

# Contact calculations of $a_2$

Ronen Weiss

The Hebrew University of Jerusalem

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# Inclusive scattering – $a_2$

**$a_2$  definition:**

$$a_2 = \frac{\sigma_A^{inc} / A}{\sigma_d^{inc} / 2}$$

$$1.5 < x_B < 1.9 \quad (Q^2 > 1.5 \text{ GeV}^2)$$

**Interpretation:**

The number of (deuteron-like) SRC pairs relative to the deuteron

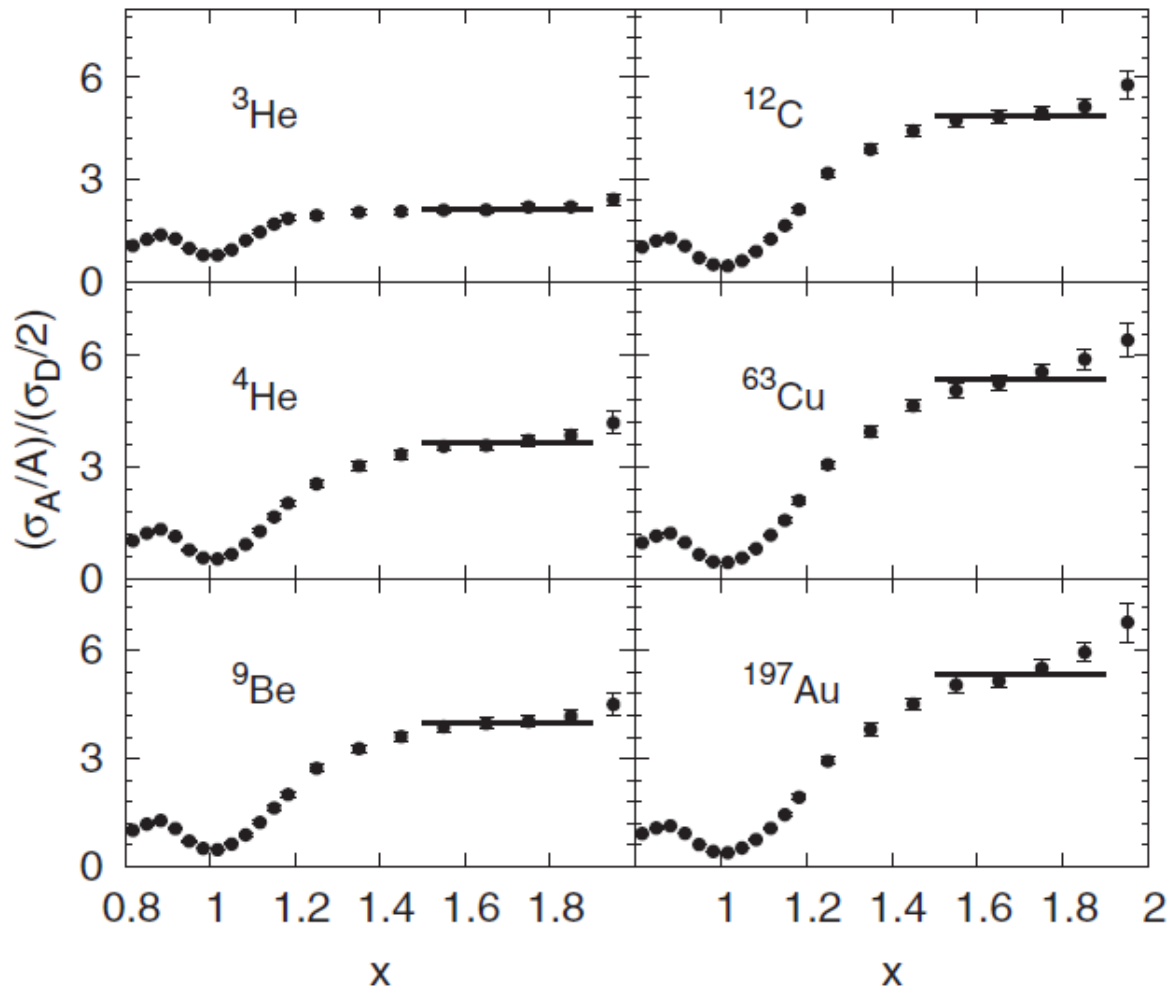
**Connection to momentum/coordinate-space densities:**

$$a_2 \approx n_A(k) / n_d(k) \quad ; \quad k \rightarrow \infty$$

$$a_2 \approx \rho_A(r_{rel}) / \rho_d(r_{rel}) \quad ; \quad r_{rel} \rightarrow 0$$

