High momentum in NN interactions

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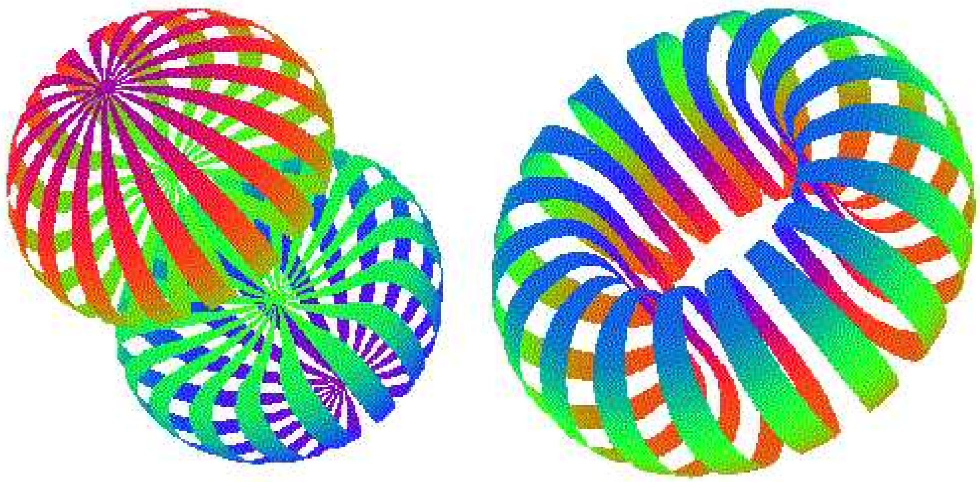

WORK NOT POSSIBLE WITHOUT EXTENSIVE COMPUTER RESOURCES

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Physics Division

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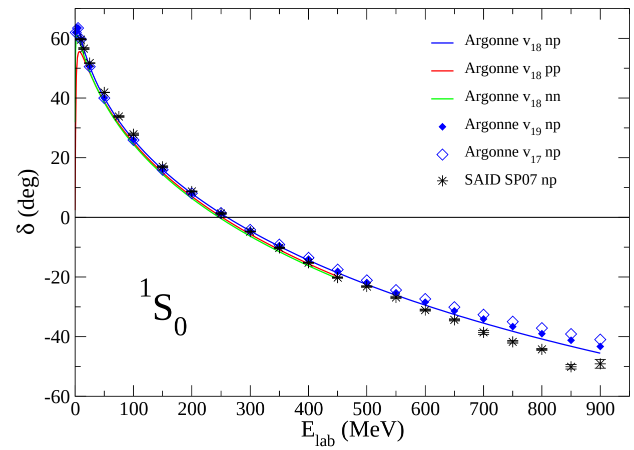
NUCLEAR HAMILTONIAN

$$H = \sum_i K_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

K_i: Non-relativistic kinetic energy, m_n-m_p effects included

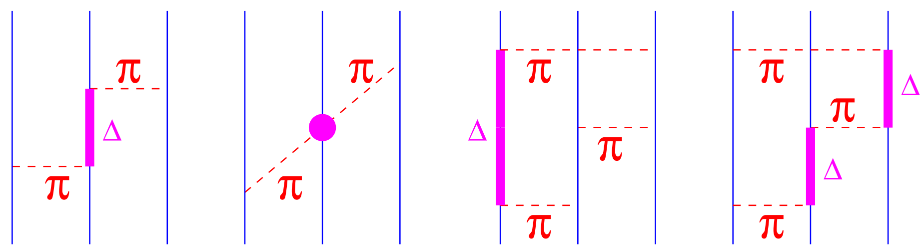
Argonne v18: $v_{ij} = v_{ij}^{\gamma} + v_{ij}^{\pi} + v_{ij}^{I} + v_{ij}^{S} = \sum v_p(r_{ij}) O_{ij}^p$

- 18 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure
- fits Nijmegen PWA93 data with χ^2/d.o.f.=1.1



Wiringa, Stoks, & Schiavilla, PRC **51**, (1995)

Urbana & Illinois: $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{3\pi} + V_{ijk}^{R}$



- Urbana has standard 2π P-wave + short-range repulsion for matter saturation
- Illinois adds 2π S-wave + 3π rings to provide extra T=3/2 interaction
- Illinois-7 has four parameters fit to 23 levels in $A \leq 10$ nuclei

Pieper, Pandharipande, Wiringa, & Carlson, PRC **64**, 014001 (2001)
Pieper, AIP CP **1011**, 143 (2008)

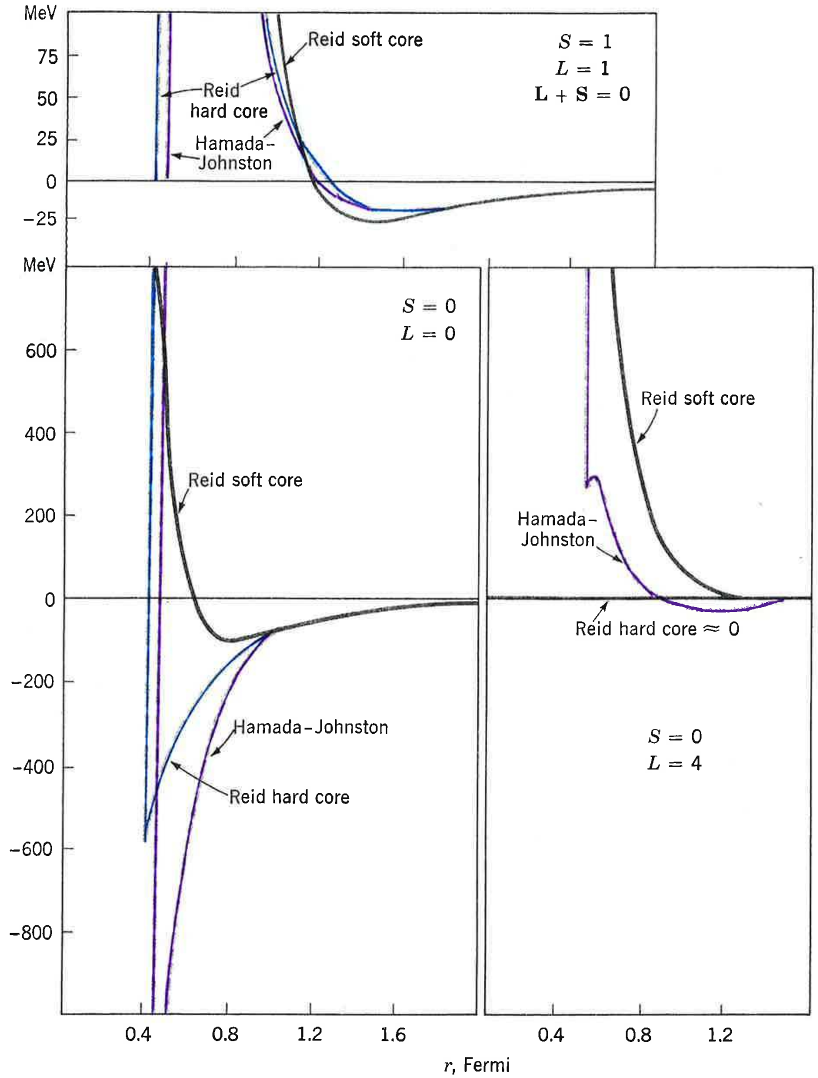
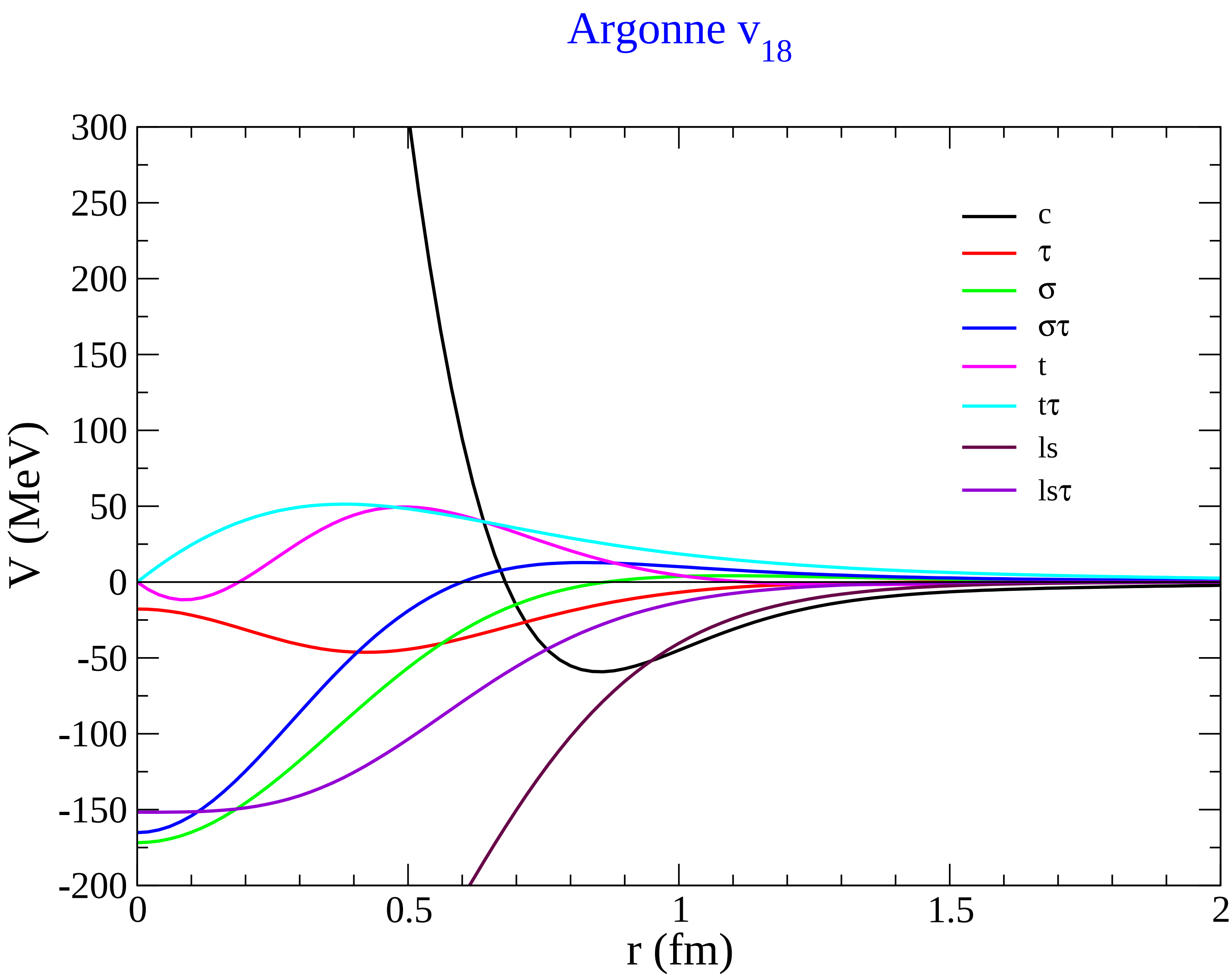


