Generalized Contact Formalism and The Spectral Function

Ronen Weiss and Nir Barnea

The Hebrew University of Jerusalem

SRC and EMC workshop - MIT - 22/3/2019

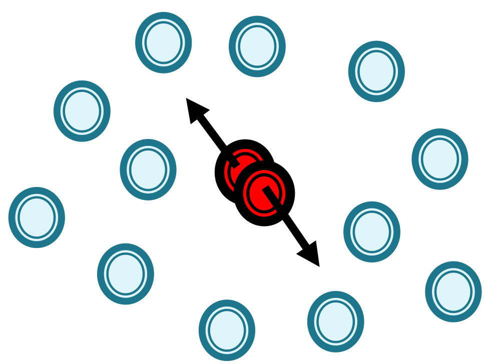
The contact formalism

- Originally developed for ultracold atomic systems
- The factorization of the wave function:

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{n_1 n_2} \to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k \neq n_1, n_2}\right)$$

Universal function
(but depends on the potential)

Nucleus-dependent function



The contact formalism

- Originally developed for ultracold atomic systems
- The factorization of the wave function:

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{n_1 n_2} \to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k \neq n_1, n_2}\right)$$

$\varphi(r) \equiv$ The **zero-energy** solution of the 2N Schrodinger Eq.

The contact formalism

- Originally developed for ultracold atomic systems
- The factorization of the wave function:

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{n_1 n_2} \to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k \neq n_1, n_2}\right)$$

$\varphi(r)$ ≡ The **zero-energy** solution of the 2N Schrodinger Eq.

- The simplest example – nn density:

$$\rho_{nn}(\boldsymbol{r}) = \langle \Psi | \delta(\boldsymbol{r}_{nn} - \boldsymbol{r}) | \Psi \rangle$$

The contact formalism

- Originally developed for ultracold atomic systems
- The factorization of the wave function:

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{n_1 n_2} \to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k \neq n_1, n_2}\right)$$

$\varphi(r) \equiv$ The **zero-energy** solution of the 2N Schrodinger Eq.

- The simplest example – nn density:

$$\rho_{nn}(\boldsymbol{r}) = \langle \Psi | \delta(\boldsymbol{r}_{nn} - \boldsymbol{r}) | \Psi \rangle \xrightarrow{r \to 0} |\varphi_{nn}(\boldsymbol{r})|^2 \frac{N(N-1)}{2} \langle A_{nn} | A_{nn} \rangle$$

The "motion" of the pair

The probability to find a correlated pair

The contact formalism

- Originally developed for ultracold atomic systems
- The factorization of the wave function:

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{n_1 n_2} \to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k \neq n_1, n_2}\right)$$

$\varphi(r) \equiv$ The **zero-energy** solution of the 2N Schrodinger Eq.

- The simplest example – nn density:

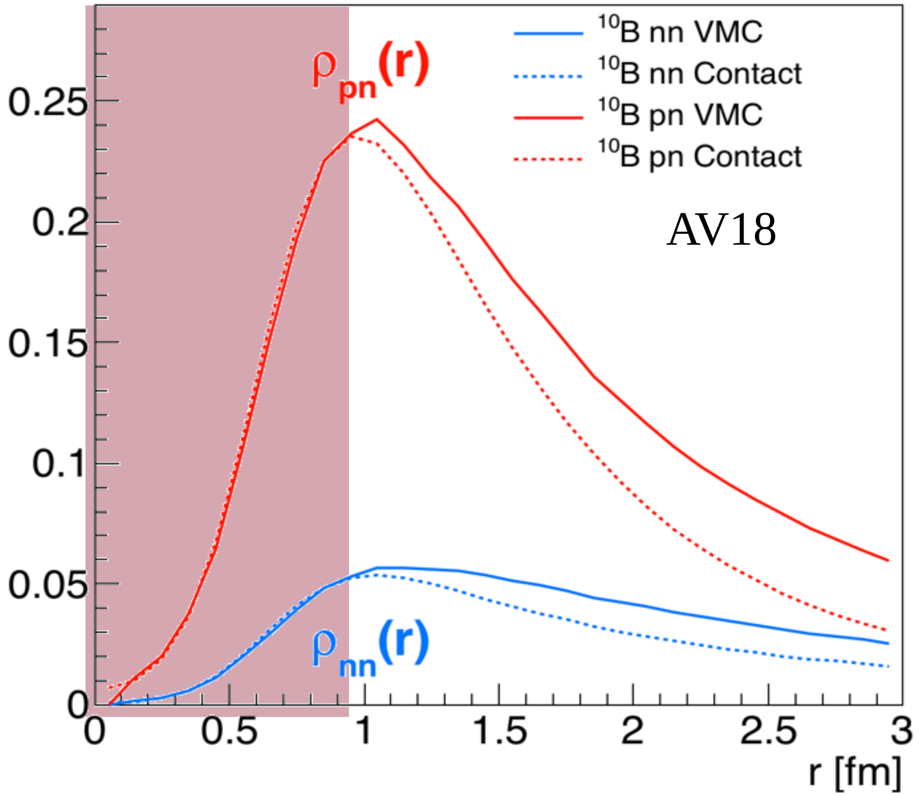
$$\rho_{nn}(\boldsymbol{r}) = \langle \Psi | \delta(\boldsymbol{r}_{nn} - \boldsymbol{r}) | \Psi \rangle \xrightarrow{r \to 0} |\varphi_{nn}(\boldsymbol{r})|^2 \frac{N(N-1)}{2} \langle A_{nn} | A_{nn} \rangle$$

The nn contact	$C_{nn} \equiv \frac{N(N-1)}{2} \langle A_{nn} \vert A_{nn} \rangle$

The contact formalism

- The simplest example – nn density:

$$\rho_{nn}(\boldsymbol{r}) \xrightarrow{r\to 0} C_{nn}|\varphi_{nn}(\boldsymbol{r})|^2$$



R.B Wiringa et. al., Phys. Rev. C 89, 024305 (2014)

The contact formalism

$$\Psi \xrightarrow{r_{nn\to 0}} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k\neq n_1,n_2}\right) \;\; ; \;\; \boldsymbol{C_{nn}} \propto \langle A_{nn}|A_{nn}\rangle$$

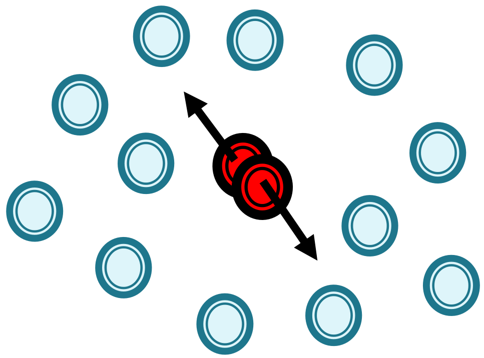
The contact formalism

$$\Psi \xrightarrow{r_{ij\to 0}} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) \times A_{ij}^{\alpha}(\boldsymbol{R}_{ij},\{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{\boldsymbol{ij}}^{\boldsymbol{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

Channels α
$= (\ell_2 S_2) j_2 m_2$

"universal" function

The pair kind
$ij \in \{pp, nn, pn\}$



The contact formalism

$$\Psi \xrightarrow{r_{ij}\to 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) \times A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C_{ij}^{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

Channels $\alpha = (\ell_2 S_2) j_2 m_2$

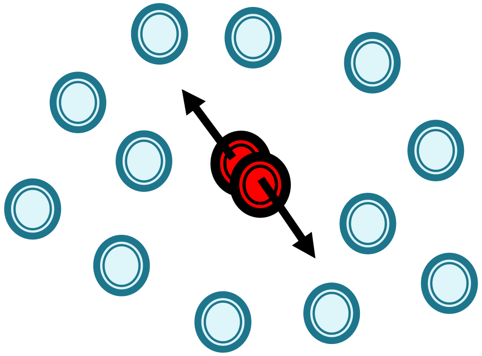
"universal" function

The pair kind $ij \in \{pp, nn, pn\}$

Main channels:

The **deuteron** channel: $\ell_2 = 0,2\,;\, s_2 = 1\,;\, j_2 = 1\,;\, t_2 = 0$

The **spin-zero** channel: $\ell_2 = 0\,;\, s_2 = 0\,;\, j_2 = 0\,;\, t_2 = 1$

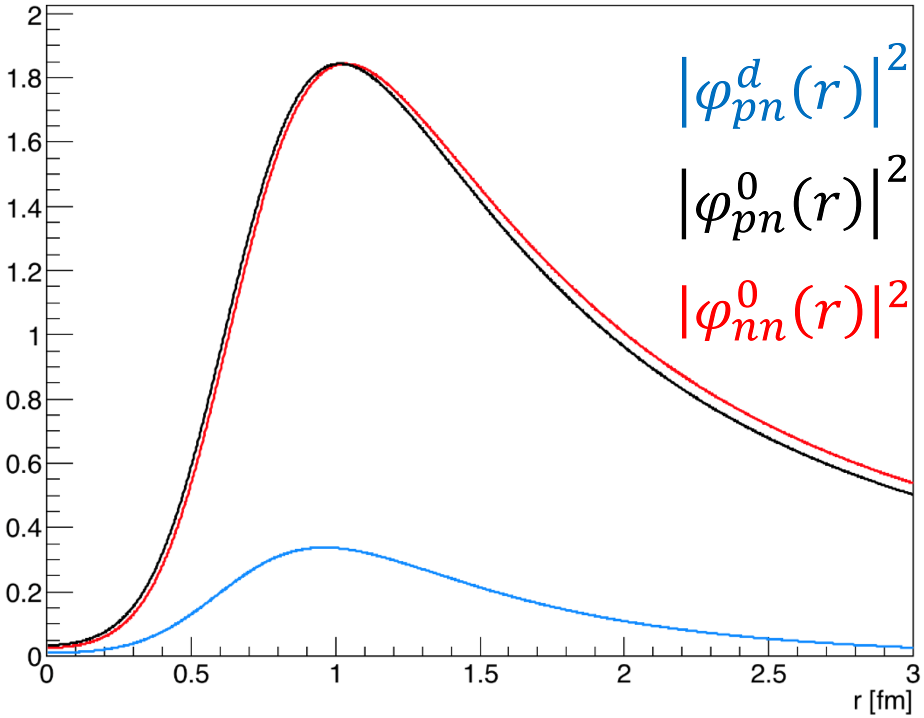


The contact formalism

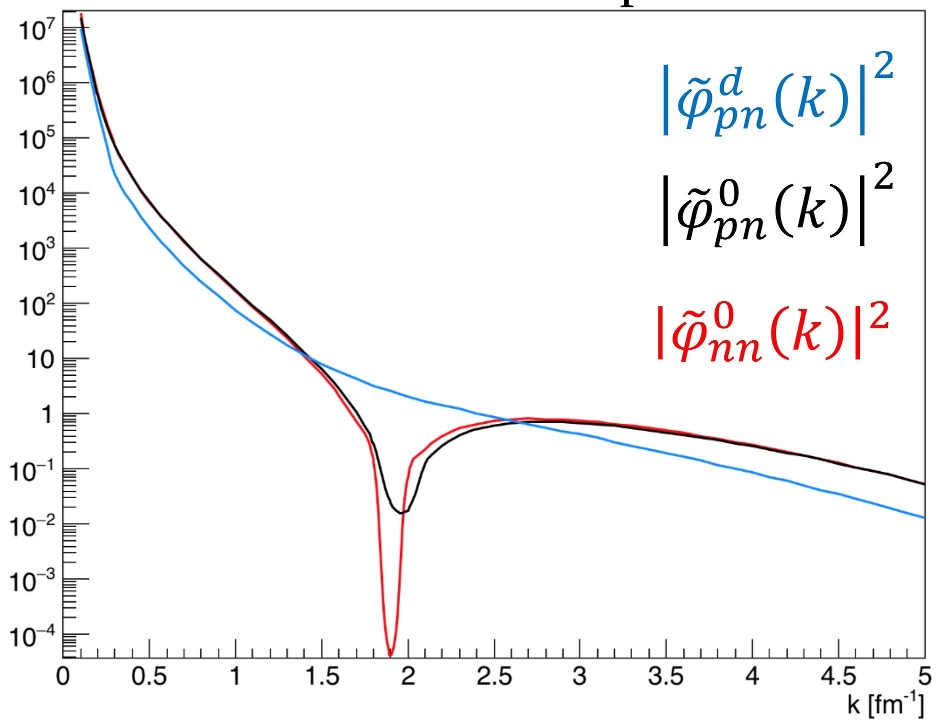
$$\psi \xrightarrow{r_{ij}\to 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) \times A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C_{ij}^{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

The universal functions using the AV18 potential

Coordinate space

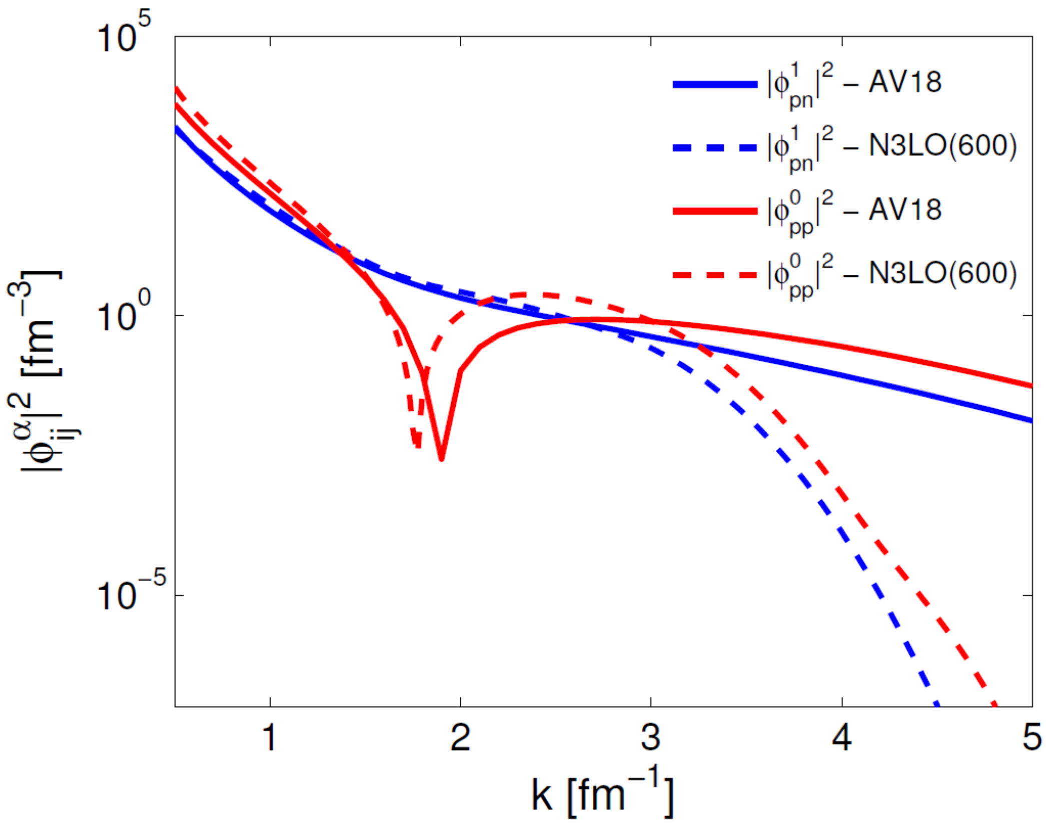


Momentum space



The contact formalism

$$\psi \xrightarrow{r_{ij}\to 0} \sum_\alpha \varphi_{ij}^\alpha(\boldsymbol{r}_{ij}) \times A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C_{ij}^{\alpha\beta}} \propto \langle A_{ij}^\alpha | A_{ij}^\beta \rangle$$