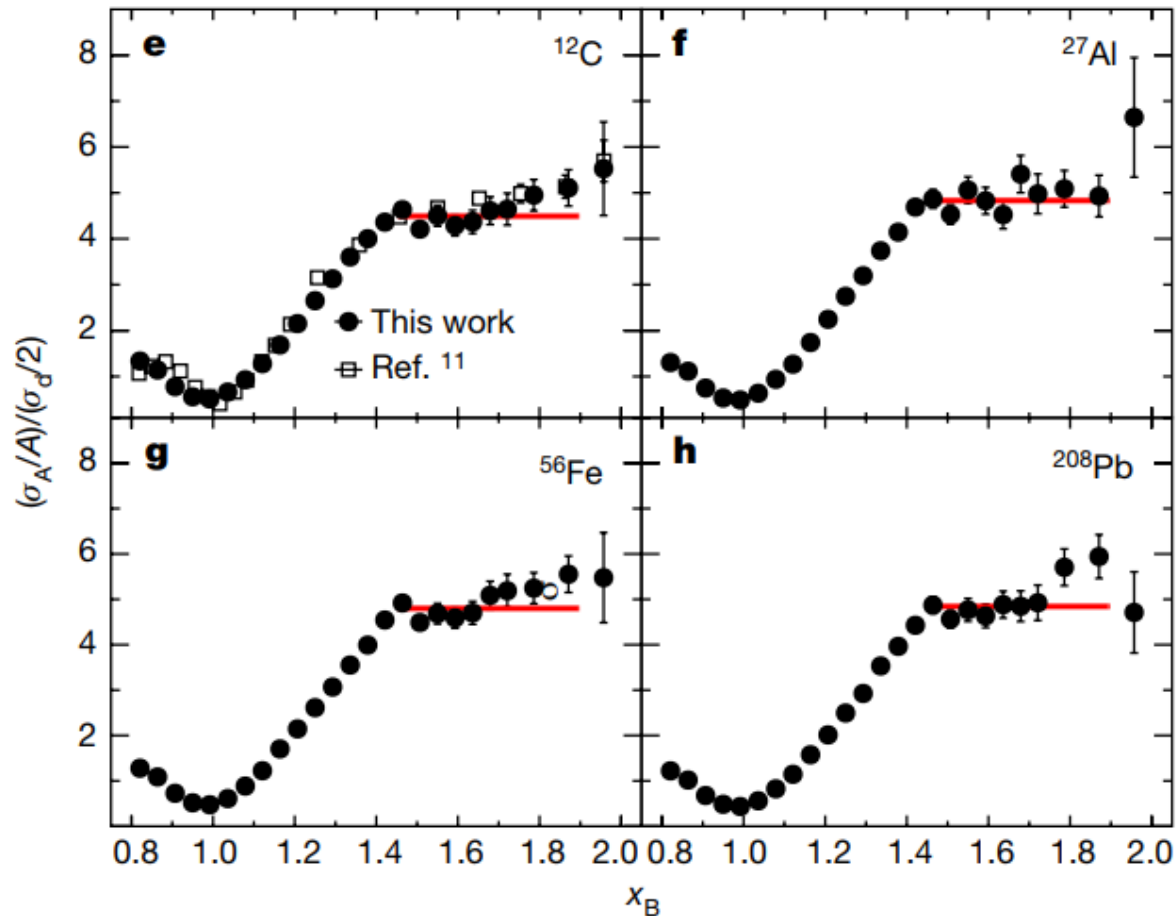
# Inclusive scattering – $a_2$

$a_2$  vs  $x_B$  at electron scattering angle  $\theta_e = 18^\circ$



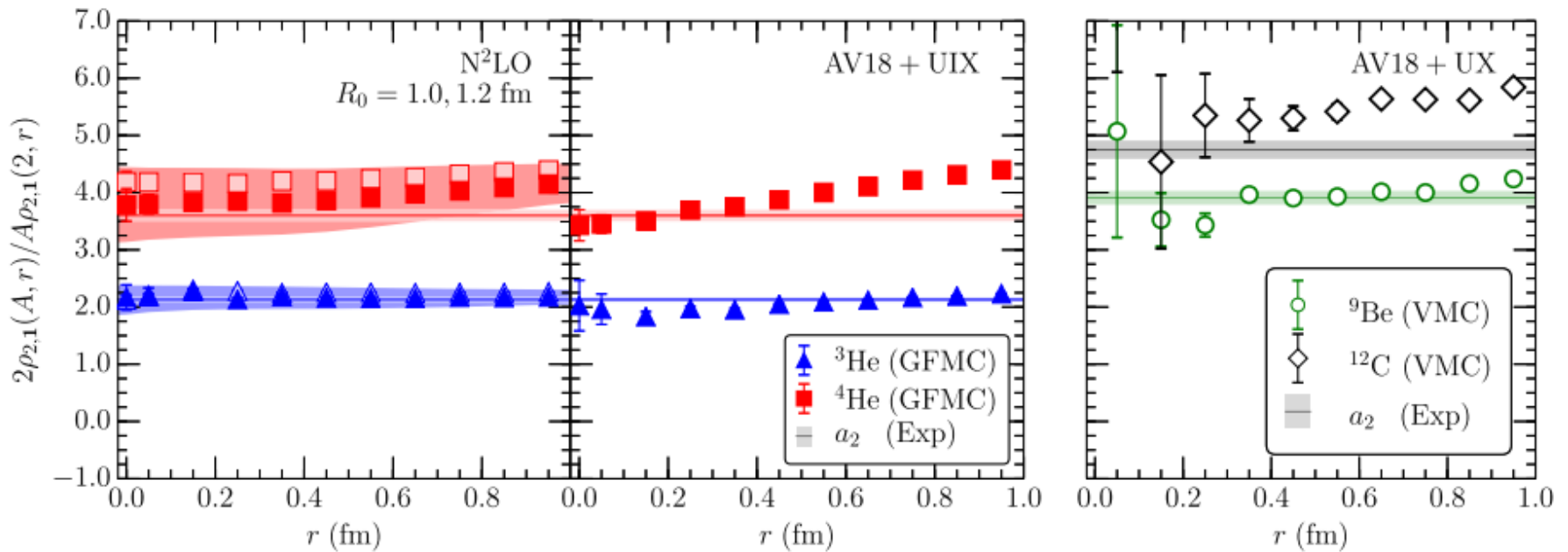
# Inclusive scattering – $a_2$

$a_2$  vs  $x_B$  for  $Q^2 > 1.5 \text{ GeV}^2$



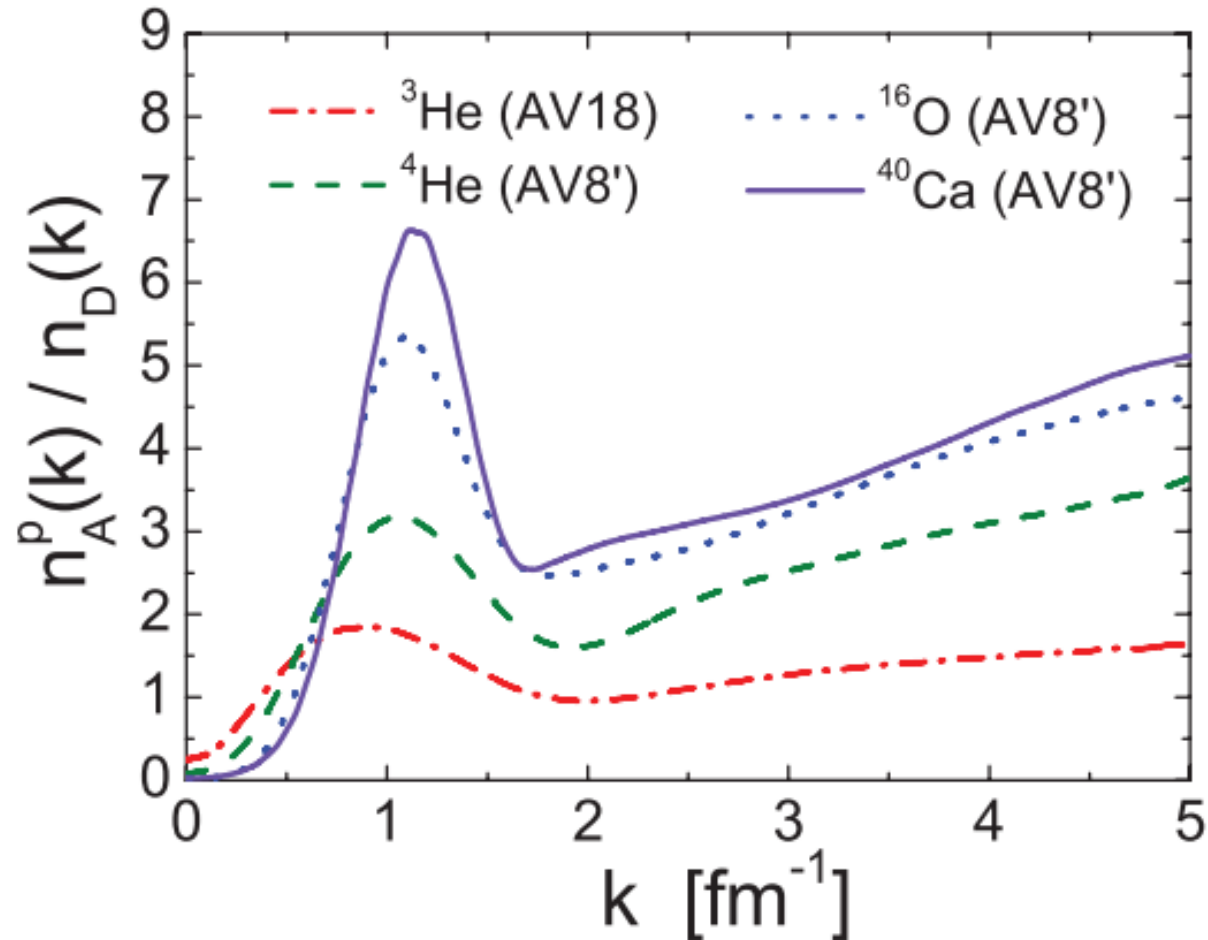
# Inclusive scattering – $a_2$

Ratios of coordinate-space densities



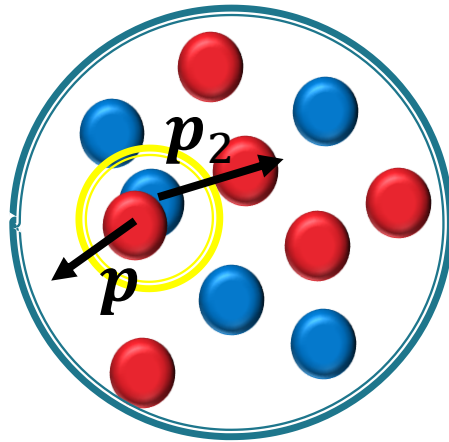
# Inclusive scattering – $a_2$

Ratios of momentum distributions



# A vs 2

Nucleus A

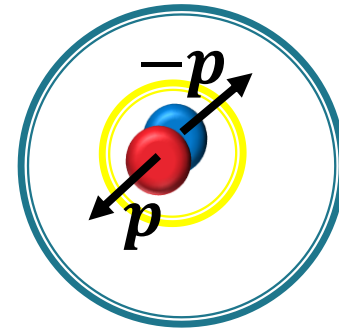


Off-shell pair

CM width  $\sim 100 - 200$  MeV/c

Complex A-2 system

Deuteron

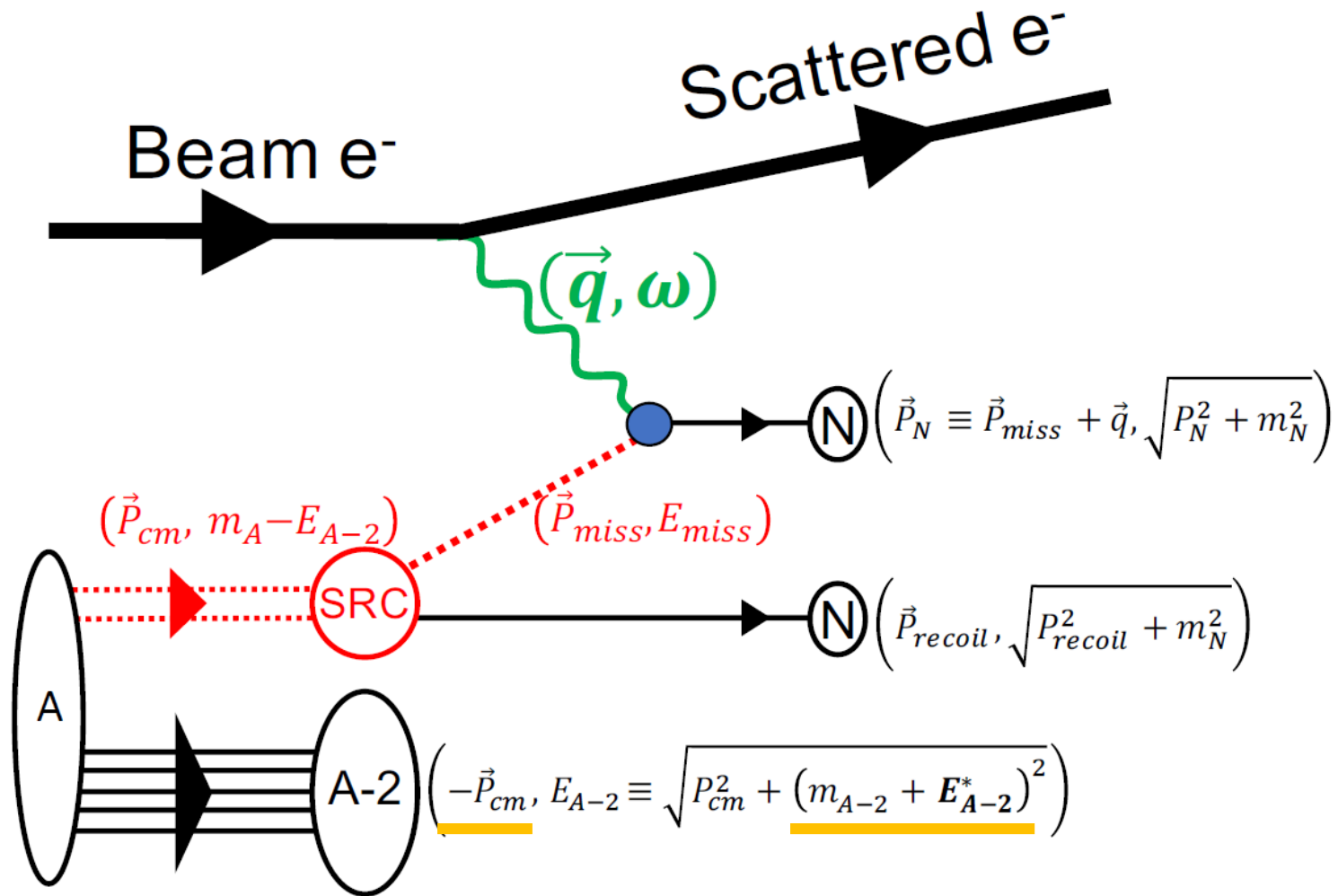


Bound deuteron

Zero CM width

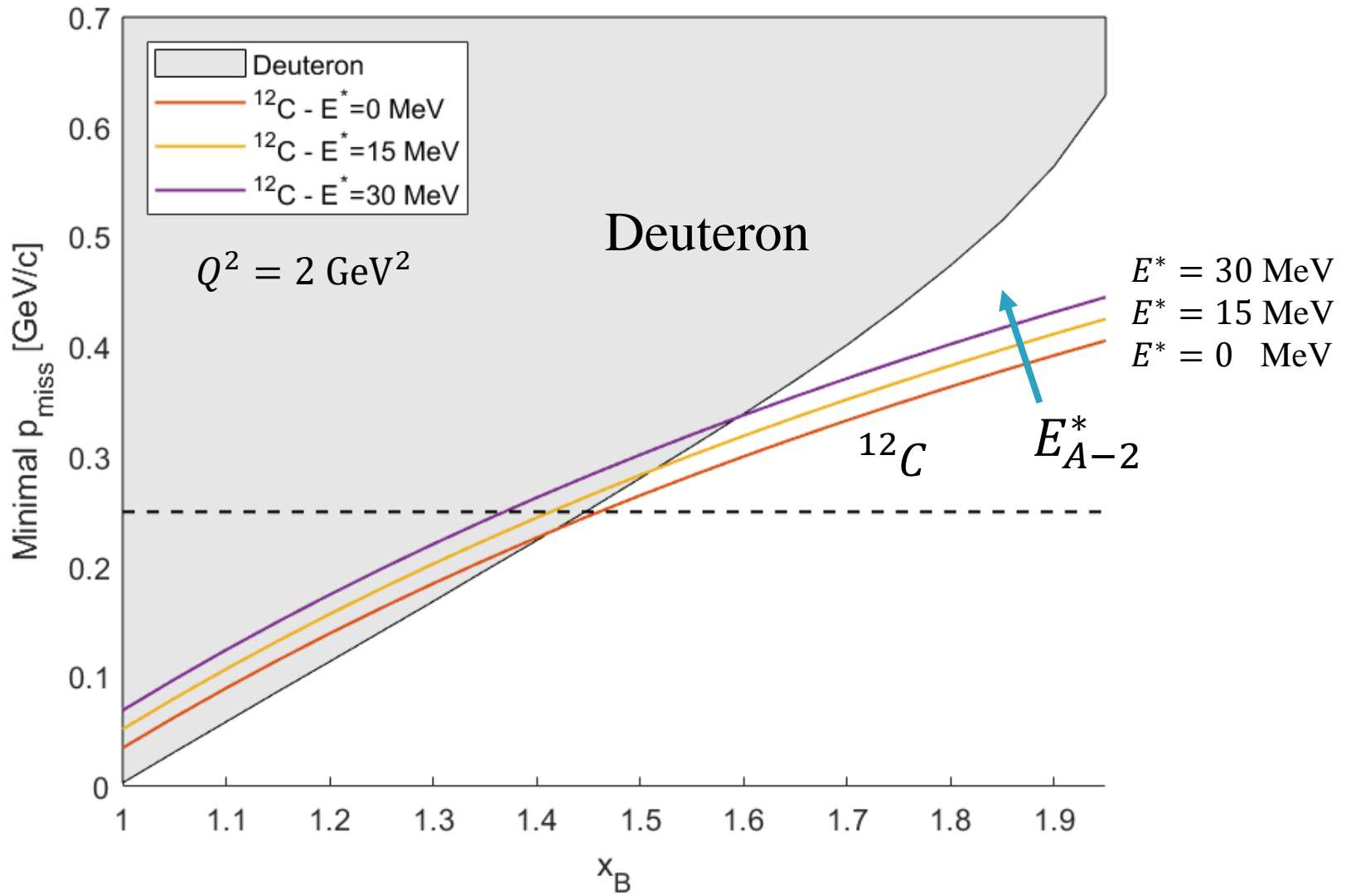
No A-2 system

# Inclusive cross section



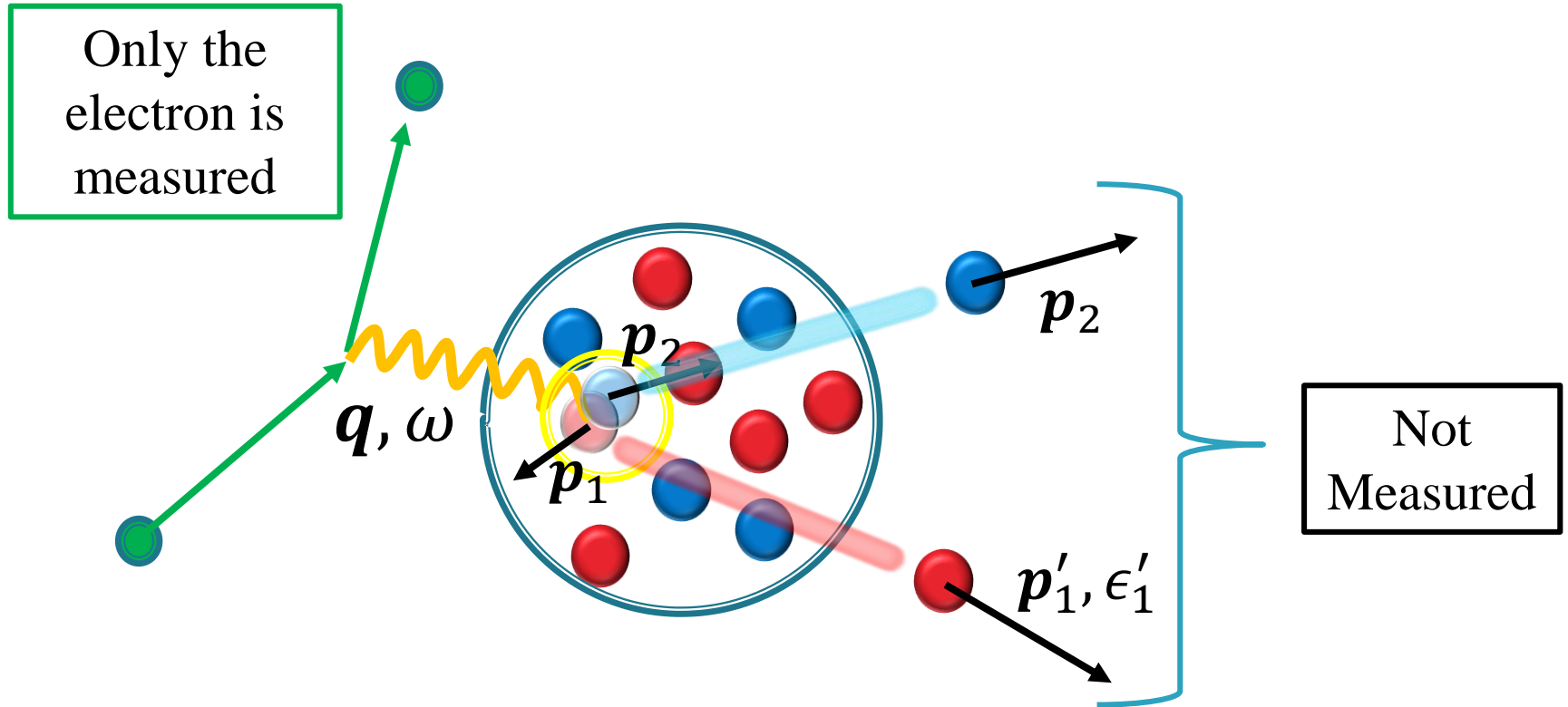


# Minimal $p_{miss}$



**Calculating  $a_2$  using  
the contact  
formalism**

# Inclusive cross section



$$\sigma_A^{inc}$$




$$a_2 \equiv \frac{\sigma_A^{inc} / A}{\sigma_d^{inc} / 2}$$

# Inclusive cross section

Integrating over the unmeasured momentum:

$$\sigma_p^{inc} = \int d^3p' \sigma_{ep} S^p(\mathbf{p}_1, \epsilon_1)$$
$$\sigma_n^{inc} = \int d^3p' \sigma_{en} S^n(\mathbf{p}_1, \epsilon_1)$$

Off-shell electron-nucleon cross section



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Off-shell electron-nucleon cross section