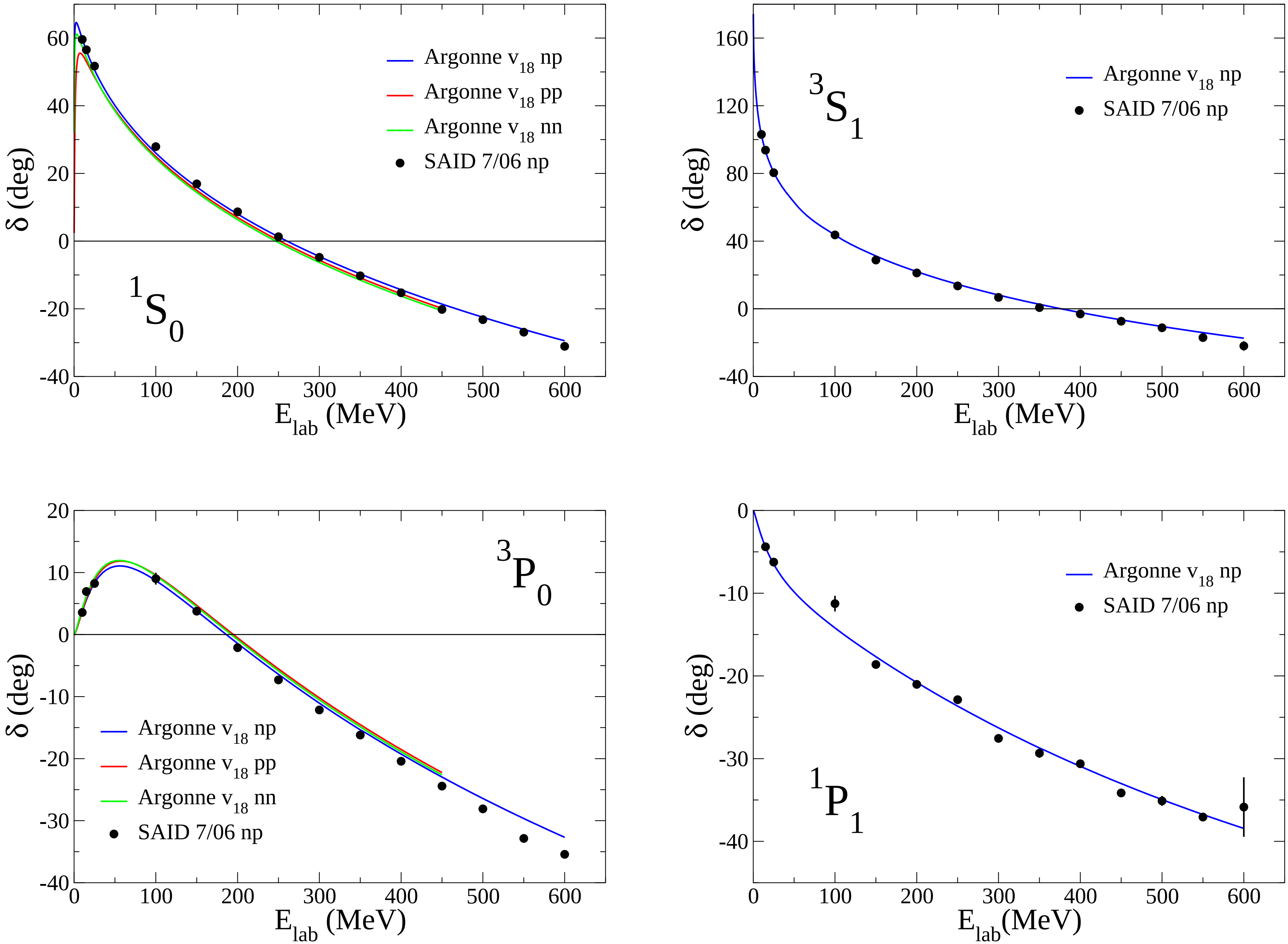
FIGURE 3-8 Comparison of the Reid hard-core and soft-core potentials with the Hamada-Johnston potential for various states of interaction. *(From R. V. Reid, Ph.D. Thesis, Cornell University, 1968.)*

CONCEPTS OF NUCLEAR PHYSICS

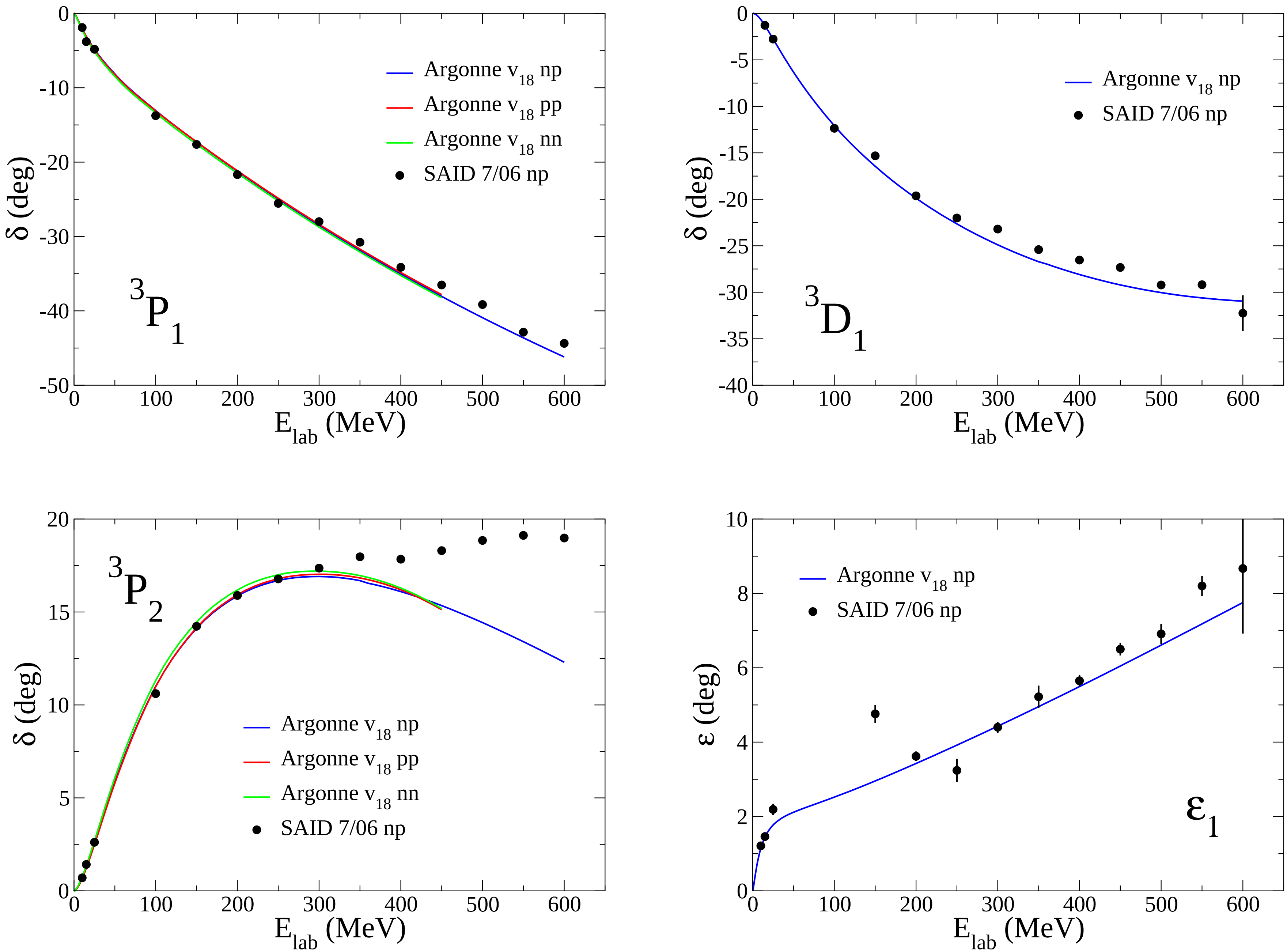
BERNARD L. COHEN (1971)



Uses 42 I^p, P^p, Q^p, R^p parameters [constrained so that $v_t(r=0) = 0$ & $\frac{\partial v_{p\neq t}}{\partial r}|_{r=0} = 0$]
plus $f_{\pi NN}$ coupling, one cutoff in $Y_\pi(r)$, $T_\pi(r)$ and the Woods-Saxon radius & diffuseness.



