High-momentum tails and lowmomentum effective theories

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High-q operators in low resolution effective theories



Consider low-k components of low-E wf's for A=2.





Consider high-k components of low-E wf's for A=2.





Consider high-k components of low-E wf's for A=2.

Scale separation ($E_{\alpha} << \Lambda^2 << q^2$)

$$\psi_{\alpha}^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^{\Lambda} d^3 p \, Z_{\Lambda} \psi_{\alpha}^{\Lambda}(\mathbf{p}) + \eta(\mathbf{q};\Lambda) \int_0^{\Lambda} d^3 p \, \mathbf{p}^2 Z_{\Lambda} \psi_{\alpha}^{\Lambda}(\mathbf{p}) \cdots$$

 $\mathbf{p}=0$

Operator **P**roduct **E**xpansion of wave function a-la Lepage

$$\begin{split} \gamma(\mathbf{q};\Lambda) &= -\int_{\Lambda}^{\Lambda_0} d\mathbf{q}' \langle \mathbf{q} | \frac{1}{QH^{\Lambda_0}Q} | \mathbf{q}' \rangle V^{\Lambda_0}(\mathbf{q}',0) \\ \beta(\mathbf{q};\Lambda) &= -\int_{\Lambda}^{\Lambda_0} d\mathbf{q}' \langle \mathbf{q} | \frac{1}{QH^{\Lambda_0}Q} | \mathbf{q}' \rangle \frac{\partial^2}{\partial p^2} V^{\Lambda_0}(\mathbf{q}',\mathbf{p}) \end{split}$$

State-independent Wilson Coefficients

Q

Λ

р



$$\langle \psi_{\alpha}^{\Lambda_{0}} | \hat{O}_{\Lambda_{0}} | \psi_{\alpha}^{\Lambda_{0}} \rangle = \int_{0}^{\Lambda} dp \int_{0}^{\Lambda} dp' \, \psi_{\alpha}^{\Lambda_{0}*}(p) O(p,p') \psi_{\alpha}^{\Lambda_{0}}(p') + \int_{0}^{\Lambda} dp \int_{\Lambda}^{\Lambda_{0}} dq \, \psi_{\alpha}^{\Lambda_{0}*}(p) O(p,q) \psi_{\alpha}^{\Lambda_{0}}(q)$$

$$+ \int_{\Lambda}^{\Lambda_0} dq \int_0^{\Lambda} dp \,\psi_{\alpha}^{\Lambda_0*}(q) O(q,p) \psi_{\alpha}^{\Lambda_0}(p) + \int_{\Lambda}^{\Lambda_0} dq \int_{\Lambda}^{\Lambda_0} dq' \,\psi_{\alpha}^{\Lambda_0*}(q) O(q,q') \psi_{\alpha}^{\Lambda_0}(q')$$

Now use:

$$\psi_{\alpha}^{\Lambda_{0}}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_{0}^{\Lambda} d^{3}p \, Z_{\Lambda} \psi_{\alpha}^{\Lambda}(\mathbf{p}) + \cdots$$
 OPE for w.f.'s
 $\psi_{\alpha}^{\Lambda_{0}}(\mathbf{p}) \approx Z_{\Lambda} \psi_{\alpha}^{\Lambda}(\mathbf{p})$ IR structure unaltered

 $O(q,p) \approx O(q,0) + \cdots$ Scale separation





Generically: $\widehat{O}_{\Lambda} = Z_{\Lambda}^2 \, \widehat{O}_{\Lambda_0} \, + \, g^{(0)}(\Lambda) \, \delta(\mathbf{r}) \, + \, g^{(2)}(\Lambda) \, \nabla^2 \delta(\mathbf{r}) \, + \, \cdots$

Scaling of high momentum tails



How does an operator that probes high-momentum w.f. components look in a low-momentum effective theory?

$$\begin{split} \langle \psi_{\alpha}^{\Lambda_{0}} | \hat{O}_{\Lambda_{0}} | \psi_{\alpha}^{\Lambda_{0}} \rangle &\approx Z_{\Lambda}^{2} \langle \psi_{\alpha}^{\Lambda} | \hat{O}_{\Lambda_{0}} | \psi_{\alpha}^{\Lambda} \rangle + g^{(0)}(\Lambda) \langle \psi_{\alpha}^{\Lambda} | \delta^{(3)}(\mathbf{r}) | \psi_{\alpha}^{\Lambda} \rangle + \cdots \\ &= 0 \text{ since } P_{\Lambda} O_{\Lambda_{0}} P_{\Lambda} = 0 \end{split}$$

E.g., momentum distribution for $q >> \Lambda$

 $\langle \psi_{\alpha}^{\Lambda_0} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | \psi_{\alpha}^{\Lambda_0} \rangle \approx \gamma^2(\mathbf{q}; \Lambda) Z_{\Lambda}^2 | \langle \psi_{\alpha}^{\Lambda} | \delta(\mathbf{r}) | \psi_{\alpha}^{\Lambda} \rangle |^2$

low-E states have the same large-q tails if leading OPE term dominates

Generalize to arbitrary **A-body** states?