Integral over the allowed phase space

CM effects

# Inclusive cross section

Integrating over the unmeasured momentum:

$$\sigma_p^{inc} = \int d^3 p' \sigma_{ep} S^p(\mathbf{p}_1, \epsilon_1)$$


Off-shell electron-nucleon cross section

$$\sigma_n^{inc} = \int d^3 p' \sigma_{en} S^n(\mathbf{p}_1, \epsilon_1)$$

$$\sigma^{inc} = \sigma_p^{inc} + \sigma_n^{inc}$$

$$S^p(\mathbf{p}_1, \epsilon_1) = C_{pn}^1 S_{pn}^1(\mathbf{p}_1, \epsilon_1) + C_{pn}^0 S_{pn}^0(\mathbf{p}_1, \epsilon_1) + 2C_{pp}^0 S_{pp}^0(\mathbf{p}_1, \epsilon_1)$$

# Inclusive cross section

Integrating over the unmeasured momentum:

Off-shell electron-nucleon cross section

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$$\frac{\sigma_A^{inc}}{\sigma_M} = C_{pn}^1 \sigma_{pn}^1(\omega, q) + C_{pn}^0 \sigma_{pn}^0(\omega, q) + 2C_{pp}^0 \sigma_{pp}^0(\omega, q) + 2C_{nn}^0 \sigma_{nn}^0(\omega, q)$$

# Results



# Results: Only deuteron pairs

The CM-width effect:

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The CM-width effect:

Input SRC ratio:  $\frac{c_{pn}^1(A)/A}{c_{pn}^1(d)/2} = 4$

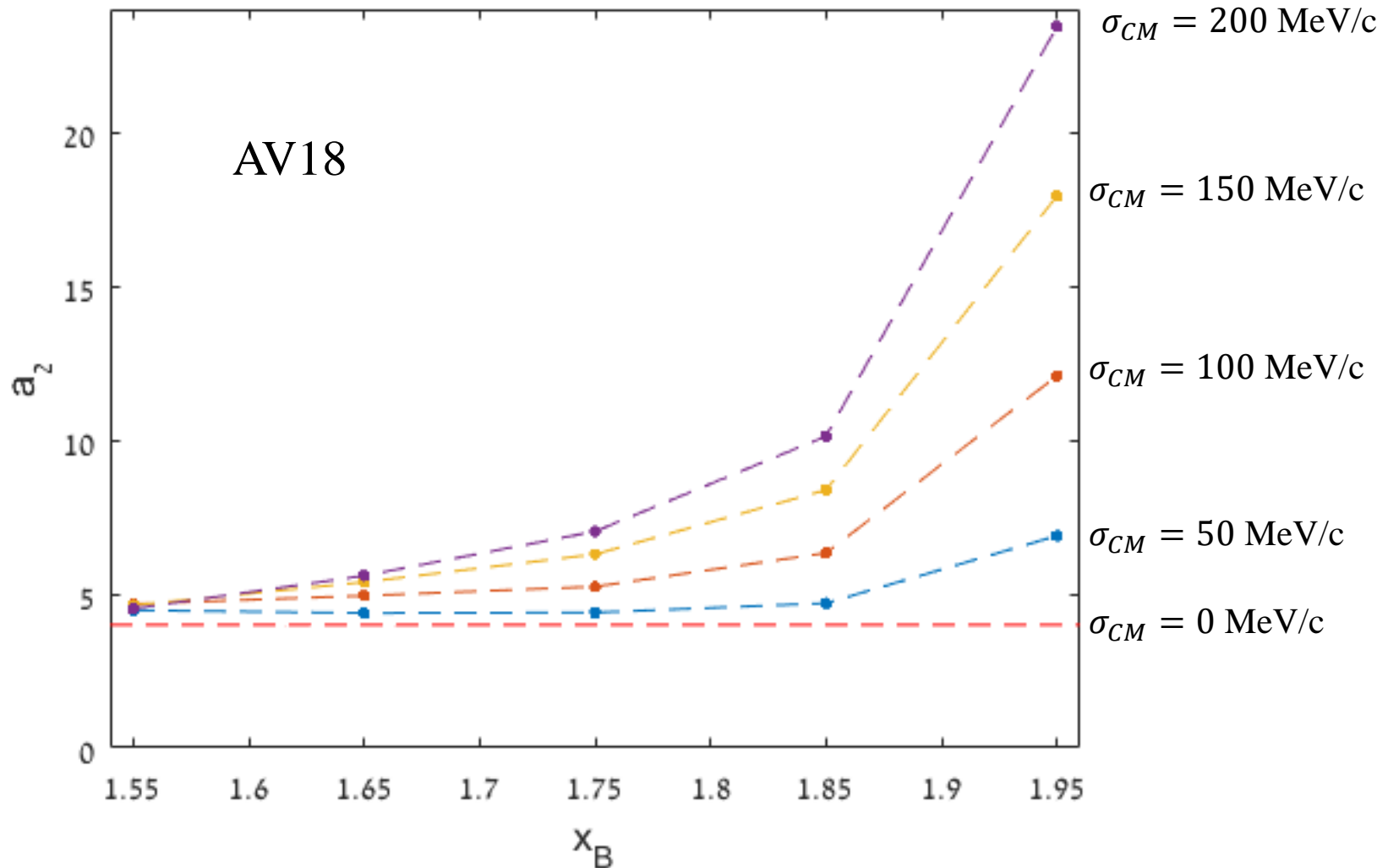
On-shell deuteron  $\left\{ \begin{array}{l} B_A - B_{A-2} = 2.224 \text{ MeV} \\ E_{A-2}^* = 0 \text{ MeV} \end{array} \right.$

$$A = 208$$

Fomin's kinematics  $\left\{ \begin{array}{l} \theta_e = 18^\circ \\ \epsilon_e = 5.766 \text{ GeV} \end{array} \right.$