Argonne v_{18} fits Nijmegen PWA93 data base of 1787 *pp* & 2514 *np* observables for $E_{lab} \leq 350$ MeV with χ^2/N_{data} = 1.1 plus *nn* scattering length & ^{2}H binding energy



Argonne v18 np
Argonne v18 pp
Argonne v18 nn
SAID 7/06 np
3P1
δ (deg)
E_lab (MeV)
Argonne v18 np
SAID 7/06 np
3D1
δ (deg)
E_lab (MeV)
3P2
Argonne v18 np
Argonne v18 pp
Argonne v18 nn
SAID 7/06 np
δ (deg)
E_lab (MeV)
Argonne v18 np
SAID 7/06 np
ε1
ε (deg)
E_lab (MeV)

Norfolk NV2: $v_{ij} = v^{\gamma}_{ij} + v^{\pi}_{ij} + v^{2\pi}_{ij} + v^{CT}_{ij} = \sum v_p(r_{ij}) O^p_{ij}$

- derived in chiral effective field theory with Δ-intermediate states
- 17 spin, tensor, spin-orbit, isospin, etc., operators
- full EM and strong CD and CSB terms included
- predominantly local operator structure suitable for quantum Monte Carlo
- multiple models with different regularization fit to Granada PWA2013 data
- Ia,b,c fit to E_{lab} = 125 MeV with χ^2/d.o.f.$\sim$1.1
- IIa,b,c fit to E_{lab} = 200 MeV with χ^2/d.o.f.$\sim$1.4

Piarulli, Girlanda, Schiavilla, Kievsky, Lovato, Marcucci, Pieper, Viviani, & Wiringa, PRC **94**, 054007 (2016)

Norfolk NV3: $V_{ijk} = V^{2\pi}_{ijk} + V^{CT}_{ijk}$

- standard 2π S-wave and 2π P-wave terms consistent with chiral NN potential
- contact terms of c_D (π-short range) and c_E (short-short range $\tau_i \cdot \tau_k$) type
- two parameters fit to ^{3}H binding and nd scattering length (NV3)
- or ^{3}H binding and β^- decay (NV3*)

Piarulli, Baroni, Girlanda, Kievsky, Lovato, Lusk, Marcucci, Pieper, Schiavilla, Viviani, Wiringa: PRL **120**, 052503 (2018)

Baroni, Schiavilla, Marcucci, Girlanda, Kievsky, Lovato, Pastore, Piarulli, Pieper, Viviani, Wiringa: PRC **98**, 044003 (2018)

QUANTUM MONTE CARLO

Variational Monte Carlo (VMC): construct Ψ_V that

- Are fully antisymmetric and translationally invariant
- Have cluster structure and correct asymptotic form
- Contain non-commuting 2- & 3-body operator correlations from v_{ij} & V_{ijk}
- Are orthogonal for multiple J^π states
- Minimize $E_V = \langle\Psi_V|H|\Psi_V\rangle \geq E$

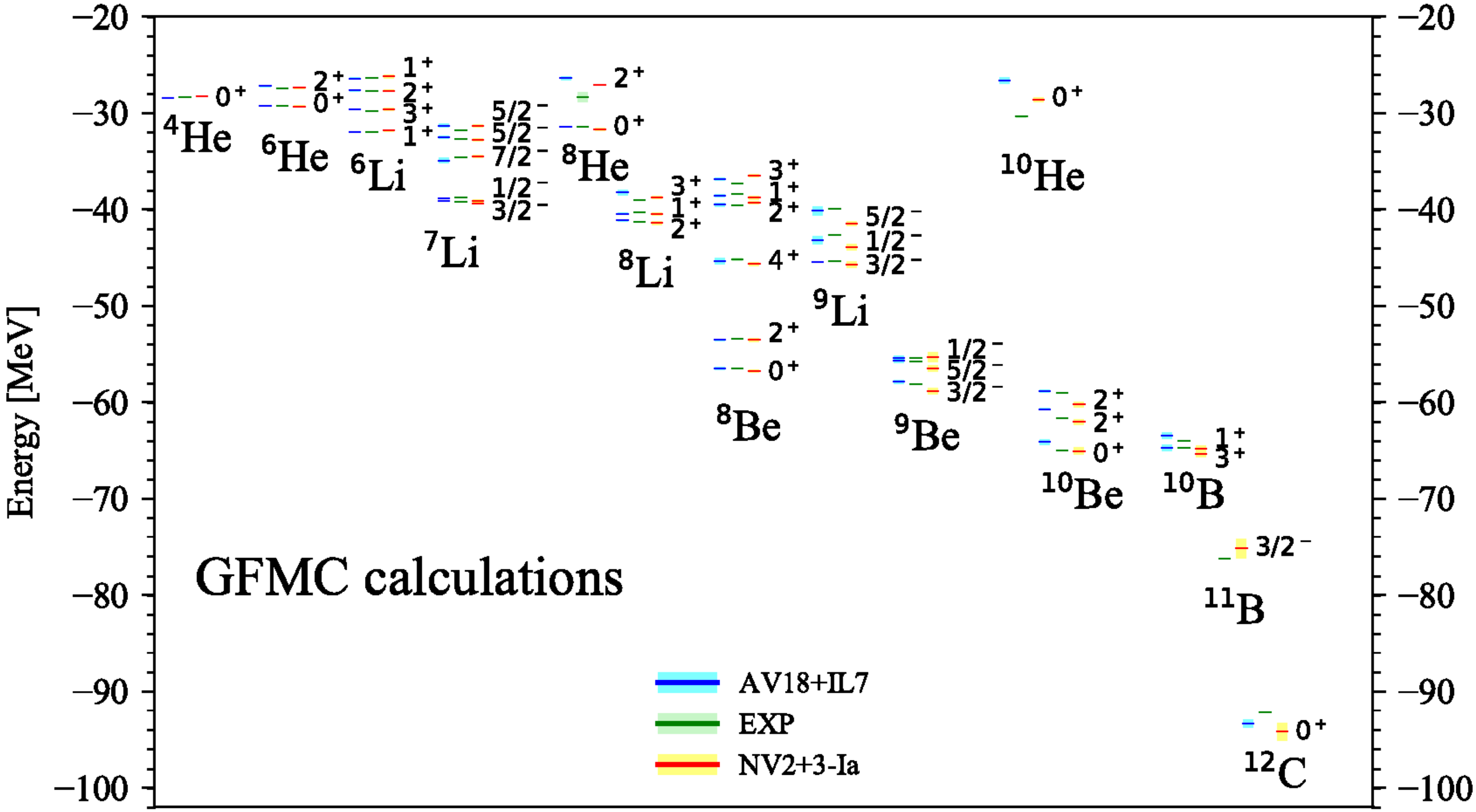
These are $\sim 2^A \binom{A}{Z}$ component (540,672 for ^{12}C) spin-isospin vectors in $3A$ dimensions

Green's function Monte Carlo (GFMC): project out the exact eigenfunction

- $\Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V = \sum_n \exp[-(E_n - E_0)\tau]a_n\Psi_n \Rightarrow \Psi_0$ at large τ
- Propagation done stochastically in small time slices $\Delta\tau$
- Exact $\langle H\rangle$ for local potentials; mixed estimates for other $\langle O\rangle$
- Constrained-path propagation controls fermion sign problem for $A \geq 8$
- Multiple excited states for same J^π stay orthogonal

Many tests demonstrate 1–2% accuracy for realistic $\langle H\rangle$

Carlson, Gandolfi, Pederiva, Pieper, Schiavilla, Schmidt, & Wiringa, RMP **87**, 1067 (2015)



RMS ΔE for 36 states: AV18+IL7 = 0.80 MeV ; NV2+3-Ia = 0.72 MeV

with signed average deviation: -0.23 MeV and +0.15 MeV

Single-Nucleon Momentum Distributions

Momentum distributions of nucleons and nucleon clusters can provide useful insights into various reactions on nuclei, such as $(e, e'p)$ and $(e, e'pN)$ electrodisintegration processes.

