EFT potentials and SRCs: Q&A

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THE OHIO STATE UNIVERSITY

U.S. DEPARTMENT OF ENERGY

NSF

NUCLEI
Nuclear Computational Low-Energy Initiative
Questions you might ask about EFT, RG, and SRCs

Why are there so many different chiral EFT interactions out there now?
What are the differences between phenomenological and EFT interactions?
If there are good phenomenological NN interactions, why use an EFT?
Do we expect high momentum distributions in nuclei to agree?
What is relevant for SRCs in the EFT paradigm?
What is an OPE and how is it relevant to SRC physics?
What happens to UV physics like SRCs with RG evolution?
What scale and scheme should we use?
Is off-shell physics measurable? No, but it can be exploited!
Is high momentum in a SRC like high density nucleonic matter?
Is there a “hard core” in chiral EFT NN interactions?
Is the EMC effect unexpected from the EFT perspective?
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### Chiral EFT expansion of nucleon-nucleon force

[from R. Machleidt]

<table>
<thead>
<tr>
<th>Order</th>
<th>2N Force</th>
<th>3N Force</th>
<th>4N Force</th>
<th>5N Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO ((Q/\Lambda_\chi)^0)</td>
<td>(\times)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NLO ((Q/\Lambda_\chi)^2)</td>
<td>(\times) (\times) (\times)</td>
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<tr>
<td>NNLO ((Q/\Lambda_\chi)^3)</td>
<td>(\times) (\times) (\times)</td>
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<tr>
<td>N^3LO ((Q/\Lambda_\chi)^4)</td>
<td>(\times) (\times) (\times) (\times)</td>
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<td>(\times) (\times) (\times) (\times)</td>
<td>(\times) (\times) (\times) (\times)</td>
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<tr>
<td>N^4LO ((Q/\Lambda_\chi)^5)</td>
<td>(\times) (\times) (\times) (\times) (\times)</td>
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<td>(\times) (\times) (\times) (\times) (\times)</td>
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</tr>
<tr>
<td>N^5LO ((Q/\Lambda_\chi)^6)</td>
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**Strong constraints from chiral symmetry**

\[
Q = \frac{\text{momentum, } m_\pi}{\Lambda_\chi} + \text{“Weinberg counting”}
\]

\[
\Lambda_\chi \approx m_\rho \approx 600–700 \text{ MeV} \quad \text{[Test??]}
\]

[This is not the regulator cutoff!]

Some stop here:
can use as local \(V\);
consistent NN+3N;
\(N^3\)LO gets hard (??)

Recent: use statistical tests of convergence patterns

Also with \(\Delta s\)
See PRC 96, 054002 for details (even more potentials now!)

\[ \text{FT}[V(r)] \rightarrow V(q = p - p') \]

Can use with QMC

Try to minimize regulator artifacts (recent soft N^4LO)

Separable: doesn’t mix partial waves and Fierz works; softer than local

<table>
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<tr>
<th>Regulator functions</th>
<th>Regulator exponent(s)</th>
<th>Chiral order/cutoff range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Local</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GT+ [22,23]</td>
<td>( \alpha e^{-\vec{p}^n} )  ( 1 - e^{-\vec{r}^n} )</td>
<td>( n = 4 )</td>
</tr>
<tr>
<td><strong>Semilocal</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EKM [9,24]</td>
<td>( e^{-\vec{p}^{n_1}} e^{-\vec{p}^{n_1}} ) ( (1 - e^{-\vec{r}^2})^{n_2} )</td>
<td>( n_1 = 2 )&lt;br&gt;( n_2 = 6 )</td>
</tr>
<tr>
<td><strong>Nonlocal</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sim [25]</td>
<td>( e^{-\vec{p}^{2n}} e^{-\vec{p}^{2n}} ) ( e^{-\vec{p}^{2n}} e^{-\vec{p}^{2n}} )</td>
<td>( n = 3 )</td>
</tr>
<tr>
<td>EMN [10]</td>
<td>( e^{-\vec{p}^{2n_1}} e^{-\vec{p}^{2n_1}} ) ( e^{-\vec{p}^{2n_2}} e^{-\vec{p}^{2n_2}} )</td>
<td>( n_1 &gt; v/2 )&lt;br&gt;( n_2 = 2(4) )</td>
</tr>
</tbody>
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Should be connected by RG (but they’re not!)
Why are there so many different chiral EFT interactions out there now?

- Most chiral EFT NN+NNN have **same** physics content and power counting N^nLO (with dissenters).
- Different regulators: non-local, local, semi-local; for technical reasons and/or minimizing artifacts.
- Regulation shouldn’t matter (at higher orders) but there are issues at present. Also fitting protocols.

What are the differences between phenomenological and EFT interactions?

If there are good phenomenological NN interactions, why use an EFT?

Do we expect high momentum distributions in nuclei to agree?
Boson exchange → model of short-distance physics
⇒ unresolved in chiral EFT (except for pion)
⇒ encoded in coefficients of contact terms

\[ \frac{g^2}{q^2 + m^2} \]
\[ \frac{g^2}{m^2} \]
\[ - \frac{g^2}{m^2} \left( \frac{q^2}{m^2} \right) \]

(Smeared) contact terms parametrize boson exchange physics and everything else.