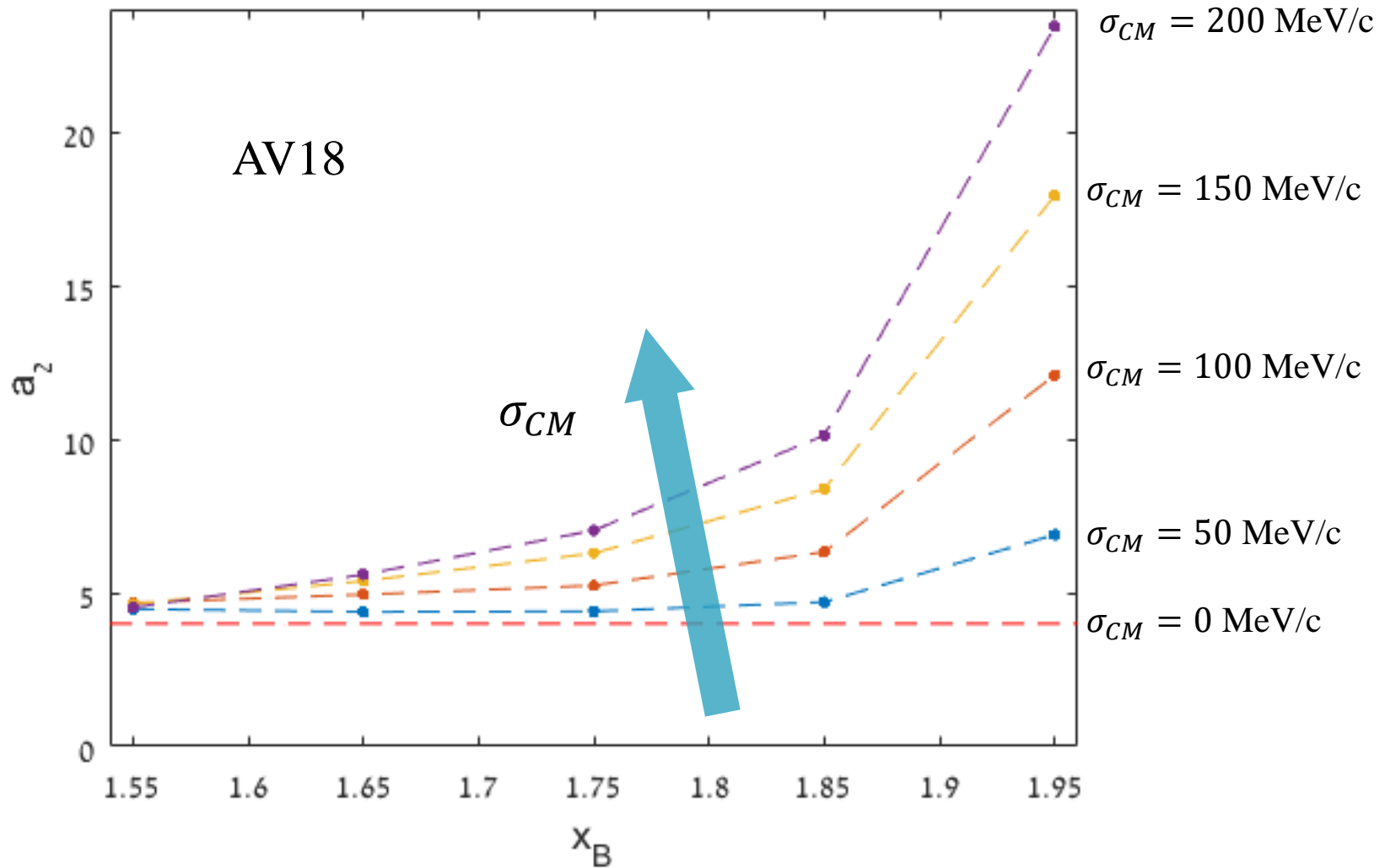
# Results: Only deuteron pairs

The CM-width effect:



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The CM-width effect:



# Results: Only deuteron pairs

## The A-2 excitation:

Input SRC ratio:  $\frac{c_{pn}^1(A)/A}{c_{pn}^1(d)/2} = 4$

Off-shell deuteron  $\left\{ \begin{array}{l} B_A - B_{A-2} = 16 \text{ MeV} \\ E_{A-2}^* = 0-30 \text{ MeV} \end{array} \right.$

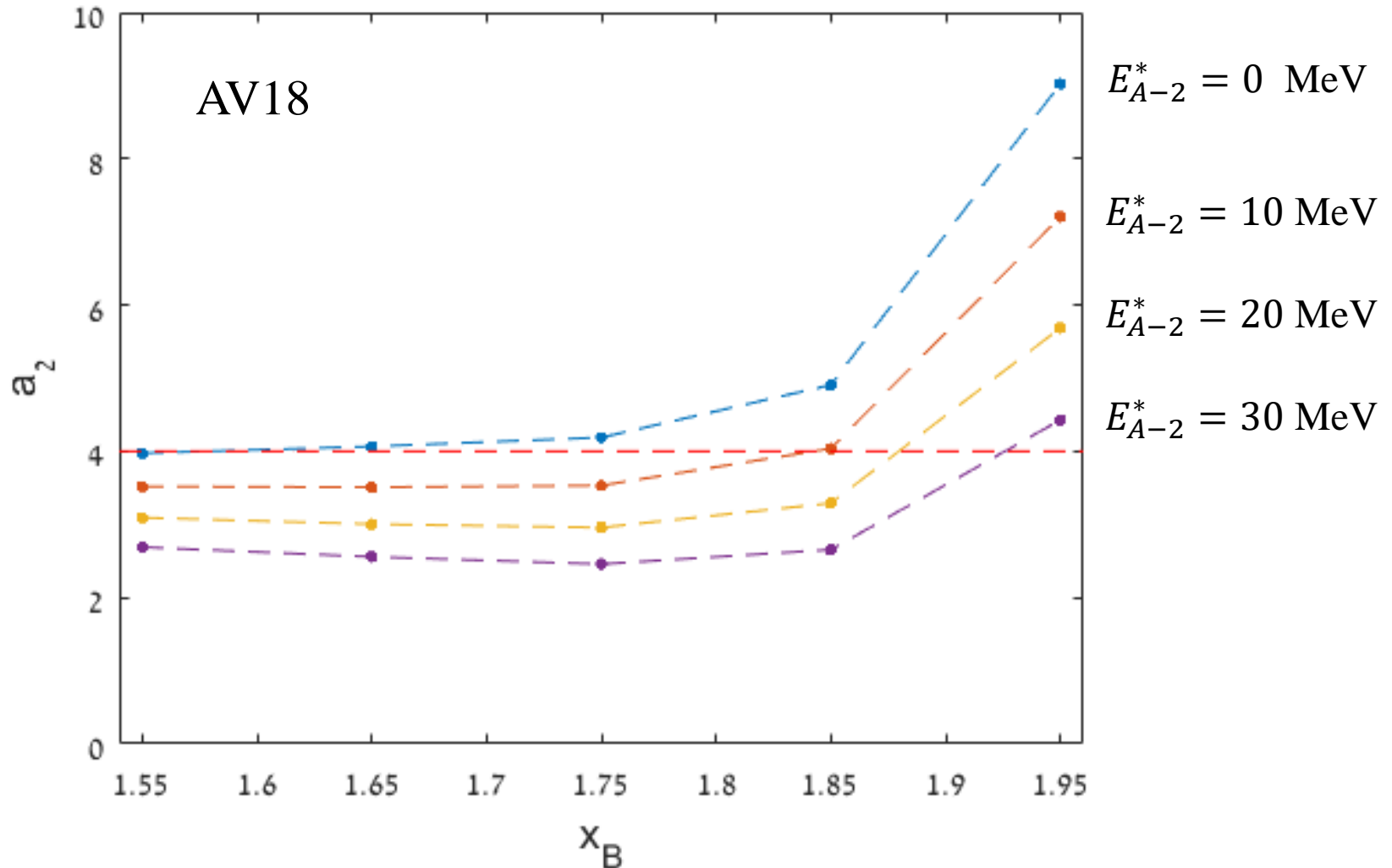
$$\sigma_{CM} = 100 \text{ MeV}/c$$

$$A = 208$$

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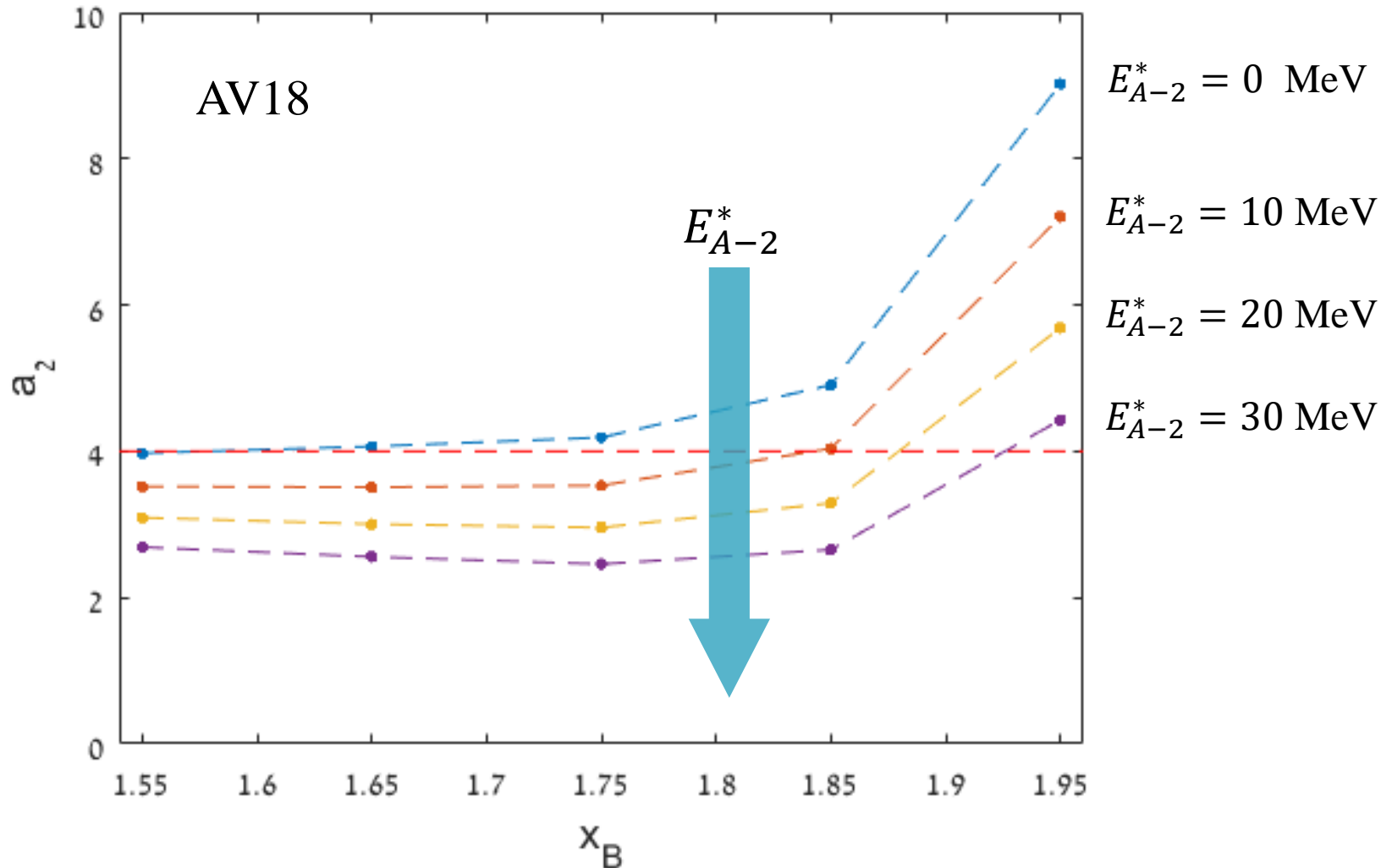
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The A-2 excitation:



# Results: Also $S = 0, T = 1$ pairs

Input SRC ratio:  $\frac{C_{pn}^1(A)/A}{C_{pn}^1(d)/2} = 4$

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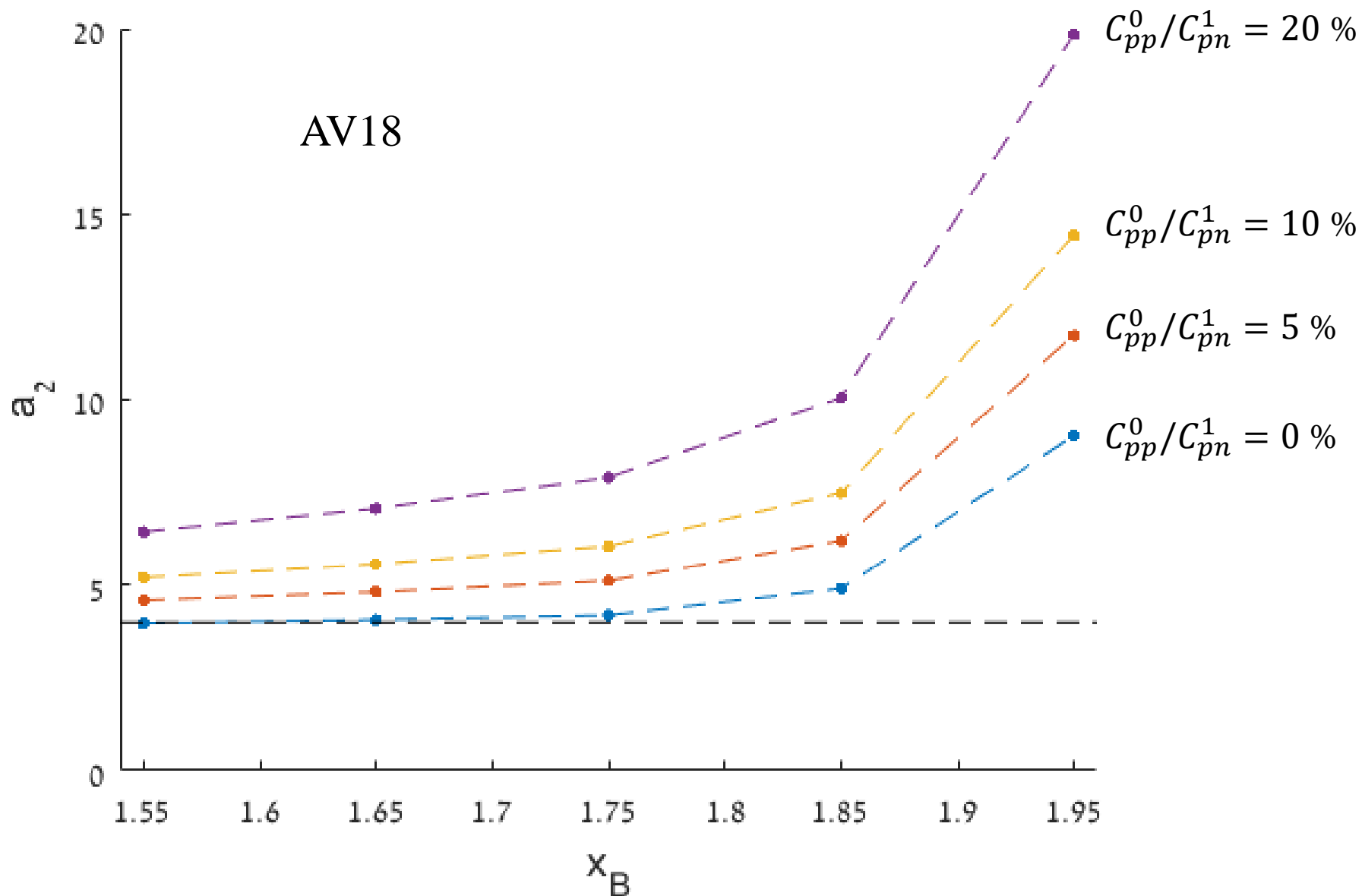
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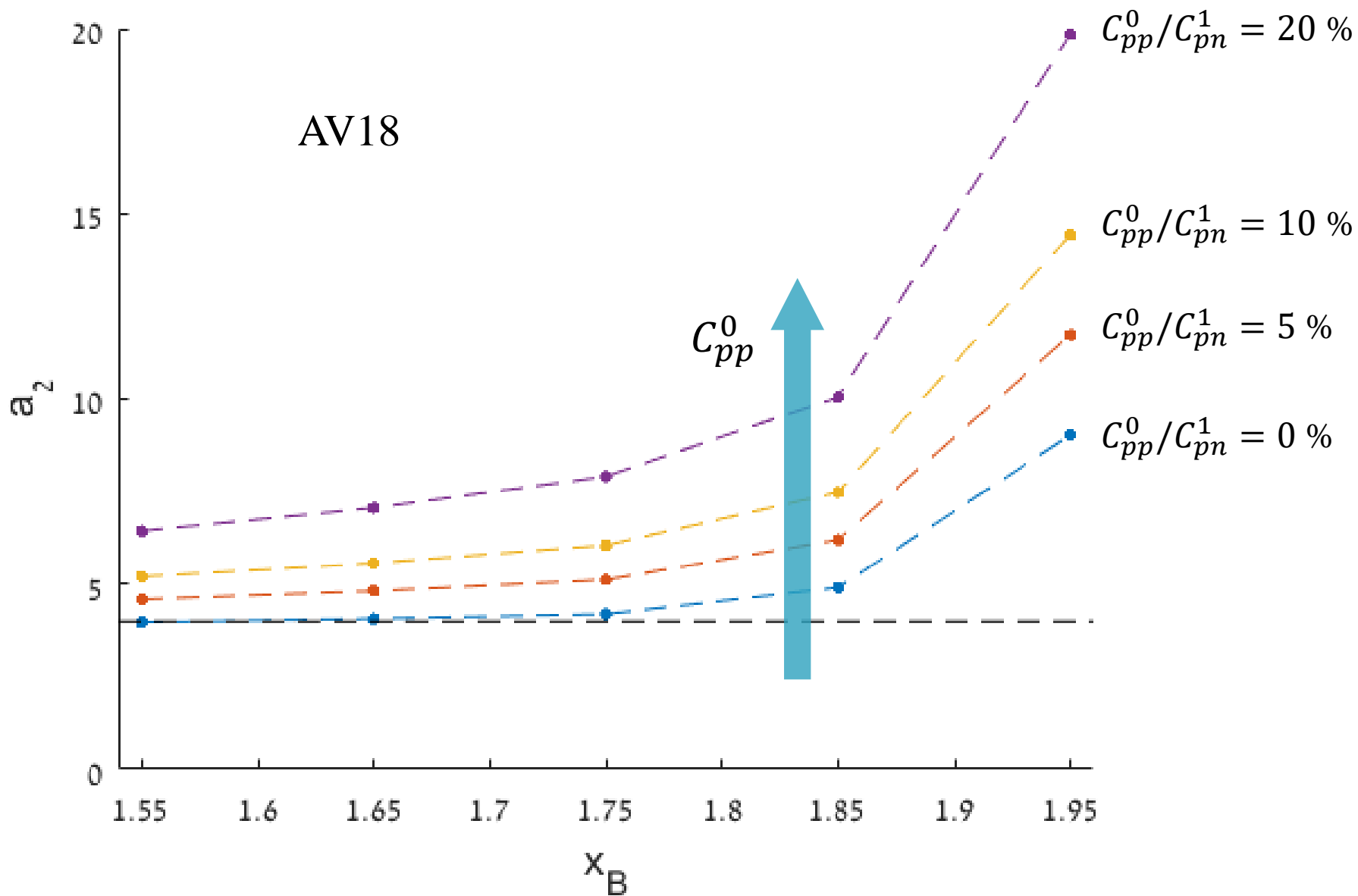
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# Results: Also $S = 0, T = 1$ pairs



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# Results

## Deviations from the $a_2$ interpretation:

- CM width – increases  $a_2$  and creates a slope
- $A - 2$  excitation – decreases  $a_2$
- $T = 1$  pairs – increase  $a_2$

# Experimental data - $^{12}\text{C}$

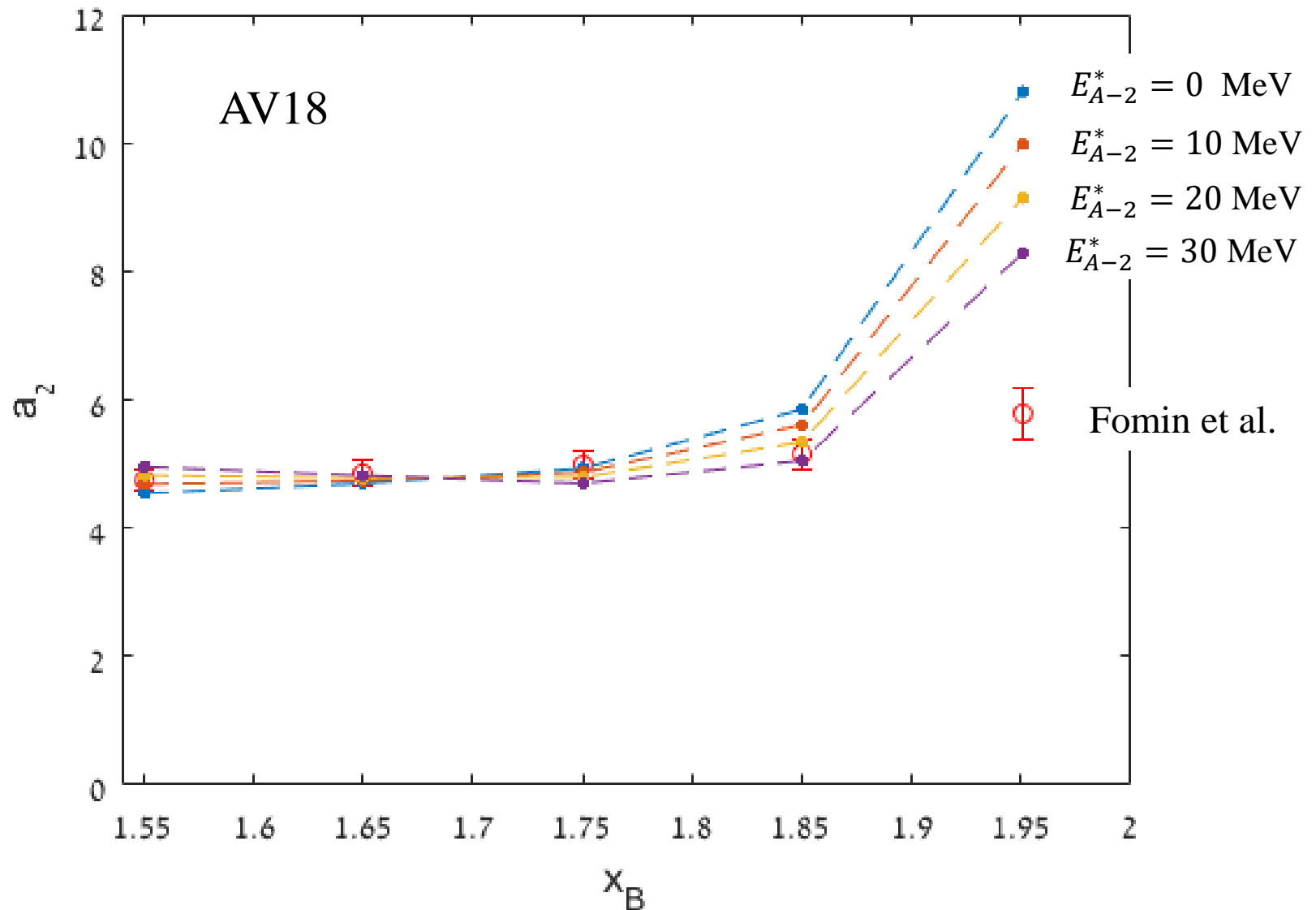
$$C_{pn}^1/C_{pp}^0 = 12.5 \text{ (fixed - VMC results)}$$