Probability of finding a nucleon in a nucleus with momentum $\mathbf{k}$ in a given spin-isospin state:

$$\rho_{\sigma\tau}(\mathbf{k}) = \int d\mathbf{r}'_1 d\mathbf{r}_1 d\mathbf{r}_2 \cdots d\mathbf{r}_A \, \psi^\dagger_{JM_J}(\mathbf{r}'_1, \mathbf{r}_2, \ldots, \mathbf{r}_A) \, e^{-i\mathbf{k}\cdot(\mathbf{r}_1 - \mathbf{r}'_1)} \, P_{\sigma\tau}(1) \, \psi_{JM_J}(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A)$$

where $P_{\sigma\tau}$ is a spin-isospin projection operator and the normalization is:

$$N_{\sigma\tau} = \int \frac{d\mathbf{k}}{(2\pi)^3} \, \rho_{\sigma\tau}(\mathbf{k})$$

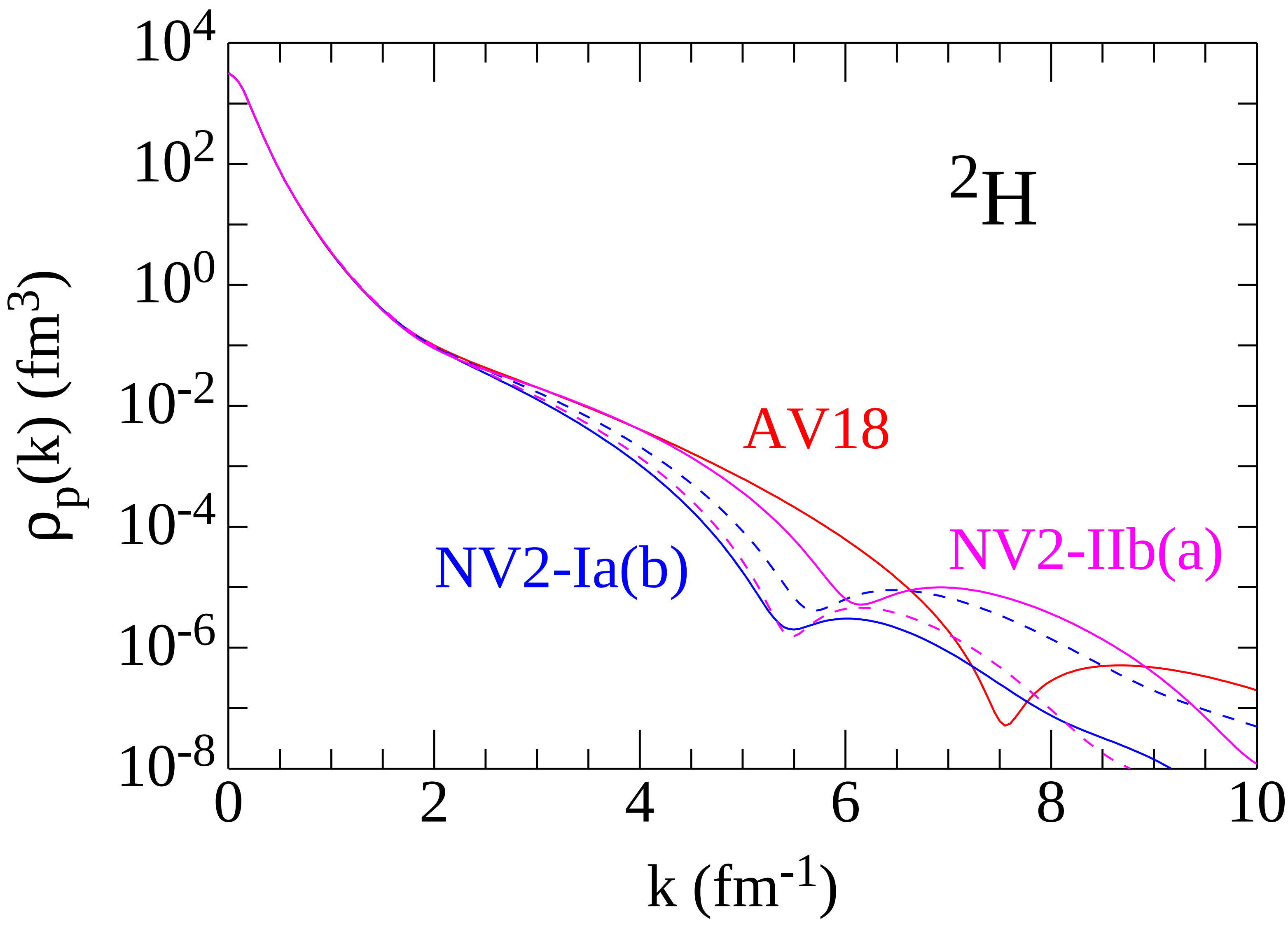
$N_{\sigma\tau}$ is the total number of nucleons with given spin-isospin.

Compute the Fourier transform by Metropolis Monte Carlo integration, using a standard Metropolis walk, guided by $|\Psi_{JM_J}(\mathbf{r}_1, \ldots, \mathbf{r}_i, \ldots, \mathbf{r}_A)|^2$ to sample configurations.
Average over all particles i in each configuration; for each particle use a grid of Gauss-Legendre points $\mathbf{x}_i$ in a randomly chosen direction and move positions in both left- and right-hand side wave functions symmetrically away from their central value:

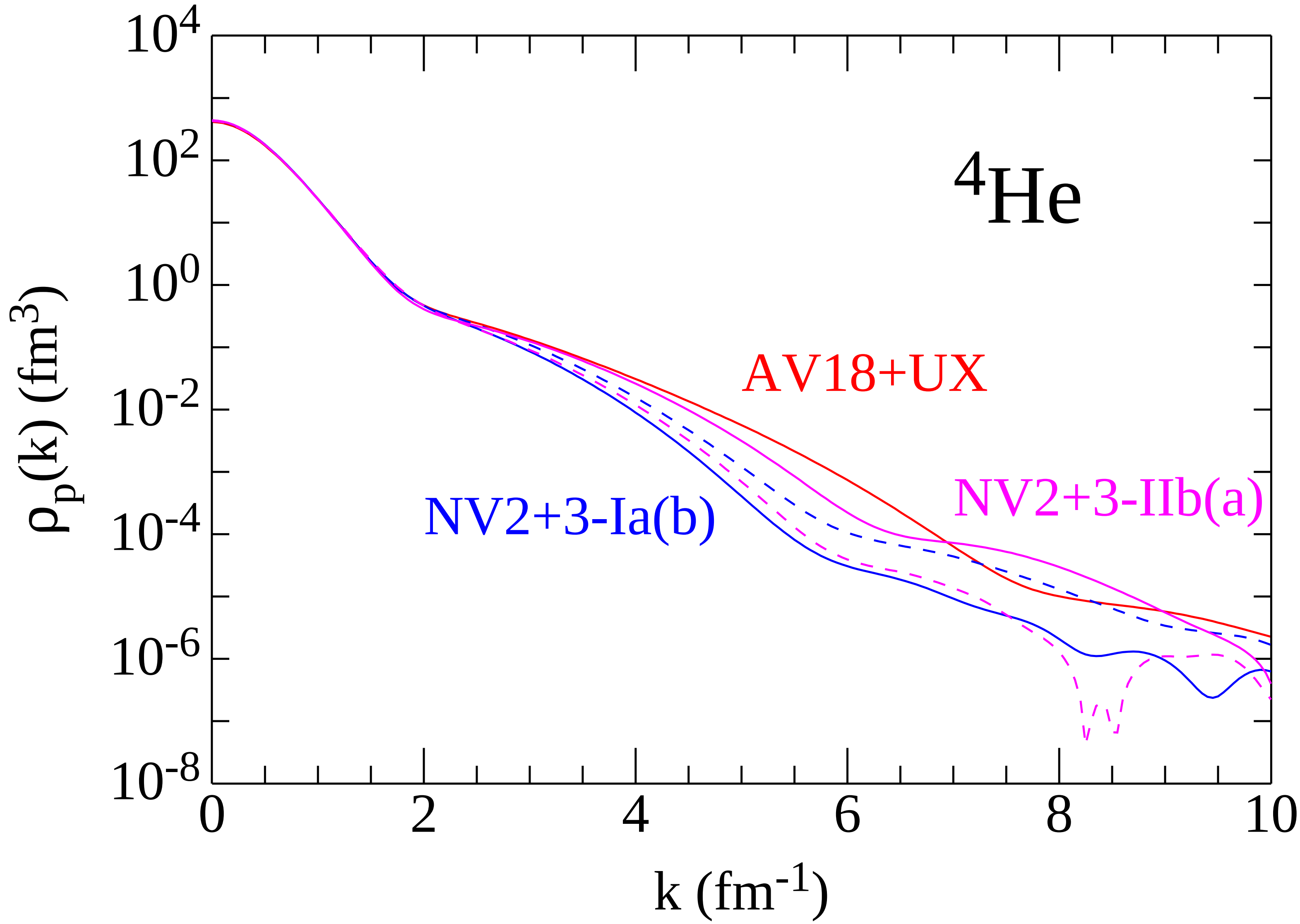
$$\rho_{\sigma\tau}(\mathbf{k}) = \frac{1}{A} \sum_i \int d\mathbf{r}_1 \cdots d\mathbf{r}_i \cdots d\mathbf{r}_A \int d\mathbf{\Omega}_x \int_0^{x_{\max}} x^2 dx \, \psi^\dagger_{JM_J}(\mathbf{r}_1, \ldots, \mathbf{r}_i + \mathbf{x}/2, \ldots, \mathbf{r}_A)$$

$$e^{-i\mathbf{k}\cdot\mathbf{x}} \, P_{\sigma\tau}(i) \, \psi_{JM_J}(\mathbf{r}_1, \ldots, \mathbf{r}_i - \mathbf{x}/2, \ldots, \mathbf{r}_A)$$