Scaling of high momentum tails



SKB and Roscher, PRC 86 (2012)

Ex1: momentum distribution from leading order OPE ($\Lambda \ll q \ll \Lambda_0$):

$$\langle \psi_{\alpha,A}^{\Lambda_{0}} | a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} | \psi_{\alpha,A}^{\Lambda_{0}} \rangle \approx \gamma^{2}(\mathbf{q};\Lambda) \times \sum_{\mathbf{k},\mathbf{k}',\mathbf{K}}^{\cdot} Z_{\Lambda}^{2} \langle \psi_{\alpha,A}^{\Lambda} | a_{\mathbf{k}+\mathbf{k}}^{\dagger} a_{\mathbf{k}-\mathbf{k}'}^{\dagger} a_{\mathbf{k}+\mathbf{k}'}^{\dagger} | \psi_{\alpha,A}^{\Lambda} \rangle$$

$$\int_{\mathbf{q}}^{\mathfrak{q}} \int_{\mathbf{q}}^{\mathfrak{q}} \int_{\mathbf{q}}^{\mathfrak{q$$

Λ

Scaling of high momentum tails



SKB and Roscher, PRC 86 (2012)

Ex2: static structure factor from leading order OPE ($\Lambda << q < \Lambda_0$):

$$\begin{split} \langle \psi_{\alpha,A}^{\Lambda_{0}} | \widehat{S}(\mathbf{q}) | \psi_{\alpha,A}^{\Lambda_{0}} \rangle &\approx & \left\{ 2\gamma(\mathbf{q};\Lambda) + \sum_{\mathbf{P}} \gamma(\mathbf{P} + \mathbf{q};\Lambda) \gamma(\mathbf{P};\Lambda) \right\} \\ & \times & \sum_{\mathbf{K},\mathbf{k},\mathbf{k}'}^{\Lambda} Z_{\Lambda}^{2} \langle \psi_{\alpha,A}^{\Lambda} | a_{\frac{\mathbf{K}}{2} + \mathbf{k}}^{\dagger} a_{\frac{\mathbf{K}}{2} - \mathbf{k}'}^{\dagger} a_{\frac{\mathbf{K}}{2} + \mathbf{k}'}^{\dagger} | \psi_{\alpha,A}^{\Lambda} \rangle \\ & \quad - \text{hard (high q) physics} \\ & \quad - \text{ lniversal (state-indep)} \\ & \quad - \text{ fixed from A=2} \\ \end{split} \qquad \mathbf{X} \qquad \begin{array}{l} - \text{ soft (low-k) m.e.} \\ & \quad - \text{ same for all high-q probes} \\ & \quad - \text{ A-dependent scale factor} \\ \end{array}$$

OPE links few- and A-body systems (cf talks of R. Weiss and J. Ryckebusch)

Correlations/scaling for 2 observables w/same leading OPE (e.g., EMC-SRC)

Subleading OPE ==> deviations from scaling calculable in principle

Ex: 0vββ and Double GT correlation





DGT and $0\nu\beta\beta$ operators main contribution for r < 2 fm

Same leading operator in OPE => linear relation

slope at ~10-15% level

Scale dependence of deuteron electrodisintegration

Test ground: ²H(e,e'p)n



 $(\omega_{
m lab}, {f q}_{
m lab})$

 $\partial \theta_{i}$

Scatteringe plane

reaction plane

z

 ϕ_p

- Simplest knockout process (no induced 3N forces/currents)
- Focus on longitudinal structure function fL

$$f_L \sim \sum_{m_s, m_I} \left| \langle \psi_f | J_0 | \psi_i \rangle \right|^2$$

$$f_L^{\lambda} \sim \left| \langle \underbrace{\psi_f | U_{\lambda}^{\dagger}}_{\psi_f^{\lambda}} \underbrace{U_{\lambda} J_0 U_{\lambda}^{\dagger}}_{J_0^{\lambda}} \underbrace{U_{\lambda} | \psi_i}_{\psi_i^{\lambda}} \rangle \right|^2; \quad U_{\lambda}^{\dagger} U_{\lambda} = I; \quad f_L^{\lambda} = f_L$$

- Components (deuteron wf, transition operator, FSI) scale-dependent, total is not.
- Are some resolutions "better" than others? E.g., in a given kinematics, can FSI be minimized with different choices of λ?? How do interpretations change with scale?

