Studying Scale Dependence of Contributions to Deuteron Electrodisintegration

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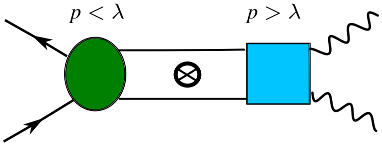
SRG makes scale dependence obvious

- Unitary transformations widely used to soften nuclear Hamiltonians leading to accelerated convergence
- SRG scale λ sets the scale for decoupling high- and low-momentum *and* separating structure and reaction



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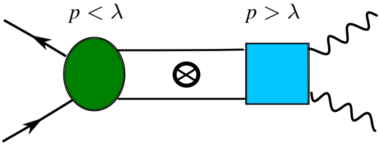
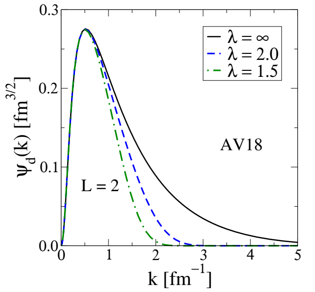
- SRG scale λ sets the scale for decoupling high- and low-momentum *and* separating structure and reaction
- Transformed wave function $\rightarrow$ no high momentum components
- $\sigma \sim \left|\langle\psi_f|\hat{O}|\psi_i\rangle\right|^2 \Rightarrow \hat{O}$ must change to keep observables invariant
- UV physics absorbed in operator (cf. Chiral EFTs)
- What about the final state interactions?





Test ground: $^2\mathrm{H}\,(e, e'\ \mathrm{p})\,\mathrm{n}$

- Use deuteron electrodisintegration to investigate scale/scheme dependence of factorization between nuclear structure and nuclear reaction
- $\dfrac{d\sigma}{d\Omega} \propto \left(v_L f_L + v_T f_T + v_{TT} f_{TT} \cos 2\phi_p + v_{LT} f_{LT} \cos \phi_p\right)$
- $v_L,\ v_T, \ldots$- electron kinematic factors. $f_L,\ f_T, \ldots$- deuteron structure functions

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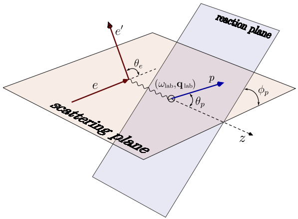
- $f_L \sim \sum_{m_s\,, m_J} \left| \langle \psi_f | J_0 | \psi_i \rangle \right|^2$
- $f_L^\lambda \sim \Big| \underbrace{\langle \psi_f | U_\lambda^\dagger}_{\psi_f^\lambda} \underbrace{U_\lambda\, J_0\, U_\lambda^\dagger}_{J_0^\lambda} \underbrace{U_\lambda\, | \psi_i \rangle}_{\psi_i^\lambda} \Big|^2; \quad U_\lambda^\dagger U_\lambda = I; \quad f_L^\lambda = f_L$

  Components depend on the scale λ. Cross section does not!

Evolutionary effects

- $^2\mathrm{H}\,(e, e'\ \mathrm{p})\,\mathrm{n}$ calculations done using AV18 potential with $\lambda = \infty$ and $\lambda = 1.5\ \mathrm{fm}^{-1}$
- $f_L \sim \sum_{m_s\ ,m_J} \left|\langle\psi_f|J_0|\psi_i\rangle\right|^2$
- Effects due to evolution of one or more components of $\langle\psi_f|J_0|\psi_i\rangle$ as a function of kinematics $\rightarrow$ scale dependence of factorization
- Proof of principle calculations using simplified J_0. Comparison to experiment not warranted



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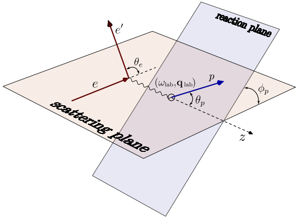

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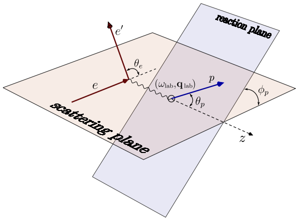
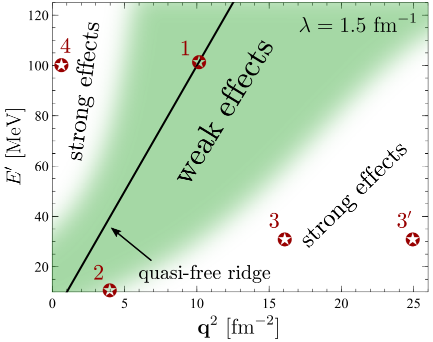
- Weak scale dependence at QFR which gets progressively stronger away from it

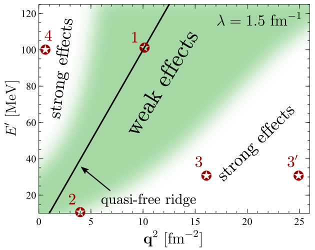


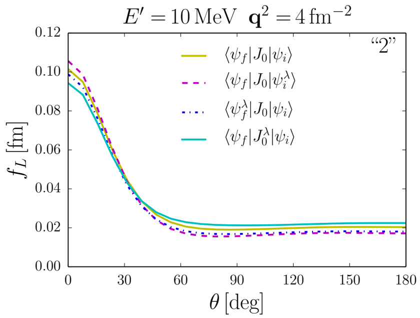


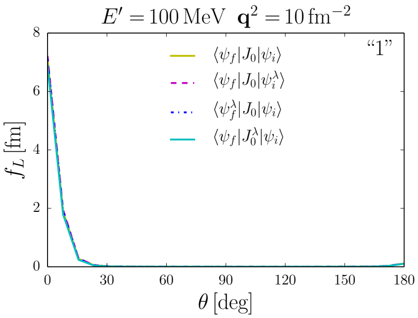
SNM et al., PRC **92**, 064002 (2015)

Results at QFR

- At the quasi-free ridge
  $E'\,(\text{in MeV}) \approx 10\,\mathbf{q}^2\,(\text{in fm}^{-2})$
- $f_L \sim \sum_{m_s\,,m_J} \left|\langle\psi_f|J_0|\psi_i\rangle\right|^2$
- Long-range part of the wave function probed at QFR $\rightarrow$ invariant under SRG evolution

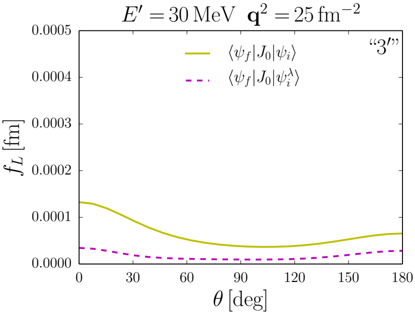


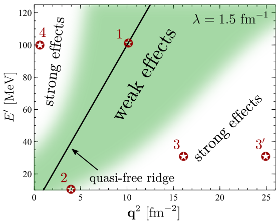


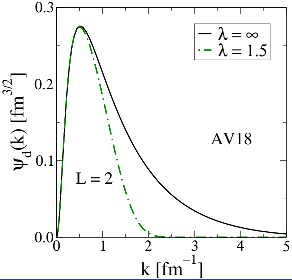


Results below QFR

- $\langle \psi_f | J_0 | \psi_i \rangle = \underbrace{\langle \phi | J_0 | \psi_i \rangle}_{\text{IA}} + \underbrace{\langle \phi | t^\dagger G_0^\dagger J_0 | \psi_i \rangle}_{\text{FSI}}$
- Below QFR two terms add constructively
- Wave function in IA probed between 2.9 and 3.4 fm^{-1} $\Rightarrow |\langle \psi_f | J_0 | \psi_i^\lambda \rangle| < |\langle \psi_f | J_0 | \psi_i \rangle|$