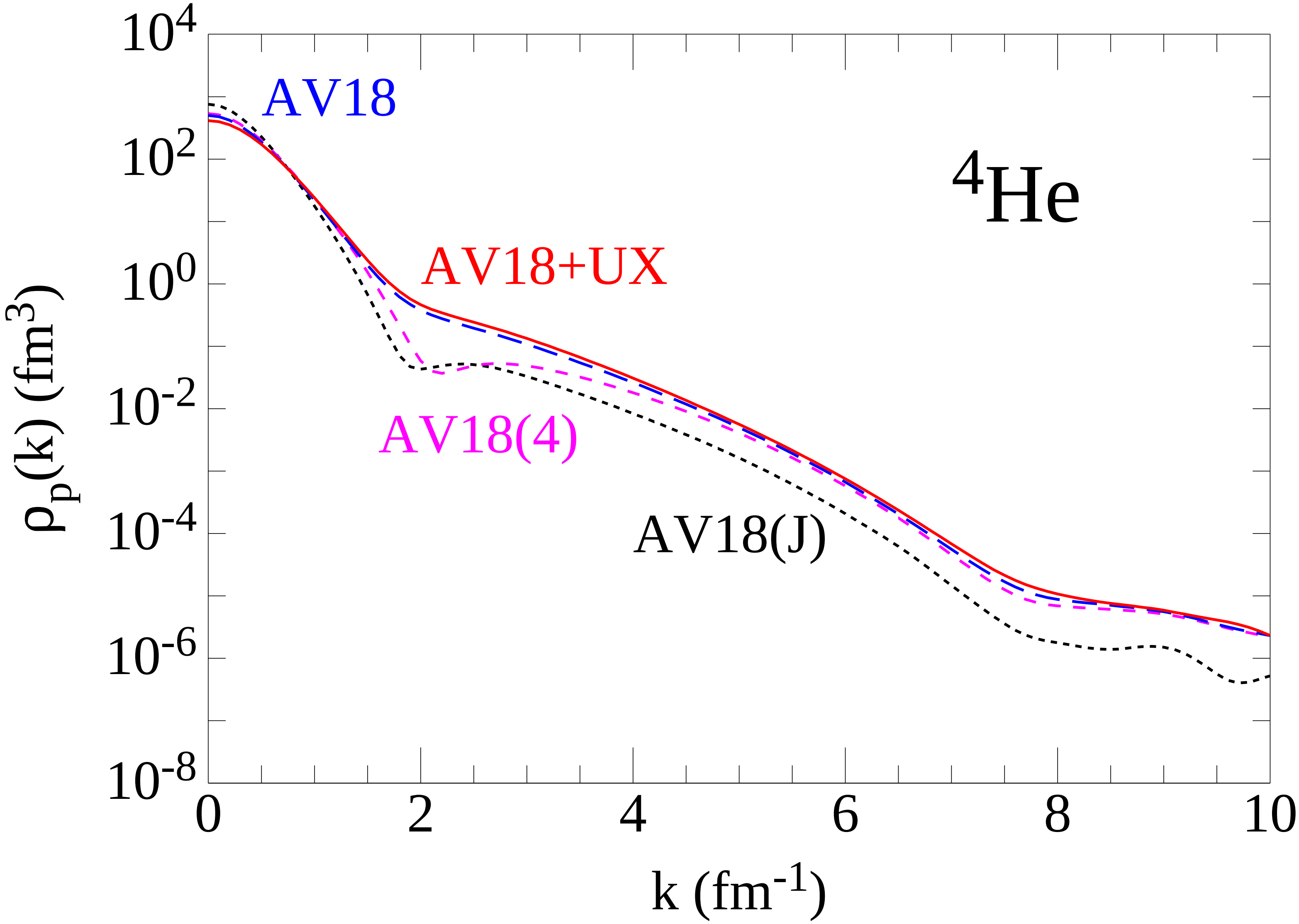
Deuteron proton momentum distribution has contributions from the S- and D-wave components of the wave function; S-wave has node at 2 fm^{-1} that D-wave fills in to give a broad high-momentum shoulder out to > 7 fm^{-1} before the second S-wave and first D-wave node.



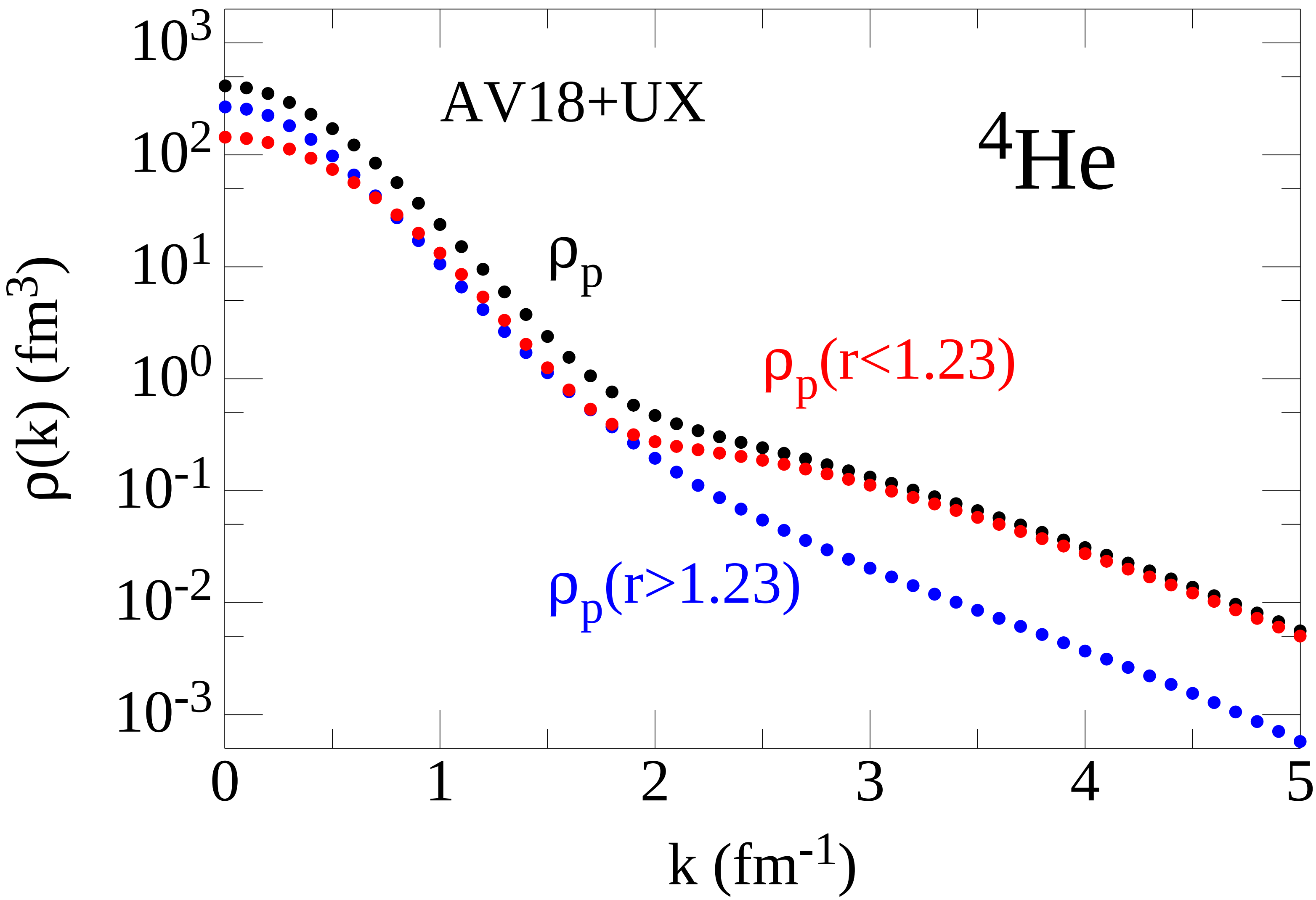
^{4}He proton momentum distribution is very similar to the deuteron, but is smaller at $k = 0$ and greater in the high-momentum shoulder because of the greater binding energy.