$$\sigma_{CM} = 100 \text{ MeV}/c$$

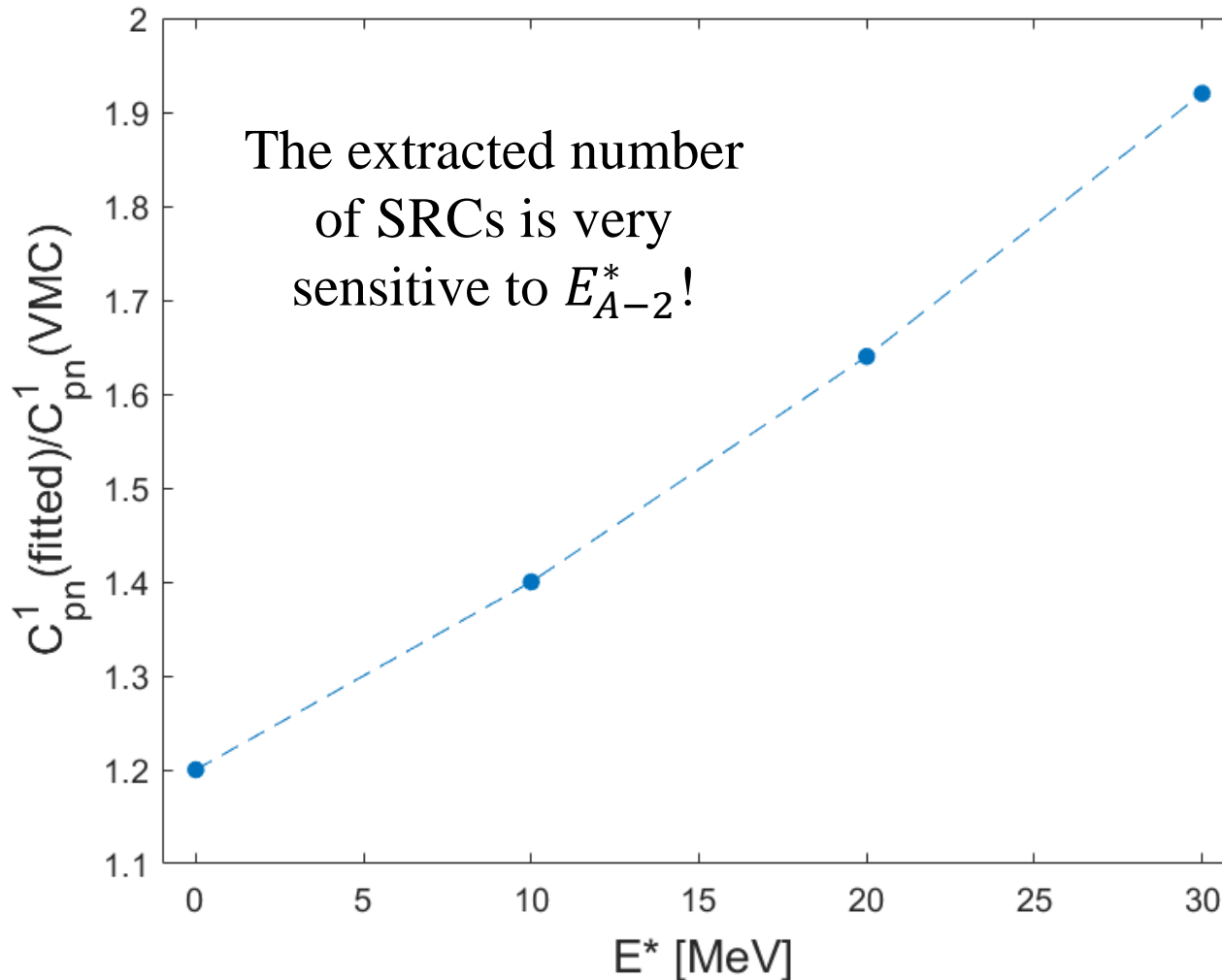
$$E_{A-2}^* = 0\text{-}30 \text{ MeV}$$

Fitting  $C_{pn}^1$

# Experimental data - $^{12}\text{C}$

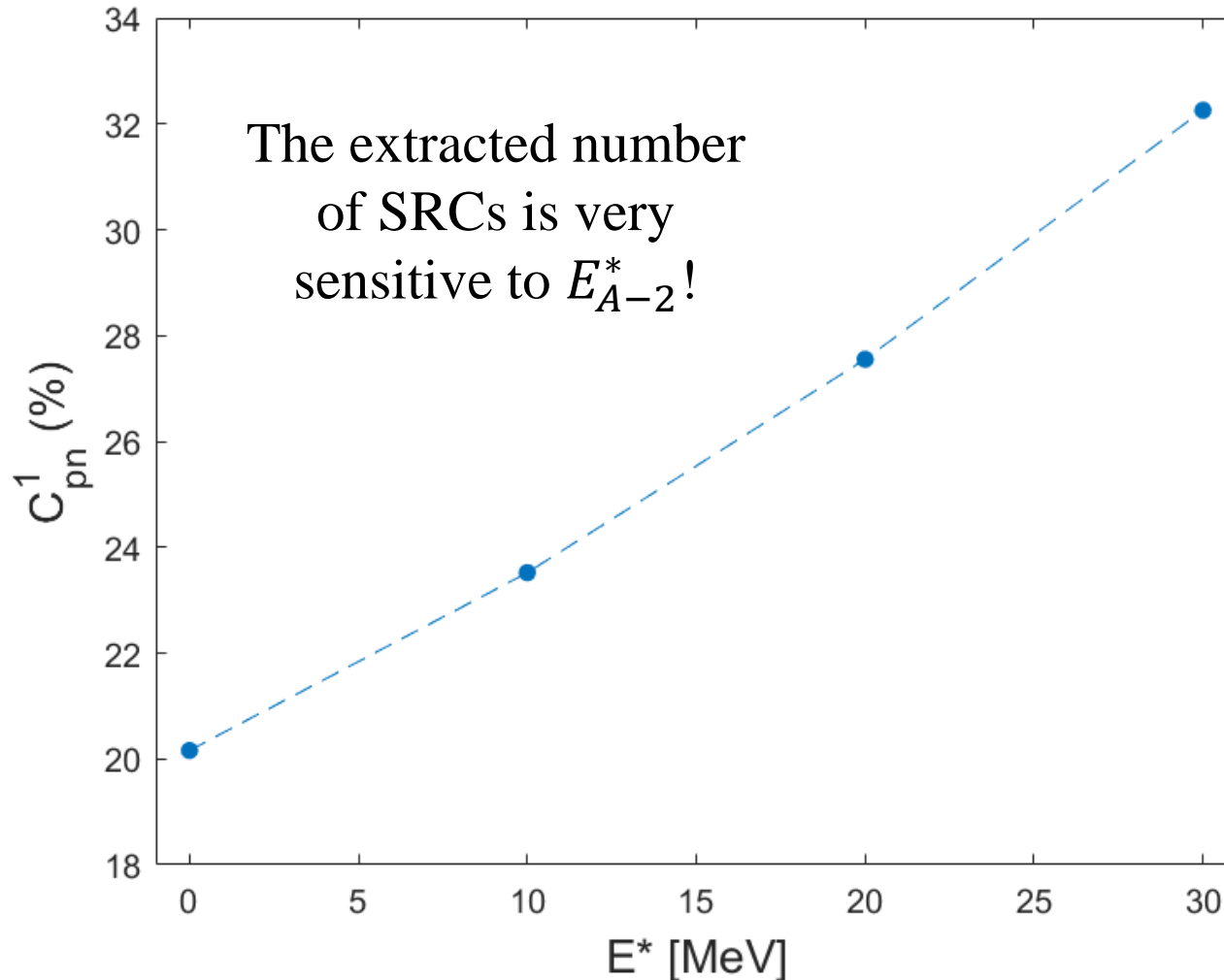


# Experimental data - $^{12}\text{C}$



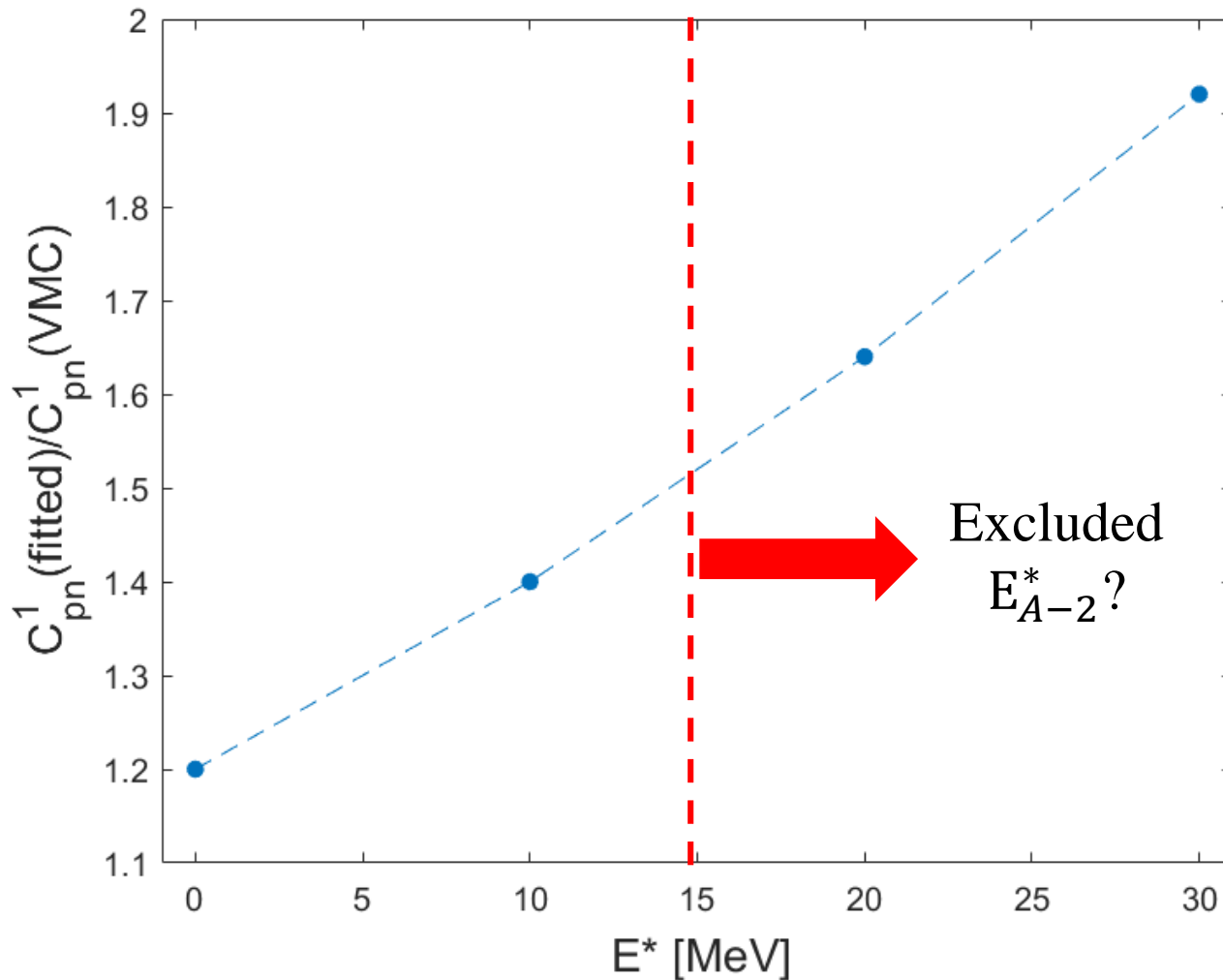
AV18 VMC results:  $C_{pn}^1(^{12}\text{C}) = 16.8 \pm 0.8$

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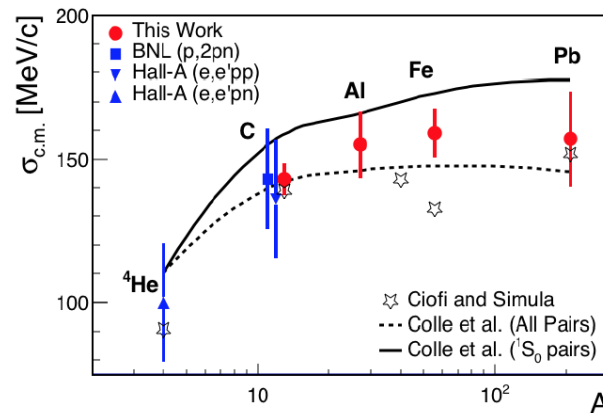
AV18 VMC results:  $C_{pn}^1(^{12}\text{C}) = 16.8 \pm 0.8$