The nuclear contact relations

The nuclear contact relations

- Momentum & coordinate-space distributions

  RW, *B. Bazak, N. Barnea, PRC* ***92***, *054311 (2015)*

  M. Alvioli, CC. Degli Atti, H. Morita, PRC ***94***, *044309 (2016)*

  RW, *R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB* ***780***, *211 (2018)*

- Photo-absorption (the Levinger constant)

  RW, *B. Bazak, N. Barnea, PRL* ***114***, *012501 (2015)*

  RW, *B. Bazak, N. Barnea, EPJA* ***52***, *92 (2016)*

- The Coulomb sum rule (and a review)

  RW, *E. Pazy, N. Barnea, Few-Body Systems* ***58***, *9 (2017)*

- Spectral function and Exclusive electron scattering

  RW, *I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB* ***791***, *242 (2019)*

- Charge density

  RW, *A. Schmidt, G. A. Miller, and N. Barnea, PLB* ***790***, *484 (2019)*

- Coupled-channels theory

  RW *and N. Barnea, PRC* ***96***, *041303(R) (2017)*

  RW *and N. Barnea, arXiv:1801.04526 [nucl-th] (2017)*

- Correlation functions

  R. Cruz-Torres, A. Schmidt, G. A. Miller, L. B. Weinstein, N. Barnea, ***RW***, *E. Piasetzky and O. Hen, PLB* ***785***, *304 (2018)*

Symmetry energy

BJ. Cai, BA. Li, PRC ***93***, *014619 (2016)*

The EMC effect

JW. Chen et al. PRL ***119***, *262502 (2017)*

The nuclear contact relations

- Momentum & coordinate-space distributions

  RW, *B. Bazak*, *N. Barnea*, *PRC* ***92***, *054311 (2015)*

  M. Alvioli, CC. Degli Atti, H. Morita, PRC ***94***, *044309 (2016)*

  RW, *R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB* ***780***, *211 (2018)*

- Photo-absorption (the Levinger constant)

  RW, *B. Bazak, N. Barnea, PRL* ***114***, *012501 (2015)*

  RW, *B. Bazak, N. Barnea, EPJA* ***52***, *92 (2016)*

- The Coulomb sum rule (and a review)

  RW, *E. Pazy, N. Barnea, Few-Body Systems* ***58***, *9 (2017)*

- Spectral function and Exclusive electron scattering

  RW, *I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB* ***791***, *242 (2019)*

- Charge density

  RW, *A. Schmidt, G. A. Miller, and N. Barnea, PLB* ***790***, *484 (2019)*

- Coupled-channels theory

  RW *and N. Barnea, PRC* ***96***, *041303(R) (2017)*

  RW *and N. Barnea, arXiv:1801.04526 [nucl-th] (2017)*

- Correlation functions

  R. Cruz-Torres, A. Schmidt, G. A. Miller, L. B. Weinstein, N. Barnea, ***RW***, *E. Piasetzky and O. Hen, PLB* ***785***, *304 (2018)*

  Symmetry energy

  BJ. Cai, BA. Li, PRC ***93***, *014619 (2016)*

  The EMC effect

  JW. Chen et al. PRL ***119***, *262502 (2017)*

Momentum tail

$$\rho_{nn}(\boldsymbol{r}) \xrightarrow{r\to 0} C_{nn}|\varphi_{nn}(\boldsymbol{r})|^2$$

Momentum tail

Relative momentum distribution

$$F_{pn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k_{rel})\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k_{rel})\right|^2$$

$$F_{nn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{nn}^{0}|\varphi_{nn}^{0}(k_{rel})|^2$$

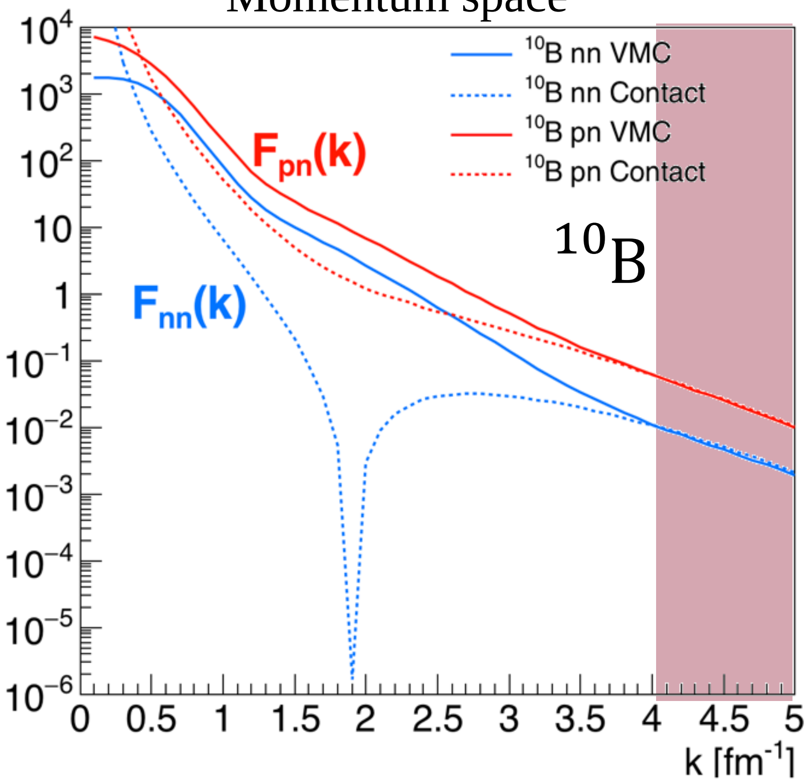
Momentum tail

Relative momentum distribution

$$F_{pn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k_{rel})\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k_{rel})\right|^2$$

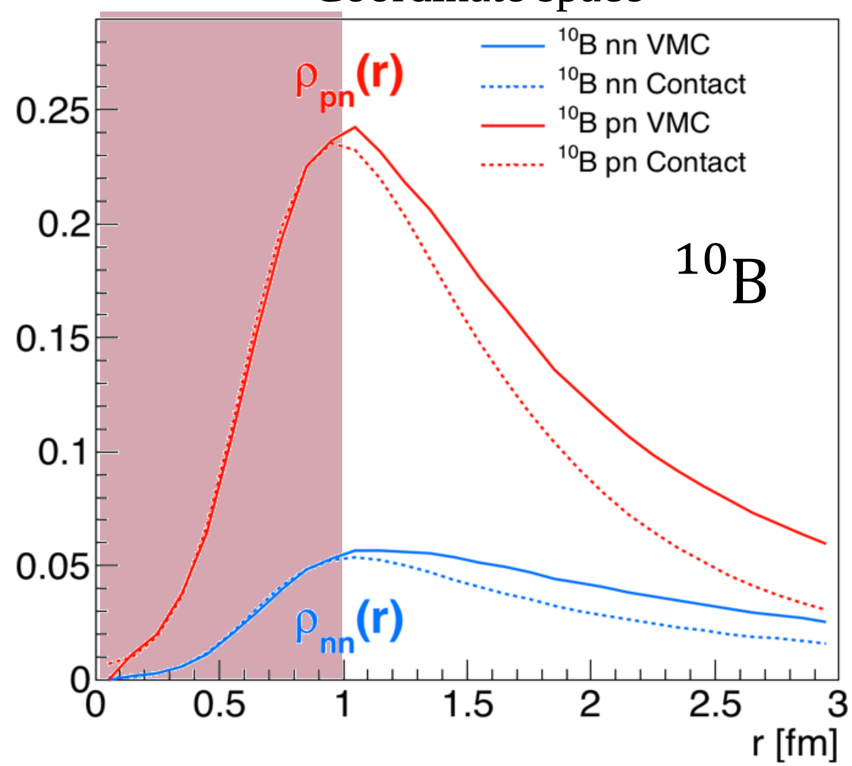
$$F_{nn}(k_{rel}) \xrightarrow[k\to\infty]{} C_{nn}^{0}|\varphi_{nn}^{0}(k_{rel})|^2$$

Momentum space



$C_{pn}^{d} \approx 11.7$; $C_{pn}^{0} \approx C_{pp}^{0} \approx 0.8$

Coordinate space



$C_{pn}^{d} \approx 10.7$; $C_{pn}^{0} \approx C_{pp}^{0} \approx 0.6$

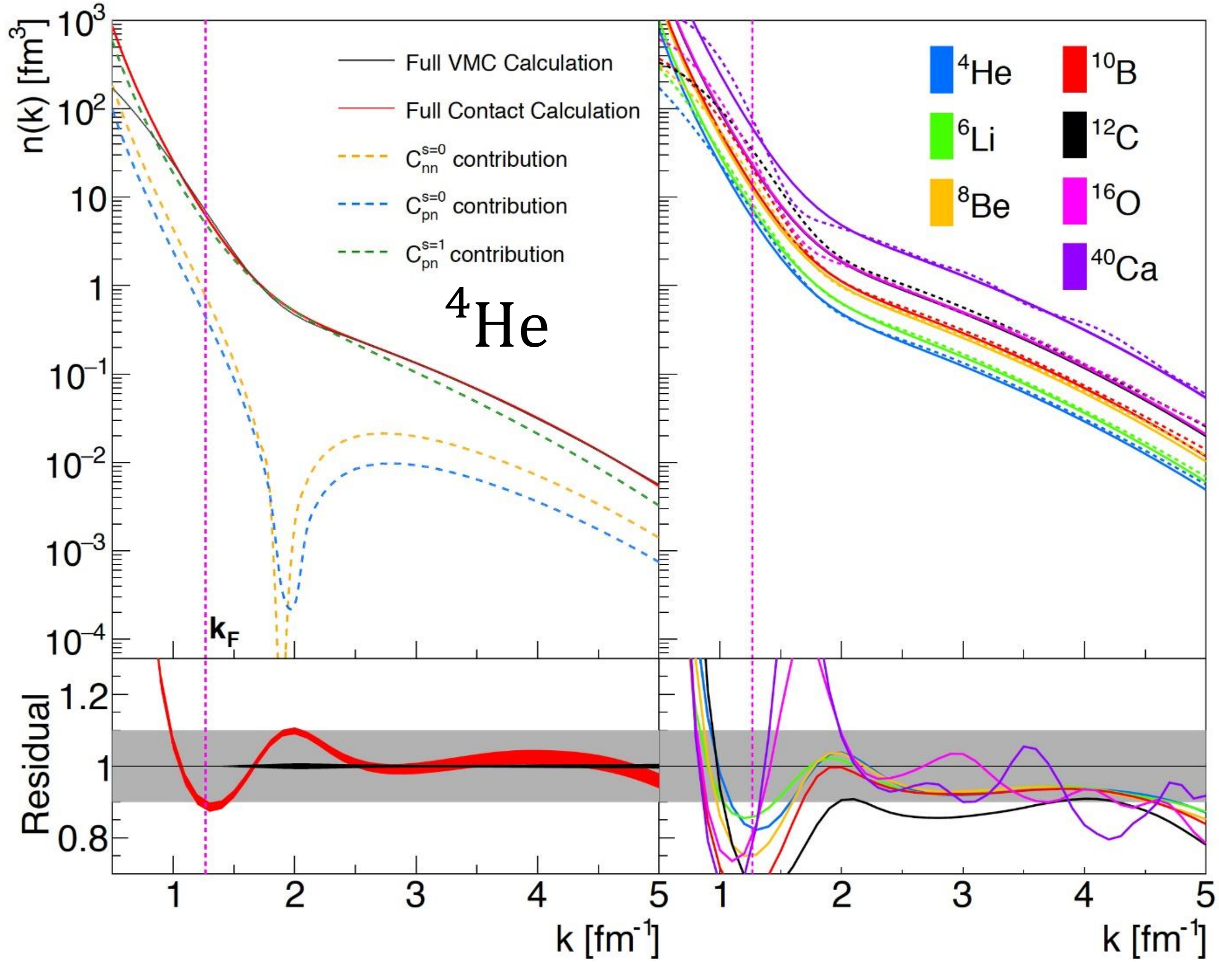
Momentum tail

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

Momentum tail

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

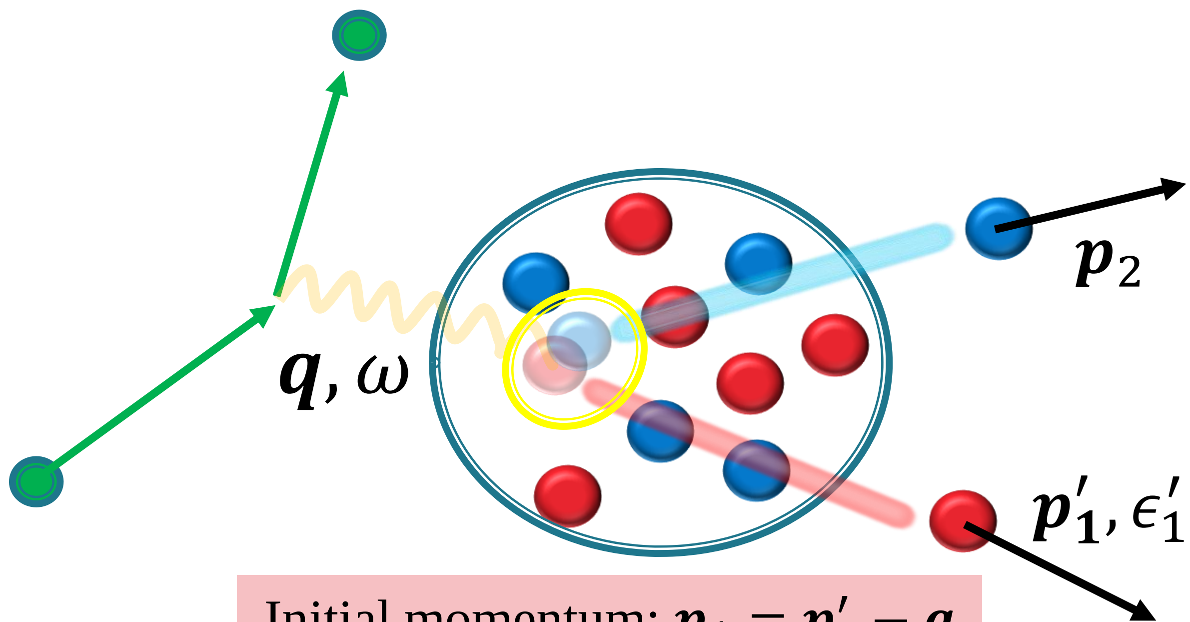
$n_p(k)$



Electron-scattering experiments

$Q^2 > 1.5\ \text{GeV}^2$

$x_B > 1.2$



Initial momentum: $\boldsymbol{p}_1 = \boldsymbol{p}_1' - \boldsymbol{q}$
Initial energy: $\epsilon_1 = \epsilon_1' - \omega$