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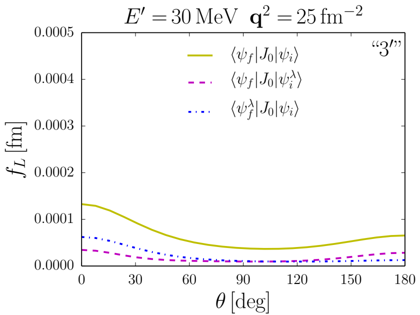

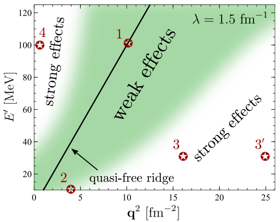

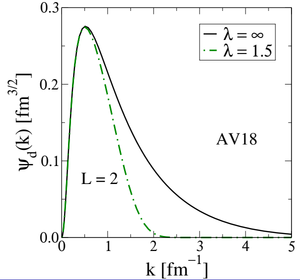

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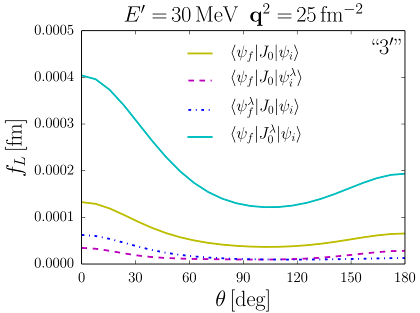

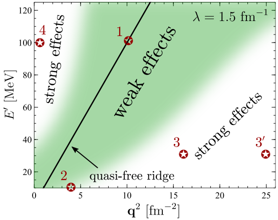

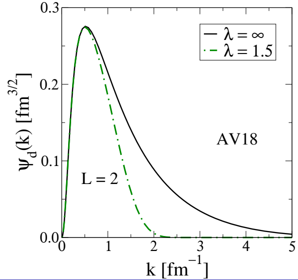

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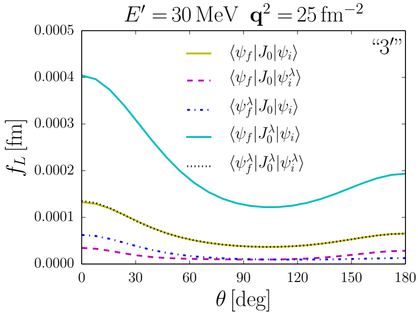
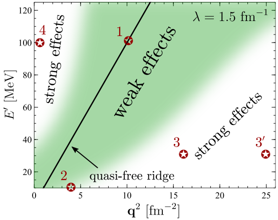
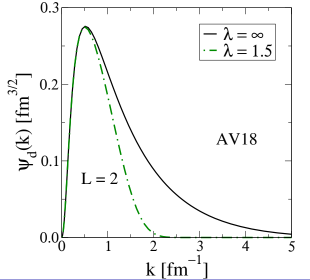
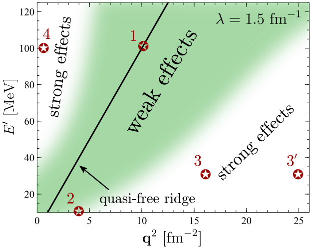


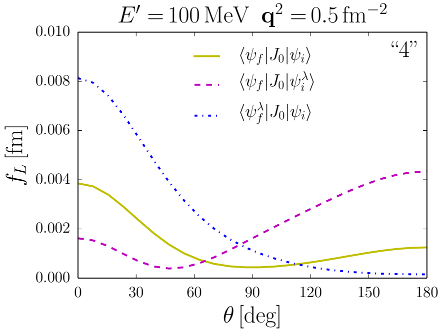




Results above QFR

- Scale dependence qualitatively different above the quasi-free ridge.
- Can be explained by looking at the effect of evolution on the overlap matrix elements
[SNM et al., PRC **92**, 064002 (2015)]





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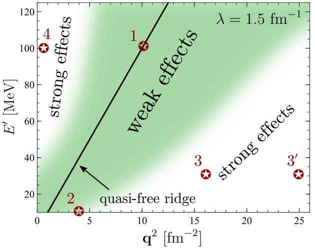
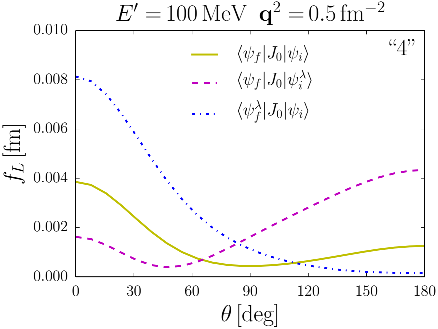
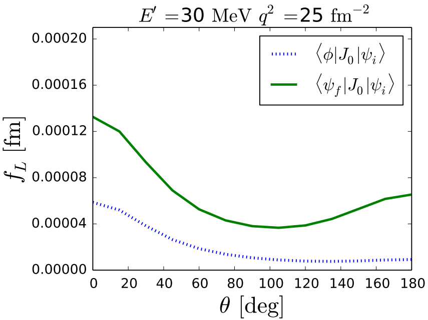
- Scale dependence depends on the kinematics, but in a *systematic* way





SRG evolution and FSI contribution



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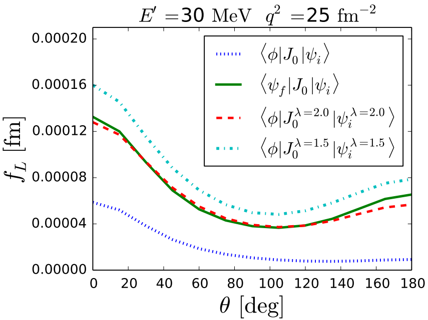

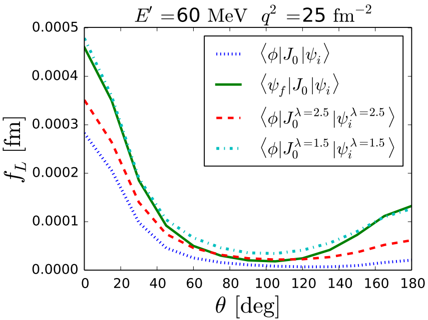

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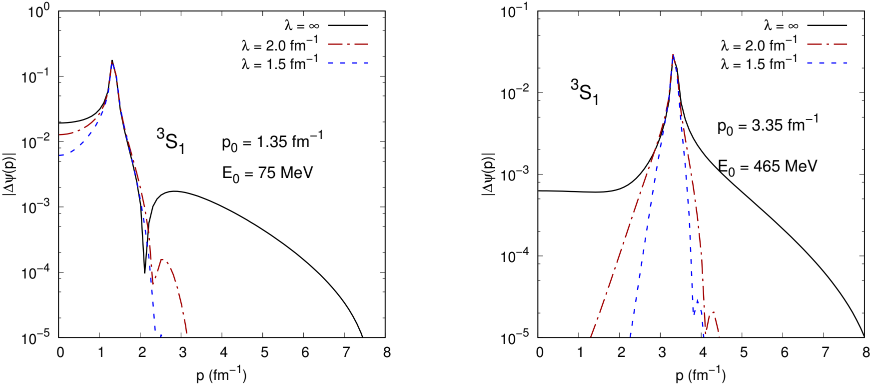
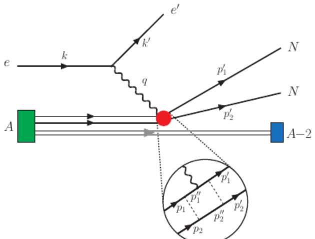
- For certain kinematics and for certain SRG scale, IA works very well!
- Intuitive explanation possible