Deuteron wave function evolution





 $k < \lambda$ components invariant <==> RG preserves long-distance physics

 $k > \lambda$ components suppressed <==> short-range correlations blurred out

Folklore: Simple wave functions at low $\lambda \leq =>$ more complicated operators? especially for high-q processes?

Final-state wave function evolution





- High-k tail suppressed with evolution
- For $p' \gtrsim \lambda$, $\Delta \psi_f^{\lambda}(p';k)$ localized around outgoing p' "local decoupling" Dainton et al. PRC 89 (2014)

Current operator evolution







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Current operator evolution







Look at kinematics relevant to SRC studies



FSI sizable at large λ but negligible at low-resolution!

Folklore:

shouldn't hard processes be complicated in low resolution $(\lambda << q)$ pictures?

Why are FSI so small at low λ ?

λ dependence of Final State Interactions







For $p' \ge \lambda$, interacting part of final state wf localized at $k \approx p'$

initial state (deuteron) wf



Dominant support of deuteron wf at $k \lesssim \lambda$

λ dependence of Final State Interactions





λ dependence of interpretations



- Analysis/interpretation of a reaction involves understanding which part of wave functions probed (highly scale dependent!)
- E.g., sensitivity to D-state w.f. in large q² processes



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 Consider large q² near threshold (small p') for θ=0 in highresolution picture (COM frame of outgoing np)



photon only couples to proton



 Consider large q² near threshold (small p') for θ=0 in highresolution picture (COM frame of outgoing np)



photon only couples to proton

• proton has large momentum => initial large relative momentum

(i.e., SRC pair)

λ dependence of SRC interpretation



 Consider large q² near threshold (small p') for θ=0 in lowresolution picture (COM frame of outgoing np)



λ dependence of SRC interpretation



 Consider large q² near threshold (small p') for θ=0 in lowresolution picture (COM frame of outgoing np)



no large relative momentum in evolved deuteron wf

1-body current makes no contribution

. 2-body current mostly stops the low-relative momentum np pair

Some questions...

Can we use RG/OPE to connect high- and low-resolution pictures? (E.g., SF's in low-resolution shell model picture and SRCs in highresolution picture)

How do interpretations vary with resolution scale? FSI? Ease of calculations (simple wf's + complicated currents vs. complicated wf's + simple currents)?

Can we use OPE + SRC/high-q measurements to extend reach of low-resolution theories ?

Can we use simpler low-resolution wf's + OPE for to do high-q electron scattering in medium mass nuclei?

Can OPE be used to calculate "subleading" corrections to GCF (R. Weiss's talk) and LCA (J. Ryckebusch's talk) approaches?

Extras

Final-state wave function evolution

Final-state wave function evolution

- Correlation "wound" at small r smoothed out under evolution
- Long-distance tail invariant (phase shifts preserved)

Deuteron electrodisintegration kinematics

Wave function factorization

LO:
$$\psi_{\alpha}^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q}; \Lambda) \int_0^{\Lambda} d^3 p \, Z_{\Lambda} \psi_{\alpha}^{\Lambda}(\mathbf{p})$$
 state for v

state-independent ratio for well-separated scales

$$\frac{\psi_{\alpha}^{\Lambda_{0}}(\mathbf{q})}{\psi_{\alpha}^{\Lambda}(\mathbf{r}=0)} \sim \gamma(\mathbf{q};\Lambda)$$
$$|E_{\alpha}| \lesssim \Lambda^{2} \quad |\mathbf{q}| \gtrsim \Lambda$$

Wave function factorization

state-independent ratio for well-separated scales

Scale dependence of short range correlations in medium-mass nuclei

in collaboration with

Nathan Parzuchowski Dick Furnstahl

EMC Effect

- Non-interacting limit ratio should be 1
- BE/A ~ 1% of $M_{N,\,}Q$
- 20% deviations from 1 = EMC effect

nucleon structure modified in-medium

Slope used to quantify size of effect

Quasi-elastic (e,e'2N)

plateaus for 1.5< x <2.0

SRC interpretation:

NN interaction scatters pair $p_1, p_2 < k_F$ to intermediate-state momenta >> k_F which are then knocked out by photon

$$a_2(A) = \frac{2}{A} \frac{\sigma_A(x_B, Q^2)}{\sigma_d(x_B, Q^2)} \approx \frac{n_A(\mathbf{q} > k_F)}{n_d(\mathbf{q} > k_F)}$$

Hen et al., Rev. Mod. Phys. 89 (2017)

NUCLEON MOMENTUM

Empirical correlation of EMC effect

Hen et al., RMP (2017); Hen et al., IJMPE (2013); Hen et al., PRC (2012); Weinstein, Piasetzky, Higinbotham, Gomez, Hen, and Shneor, PRL (2011).