Cross section $\propto$ Probability to find a nucleon with momentum $\boldsymbol{p}_1$ and energy ϵ_1 $\equiv$ The spectral function $S(\boldsymbol{p}_1, \epsilon_1)$

The spectral function

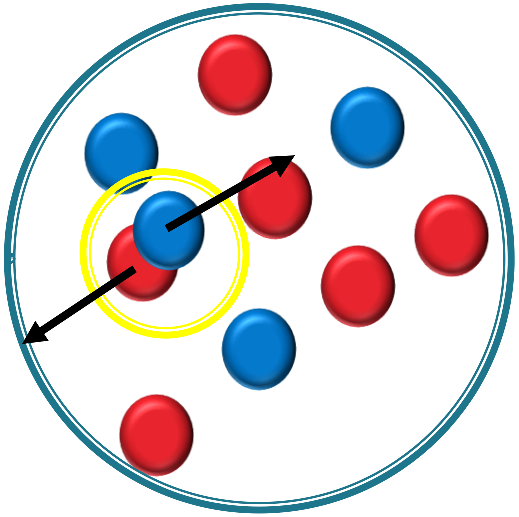
$$S(\boldsymbol{p_1}, \epsilon_1) = \sum_{s} \sum_{f_{A-1}} \delta(\epsilon_1 + E_f^{A-1} - E_0) \left| \langle f_{A-1} | a_{\boldsymbol{p_1},s} | \psi_0 \rangle \right|^2$$

The spectral function

$$S(\boldsymbol{p_1}, \epsilon_1) = \sum_s \sum_{f_{A-1}} \delta(\epsilon_1 + E_f^{A-1} - E_0) \left| \langle f_{A-1} | a_{\boldsymbol{p_1},s} | \psi_0 \rangle \right|^2$$

The initial wave function

$$\psi_0 \to \sum_\alpha \varphi_{ij}^\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$



The spectral function

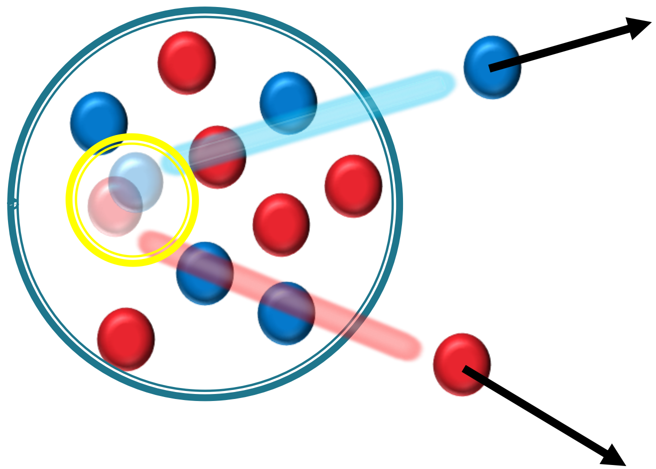
$$S(\boldsymbol{p_1}, \epsilon_1) = \sum_{s} \sum_{f_{A-1}} \delta(\epsilon_1 + E_f^{A-1} - E_0) \left| \langle f_{A-1} | a_{\boldsymbol{p_1},s} | \psi_0 \rangle \right|^2$$

The initial wave function

$$\psi_0 \rightarrow \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$

The final wave function

$$|\psi_f^{12}\rangle = a_{\boldsymbol{p_1},s}^{\dagger} |f_{A-1}\rangle \propto |\Psi_{\nu}^{A-2}\rangle e^{i\boldsymbol{p_1}\cdot r_1 + i\boldsymbol{p}_2\cdot r_2} \chi_{s_1} \chi_{s_2}$$



The spectral function

$$S(\boldsymbol{p_1},\epsilon_1) = \sum_{s}\sum_{f_{A-1}} \delta(\epsilon_1 + E_f^{A-1} - E_0) \left|\langle f_{A-1}|a_{\boldsymbol{p_1},s}|\psi_0\rangle\right|^2$$

The initial wave function

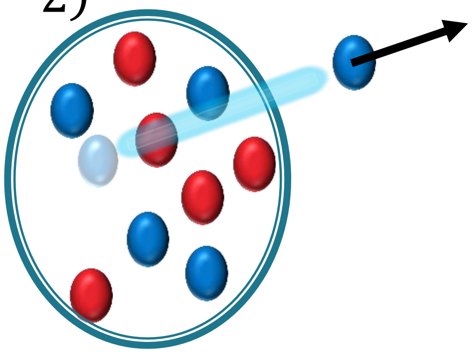
$$\psi_0 \to \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij},\{\boldsymbol{r}_k\}_{k\neq i,j})$$

The final wave function

$$|\psi_f^{12}\rangle = a^{\dagger}_{\boldsymbol{p_1},s}|f_{A-1}\rangle \propto |\Psi_\nu^{A-2}\rangle e^{i\boldsymbol{p_1}\cdot r_1 + i\boldsymbol{p}_2\cdot r_2}\chi_{s_1}\chi_{s_2}$$

Energy conservation:

$$E_f^{A-1} = \epsilon_2 + (A-2)m - B_f^{A-2} + \frac{P_{12}^2}{2m(A-2)}$$



The spectral function

$$S(\boldsymbol{p_1},\epsilon_1) = \sum_{s}\sum_{f_{A-1}} \delta(\epsilon_1 + E_f^{A-1} - E_0)\left|\langle f_{A-1}|a_{\boldsymbol{p_1},s}|\psi_0\rangle\right|^2$$

The initial wave function

$$\psi_0 \to \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij},\{\boldsymbol{r}_k\}_{k\neq i,j})$$

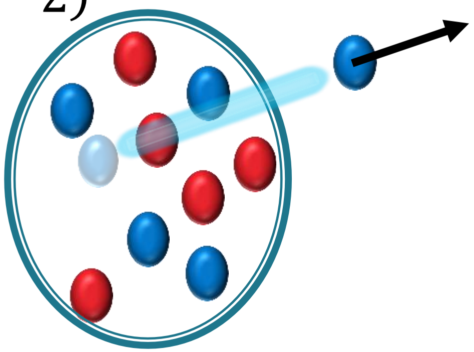
The final wave function

$$|\psi_f^{12}\rangle = a^{\dagger}_{\boldsymbol{p_1},s}|f_{A-1}\rangle \propto |\Psi_\nu^{A-2}\rangle e^{i\boldsymbol{p_1}\cdot r_1 + i\boldsymbol{p}_2\cdot r_2}\chi_{s_1}\chi_{s_2}$$

Energy conservation:

$$E_f^{A-1} = \epsilon_2 + (A-2)m - B_f^{A-2} + \frac{P_{12}^2}{2m(A-2)}$$

$$B_f^{A-2} \approx \langle B_f^{A-2}\rangle = \mathrm{B}^{\mathrm{A-2}} - \mathrm{E}^*$$



The spectral function

$p_1 > k_F$

$$S^p(\boldsymbol{p_1}, \epsilon_1) = C^1_{pn} S^1_{pn}(\boldsymbol{p_1}, \epsilon_1) + C^0_{pn} S^0_{pn}(\boldsymbol{p_1}, \epsilon_1) + 2C^0_{pp} S^0_{pp}(\boldsymbol{p_1}, \epsilon_1)$$

The spectral function

$p_1 > k_F$

$$S^p(\boldsymbol{p_1}, \epsilon_1) = C^1_{pn} S^1_{pn}(\boldsymbol{p_1}, \epsilon_1) + C^0_{pn} S^0_{pn}(\boldsymbol{p_1}, \epsilon_1) + 2C^0_{pp} S^0_{pp}(\boldsymbol{p_1}, \epsilon_1)$$

$$S^\alpha_{ab}(\boldsymbol{p_1}, \epsilon_1) = \frac{1}{4\pi} \int \frac{d^3 p_2}{(2\pi)^3} \delta(f(\boldsymbol{p}_2))\, n_{CM}(\boldsymbol{p_1} + \boldsymbol{p_2}) |\tilde{\varphi}^\alpha_{ab}(|\boldsymbol{p_1} - \boldsymbol{p_2}|/2|)|^2$$

$$f(\boldsymbol{p}_2) \equiv \epsilon_1 + \sqrt{p_2^2 + m^2} - 2m + \left(B_i^A - \langle B_f^{A-2} \rangle\right) + \frac{(\boldsymbol{p_1} + \boldsymbol{p_2})^2}{2m(A-2)}$$

$$n_{CM}(\boldsymbol{K}) \propto e^{-\frac{K^2}{2\sigma_{CM}^2}}$$

Similar to the convolution model

C. Ciofi degli Atti, S. Simula, L. L. Frankfurt, and M. I. Strikman, Phys. Rev. C 44, R7(R) (1991),
C. Ciofi degli Atti and S. Simula PRC ***53,*** *1689 (1996)*

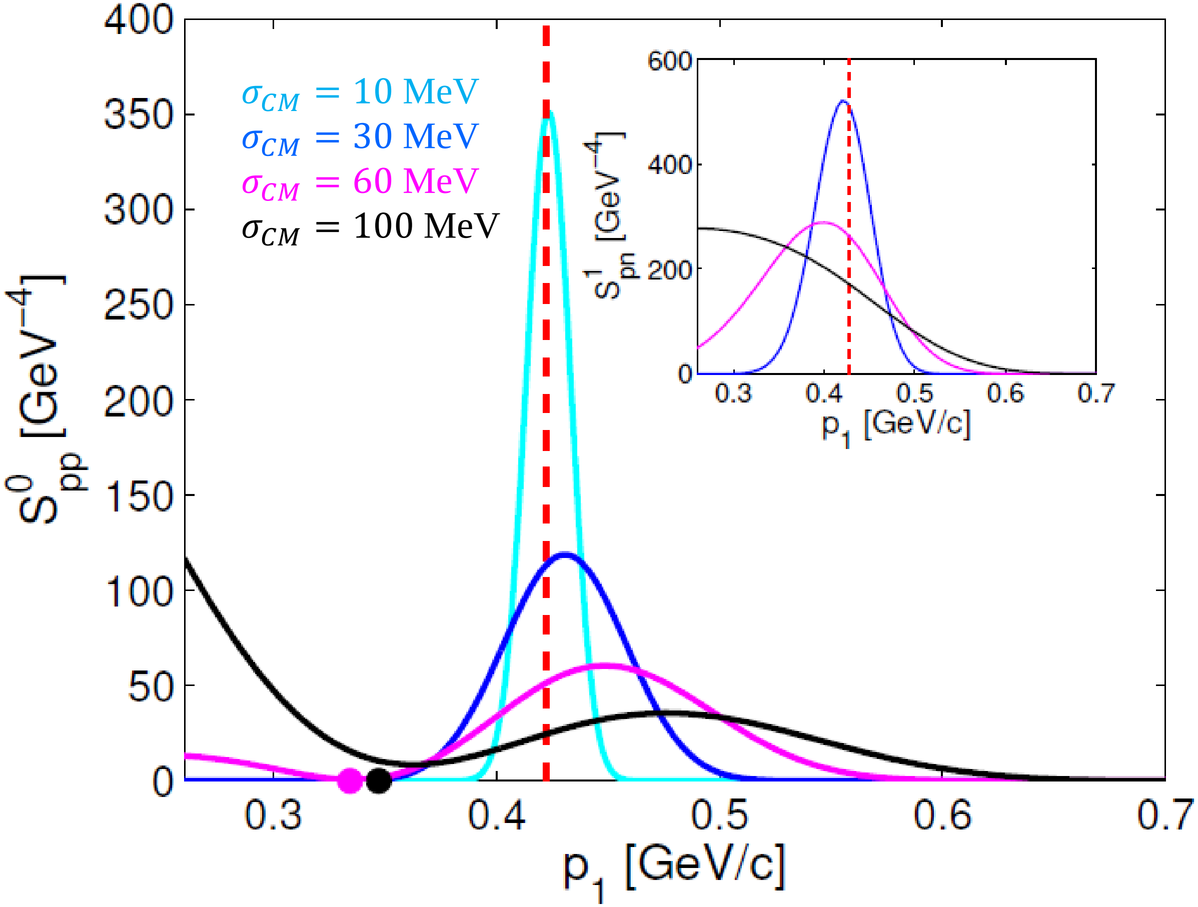
The spectral function

$$S^p(\mathbf{p_1}, \epsilon_1) = C^1_{pn} S^1_{pn}(\mathbf{p_1}, \epsilon_1) + C^0_{pn} S^0_{pn}(\mathbf{p_1}, \epsilon_1) + 2C^0_{pp} S^0_{pp}(\mathbf{p_1}, \epsilon_1)$$