Contributions from loop integrals with high \( q \) absorbed in derivative expansion.


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What are the differences between phenomenological and EFT interactions?

- EFT model independence from completeness of operators; phenom. like Taylor expansion missing missing terms.
- But for NN, not much difference at low $E$ beyond fine details of chiral symmetry (hard to see in nuclei).
- Breakdown scale of EFT is physical; not the same as the cutoff, but often taken comparable (not fit!).

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If there are good phenomenological NN interactions, why use an EFT?

- If you only care about NN, you might be better off with phenomenological modeling of high energy.
- Why EFT? QCD, consistent many-body forces and currents; connect pi-N, NN, NNN, ...; UQ enabled.

Do we expect high momentum distributions in nuclei to agree?

- If you only care about NN, you might be better off with phenomenological modeling of high energy.
- Why EFT? QCD, consistent many-body forces and currents; connect pi-N, NN, NNN, ...; UQ enabled.
Only now are all ingredients (e.g., consistent currents, UQ) being put together. Here: Epelbaum et al. from TRIUMF 2019 deuteron form factors (preliminary!)

- Consistent regularization for potential and two-body current respects chiral symmetry.
- Two-pion exchange NN couplings from rigorous pi-N scattering (long-range 3N, too!).
- Bayesian uncertainty quantification based on convergence pattern (68% band).

Key goal: understand emergence of nuclear saturation (connected to radii, b.e.’s)
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Do we expect high momentum distributions in nuclei to agree?

• No, if you define a momentum distribution as probabilities of finding momentum $q$: $\langle a_q^\dagger a_q \rangle$
• Different EFT schemes will differ, as will unitary RG evolution of wfs without evolving operator.
• This is not from lack of information; these distributions are scale and scheme dependent. \textbf{Can we relate?}
Parton distributions as paradigm

\[ F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \hat{F}_2^a(x, Q/\mu_f) \]

Separation between long- and short-distance physics is not unique!

Observable (e.g. form factor) is independent of factorization scale, but pieces are not.

- Deuteron momentum distribution is scale and scheme dependent
- Initial AV18 potential evolved with SRG from \( \lambda = \infty \) to \( \lambda = 1.5 \text{ fm}^{-1} \)
- High momentum tail shrinks as \( \lambda \) decreases (lower resolution)

But exactly the same result with softest potential if you evolve a\( ^+a \)
What is relevant for SRCs in the EFT paradigm?

What is an OPE and how is it relevant to SRC physics?

What happens to UV physics like SRCs with RG evolution?

What scale and scheme should we use?
What is relevant for SRCs in the EFT paradigm?

- EFT has a separation into long-distance physics, which is calculated explicitly order-by-order, and general parameterization of short-distance physics at matching scale. Evolve by RG to other scales.
- Tensor from pion exchange is long-range chiral EFT physics; “core” is unresolved ⇒ contact operators.

What is an OPE and how is it relevant to SRC physics?

What happens to UV physics like SRCs with RG evolution?

What scale and scheme should we use?
Cold atoms near unitarity: an OPE-RG-EFT perspective

E. Braaten et al., arXiv:1008.2922 + ...

System of fermions with short-range interactions with large scattering length $a$.

Described by QFT formulation of Zero-Range Model $\rightarrow$ "pionless EFT":

$$\mathcal{H} = \sum_{\sigma} \frac{1}{m} \nabla \psi_{\sigma}^\dagger \cdot \nabla \psi_{\sigma}^{(\Lambda)} + \frac{g(\Lambda)}{m} \psi_1^\dagger \psi_2^\dagger \psi_2^{(\Lambda)} + V_{\text{external}} \quad g(\Lambda) = \frac{4\pi a}{1 - 2a\Lambda/\pi}$$

UV cutoff $\Lambda$ is required to make matrix elements of these operators well-defined.

The short-distance OPE is an operator identity with $|r| \rightarrow 0$; for example:

$$\psi_{\sigma}^\dagger (R - \frac{1}{2} r) \psi_{\sigma} (R + \frac{1}{2} r) = \psi_{\sigma}^\dagger \psi_{\sigma} (R) + \frac{1}{2} r \cdot [\psi_{\sigma}^\dagger \nabla \psi_{\sigma} (R) \nabla \psi_{\sigma}^\dagger \psi_{\sigma} (R)] - \frac{r}{8\pi} \frac{g(\Lambda)^2 \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1^{(\Lambda)}}{\text{finite}} + \cdots$$

Contact $C = \int d^3R g(\Lambda)^2 \langle \psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1^{(\Lambda)} (R) \rangle$ is in many universal relations because dominant op.

E.g., momentum density at large $k$ from small $|r|$: $n_{\sigma}(k) \rightarrow C/k^4$ [from non-analytic $r$!]

Note that the ratio of $\psi_1^\dagger \psi_2^\dagger \psi_2 \psi_1^{(\Lambda)} (R)$ in different states will be finite from cancellation!
What is relevant for SRCs in the EFT paradigm?