Final state story



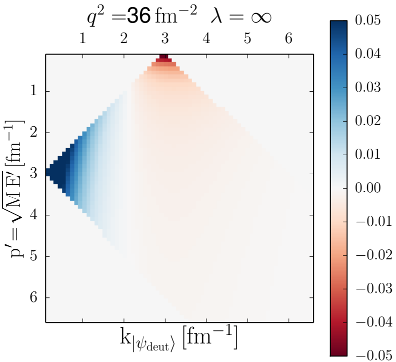
- $|\psi_f^{p_0}\rangle = \underbrace{|\phi_{p_0}\rangle}_{\text{plane wave}} + \overbrace{G_{p_0}}^{\text{Green's fn.}} \underbrace{t_{p_0}}_{t-\text{matrix}} |\phi_{p_0}\rangle$
- $|\psi_f^{p_0}{}_\lambda\rangle = |\phi_{p_0}\rangle + G_{p_0}\, t_{p_0\,\lambda}|\phi_{p_0}\rangle$
- Subtracting off the plane wave part: $\Delta\psi(p) = \langle p|G_{p_0}\, t_{p_0}|\phi_{p_0}\rangle$

Current evolution story



at high resolution

Integrand of $\langle p';^3S_1|J_0^\lambda(q)|\psi^\lambda_{^3S_1}\rangle$



- Varying λ shuffles the physics between short- and long-distance parts
- λ decreases $\to$ blob size increases. One-body current operator develops two and higher body components

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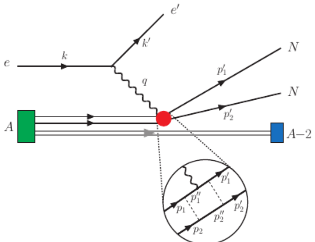

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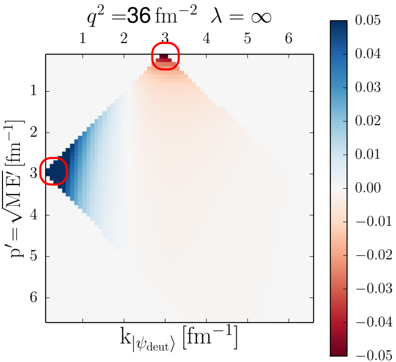

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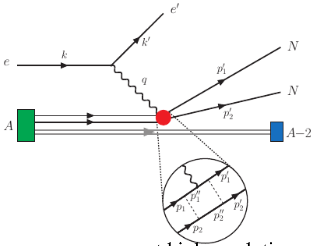

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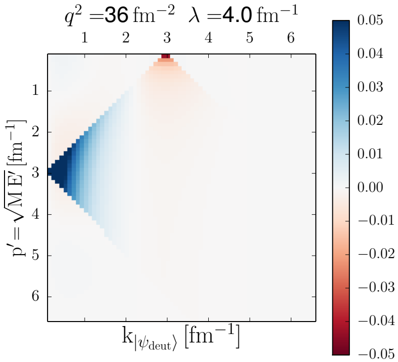

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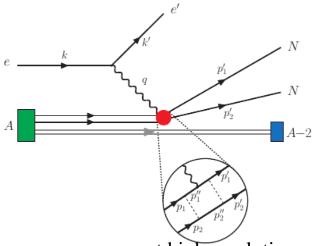

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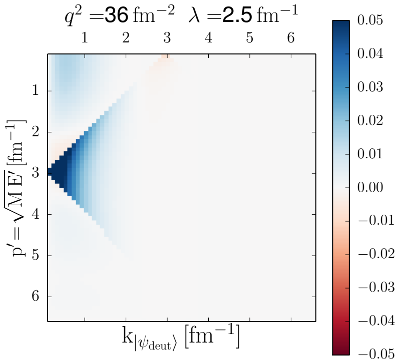

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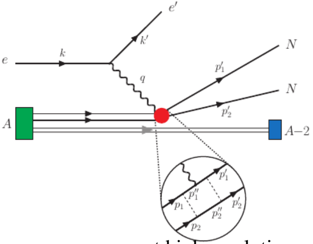

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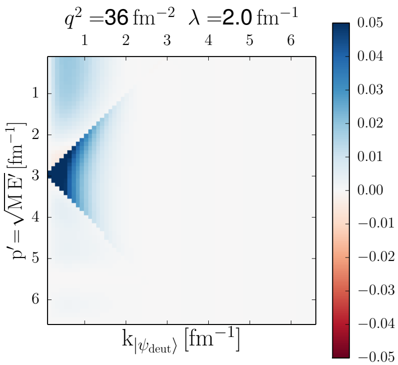

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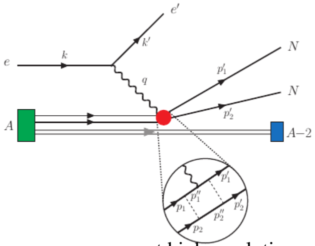

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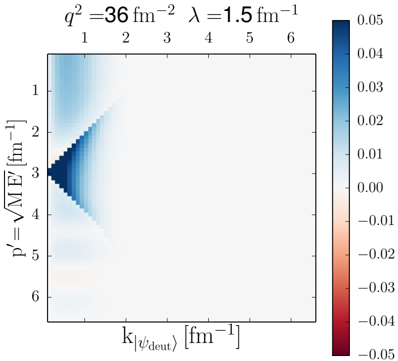

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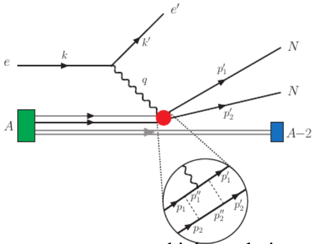

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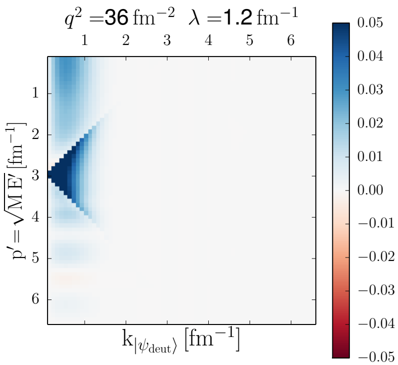


- Varying λ shuffles the physics between short- and long-distance parts
- λ decreases $\rightarrow$ blob size increases. One-body current operator develops two and higher body components
- At low-momentum, the SRG-induced two-body currents are of the form of regulated contact terms

Summary and Moving Forward

- Scale dependence abounds... in a systematic way which can be accounted for.
- The evolved picture helpful in understanding the role of FSI.
- Changes to the current operator due to evolution for certain kinematics factorize into components that depend on high and low momentum
$\rightarrow$ ripe for OPE analysis

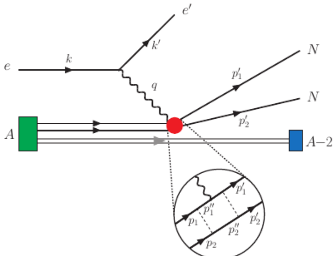
To do:

- Investigate high-momentum processes
- Consistently extract process-independent quantities from experiments
$\rightarrow$ What is the best scale to use?
$\rightarrow$ What are the controlled approximations that we can make?
$\rightarrow$ Model dependence of SRC, spectroscopic factors, . . .