^{4}He

$\epsilon_1 = 0.82$ GeV

AV18 potential



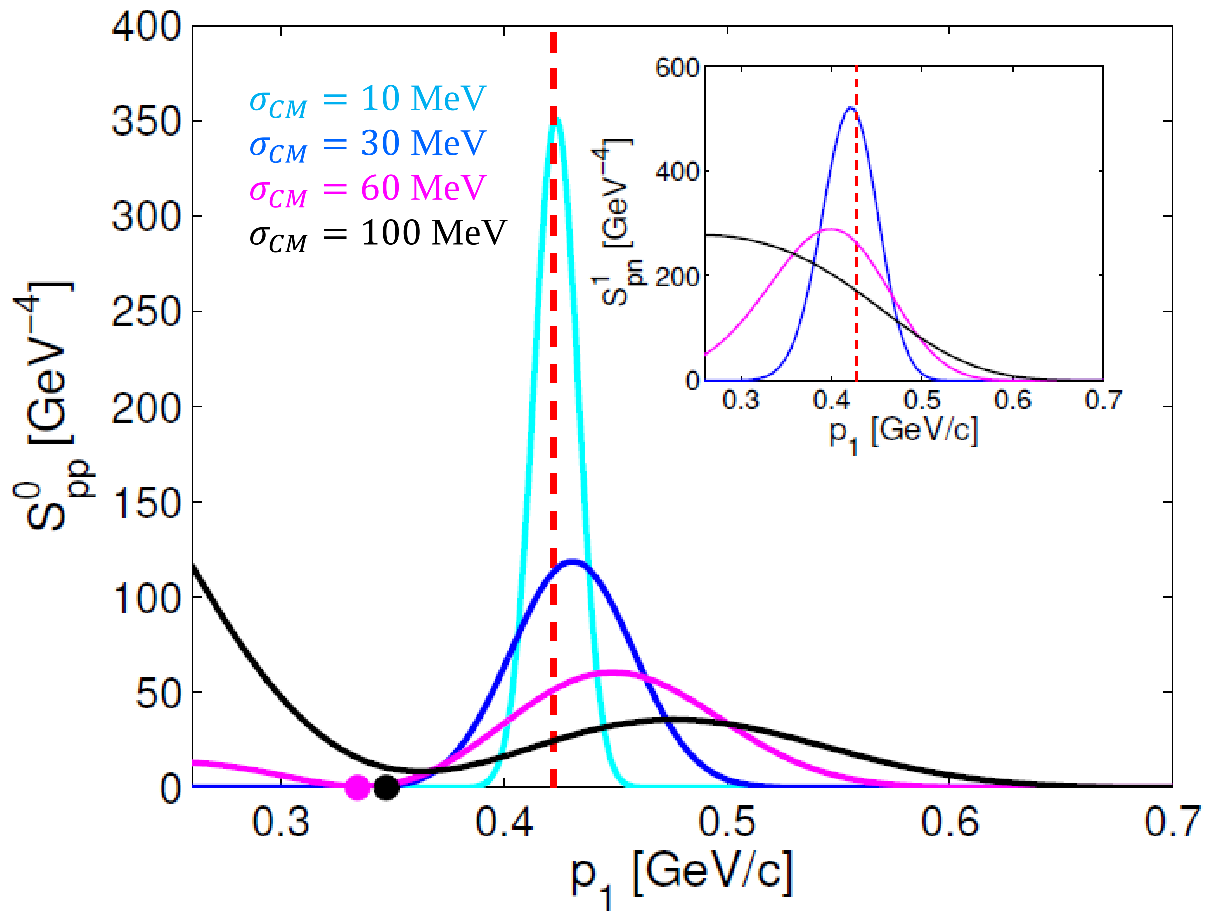
The spectral function

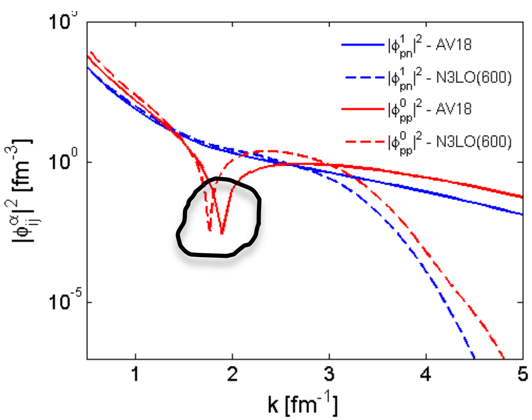
$$S^p(\boldsymbol{p_1}, \epsilon_1) = C^1_{pn} S^1_{pn}(\boldsymbol{p_1}, \epsilon_1) + C^0_{pn} S^0_{pn}(\boldsymbol{p_1}, \epsilon_1) + 2C^0_{pp} S^0_{pp}(\boldsymbol{p_1}, \epsilon_1)$$

^{4}He

$\epsilon_1 = 0.82$ GeV

AV18 potential





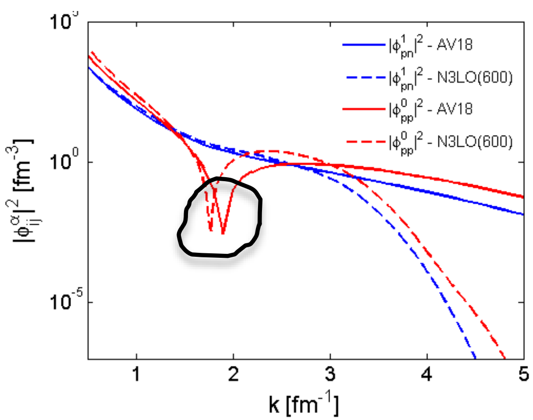
The spectral function

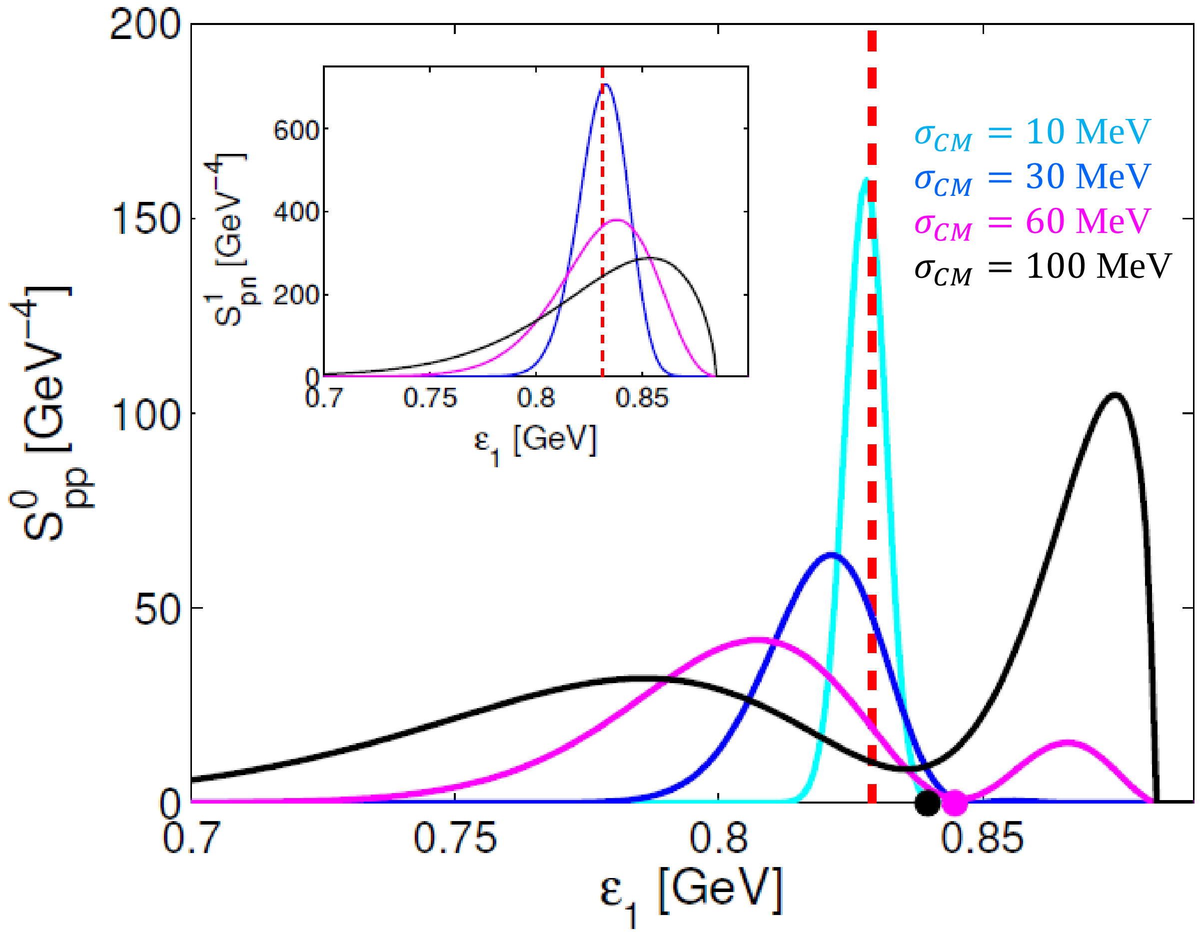
$$S^p(\boldsymbol{p_1}, \epsilon_1) = C^1_{pn} S^1_{pn}(\boldsymbol{p_1}, \epsilon_1) + C^0_{pn} S^0_{pn}(\boldsymbol{p_1}, \epsilon_1) + 2C^0_{pp} S^0_{pp}(\boldsymbol{p_1}, \epsilon_1)$$

^{4}He

$p_1 = 400$ MeV/c

AV18 potential





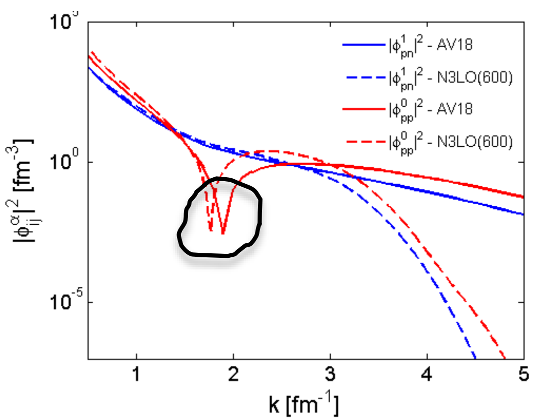
The spectral function

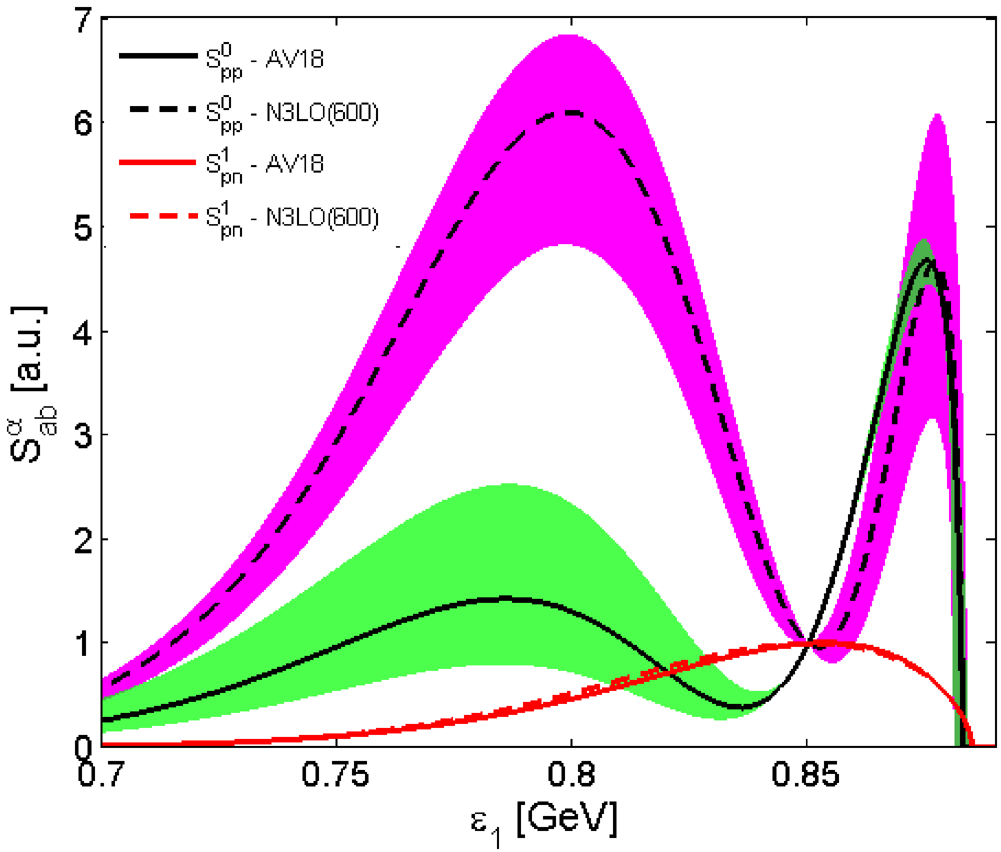
$$S^p(\boldsymbol{p_1},\epsilon_1) = C^1_{pn}S^1_{pn}(\boldsymbol{p_1},\epsilon_1) + C^0_{pn}S^0_{pn}(\boldsymbol{p_1},\epsilon_1) + 2C^0_{pp}S^0_{pp}(\boldsymbol{p_1},\epsilon_1)$$