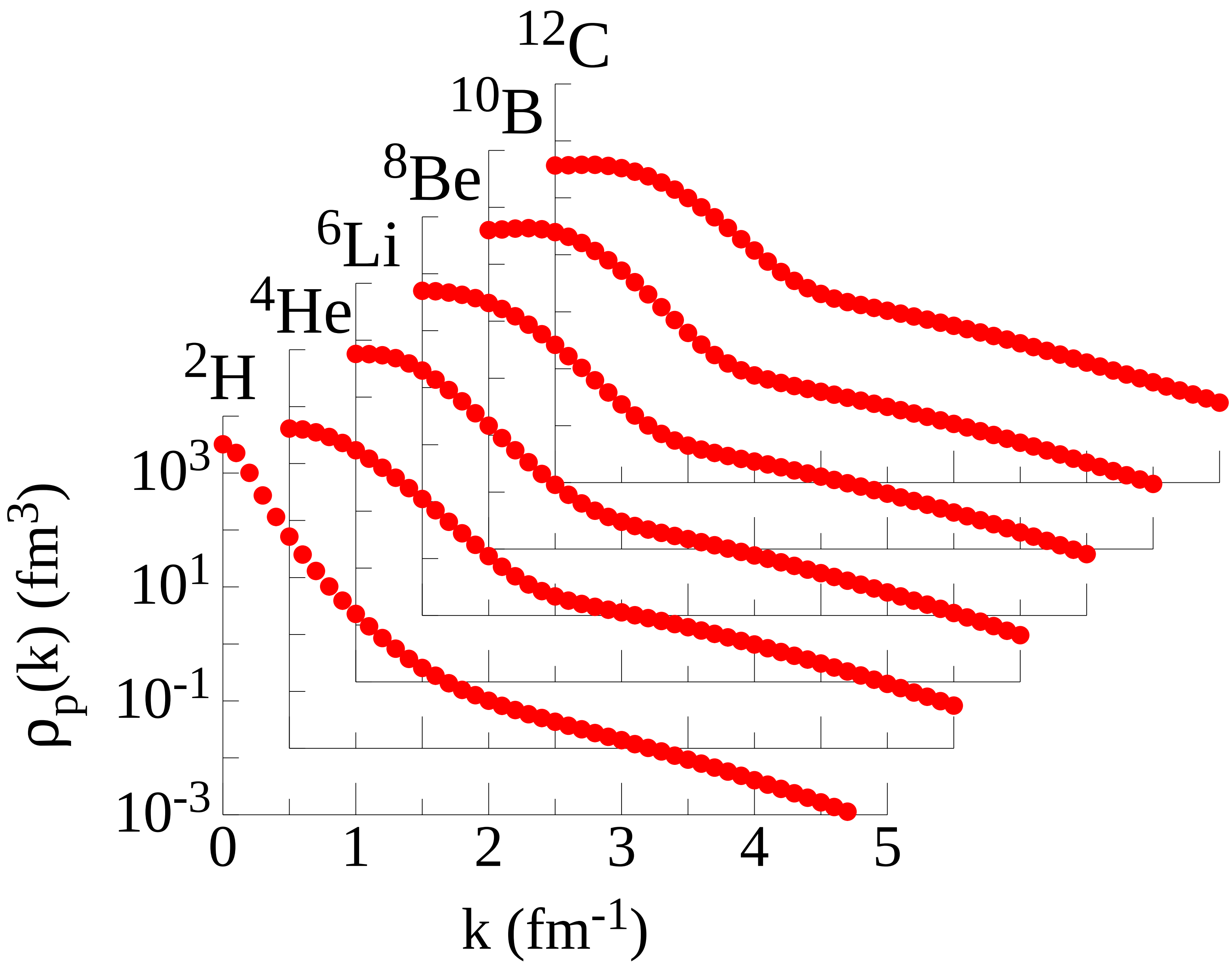
To study source of higher momentum densities, we show calculations for AV18+UX, for AV18 alone, for AV18(4) with tensor correlations removed, and for AV18(J) with only cental Jastrow correlations. The numbers of protons above $k = 1.4\ \mathrm{fm}^{-1}$ (out of 2 total) for these cases are 0.25, 0.20, 0.10, and 0.05, respectively.



To study the dependence on the position of the nucleons within the nucleus, we can count only those which are within a certain distance of the center of mass. For the AV18+UX ^{4}He wave function, half the nucleons are inside $r = 1.23$ fm, and they provide the bulk of the high-momentum nucleons.



The proton momentum distributions in $T = 0$ light nuclei all have similar shapes, with the $k = 0$ peak decreasing in magnitude and broadening out as the binding increases and p-shell nucleons are added. The high momentum shoulder due to pion-exchange is universal.



NUCLEON-PAIR MOMENTUM DISTRIBUTIONS

Probability of finding two nucleons in a nucleus with relative momentum $\mathbf{q} = (\mathbf{q}_1 - \mathbf{q}_2)/2$ and total center-of-mass momenta $\mathbf{Q} = (\mathbf{q}_1 + \mathbf{q}_2)$ in a given spin-isospin state:

$$\rho_{ST}(\mathbf{q},\mathbf{Q}) = \int d\mathbf{r}'_1 d\mathbf{r}_1 d\mathbf{r}'_2 d\mathbf{r}_2 d\mathbf{r}_3 \cdots d\mathbf{r}_A \, \psi^{\dagger}_{JM_J}(\mathbf{r}'_1,\mathbf{r}'_2,\mathbf{r}_3,\ldots,\mathbf{r}_A)\, e^{-i\mathbf{q}\cdot(\mathbf{r}_{12}-\mathbf{r}'_{12})}$$

$$e^{-i\mathbf{Q}\cdot(\mathbf{R}_{12}-\mathbf{R}'_{12})}\, P_{ST}(12)\, \psi_{JM_J}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\ldots,\mathbf{r}_A)$$

where P_{ST} is a pair spin-isospin projection operator and the normalization is:

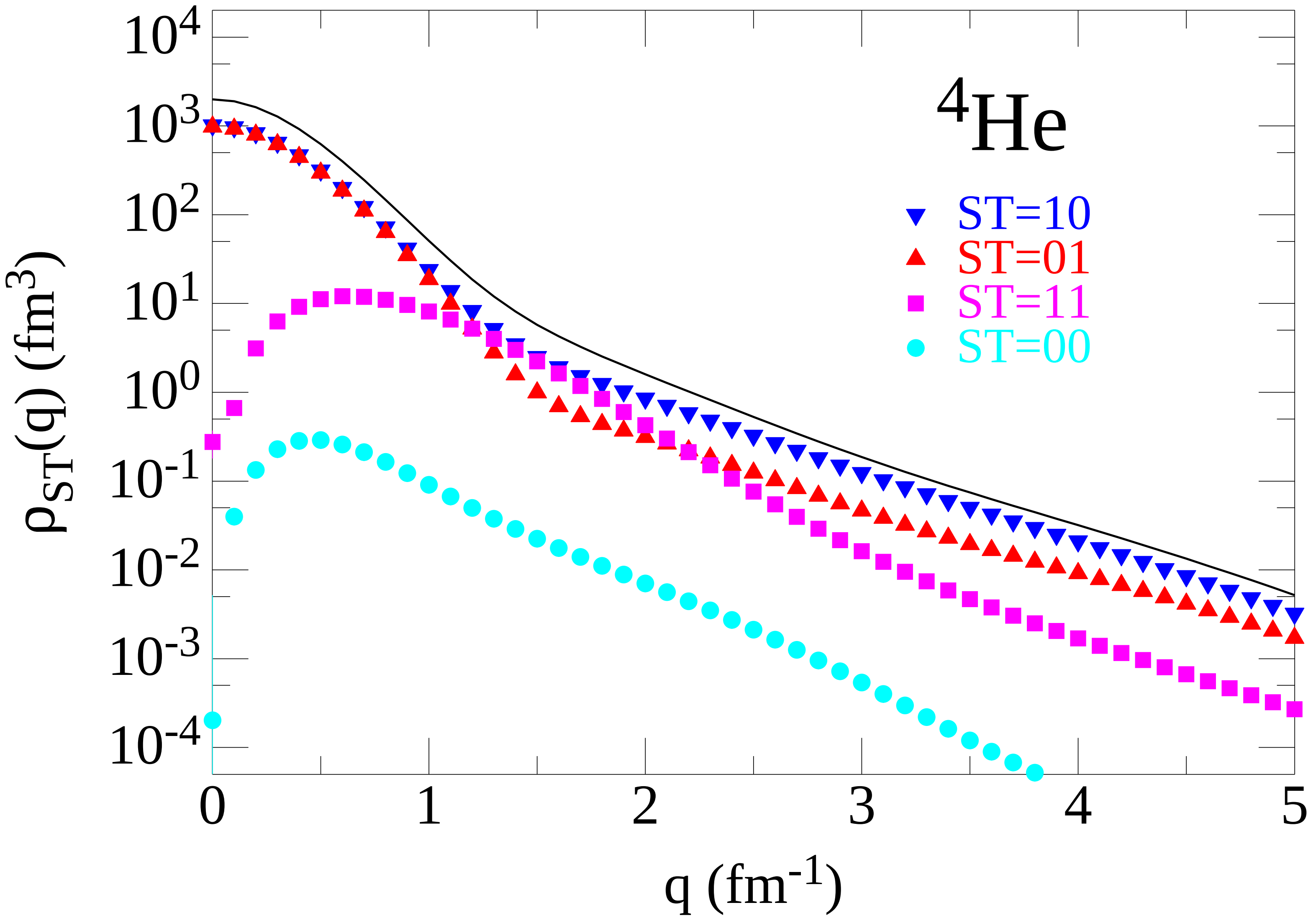
$$N_{ST} = \int \frac{d\mathbf{q}}{(2\pi)^3}\frac{d\mathbf{Q}}{(2\pi)^3}\rho_{ST}(\mathbf{q},\mathbf{Q})$$

and N_{ST} is the total number of nucleon pairs with given spin-isospin.