Back up

SRG evolution and FSI contribution: physical picture

- momentum transfer **q** large, momentum of outgoing nucleons $\mathbf{p}'$ small
- Evolved picture: the one-body current couples with the proton to kick it into high momentum state
- FSI necessary to share the high momentum with neutron, such that $\mathbf{p}'$ small
- Evolved picture: Two-body current couples with both proton and neutron such that final momentum is small



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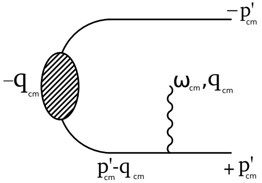

Changes as a function of λ

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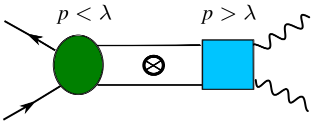

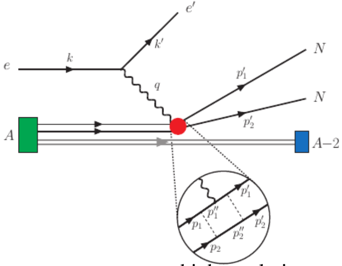

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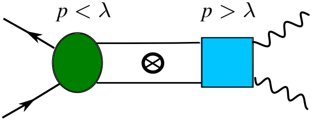

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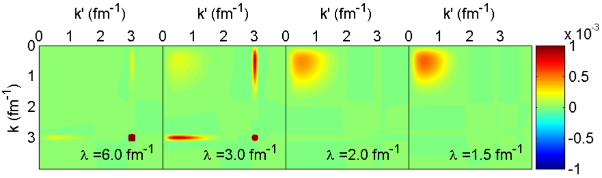
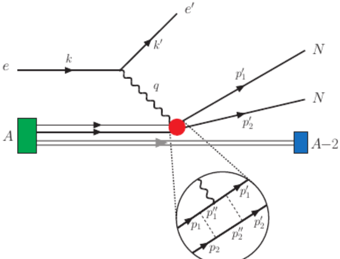


Anderson et al., PRC **82**, 054001 (2010)



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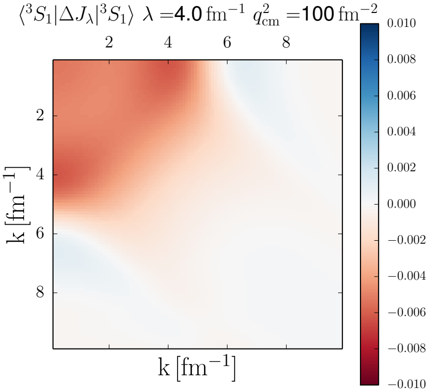
Current evolution story



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- For low-momentum, the SRG-induced two-body currents are of the form of regulated contact terms

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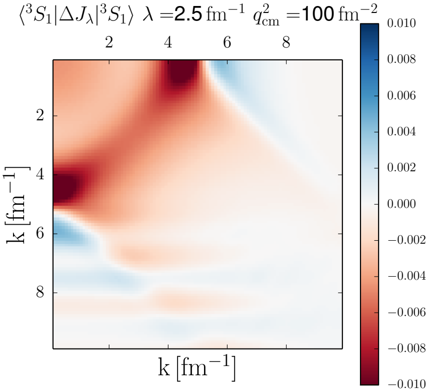

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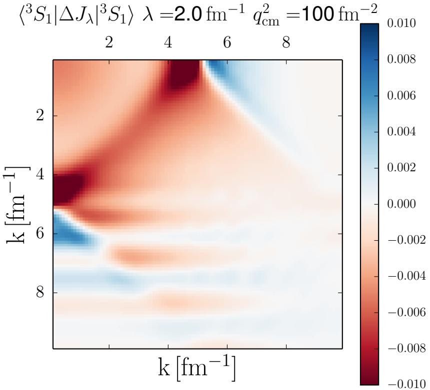

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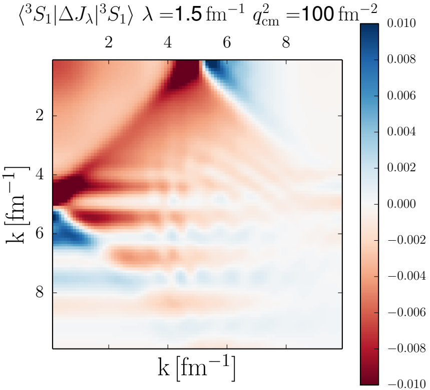

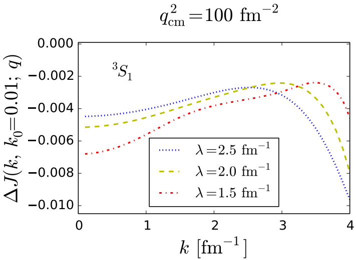

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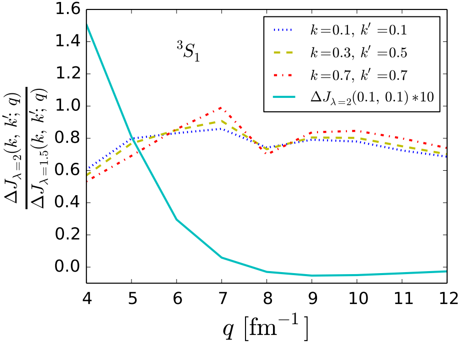
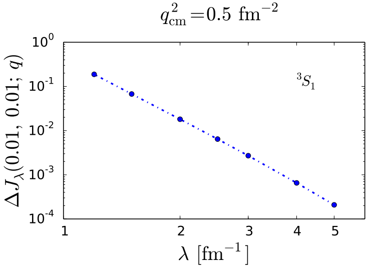
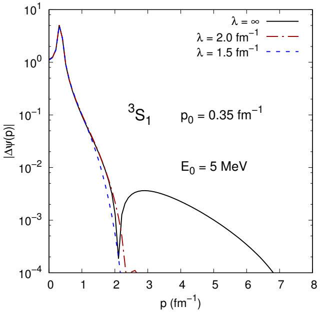


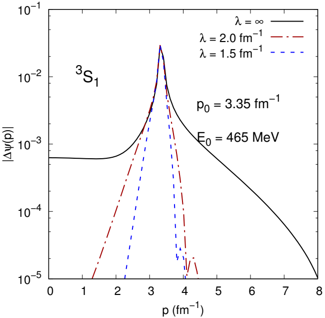


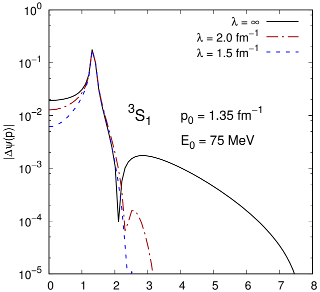
- $\Delta J_\lambda(k,k';q) \xrightarrow[q/2>\lambda]{k,k'\approx 0} \Lambda(\lambda)\,Q(q)$ with $\Lambda(\lambda) \sim \dfrac{1}{\lambda}$.
- $\Delta J_\lambda(k,k';q) \xrightarrow[q\ll\lambda]{k,k'\approx 0} \dfrac{1}{\lambda^4}$
- Both factorization and $\frac{1}{\lambda}$, $\frac{1}{\lambda^4}$ dependence can be motivated from perturbation theory starting from the SRG flow equation: $\dfrac{dV_\lambda}{d\lambda} \propto [\eta_\lambda,\ V_\lambda]$