^{4}He

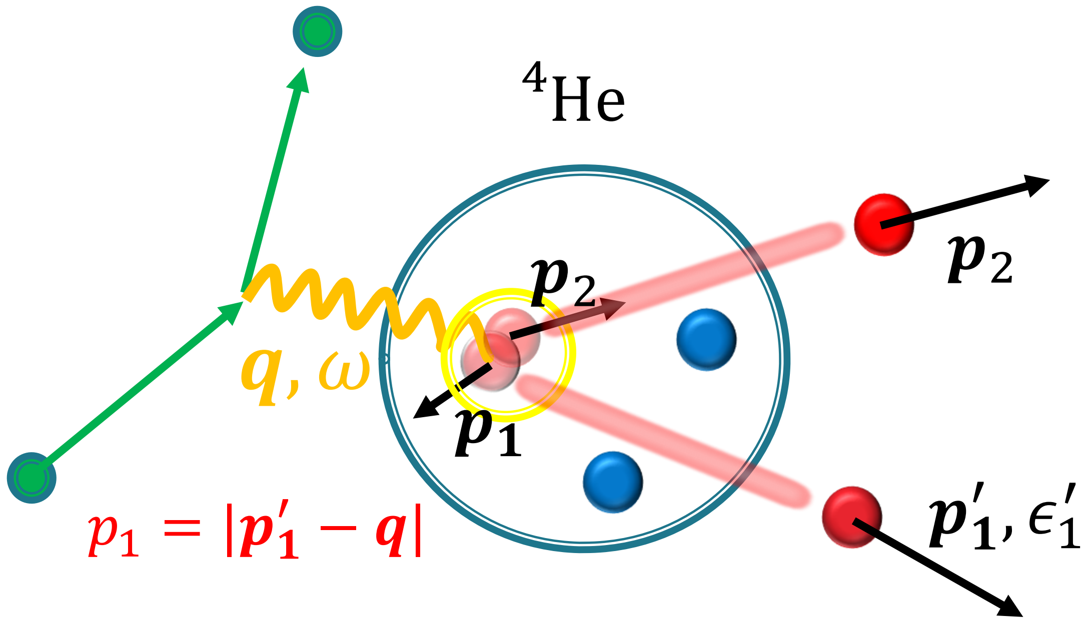
$p_1 = 390 - 410$ MeV/c

$\sigma_{CM} = 100$ MeV





Exclusive experiments

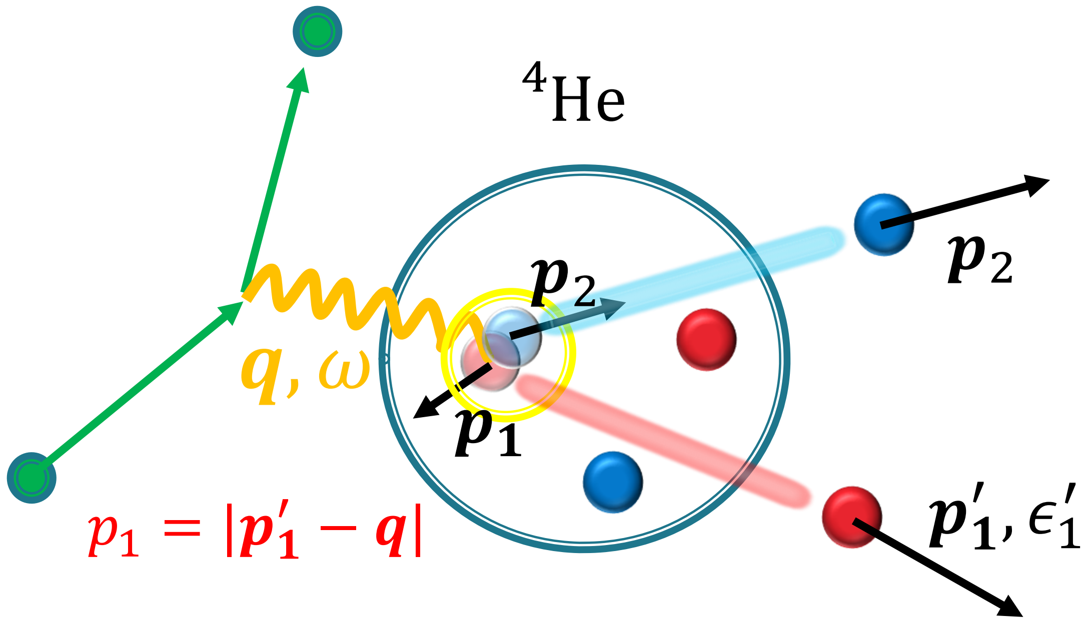


$$\frac{{}^4He(e,e'pp)/2}{{}^4He(e,e'pn)}$$

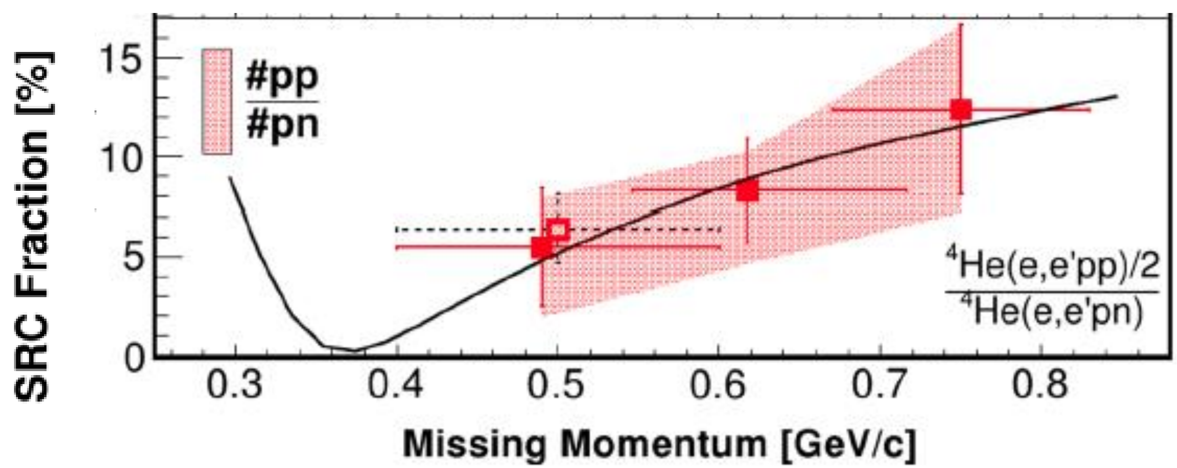
FSI corrections →

$$\frac{\#pp}{\#pn}(\boldsymbol{p}_1, \epsilon_1)$$

Exclusive experiments



The experiment by Korover et. al. *[PRL 113, 022501 (2014)]:*



$\epsilon_1 \approx 0.66$ GeV

$\epsilon_1 \approx 0.74$ GeV

$\epsilon_1 \approx 0.81$ GeV

Exclusive experiments

Using the spectral function

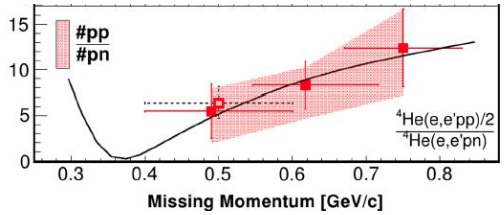
$$\frac{\#pp}{\#pn} = \frac{S^0_{pp}(p_1,\epsilon_1)}{\frac{\boldsymbol{C^1_{pn}}}{\boldsymbol{C^0_{pp}}} S^1_{pn}(p_1,\epsilon_1) + S^0_{pn}(p_1,\epsilon_1)}$$

Assuming isospin symmetry for symmetric nuclei $C^0_{pp} \approx C^0_{pn}$

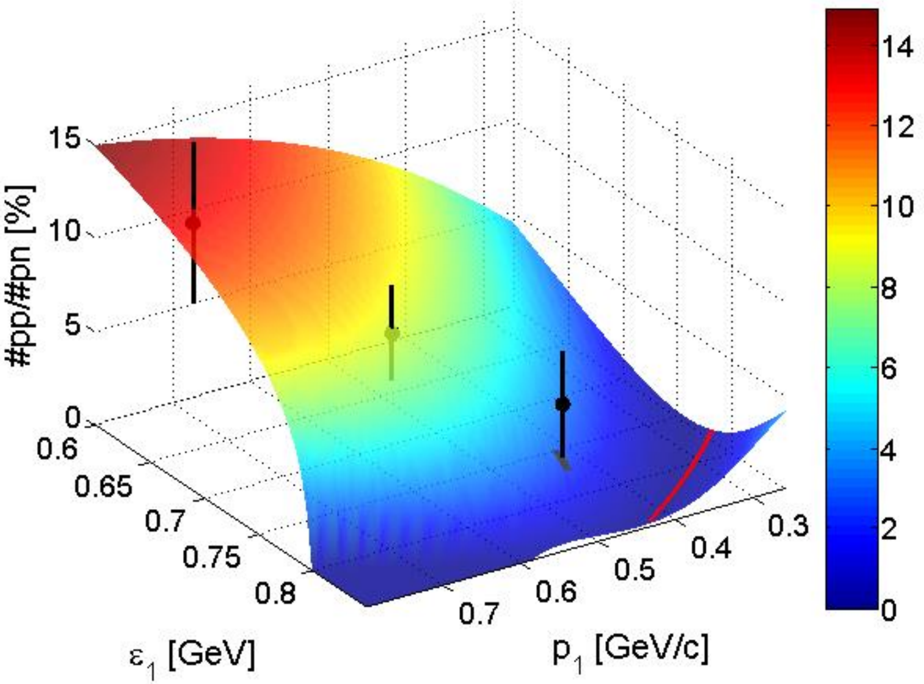
Exclusive experiments

Using the spectral function

$$\frac{\#pp}{\#pn} = \frac{S^0_{pp}(p_1,\epsilon_1)}{\frac{\boldsymbol{C^1_{pn}}}{\boldsymbol{C^0_{pp}}} S^1_{pn}(p_1,\epsilon_1) + S^0_{pn}(p_1,\epsilon_1)}$$



AV18 ^{4}He



$$\frac{\boldsymbol{C^d_{pn}}}{\boldsymbol{C^0_{pp}}}(\,^4He) = 20 \pm 5$$

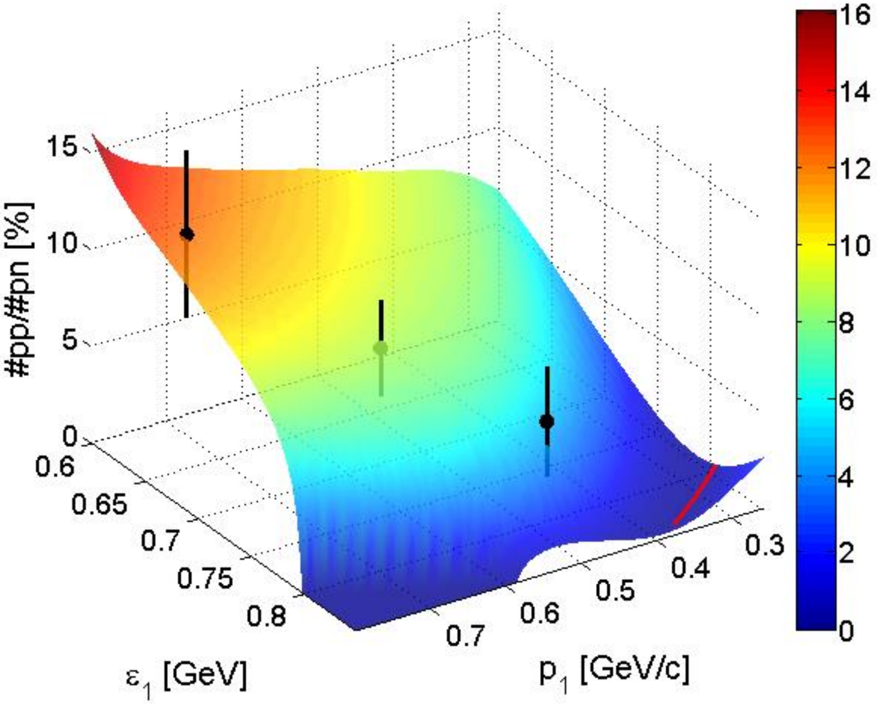
Previous results $\frac{C^d_{pn}}{C^0_{pp}}(\,^4He) = 17 - 21$

Exclusive experiments

Using the spectral function

$$\frac{\#pp}{\#pn} = \frac{S^0_{pp}(p_1,\epsilon_1)}{\frac{\boldsymbol{C^1_{pn}}}{\boldsymbol{C^0_{pp}}} S^1_{pn}(p_1,\epsilon_1) + S^0_{pn}(p_1,\epsilon_1)}$$

N3LO(600) ^{4}He

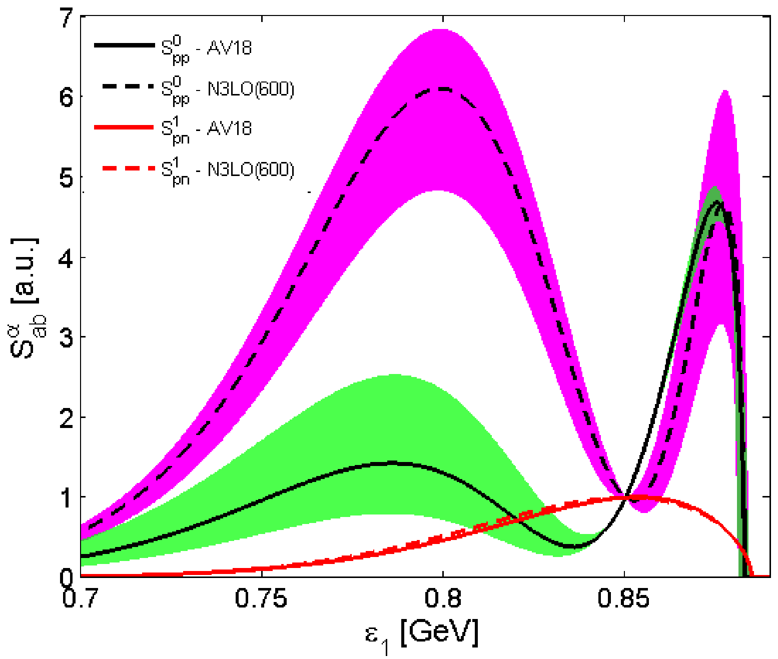


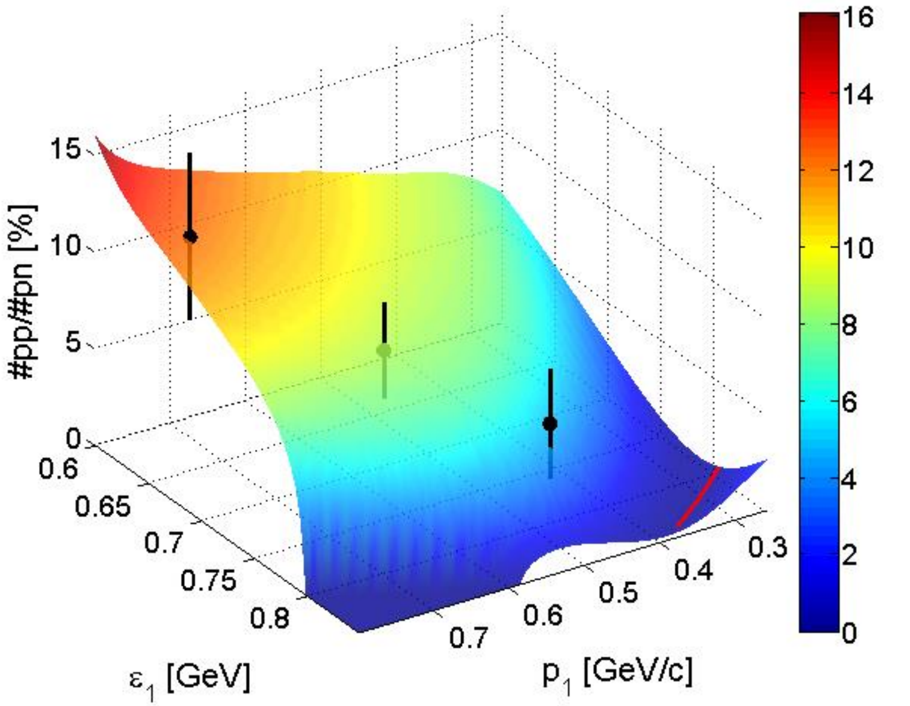
Exclusive experiments

Using the spectral function

$$\frac{\#pp}{\#pn} = \frac{S^0_{pp}(p_1, \epsilon_1)}{\frac{\boldsymbol{C^1_{pn}}}{\boldsymbol{C^0_{pp}}} S^1_{pn}(p_1, \epsilon_1) + S^0_{pn}(p_1, \epsilon_1)}$$