Why should 2 seemingly unrelated processes be linearly related?

EFT explanation

Match isoscalar twist-2 quark operators to LO nucleon operators

$$\underbrace{ \left(\begin{array}{c} \bullet \\ \bullet \end{array} \right)}_{\square} \xrightarrow{ } \left(x^n \right)_A(Q) \approx \langle x^n \rangle_N(Q) \left[A + \alpha_n(\Lambda) \langle A | : (N^{\dagger}N)^2 : |A \rangle \right] + \cdots$$

DIS: $F_2^A(x,Q^2)/A \approx F_2^N(x,Q^2) + g_2(A,\Lambda)f_2(x,Q^2,\Lambda)$ $g_2(A,\Lambda) = \frac{1}{2A}\langle A| : (N^{\dagger}N)^2 : |A\rangle_{\Lambda}$

QES: $\sigma_A/A \approx \sigma_N + g_2(A, \Lambda)\sigma_2(\Lambda)$

$$a_2(A, 1 < x < 2) \approx \frac{g_2(A, \Lambda)}{g_2(2, \Lambda)}$$
$$\frac{dR_{EMC}(A, x)}{dx} \approx C(x) [a_2(A) - 1]$$

scale dependence cancels in ratio! linear relation between EMC/SRC!

QMC results for a₂

$$\rho_{2,1}(A,r) = \frac{1}{4\pi r^2} \langle \Psi | \sum_{i< j}^A \delta(r - |\mathbf{r}_i - \mathbf{r}_j|) | \Psi \rangle$$

Scale and scheme dependent

...But ratio is ~ independent of scale and scheme!!

IM-SRG(2) calculations of a₂

Opportunities

access heavier nuclei (here up to A = 40) access wider range of interactions^{**} (not limited to local interactions)

Challenges

QMC cleanly extrapolates to r = 0 (vs. implicit smearing due to truncated HO basis) impact of IM-SRG(2) truncation errors?

** here we use the semi-local n4lo NN interaction of Reinert, Krebs, and Epelbaum

arXiv:1711.08821

IR extrapolations of $\rho_{2,1}$

Truncated HO basis ==> IR cutoff (box size L_{IR}) and UV cutoff

$$L_{IR} \sim \sqrt{2(2n+l)_{\max}+3} \ b$$

$$\rho_{2,1}(L_{IR}) = \rho_{2,1}(\infty) + a_0 e^{-k_\infty L_{IR}} \qquad \Lambda_{UV} \sim \sqrt{2(2n+l)_{\max} + 3} b^{-1}$$

"motivated" by energy/radii formulae More et al. PRC87 (2013)

UV convergence of $\rho_{2,1}$

- No well-founded UV extrapolation formula
- UV convergence reasonable (w/IR correction added) for Λ= 400, 450 MeV
- 500 MeV convergence not so nice

UV convergence of $\rho_{2,1}$

- No well-founded (yet) UV extrapolation formula
- UV convergence not as clean for A > 2, not fully understood

Results for a₂

Need to disentangle UV/IR convergence and IM-SRG(2) truncation errors (and estimate EFT truncation error) before concluding if a_2 scale-independent

One possible hint

- ρ_{21} normalized to Λ =400. Different A-values should be equal
- A=2 systematically off from other A-values
- A=2 done in FCI, while A>2 in IM-SRG(2)
- Maybe better to normalize a_2 to ⁴He instead ?

Results for a₂ (normalized to A=4)

Looks a little more systematic...

... but need better control on IR/UV convergence, IM-SRG(2) truncation errors, etc. to say more

- First IM-SRG calculations of SRC factor a₂(A) carried out for closed-shell systems thru ⁴⁰Ca
- Results "not crazy" [reasonably scale-independent and close to experimental values], but much more work needed to disentangle systematics of IR/UV convergence and IM-SRG(2) truncation errors
- Near term todo list:
 - other EFT interactions (local, semi-local, nonlocal, different chiral orders, different cutoffs, etc.) and SRG-evolved ones to access wider range of resolution scales