Final state story

- $|\psi_f^{p_0}\rangle = \underbrace{|\phi_{p_0}\rangle}_{\text{plane wave}} + \overbrace{G_{p_0}}^{\text{Green's fn.}} \underbrace{t_{p_0}}_{t-\text{matrix}} |\phi_{p_0}\rangle$
- $|\psi_f^{p_0}{}_\lambda\rangle = |\phi_{p_0}\rangle + G_{p_0}\, t_{p_0\,\lambda} |\phi_{p_0}\rangle$
- Subtracting off the plane wave part: $\Delta\psi(p) = \langle p | G_{p_0}\, t_{p_0} | \phi_{p_0}\rangle$



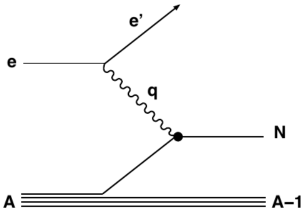




Back up: Basic Problem

- Goal: Extract nuclear properties from experiments and predict them with theory
- $\dfrac{d\sigma}{d\Omega} \propto \left| \langle \psi_{\text{final}} \,|\, \hat{O}(q) \,|\, \psi_{\text{initial}} \rangle \right|^2$
- $\langle \underbrace{\psi_{\text{final}}}_{\text{structure}} \,|\, \overbrace{\hat{O}(q)}^{\text{reaction}} \,|\, \underbrace{\psi_{\text{initial}}}_{\text{structure}} \rangle$

Nucleon knockout reaction



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- $$\langle \underbrace{\psi_{\text{final}}}_{\text{structure}} \,|\, \overbrace{\hat{O}(q)}^{\text{reaction}} \,|\, \underbrace{\psi_{\text{initial}}}_{\text{structure}} \rangle$$
- Use factorization to isolate individual components and extract process-independent nuclear properties

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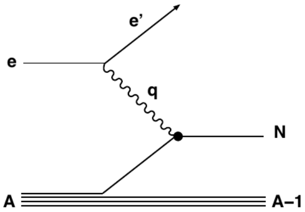
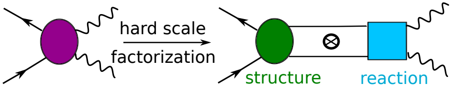
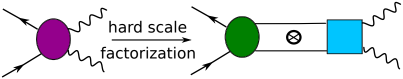


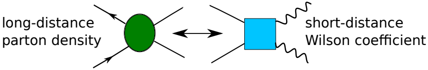


Factorization: Examples

High-E QCD



$$F_2(x,Q^2) \sim \sum_a f_a(x,\mu_f) \otimes \widehat{F}_2^a(x,Q/\mu_f)$$



- Separation between long- and short-distance physics is not unique, but defined by the scale μ_f
- Form factor F_2 is independent of μ_f, but pieces are not
- $f_a(x,Q^2)$ runs with Q^2 but is process independent

Low-E Nuclear

Observable: cross section

Structure model: spectroscopic factor

Reaction model: single-particle cross section

$$\sigma^{if} = \sum_{|J_i - J_f| \le j \le J_i + J_f} S_j^{if}\,\sigma_{sp}$$

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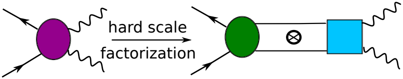

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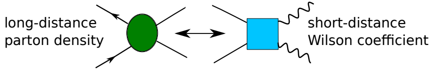

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Open questions

- When does factorization hold?
- Which process-independent nuclear properties can we extract?
- What is the scale/scheme dependence of the extracted properties?

Back up: Numerical implementation

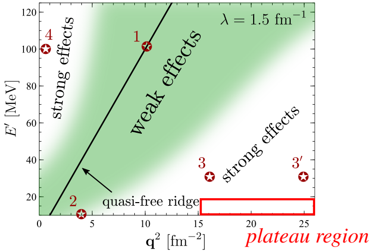
- $\langle \phi | t_\lambda^\dagger \, G_0^\dagger \, J_0^\lambda | \psi_i^\lambda \rangle = \langle \phi | t_\lambda^\dagger \, G_0^\dagger \, \widetilde{U} \, J_0 \, \widetilde{U}^\dagger | \psi_i^\lambda \rangle + \cdots$
- $U = I + \widetilde{U}$. Smooth $\widetilde{U}$ amenable to interpolation.
- Insert complete set of partial wave basis of the form
  $$1 = \frac{2}{\pi} \sum_{\substack{L,S \\ J,m_J}} \sum_{T=0,1} \int dp \, p^2 \, |p \, J \, m_J \, L \, S \, T\rangle \, \langle p \, J \, m_J \, L \, S \, T| \ .$$
- Large number of nested sums and integrals. Caching techniques used to avoid recalculation of t-matrix.
- Parallelization implemented using TBB library. Run on a node with 48 cores.

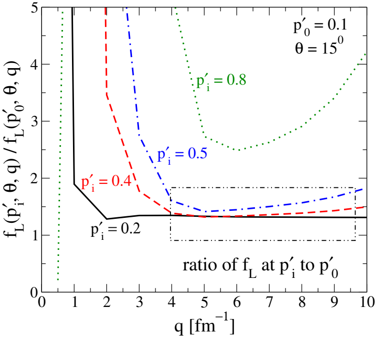
Back up: Numerical implementation – representative term

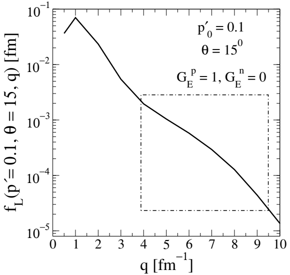
$$
\begin{aligned}
\langle \phi | t_\lambda^\dagger\, G_0^\dagger\, \widetilde{U}\, J_0\, \widetilde{U}^\dagger | \psi_i^\lambda \rangle &= \frac{8}{\pi^2} \sqrt{\frac{2}{\pi}}\, \frac{M}{\hbar c} \int \frac{\mathrm{d}k_2\, k_2^2}{(p' + k_2)(p' - k_2 - i\epsilon)} \sum_{T_1=0,1} \left(G_E^p + (-1)^{T_1}\, G_E^n\right) \\
&\times \sum_{L_1=0}^{L_{\max}} \left(1 + (-1)^{T_1} (-1)^{L_1}\right) \times Y_{L_1, m_{J_d} - m_{s_f}}(\theta', \varphi') \sum_{J_1 = |L_1 - 1|}^{L+1} \langle L_1\, m_{J_d} - m_{s_f}\, S = 1\, m_{s_f} | J_1\, m_{J_d} \rangle \\
&\times \sum_{L_2=0}^{L_{\max}} t_\lambda^*(k_2, p', L_2, L_1, J_1, S = 1, T_1) \sum_{L_3=0}^{L_{\max}} \sum_{\widetilde{m}_s=-1}^{1} \langle J_1\, m_{J_d}\, L_3\, m_{J_d} - \widetilde{m}_s | S = 1\, \widetilde{m}_s \rangle \\
&\times \sum_{L_4=0}^{L_{\max}} \langle L_4\, m_{J_d} - \widetilde{m}_s\, S = 1\, \widetilde{m}_s | J = 1\, m_{J_d} \rangle \int \mathrm{d}k_4\, k_4^2\, \widetilde{U}(k_2, k_4, L_2, L_3, J_1, S = 1, T_1) \\
&\times \int \mathrm{d}\cos\theta\, P_{L_3}^{m_{J_d} - \widetilde{m}_s}(\cos\theta)\, P_{L_4}^{m_{J_d} - \widetilde{m}_s}\left(\cos\alpha'(k_4, \theta)\right) \\
&\times \int \mathrm{d}k_6\, k_6^2 \sum_{L_d=0,2} \widetilde{U}\left(k_6, \sqrt{{k_4}^2 - k_4 q \cos\theta + q^2/4}, L_d, L_4, J = 1, S = 1, T = 0\right) \psi_{L_d}^\lambda(k_6)\,.
\end{aligned}
$$