N3LO(600) ^{4}He





Exclusive experiments

Using the spectral function

$$\frac{\#pp}{\#p} = \frac{S^0_{pp}(p_1,\epsilon_1)}{\frac{\boldsymbol{C^1_{pn}}}{\boldsymbol{C^0_{pp}}} S^1_{pn}(p_1,\epsilon_1) + S^0_{pn}(p_1,\epsilon_1) + 2S^0_{pp}(p_1,\epsilon_1)}$$

Exclusive experiments

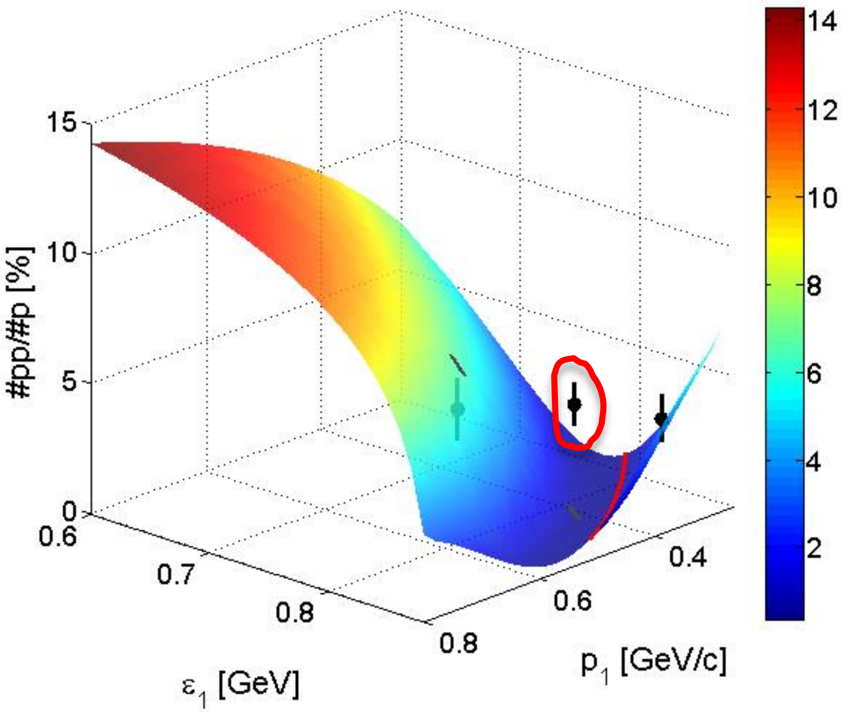
Using the spectral function

$$\frac{\#pp}{\#p} = \frac{S^0_{pp}(p_1, \epsilon_1)}{\frac{\boldsymbol{C^1_{pn}}}{\boldsymbol{C^0_{pp}}} S^1_{pn}(p_1, \epsilon_1) + S^0_{pn}(p_1, \epsilon_1) + 2S^0_{pp}(p_1, \epsilon_1)}$$

AV18 $\quad ^{12}\mathrm{C}$

$$\frac{\boldsymbol{C^d_{pn}}}{\boldsymbol{C^0_{pp}}}(^{12}C) = 14 \pm 3$$

Previous results $\quad \frac{C^d_{pn}}{C^0_{pp}}(^{12}C) = 11 - 18$



Experimental data:
R. Shneor et al., Phys. Rev. Lett. 99, 072501 (2007)

Summary

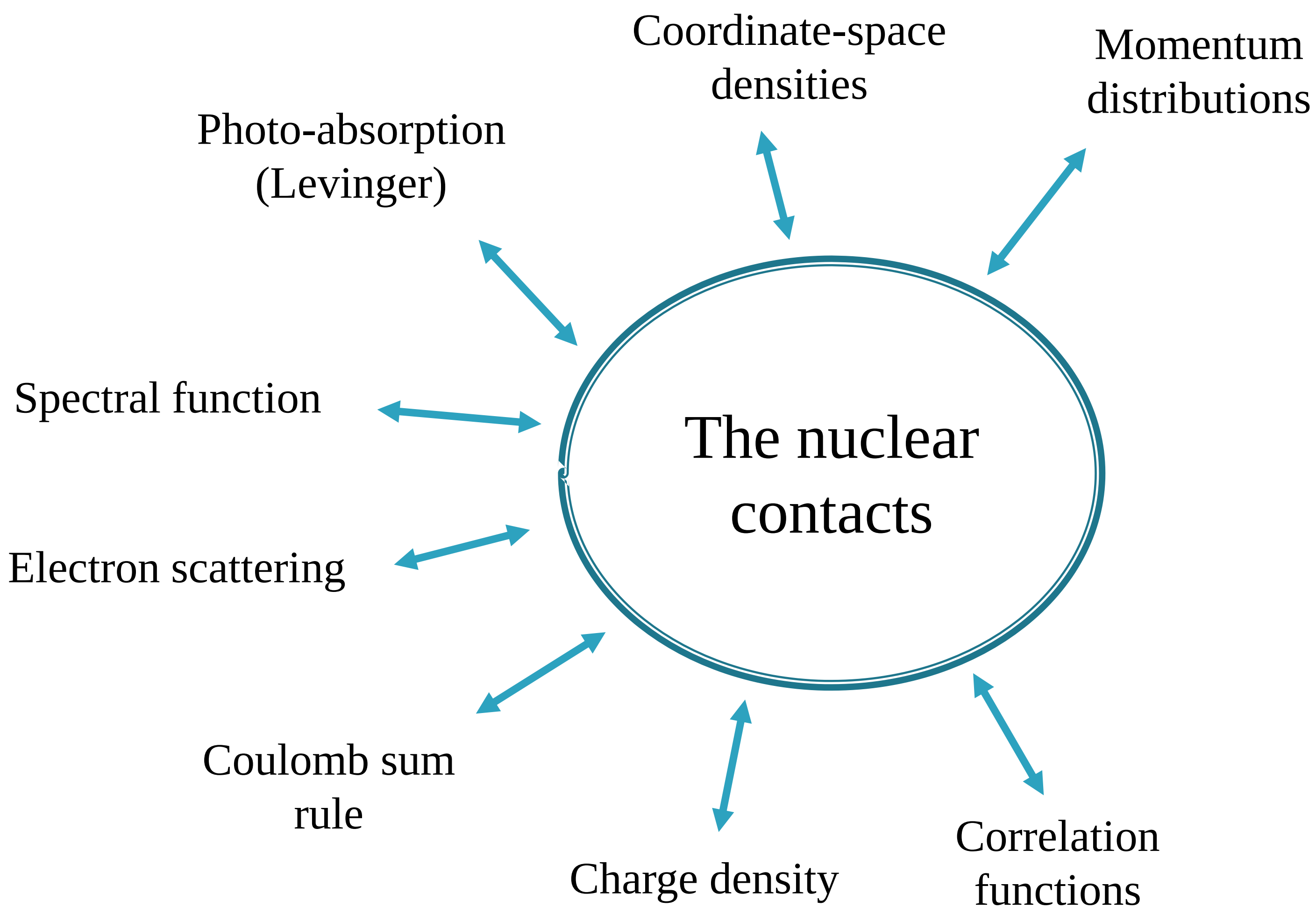
1. The **nuclear contacts -** the probability of 2N-SRCs.

2. High**-momentum tails** and short-range densities are described well by the contact relations.

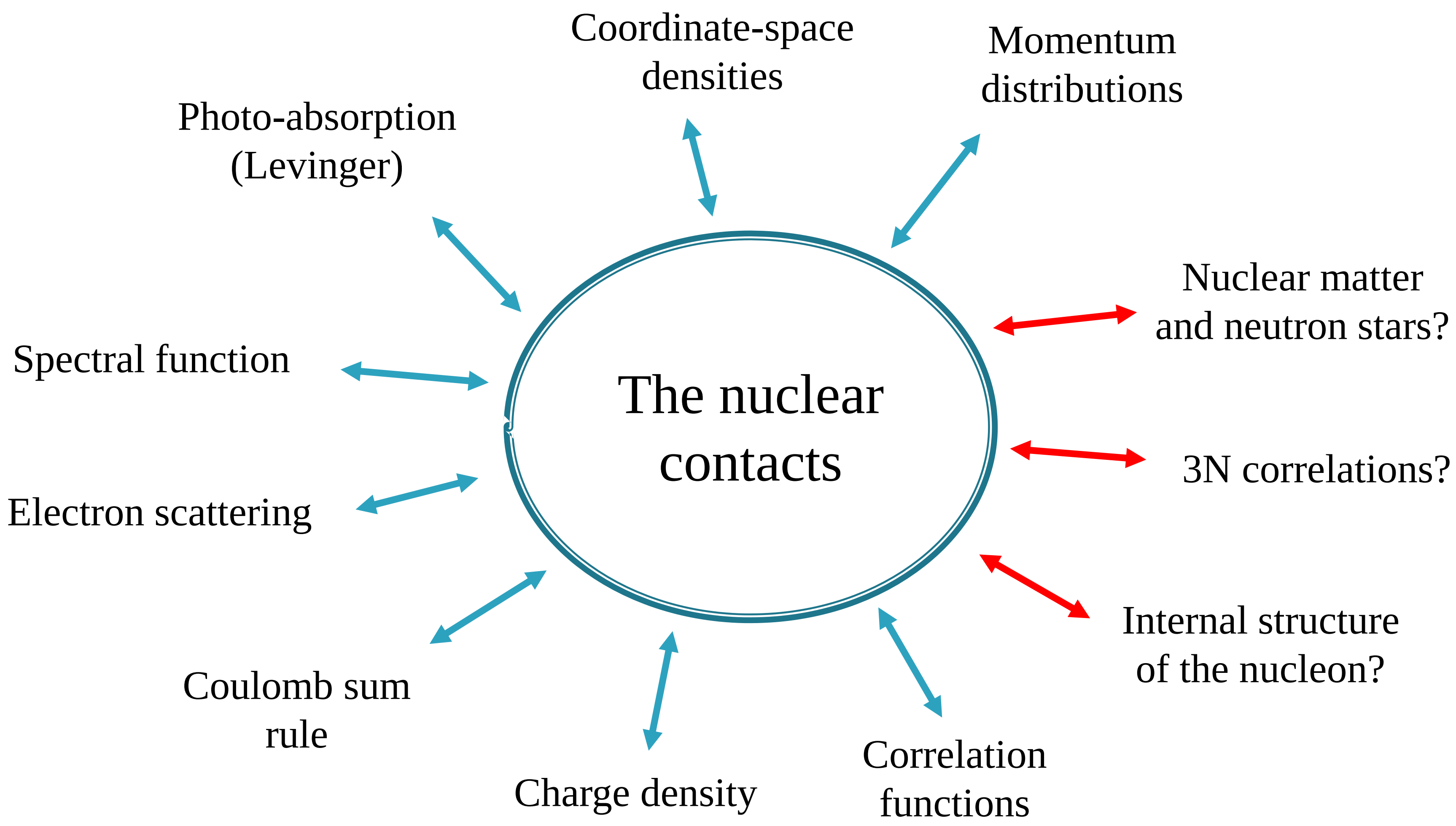
3. The high-momentum **spectral function** is calculated using the contact formalism.

