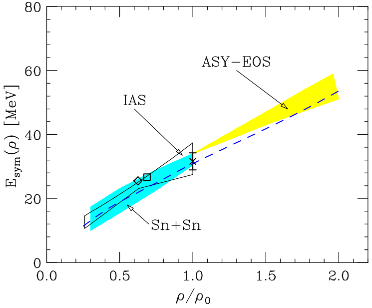
- The same approximation can be employed to obtain the single-nucleon spectrum and the effective masses

$$e_\lambda(\mathbf{k}) = \frac{\mathbf{k}^2}{2m} + \sum_{\mathbf{k}'\lambda'} \langle \mathbf{k}\lambda\ \mathbf{k}'\lambda' | v^{\rm eff} | \mathbf{k}\lambda\ \mathbf{k}'\lambda' \rangle_A \ n_\lambda(\mathbf{k}) \quad , \quad \frac{1}{m^\star} = \frac{1}{|\mathbf{k}|} \frac{d e_\lambda(\mathbf{k})}{d|\mathbf{k}|}$$

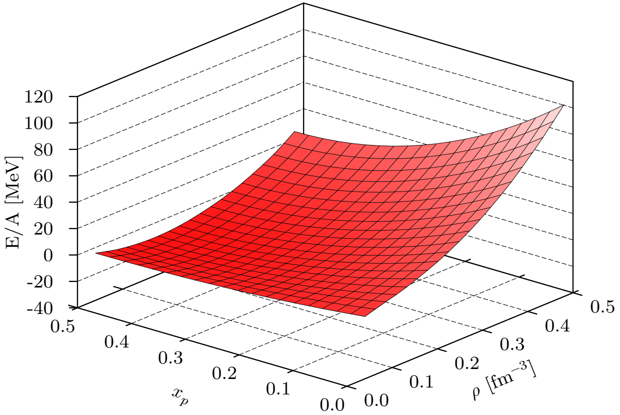
Pressure of SNM and Symmetry Energy

$$P = -\left(\frac{\partial E}{\partial V}\right)_N \quad ; \quad E_{\text{sym}}(\rho) = \left[\frac{\partial^2 (E(\rho,\delta)/N)}{\partial \delta^2}\right]_{\delta=0} \quad , \quad \delta = 1 - 2x_p$$





⋆ Energy of unpolarized nuclear matter as a function of baryon density and proton fraction $0 \leq x_p \leq 0.5$. The generalization to spin-polarized matter is straightforward.

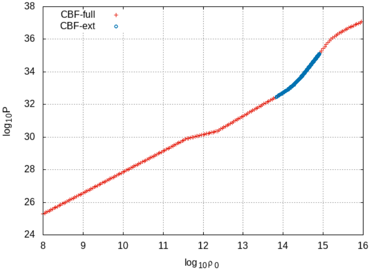


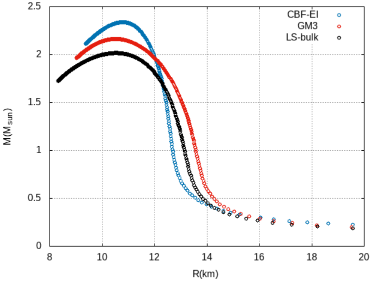
Summary & Outlook

- ⋆ Screening of nucleon-nucleon interactions in mater can be efficiently described in coordinate space using the formalism based on correlated states
- ⋆ This formalism can be employed to derive a density-dependent effective interaction—suitable to carry out calculations in many-body perturbation theory—from a realistic phenomenological Hamiltonian
- ⋆ The ability of this approach to describe quantities other than the ground-state energy has been tested extensively in the fermion hard-shere system, comparing to the results of low-density expansions
- ⋆ Early results of calculations of nuclear matter properties relevant to neutron stars look promising