q-factorization of f_L

- $f_L \equiv f_L(p', \theta; q)$
  p' and θ: outgoing nucleon
  q: momentum transfer
- For $p' \ll q, f_L$ scales with q
  $f_L(p', \theta; q) \to g(p', \theta)B(q)$
- Note that f_L is a strong function of q

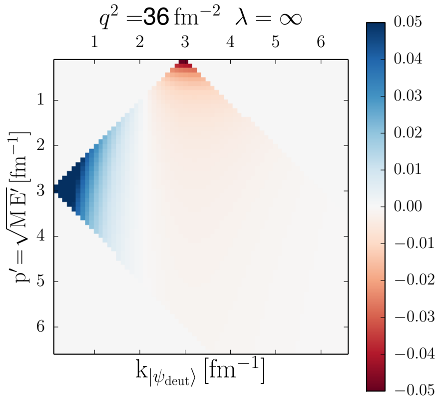






Exploring the evolution of the current

Integrand of $\langle p';{}^3S_1|J_0^\lambda(q)|\psi^\lambda_{{}^3S_1}\rangle$



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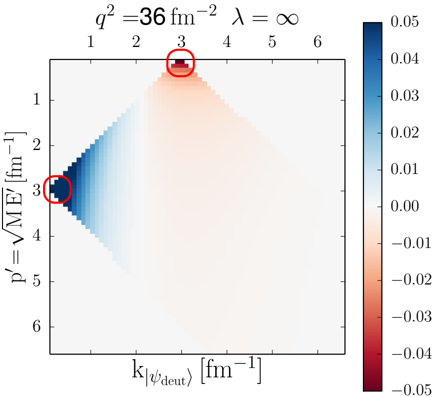

- The unevolved current is one-body peaked at $q/2$.

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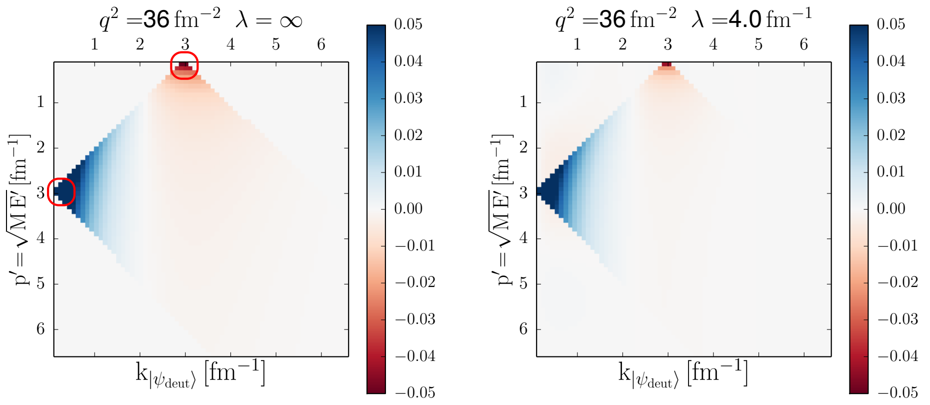

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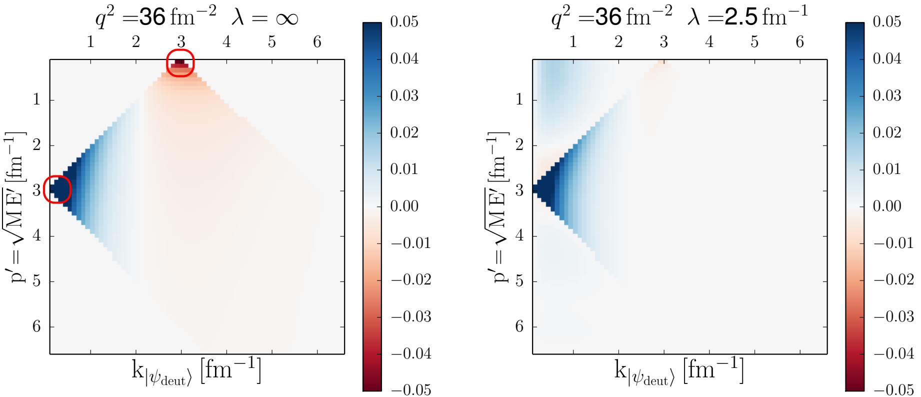

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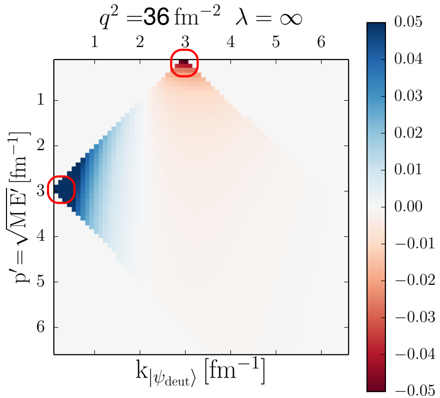

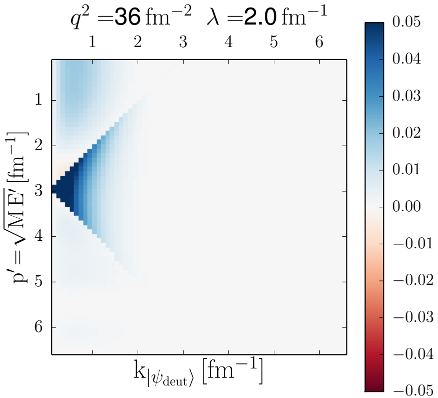

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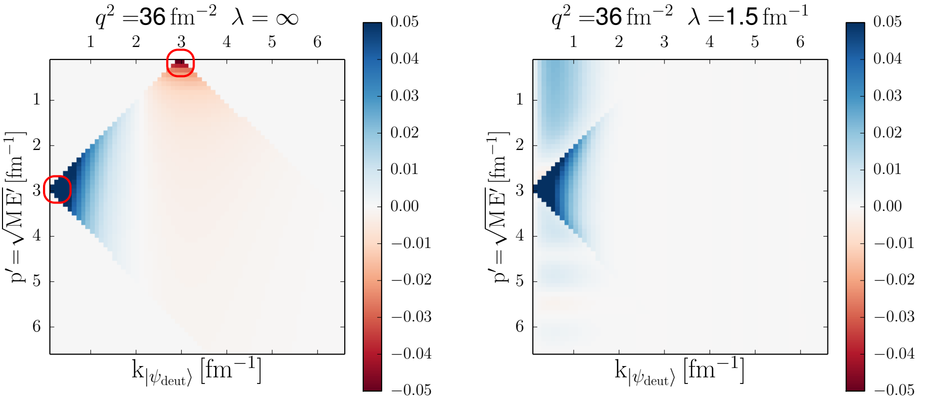