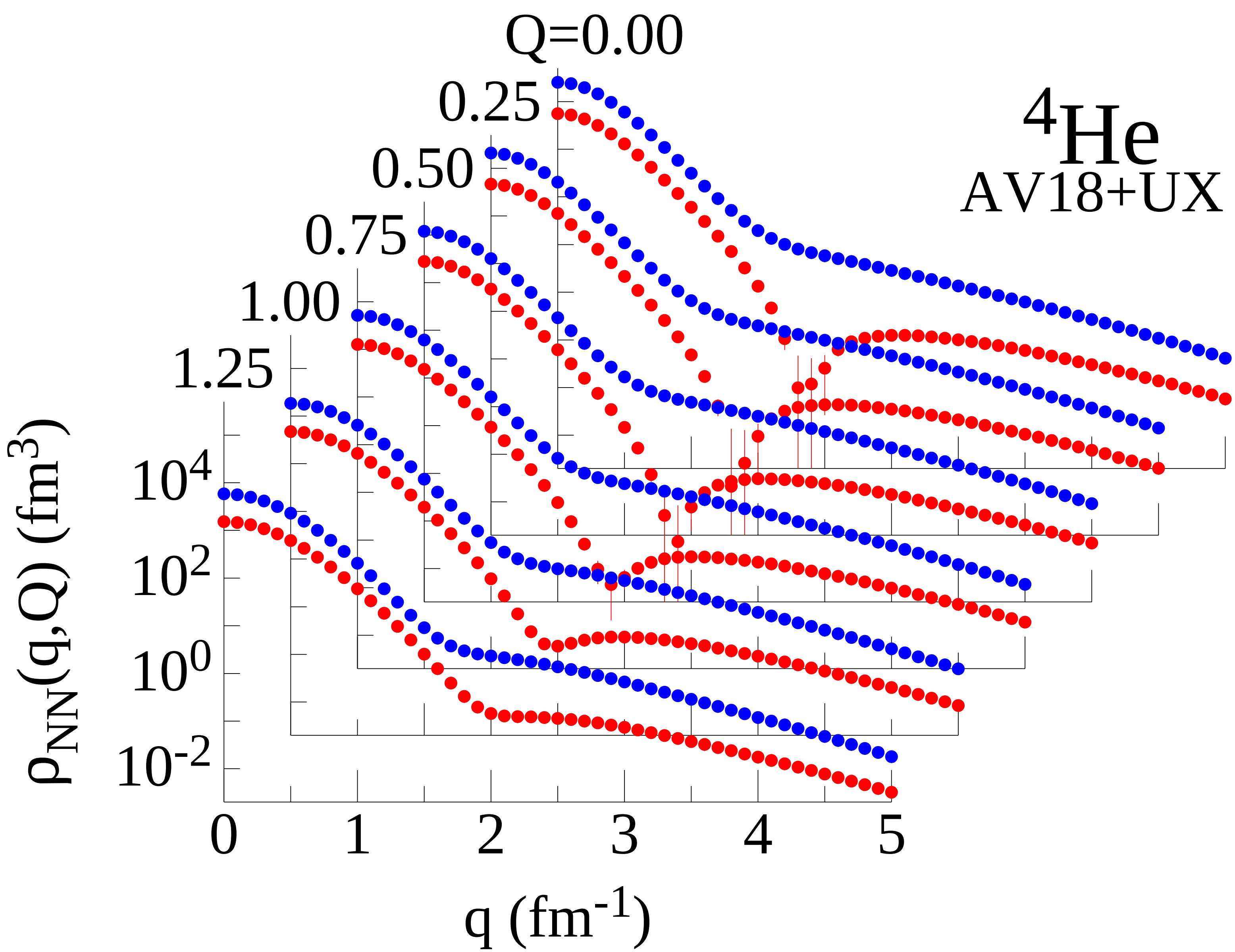
This double Fourier transform is evaluated by a double Gauss-Legendre integration over each pair of nucleons in each configuration sampled by the variational Monte Carlo wave function. The angle between $\mathbf{q}$ and $\mathbf{Q}$ can be randomly chosen or at a fixed angle, such as $\mathbf{q}||\mathbf{Q}$.

The nucleon-pair momentum distribution can be projected in various ways. Here is ρ_{ST}, as a function of q, integrated over all Q, with projection into states of total S=0,1 and T=0,1

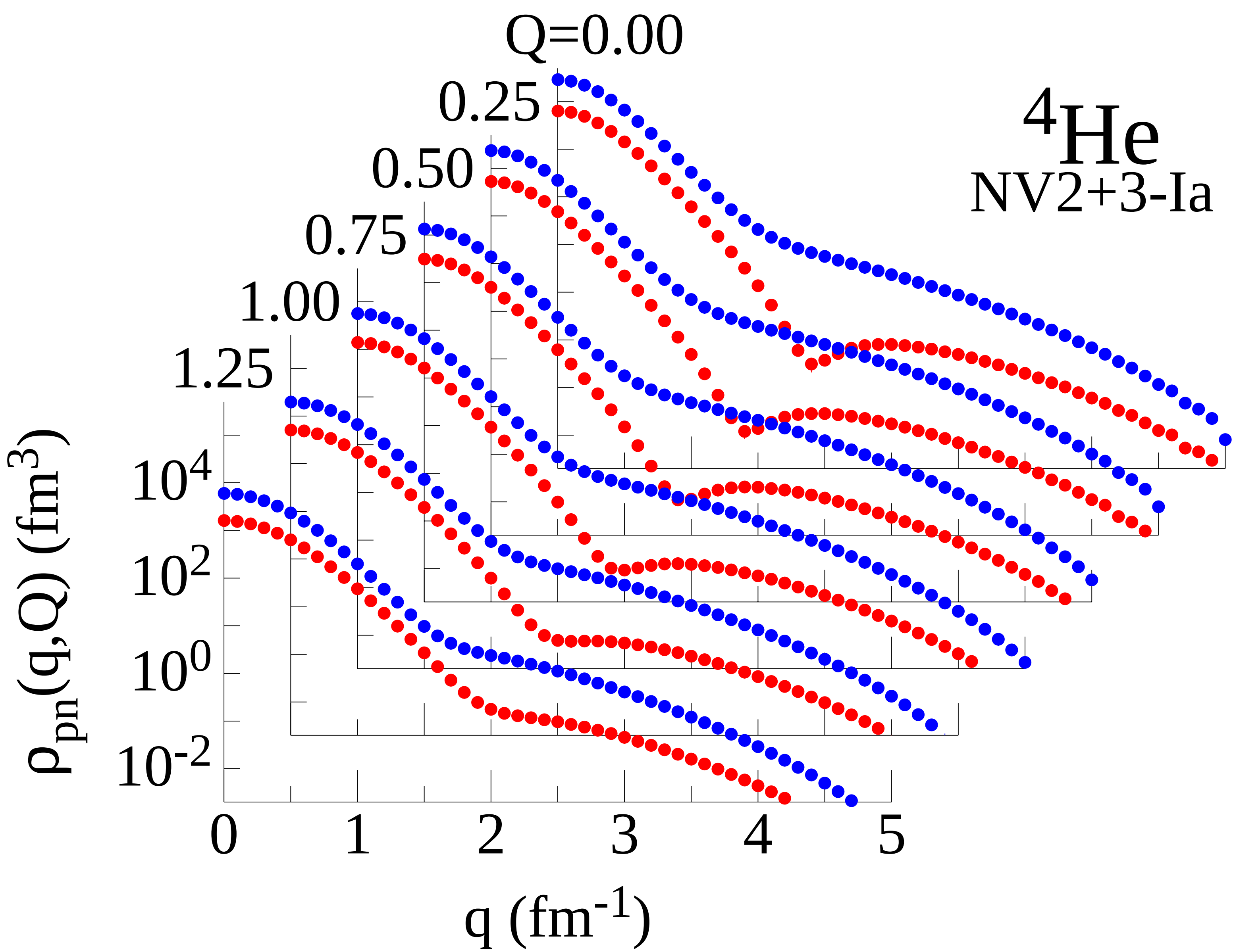
In an independent-pair model, ^{4}He would have 3 pairs each of ST=10 and 01, but tensor forces shift $^1/_2$ pair of ST=01 to 11.