# Light vs Heavy nuclei

## For heavy nuclei:

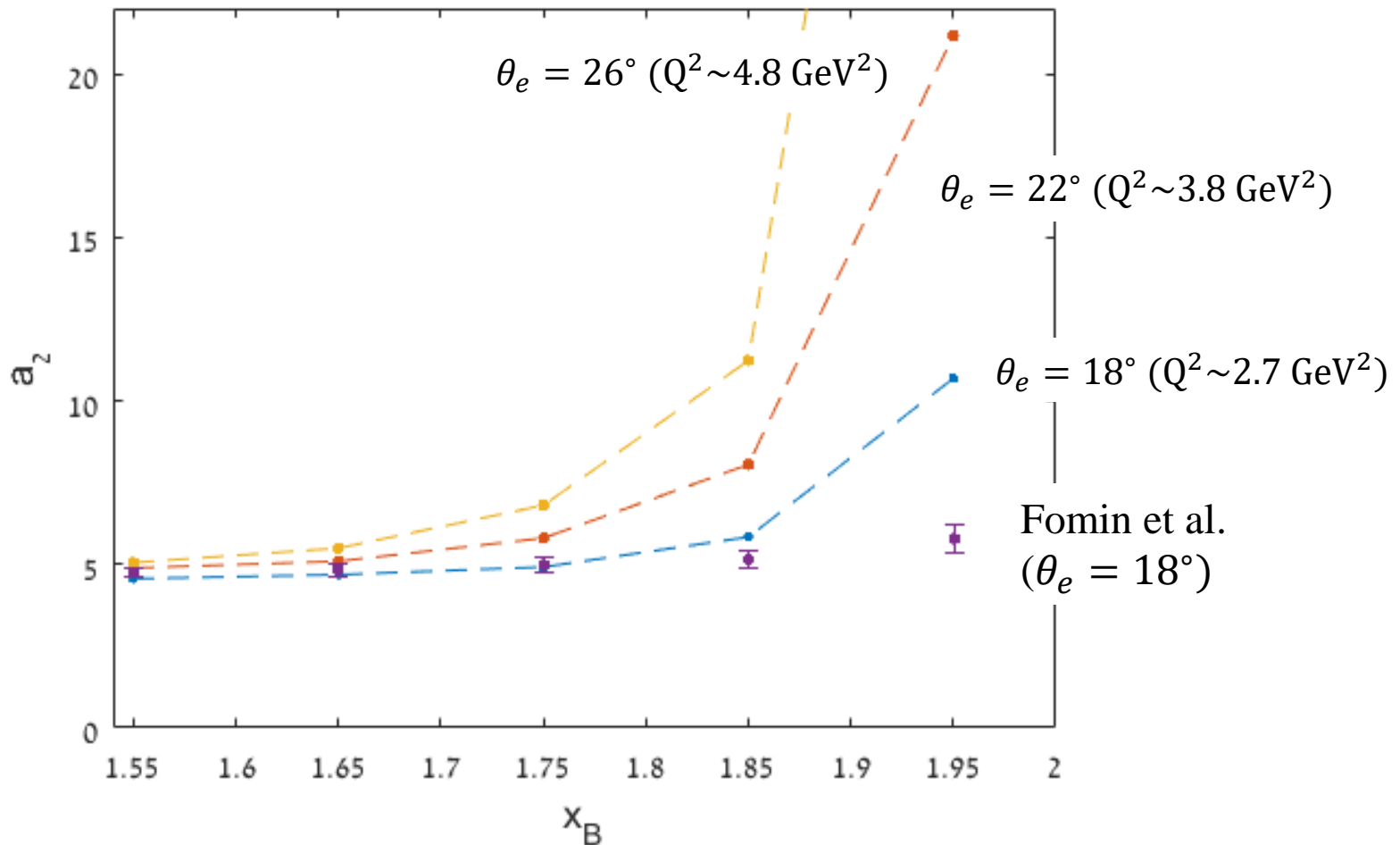
- $\sigma_{CM}$ ,  $E^*$ , and  $T = 0$  – expected to saturate
- leads to an (unknown) **constant correction** ( $E^*$  is unknown)



## For light nuclei:

- $\sigma_{CM}$ ,  $E^*$ , and  $T = 0$  – expected to depend on  $A$
- Leads to **difficulties comparing between light nuclei**

# Results: $Q^2$ dependence

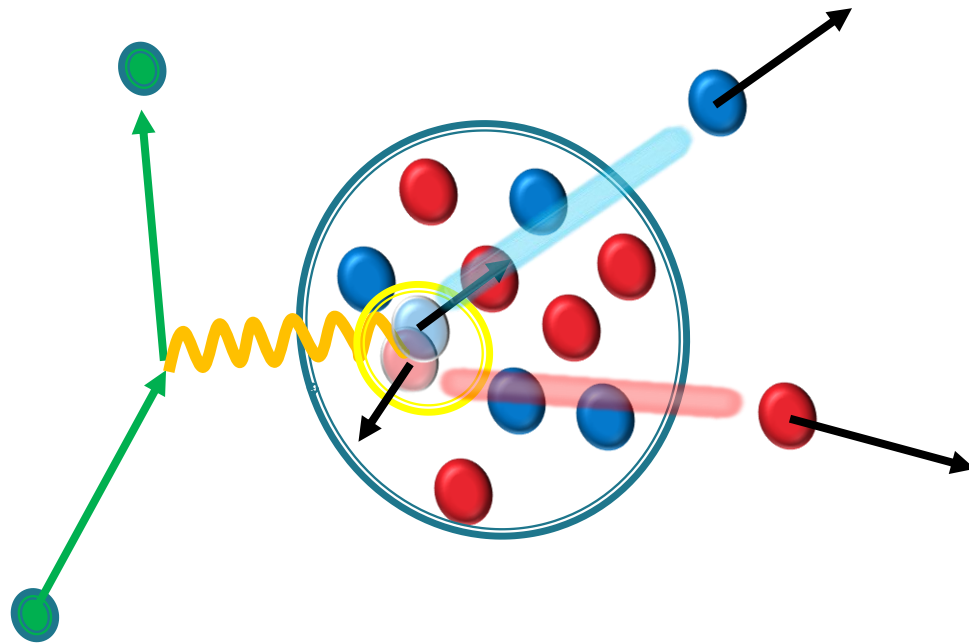


Using the fitted contact values for  $\sigma_{CM} = 100 \text{ MeV}/c$ ,  $E^* = 0$

# Summary

1. Experimental data of  $a_2$  can be described using **contact-formalism cross-section calculations**
2. The value of  $a_2$  is **sensitive** to  $\sigma_{CM}$  and  $E_{A-2}^*$
3. External inputs are necessary to extract the amount of SRCs from  $a_2$
4. Important for the interpretation of  $a_2$  of **light nuclei**
5. We can use  $a_2$  to learn about the (A-2) system ( $E_{A-2}^*$ )
6. We can provide  **$Q^2$ -dependence** predictions

# Questions?



# The spectral function - Reminder

$$S^p(\mathbf{p}_1, \epsilon_1) = C_{pn}^1 S_{pn}^1(\mathbf{p}_1, \epsilon_1) + C_{pn}^0 S_{pn}^0(\mathbf{p}_1, \epsilon_1) + 2C_{pp}^0 S_{pp}^0(\mathbf{p}_1, \epsilon_1)$$

$$S_{ab}^\alpha(\mathbf{p}_1, \epsilon_1) = \frac{1}{4\pi} \int \frac{d^3 p_2}{(2\pi)^3} \delta(f(\mathbf{p}_2)) n_{CM}(\mathbf{p}_1 + \mathbf{p}_2) |\tilde{\varphi}_{ab}^\alpha(|\mathbf{p}_1 - \mathbf{p}_2|/2)|^2$$

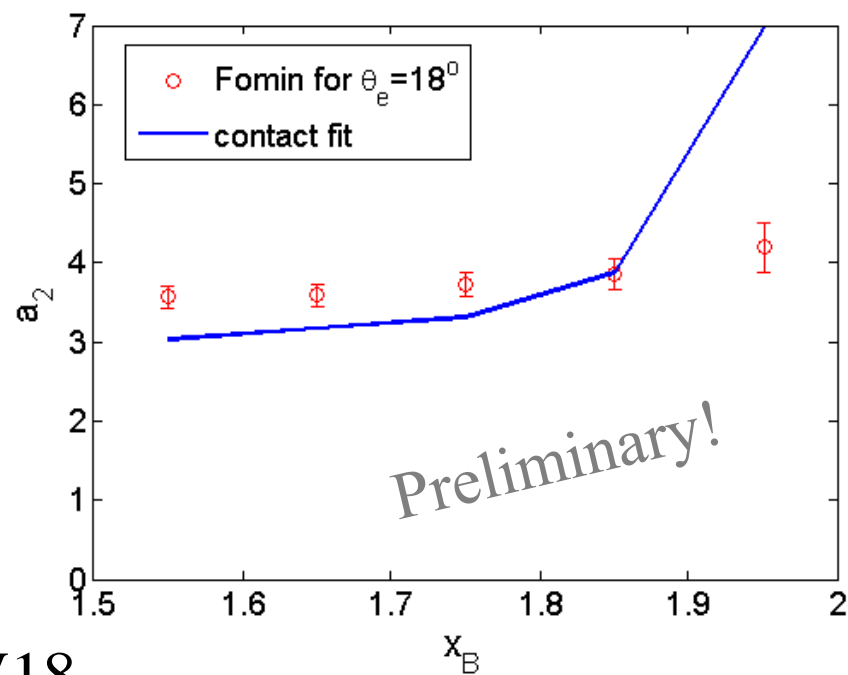
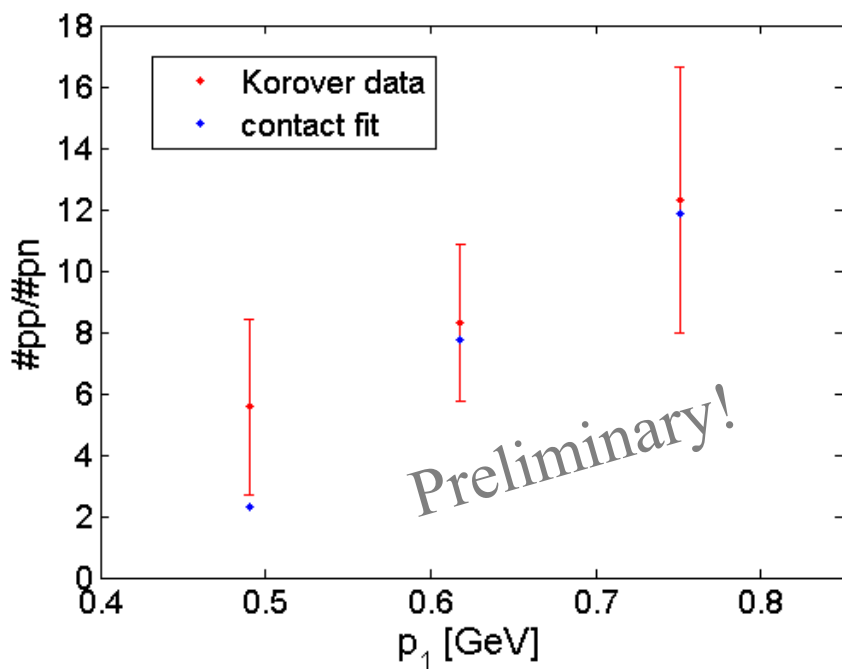
$$f(\mathbf{p}_2) \equiv \epsilon_1 + \sqrt{p_2^2 + m^2} - 2m + (B_i^A - \langle B_f^{A-2} \rangle) + \frac{(\mathbf{p}_1 + \mathbf{p}_2)^2}{2m(A-2)}$$

$$n_{CM}(\mathbf{K}) \propto e^{-\frac{K^2}{2\sigma_{CM}^2}}$$

# Inclusive experiments

Fitting inclusive and exclusive experiments simultaneously -  ${}^4\text{He}$

$$\sigma_{CM} = 100 \text{ MeV and } B_f^{A-2} = \text{ground state}$$



$$C_{pn}^d({}^4\text{He}) \times \frac{100}{A/2} = 16.8 \quad ; \quad C_{pp}^0({}^4\text{He}) \times \frac{100}{A/2} = 0.75$$