- EFT has a separation into long- and short-distance, which is calculated explicitly order-by-order.
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What is an OPE and how is it relevant to SRC physics?

- Operator product expansion (OPE) manifests factorization into state-independent coefficients for physics above matching scale and operators whose matrix elements are below matching scale.
- QFT basis for contact formalism. If leading operator dominates, then correlations! E.g., SRC/EMC.

What happens to UV physics like SRCs with RG evolution?

What scale and scheme should we use?
General result: separation of scales in loop integrals allows derivative expansion.

Factorization: \( \Delta V_\lambda(k, k') = \int U_\lambda(k, q) V_\lambda(q, q') U_\lambda^\dagger(q', k') \) for \( k, k' < \lambda, \ q, q' \gg \lambda \)

\[ U_\lambda \rightarrow^{K \cdot Q} K(k)[\int Q(q) V_\lambda(q, q') Q(q')]K(k') \text{ with } K(k) \approx 1! \]

See Scott Bogner’s talk for details
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**What happens to UV physics like SRCs with RG evolution?**

- SRG makes unitary transformations, so no physics is lost, but reshuffled (same flaws as original!).
- Leading changes from UV physics can be expanded in contact operators, as with EFT (but no truncation).

**What scale and scheme should we use?**
Current operator evolution

\[ q^2 = 36 \text{ fm}^{-2} \]

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What scale and scheme should we use?

• In QCD, use RG change of scale to ensure perturbation theory is optimal; key is ability to evolve.
• More perturbative interactions are important for some ab initio many-body methods (e.g., coupled cluster or IM-SRG) but not others (QMC). Evolve to low scale for former, latter can use high scale.
• Is interpretation / extraction from experiment better for high scale? If so, can we evolve results?
Is high momentum in a SRC like high density nucleonic matter?

Is there a “hard core” in chiral EFT NN interactions?

Is the EMC effect unexpected from the EFT perspective?

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- Long history in nuclear physics of trying to measure off-shell physics (D-state prob., NNγ, ...).
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- A hard core is required for **local** potentials to explain NN phase shifts (e.g., S-wave change of sign).
- Chiral EFT with local regulators develop strong short-range repulsion from leading contact, but with non-local regulators comes from nucleon momentum dependence. Alternative pictures!

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Match OPE matrix elements: LO nucleon operators to isoscalar twist-two quark operators

\[ R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)G(A) \quad \text{where} \quad G(A) = \langle A| (N^\dagger N)^2 |A\rangle / A\Lambda_0 \]

\[ \implies \text{the slope } \frac{dR_A}{dx} \text{ scales with } G(A) \]

- For chiral EFT, can apply NDA (naïve dimensional analysis) to estimate coefficients:

\[ \mathcal{L}_{\chi\text{eft}} = c_{lmn} \left( \frac{N^\dagger (\cdots) N}{f_\pi^2 \Lambda_\chi} \right)^l \left( \frac{\pi}{f_\pi} \right)^m \left( \frac{\partial^\mu, m_\pi}{\Lambda_\chi} \right)^n f_\pi^2 \Lambda_\chi^2 \]

\[ f_\pi \sim 100 \text{ MeV}, \quad 1000 \geq \Lambda_\chi \geq 500 \implies \frac{1}{7} \leq \frac{\rho_0}{f_\pi^2 \Lambda} \leq \frac{1}{4} \]

- NDA works for fits to $\chi$PT, NN scattering, ... Does it work for EMC coefficient?
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- EFT for EMC: OPE first at QCD level then match to EFT for matrix elements; 2-body op. must be there!
- If mean-field estimate + chiral naturalness valid, then expected EMC slope is consistent with experiment.
- Dominance of leading two-body contact accounts for EMC-SRC correlation. Loopholes?
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Extras
Matching chiral effective field theory to QCD

\[ \langle \Psi | O_{\text{QCD}} | \Psi' \rangle = \langle \Psi | \sum_i O_{\text{EFT}}^{(i)} | \Psi' \rangle \]

- Operators not forbidden are compulsory
- Symmetries limit what is allowed
- Complete set of operators order by order
- Power counting based on naturalness

For chiral EFT, apply NDA (naive dimensional analysis) to estimate coefficients:

- NDA works for fits to \( \chi \)PT, NN scattering, ...
- Always have 1-, 2-, many-body operators!

Many-body currents are inevitable!

\[ \mathcal{L}_{\text{\chi\text{eft}}} = \sum_{n} \frac{m_{\pi}}{\chi} f_{\pi}^2 \Lambda^2 (\frac{\rho_0}{f_{\pi}^2 \Lambda}) \]

\( f_{\pi} \sim 100 \text{ MeV} \)

- \( \rho_0 \leq \frac{1}{4} \)
- NDA leading, ...
SRCs and the EMC effect in EFT


\[ a_2(A) = \frac{2 \rho_{2,1}(A, 0)}{A \rho_{2,1}(2, 0)} \]

\[ \chi^2 / \text{ndf} = 0.7688 / 3 \]

\[ a = -0.07879 \pm 0.006376 \]