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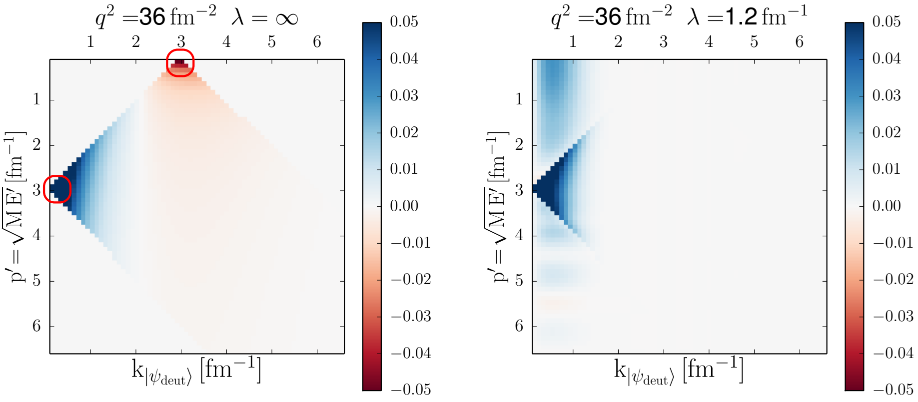

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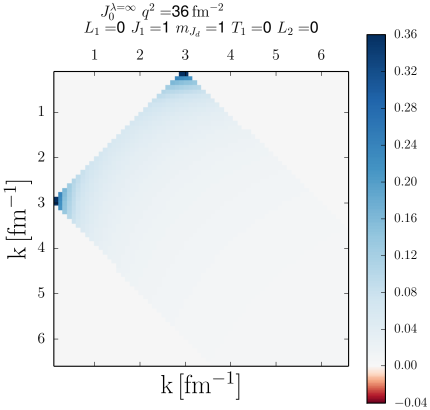

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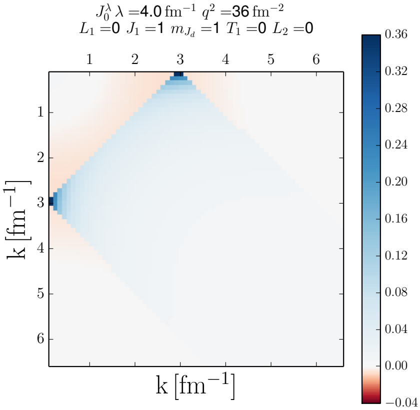

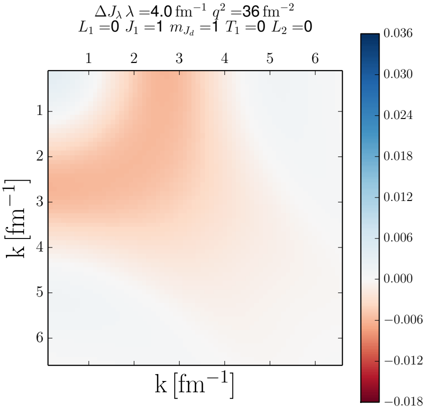

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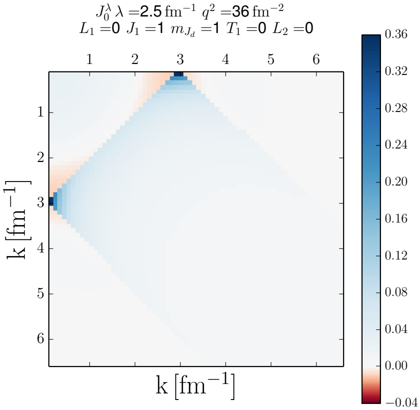

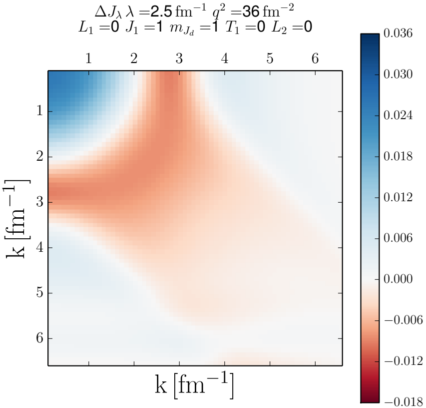

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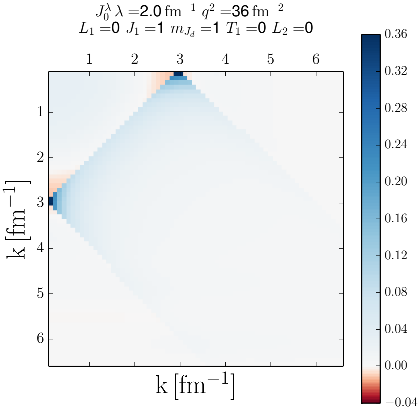

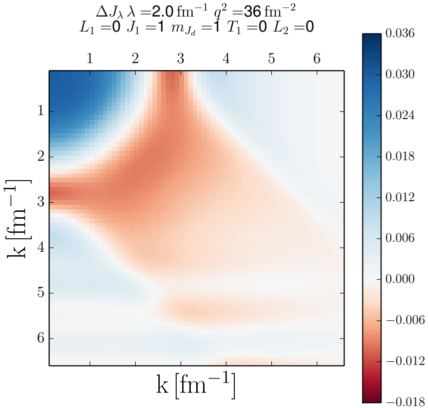

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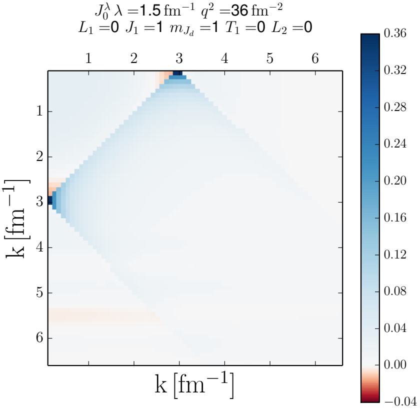

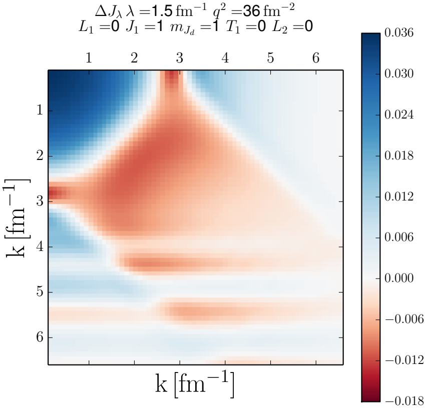

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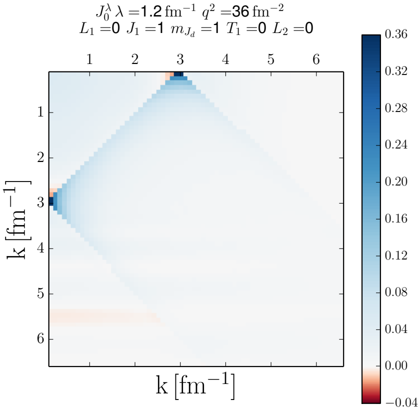