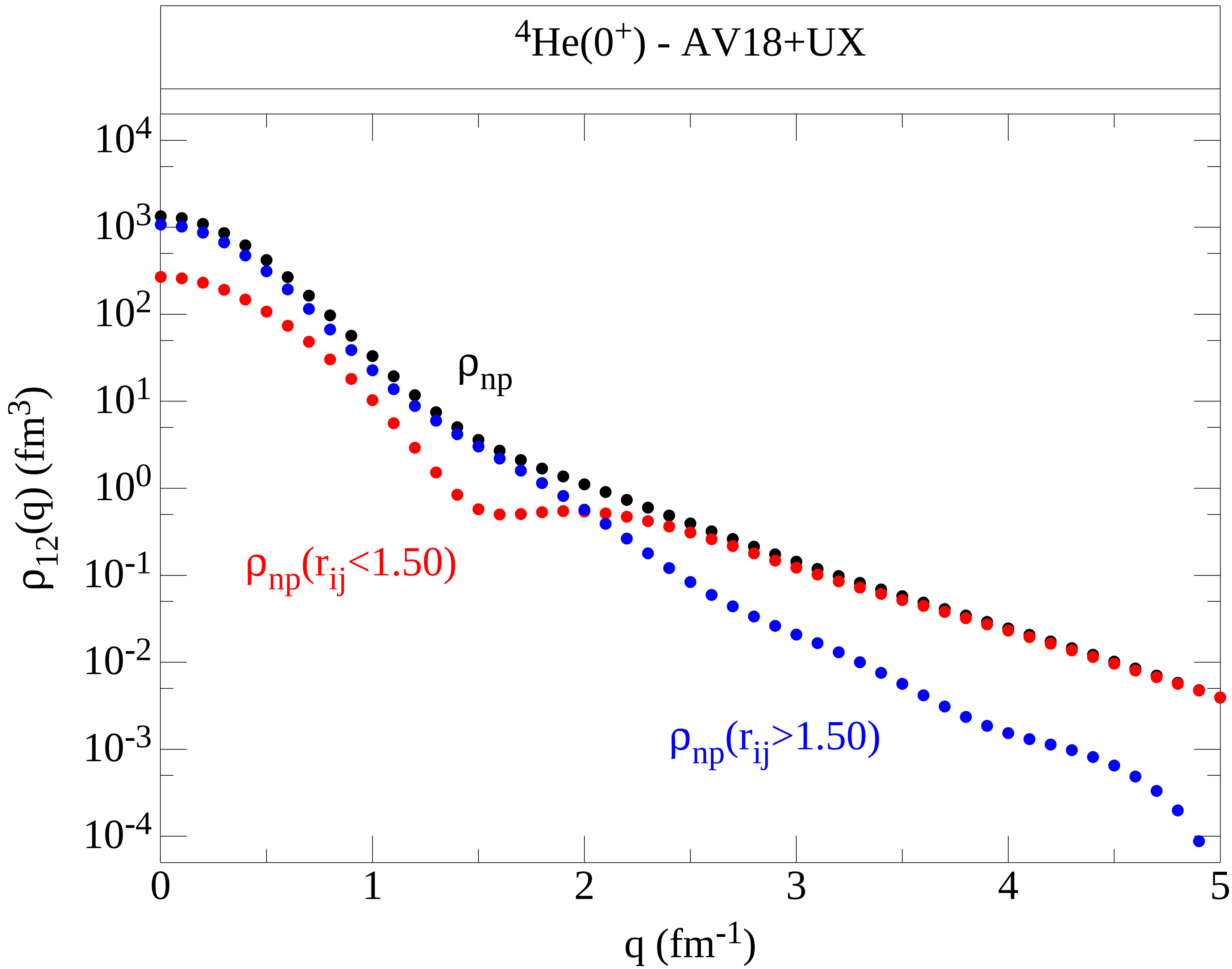
The *np* and *pp* momenta in ^{4}He for AV18+UX averaged over all angles between **q** and **Q** shown as a function of q for fixed values of Q. The *pp* pairs are primarily in relative $^1S_0$ states and show a typical S-wave node at $Q = 0$, filled in as Q increases; *np* pairs have no node.



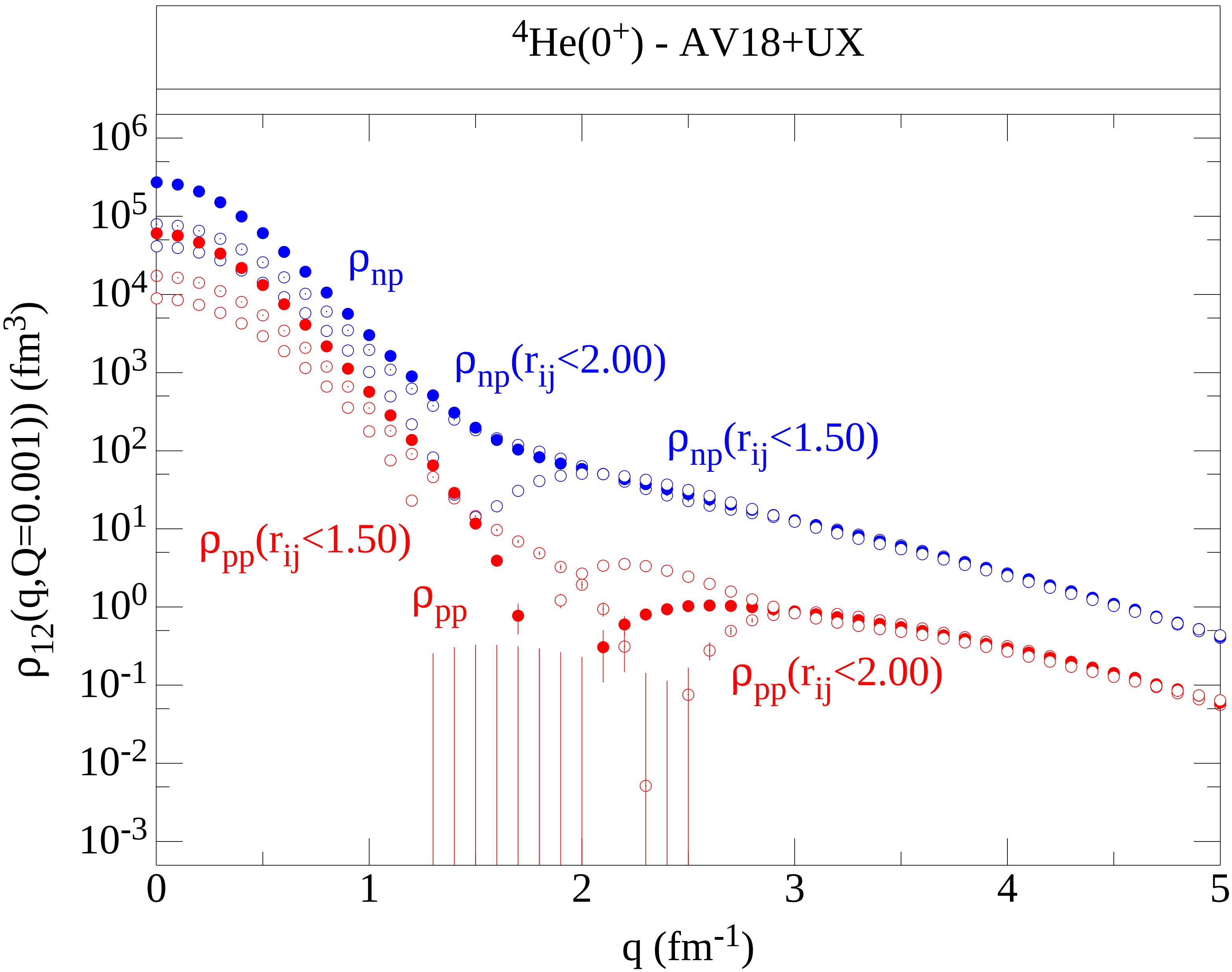
The *np* and *pp* momenta in ^{4}He for NV2+3-Ia averaged over all angles between **q** and **Q** shown as a function of q for fixed values of Q. The *pp* minimum is again present although not as sharp, and both *pp* and *np* fall off more rapidly at high q.



The *np* momenta in ^{4}He for AV18+UX averaged over all angles between $\mathbf{q}$ and $\mathbf{Q}$ and integrated over Q, and broken into contributions from pairs less than or greater than $r_{ij} = 1.5$ fm.



Total np and pp momentum distributions in ^{4}He for AV18+UX for $Q = 0$ fm and with contributions from pairs less than $r_{ij} = 1.5$ or $r_{ij} = 2.0$ fm.



TABULATIONS

Single-nucleon momentum distributions are available for many additional cases, including ^{3}H, ^{8}He, ^{8}Li, ^{9}Li, ^{9}Be, ^{10}Be, ^{10}B, ^{11}B and two-nucleon distributions for ^{3}He
Results of CVMC calculations for ^{16}O and ^{40}Ca are also posted.

For anyone who wishes to use these momentum distributions, they are available on-line:

For single-nucleon momentum distributions: www.phy.anl.gov/theory/research/momenta

(single-nucleon density distributions are at www.phy.anl.gov/theory/research/density)

For two-nucleon momentum distributions: www.phy.anl.gov/theory/research/momenta2

More calculations will be posted as they become available; requests will be entertained.