4. Provide predictions for the energy and momentum dependence of **exclusive scattering experiments**.

Summary

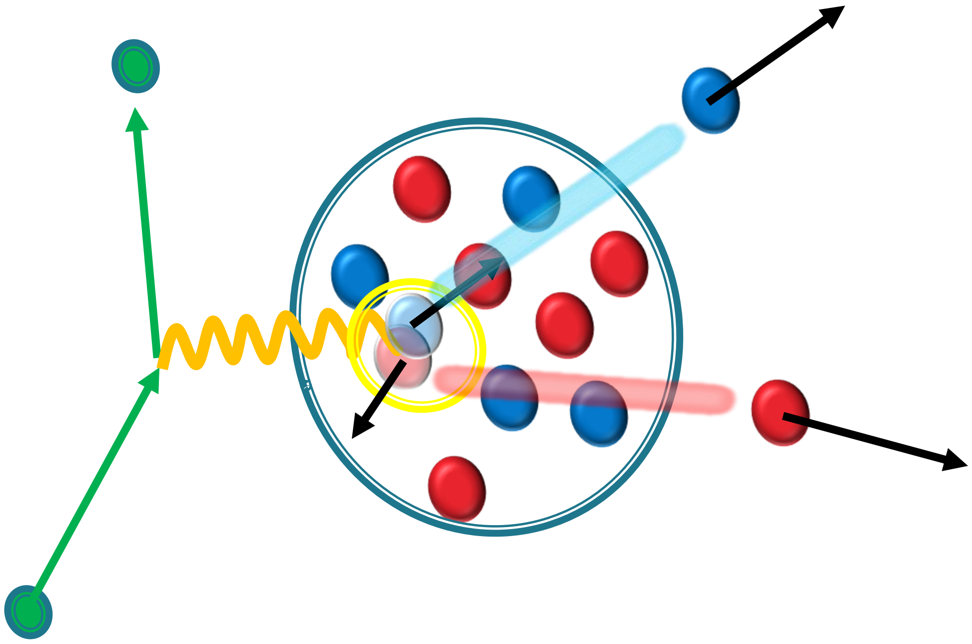

Coordinate-space densities
Momentum distributions
Photo-absorption (Levinger)
Spectral function
The nuclear contacts
Electron scattering
Coulomb sum rule
Charge density
Correlation functions

Summary

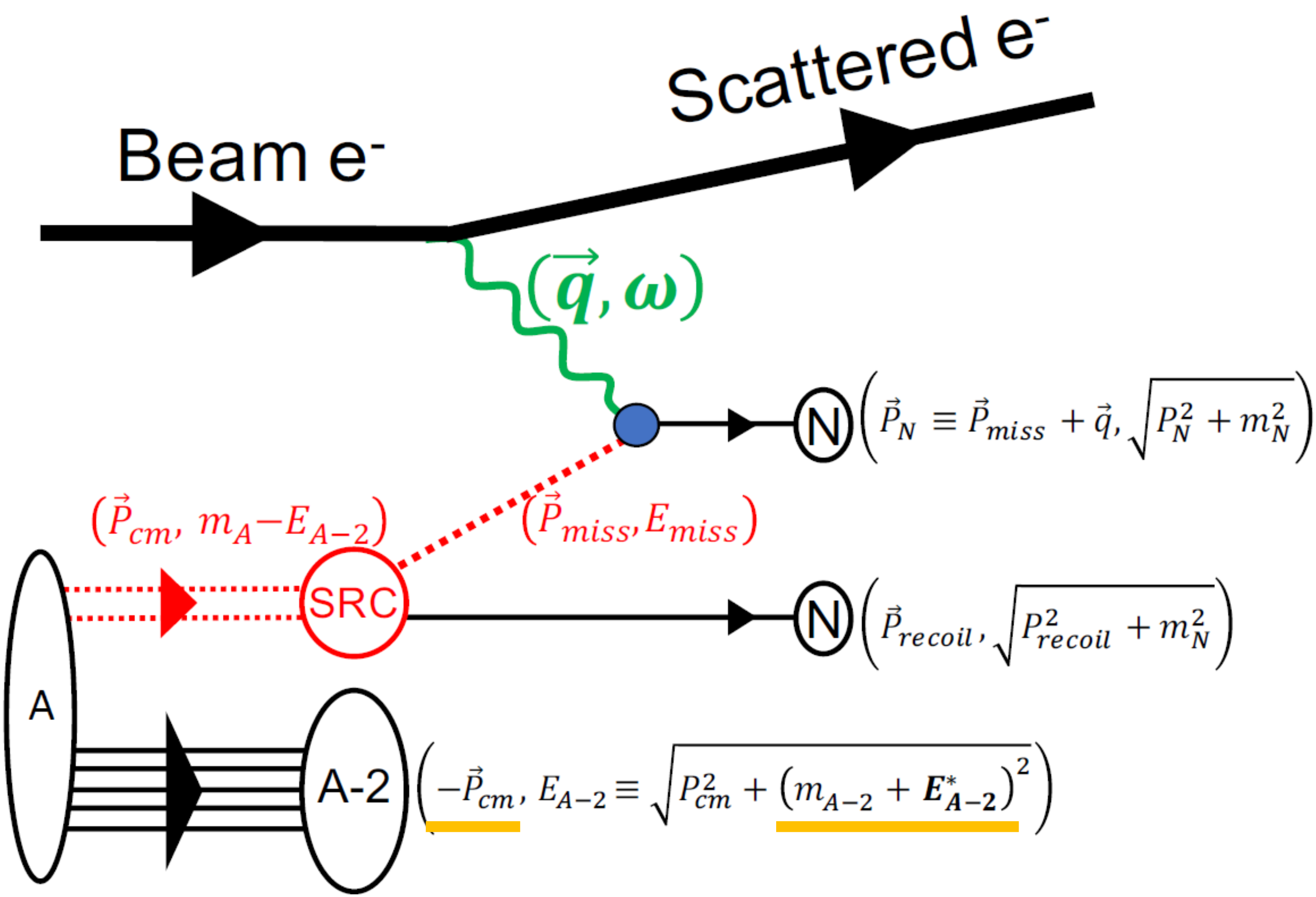


Coordinate-space densities
Momentum distributions
Photo-absorption (Levinger)
Nuclear matter and neutron stars?
Spectral function
The nuclear contacts
3N correlations?
Electron scattering
Internal structure of the nucleon?
Coulomb sum rule
Charge density
Correlation functions

Questions?



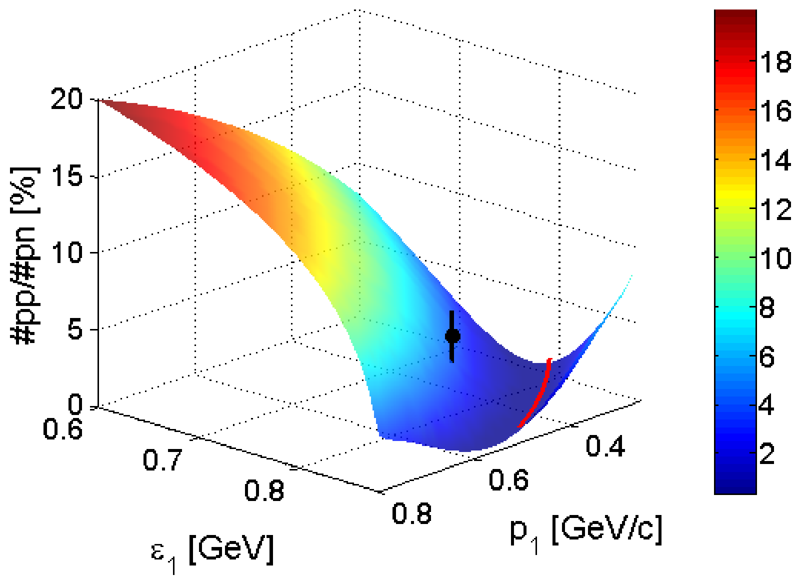
Inclusive cross section



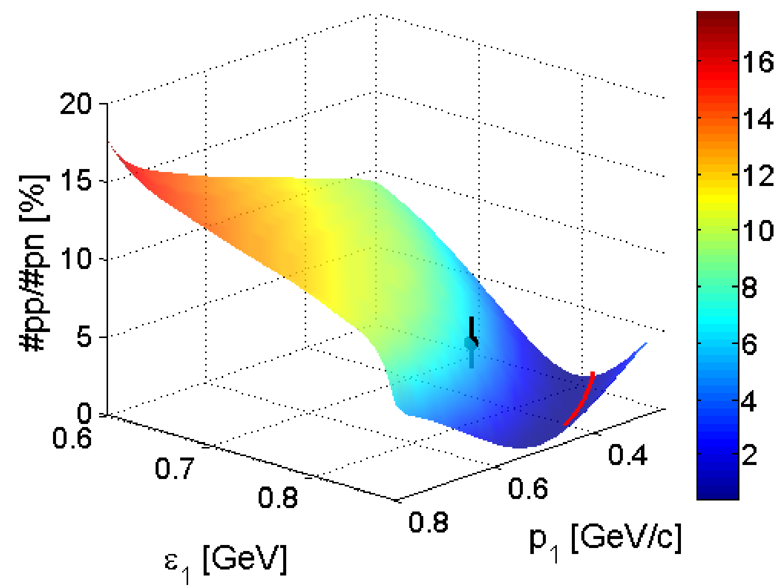
Additional figures

^{12}C $\#pp/\#pn$

AV18



N3LO

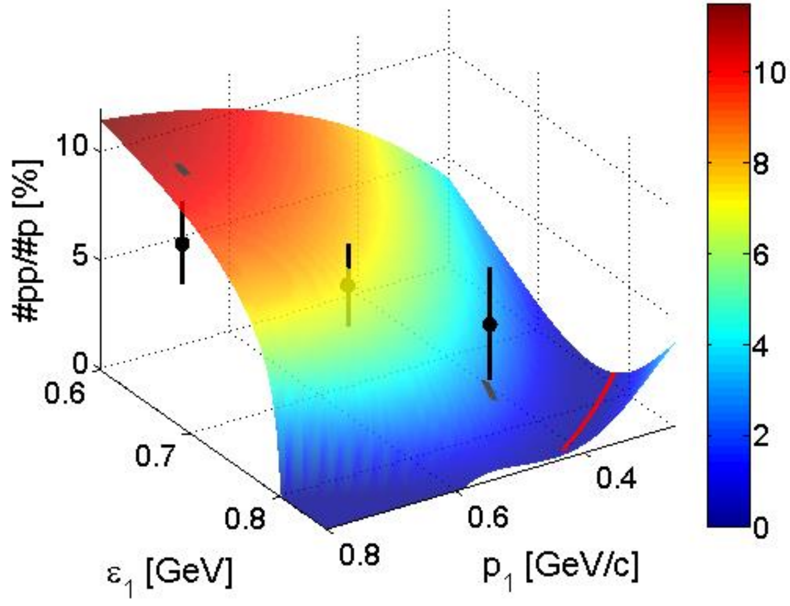


Experimental data:
R. Shneor et al., Phys. Rev. Lett. 99, 072501 (2007)

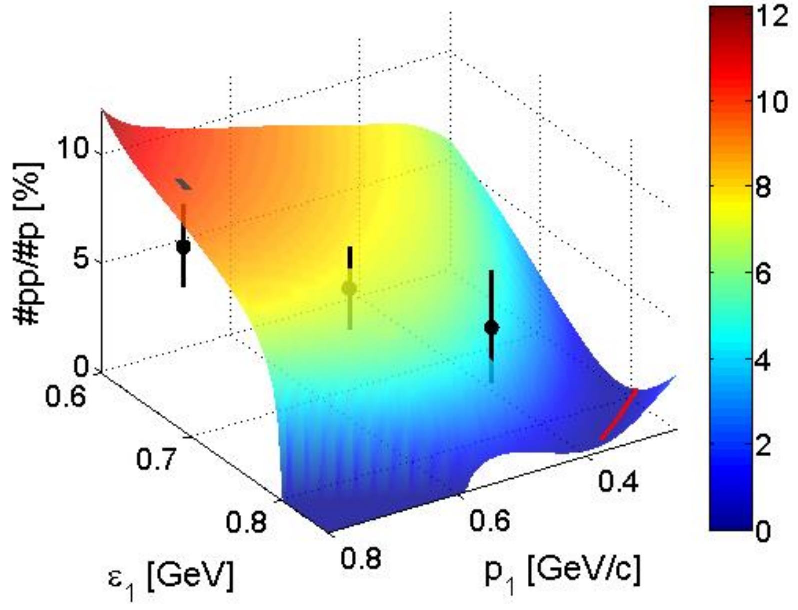
Additional figures

^{4}He $\#pp/\#p$

AV18



N3LO

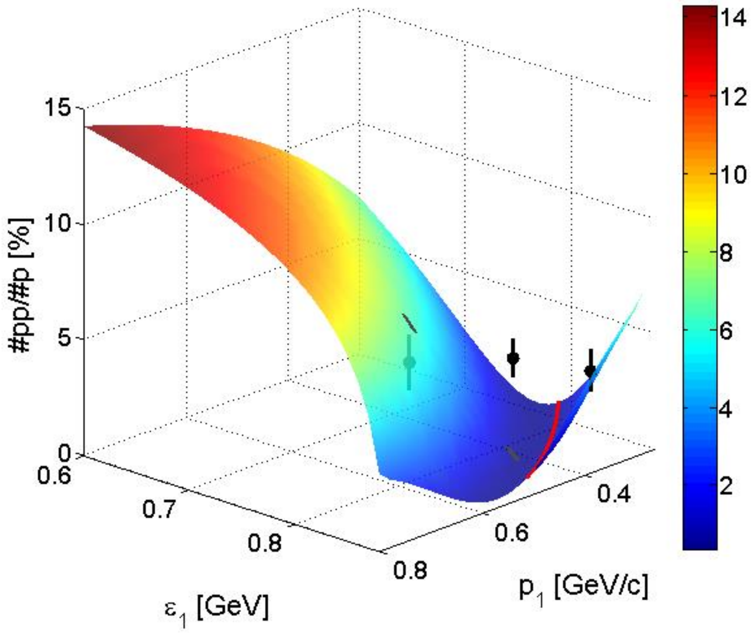


Experimental data from Korover et. al. [PRL 113, 022501 (2014)]:

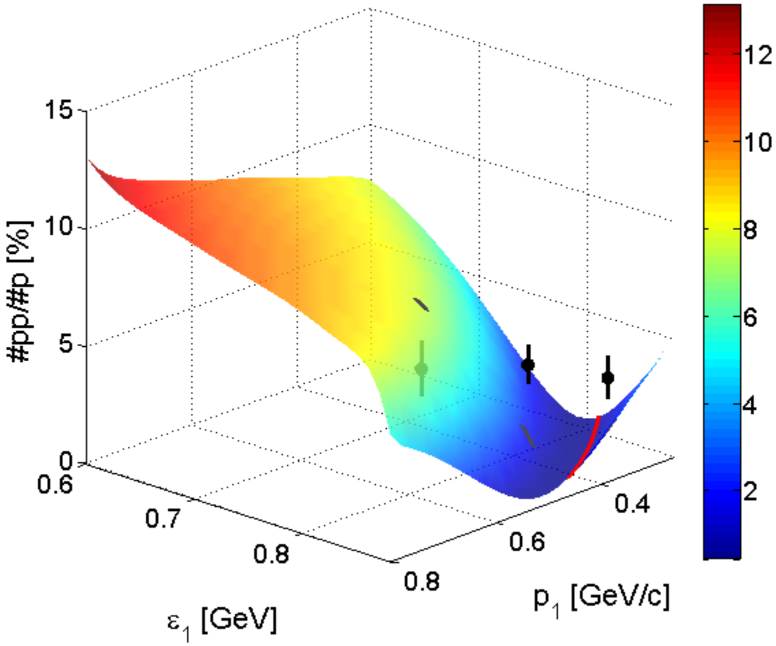
Additional figures

^{12}C $\#pp/\#p$

AV18



N3LO

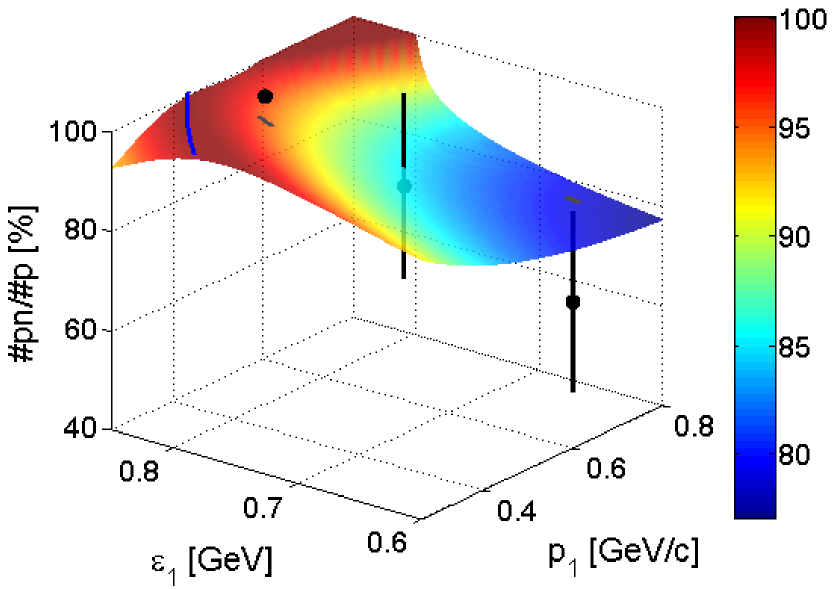


Experimental data:
R. Shneor et al., Phys. Rev. Lett. 99, 072501 (2007)

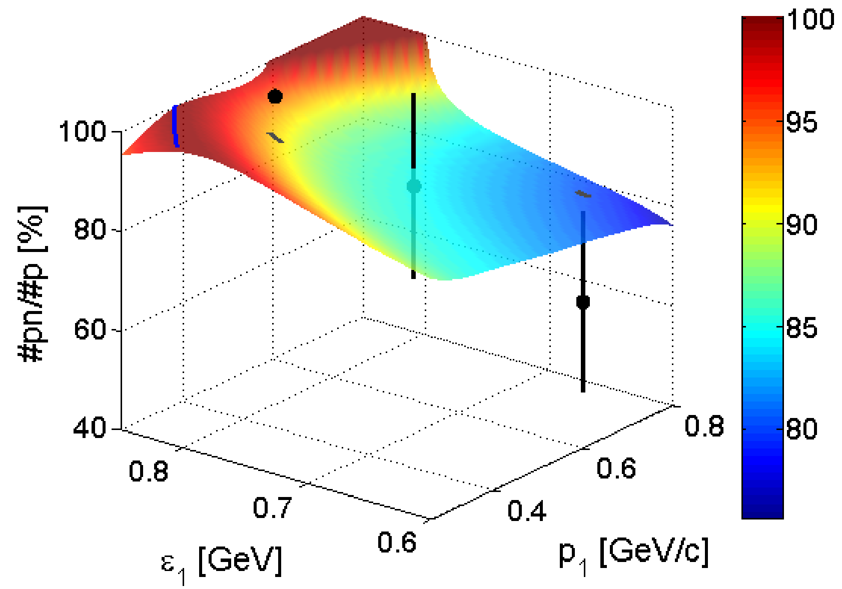
Additional figures

^{4}He $\#pn/\#p$

AV18



N3LO

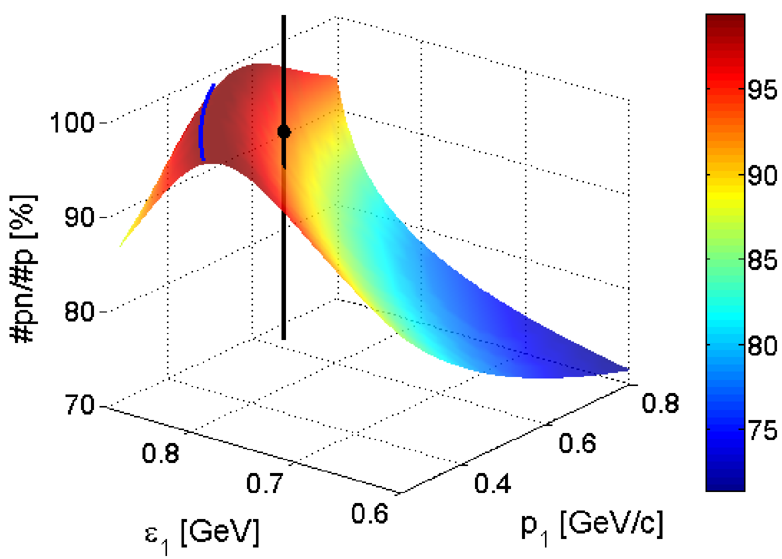


Experimental data from Korover et. al. [PRL 113, 022501 (2014)]:

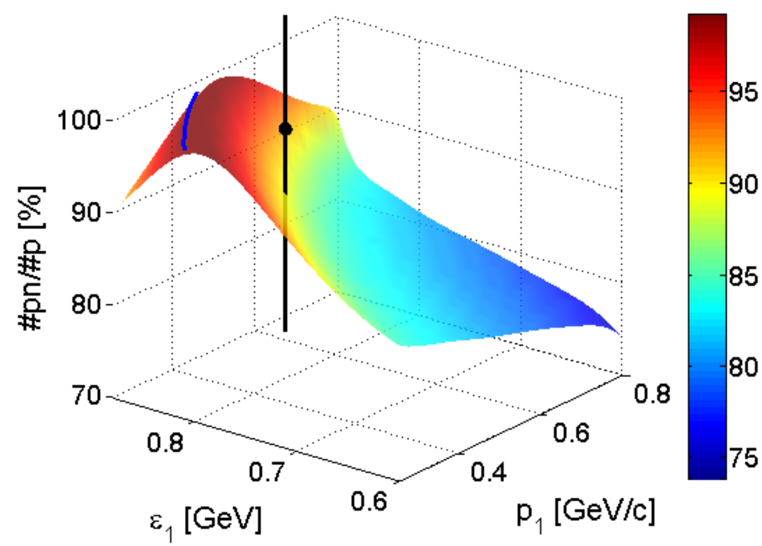
Additional figures

^{12}C $\#pn/\#p$

AV18



N3LO

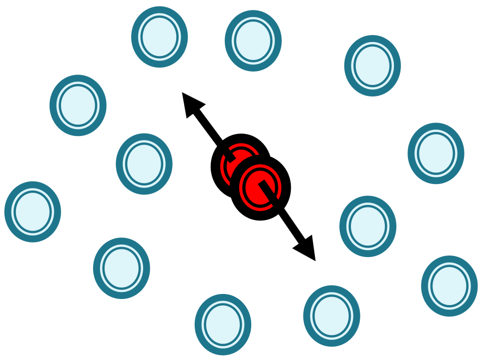


Experimental data:
R. Shneor et al., Phys. Rev. Lett. 99, 072501 (2007)

The starting point - Factorization

- The factorization of the wave function:

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{n_1 n_2} \to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k \neq n_1, n_2}\right)$$



The starting point - Factorization

- The factorization of the wave function:

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{n_1 n_2} \to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k \neq n_1, n_2}\right)$$

- The two-body system:

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi = E\varphi$$

For $r \to 0$: The energy becomes negligible $E \ll \frac{\hbar^2}{mr^2}$

$\Rightarrow$ $\varphi(r) \equiv$ The **zero-energy** solution of the Schrodinger Eq.

The starting point - Factorization

- The factorization of the wave function:

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{n_1 n_2} \to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k \neq n_1, n_2}\right)$$

- The simplest example – nn density:

$$\rho_{nn}(\boldsymbol{r}) = \langle \Psi | \delta(\boldsymbol{r}_{nn} - \boldsymbol{r}) | \Psi \rangle$$

$$\rho_{nn}(\boldsymbol{r}) \xrightarrow{r \to 0} \langle \varphi_{nn} | \delta(\boldsymbol{r}_{nn} - \boldsymbol{r}) | \varphi_{nn} \rangle \frac{N(N-1)}{2} \langle A_{nn} | A_{nn} \rangle = |\varphi_{nn}(\boldsymbol{r})|^2 \frac{N(N-1)}{2} \langle A_{nn} | A_{nn} \rangle$$

The starting point - Factorization

- The factorization of the wave function:

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{n_1 n_2}\to 0} \varphi_{nn}(r_{nn}) \times A_{nn}\left(\boldsymbol{R}_{nn}, \{\boldsymbol{r}_k\}_{k\neq n_1,n_2}\right)$$

- The simplest example – nn density:

$$\rho_{nn}(\boldsymbol{r}) \xrightarrow{r\to 0} |\varphi_{nn}(\boldsymbol{r})|^2 \frac{N(N-1)}{2} \langle A_{nn}|A_{nn}\rangle$$

- Universal for all nuclei
- Simply calculated

The probability to find a correlated pair

The nn contact $C_{nn} \equiv \frac{N(N-1)}{2} \langle A_{nn}|A_{nn}\rangle$

The atomic contact

- Zero-range condition: $r_0 \ll a, d$

Interaction range → r_0; Scattering length → a; Distance between particles → d

- Many quantities are connected to the ***contact C***:

$$n(k) = \boldsymbol{C}/k^4 \text{ for } k \to \infty$$

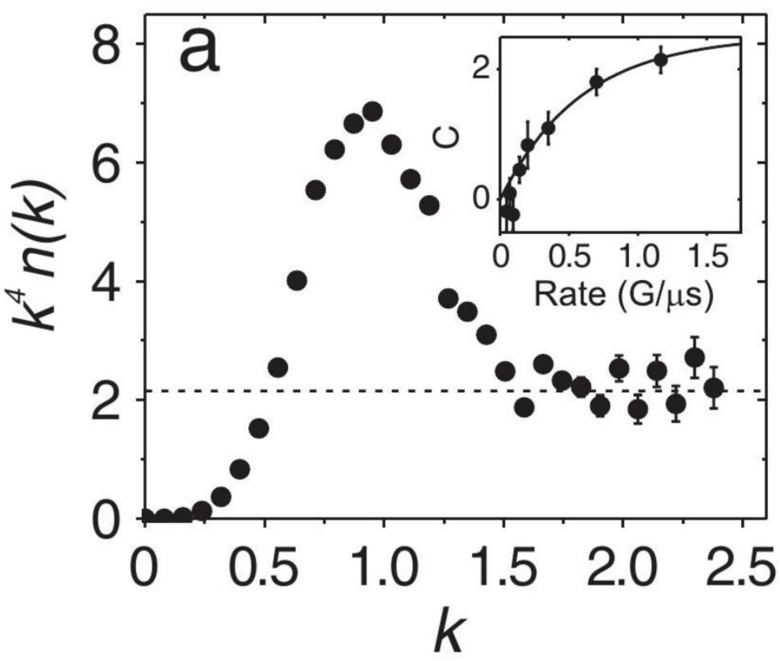
$$T + U = \frac{\hbar^2}{4\pi m a}\boldsymbol{C} + \sum_\sigma \frac{d^3k}{(2\pi)^3}\frac{\hbar^2 k^2}{2m}\left(n_\sigma(\boldsymbol{k}) - \frac{\boldsymbol{C}}{k^4}\right)$$

and many more…

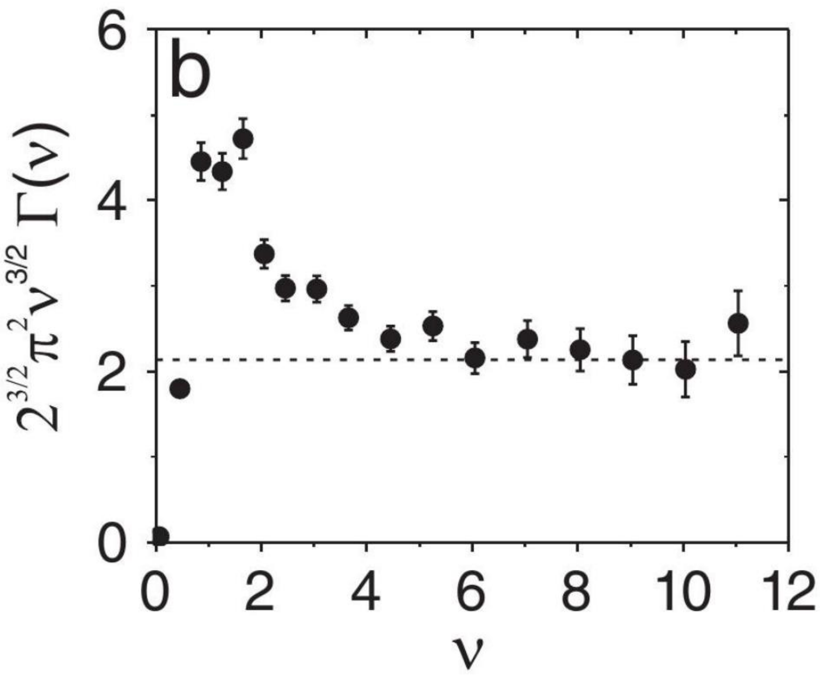
S. Tan, Ann. Phys. (N.Y.) 323, 2952 (2008); Ann. Phys. (N.Y.) 323, 2971 (2008); Ann. Phys. (N.Y.) 323, 2987 (2008)

The atomic contact

Momentum distribution



RF line shape



J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)

From atoms to nucleons

$$\Psi \xrightarrow{r_{ij}\to 0} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right) \qquad r_0 \ll d, a$$

$$\Psi \xrightarrow{r_{ij}\to 0} \varphi_{ij}(r_{ij}) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

~~$r_0 \ll d, a$~~

$$\Psi \xrightarrow{r_{ij}\to 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) \times A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{ij}^{\alpha\beta} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

Channels $\alpha = (\ell_2 S_2) j_2 m_2$

"universal" function

The pair kind $ij \in \{pp, nn, pn\}$

3 matrices