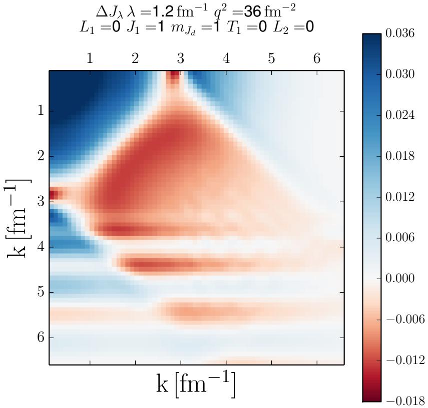

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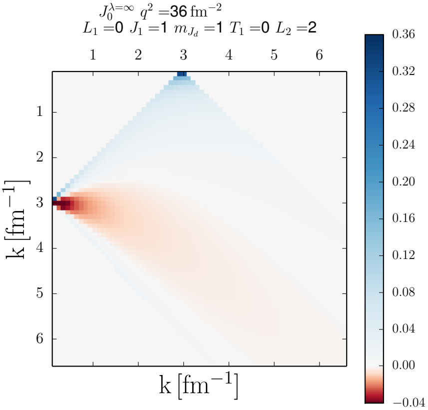
Isolating the evolution of current: $L = 0$, $L' = 2$



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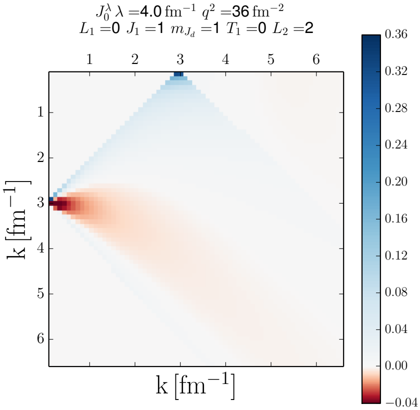

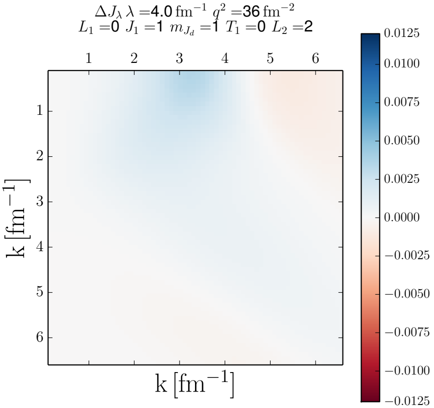

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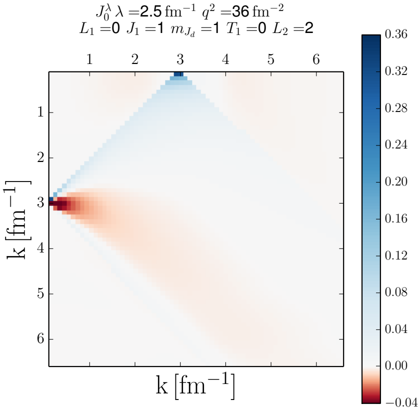

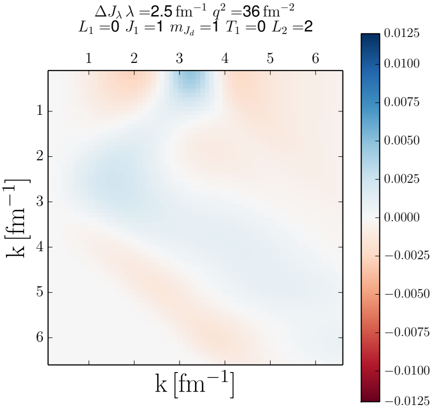

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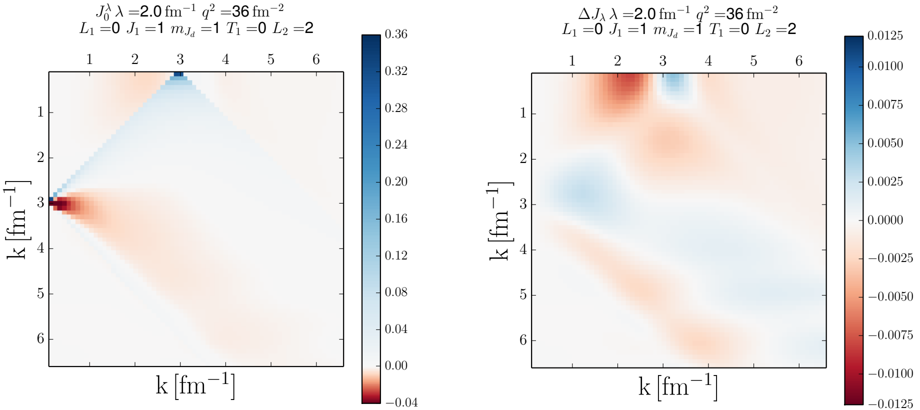

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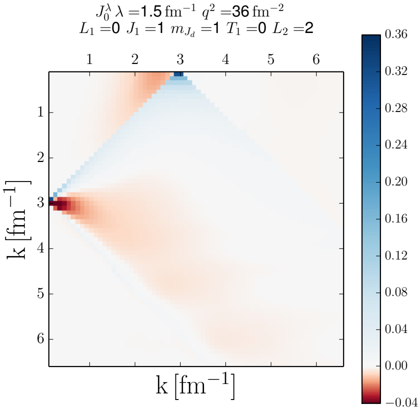

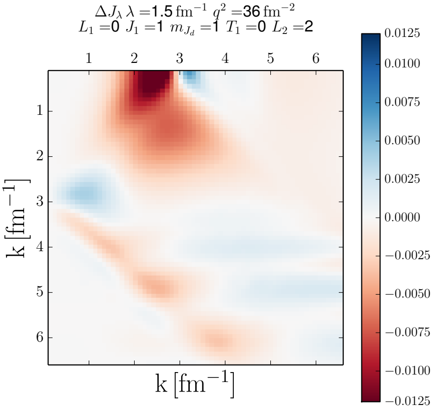

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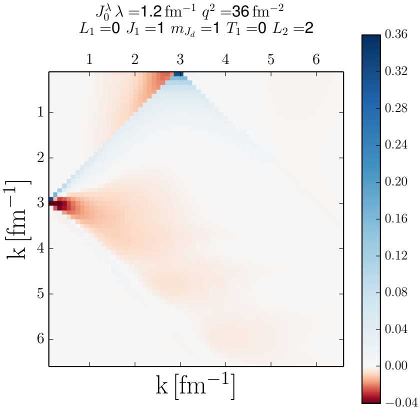

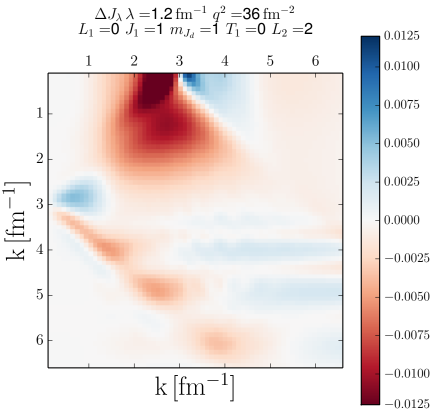

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