Backup slides

EoS and Mass-Radius Relation

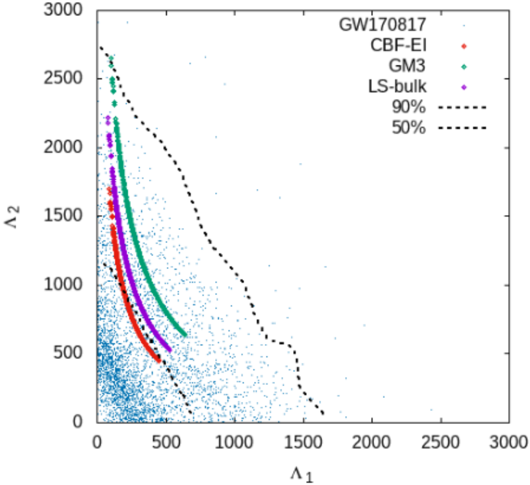




⋆ The information obtained from GW170817 suggests that nuclear matter cannot be very stiff, and that the radius of a neutron star with $M \approx 1.35\ M_{\odot}$ can not exceed $\sim$ 14 Km

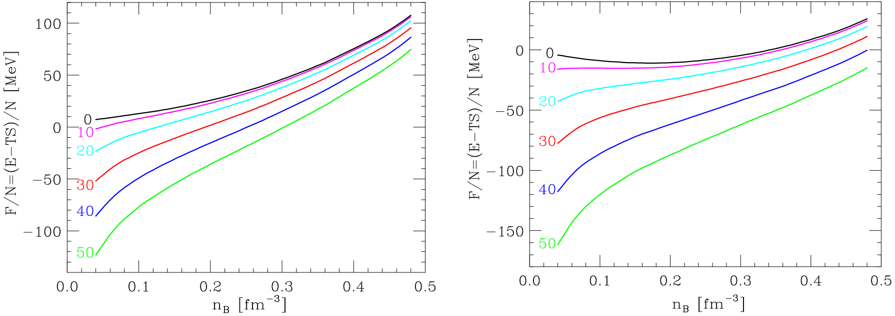
TIDAL DEFORMATION FROM GW170817

- From the MSc Thesis of A. Sabatucci

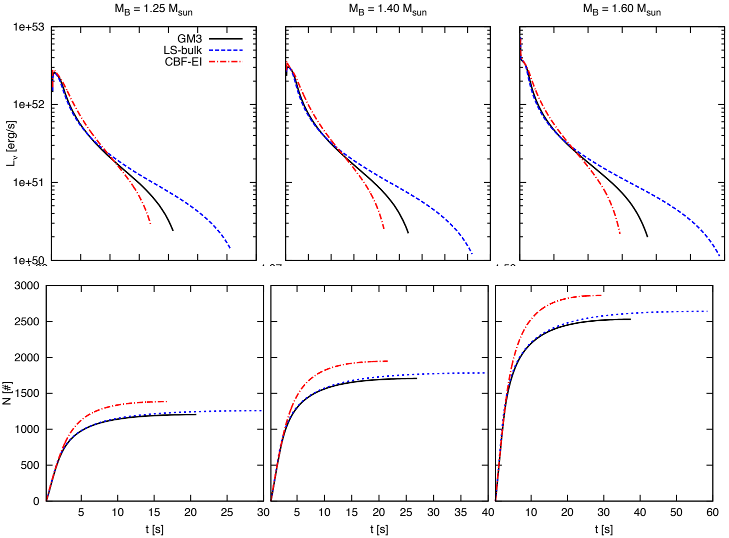


Extension to $T > 0$

- Assuminhg that thermal effect do not significantly affect the dynamics of strong interactions, the effective interaciotns can be used to obtain the properties of nuclear matter at $T > 0$
- Replace $\theta(k_F - k) \rightarrow \{1 + \exp[e(k) - \mu]/T\}^{-1}$



Neutrino Luninosity of Proto Neutron Stars (PNS)



Frequencies of Quasi Normal Modes of PNS

★ G. Camelio PhD Thesis and PRD (2017)