Previous results:

11-16

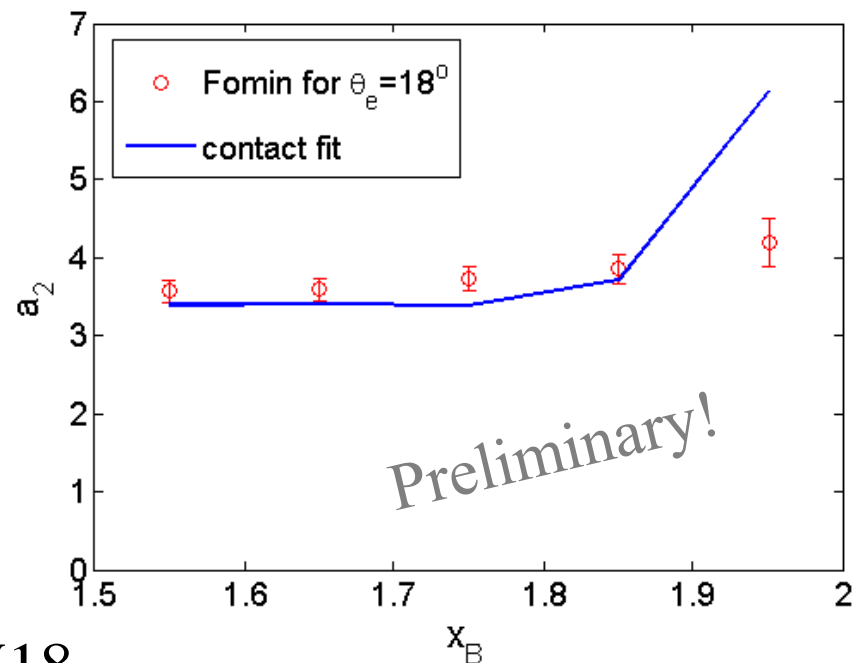
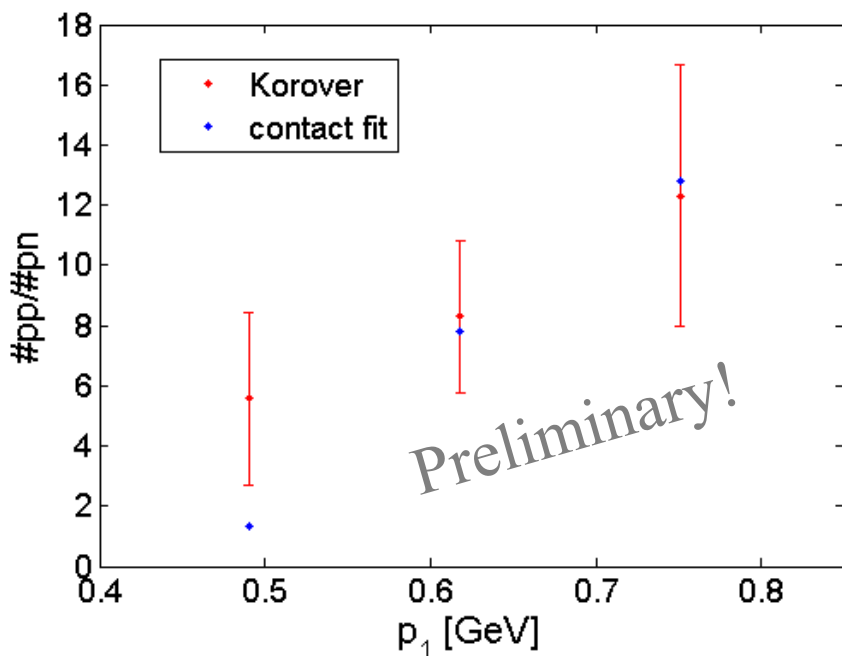
;

0.5-1

# Inclusive experiments

Fitting inclusive and exclusive experiments simultaneously -  ${}^4\text{He}$

$$\sigma_{CM} = 100 \text{ MeV} \text{ and } B_f^{A-2} = \text{ground state} + 20 \text{ MeV}$$



AV18

$$C_{pn}^d({}^4\text{He}) \times \frac{100}{A/2} = 24.3 ; C_{pp}^0({}^4\text{He}) \times \frac{100}{A/2} = 1.25$$

Previous results:

11-16

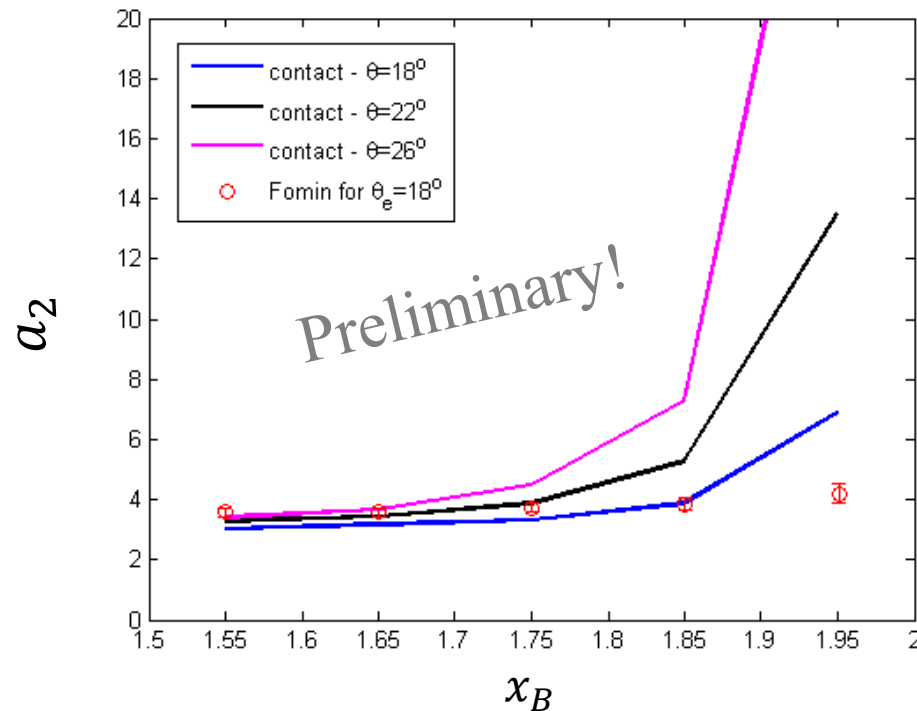
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# Inclusive experiments

$Q^2$ -dependence -  ${}^4\text{He}$

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$$C_{pn}^d({}^4\text{He}) \times \frac{100}{A/2} = 16.8 \quad ; \quad C_{pp}^0({}^4\text{He}) \times \frac{100}{A/2} = 0.75$$

Previous results:

11-16

;

0.5-1



# Inclusive cross section

The inclusive cross section for nucleus A:

$$C_{pn}^d(A)\sigma_{pn}^d(\omega, q; A) + C_{pn}^0(A)\sigma_{pn}^0(\omega, q; A) + 2C_{pp}^0(A)\sigma_{pp}^0(\omega, q; A) + 2C_{nn}^0(A)\sigma_{nn}^0(\omega, q; A)$$

For the deuteron

$$C_{pn}^d(d)\sigma_{pn}^d(\omega, q, A = 2)$$

If the  $T = 1$  contributions are negligible:

$$\left(\frac{A}{2}\right) a_2 = \frac{C_{pn}^d(A)\sigma_{pn}^d(\omega, q; A)}{C_{pn}^d(d)\sigma_{pn}^d(\omega, q; d)}$$

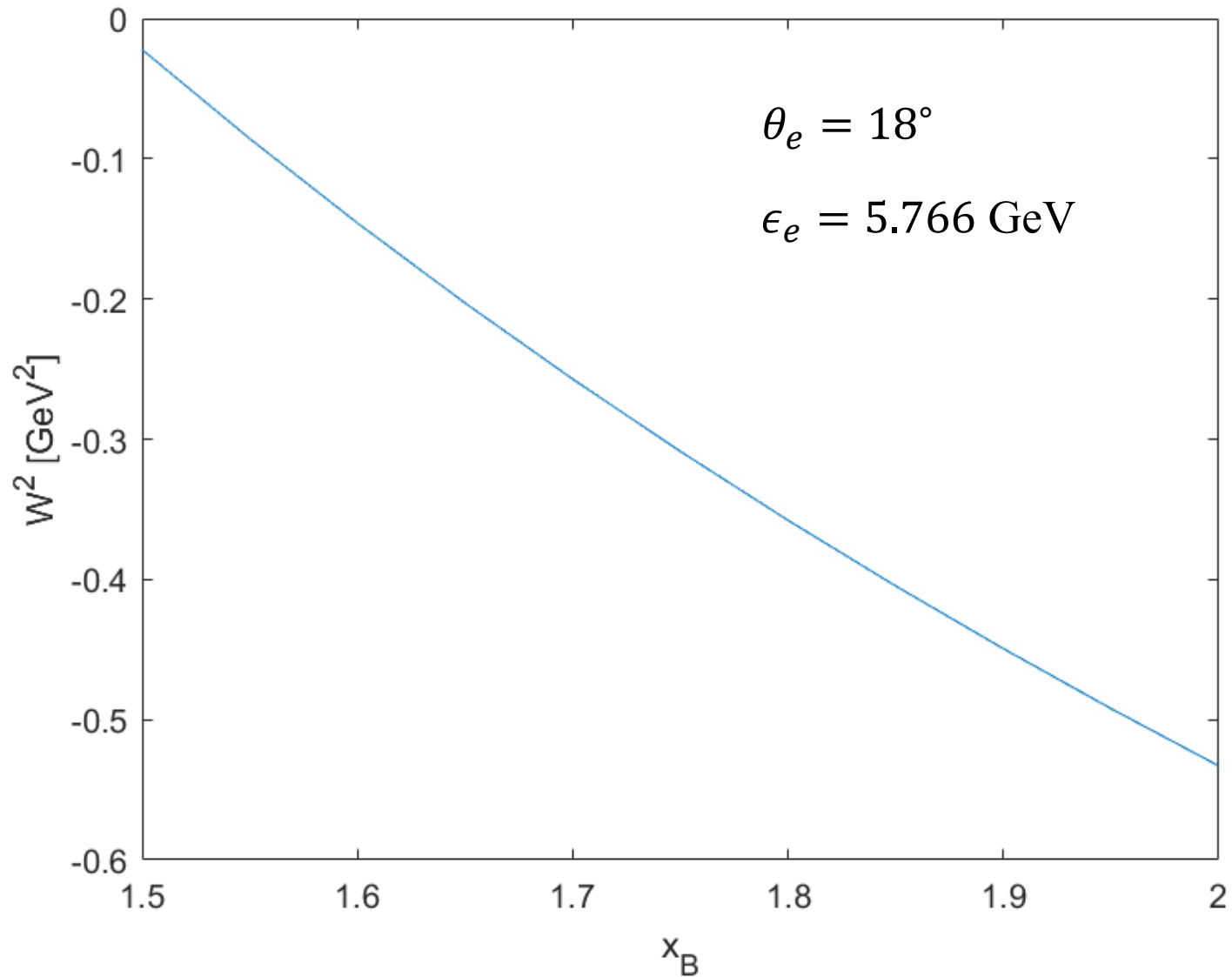
$$\text{If } \left\{ \begin{array}{l} \sigma_{CM}(A) = 0 \\ B_A - B_{A-2}^* = 2.224 \text{ MeV} \end{array} \right.$$



$$\left(\frac{A}{2}\right) a_2 = \frac{C_{pn}^d(A)}{C_{pn}^d(d)}$$

The regular interpretation!

$$W^2 = m_p^2 + 2m_p\omega - Q^2$$



$$W^2 = m_p^2 + 2m_p\omega - Q^2$$

