Nuclear RG perspective on SRC and EMC physics

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NUCLEI
Nuclear Computational Low-Energy Initiative

MIT Workshop on SRC and EMC Physics

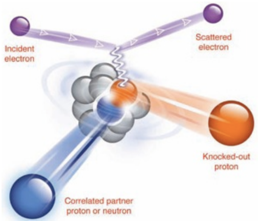
December, 2016

NSF

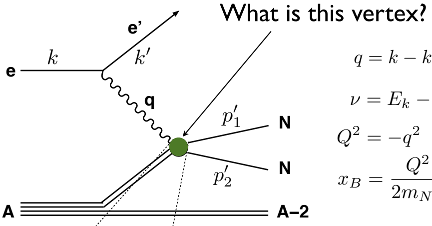
LENPIC

Collaborators: S. Bogner (MSU), K. Hebeler (TU Darmstadt), S. König (TU Darmstadt), S. More (MSU)

Large Q^2 scattering at different RG decoupling scales



Subedi et al., Science 320, 1476 (2008)



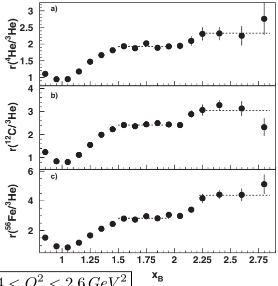
$$q = k - k'$$

$$\nu = E_k - E_{k'}$$

$$Q^2 = -q^2$$

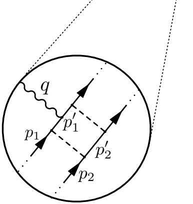
$$x_B = \frac{Q^2}{2m_N\nu}$$

Higinbotham, arXiv:1010.4433



$1.4 < Q^2 < 2.6\,GeV^2$

Egiyan et al. PRL 96, 1082501 (2006)



SRC interpretation:
NN interaction can scatter states with $p_1, p_2 \lesssim k_F$ to intermediate states with $p_1', p_2' \gg k_F$ which are knocked out by the photon

SRC explanation relies on high-momentum nucleons in structure

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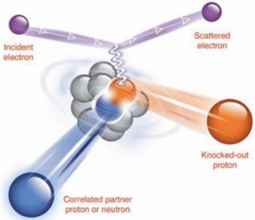

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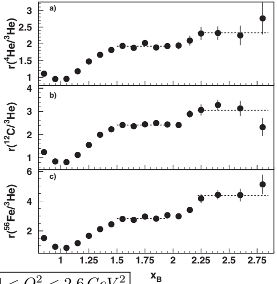

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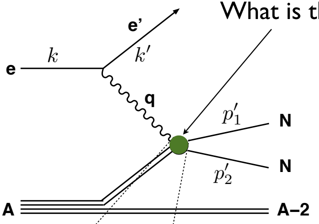
What is this vertex?



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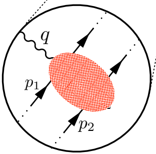

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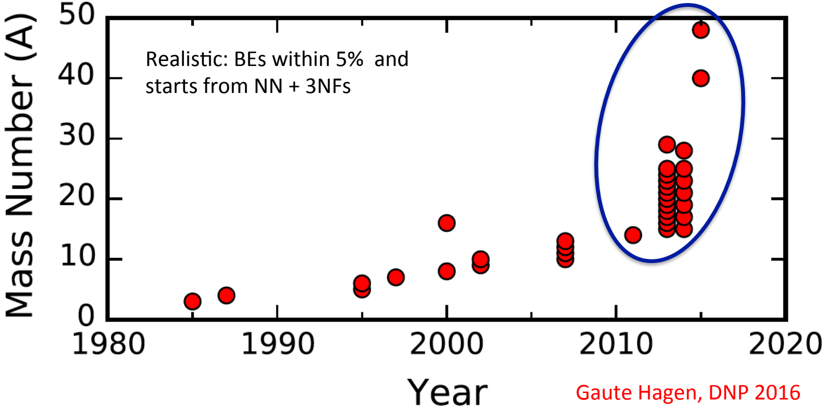
How to explain cross sections in terms of low-momentum interactions?

Vertex depends on the resolution!

RG evolution changes physics *interpretation* but not cross section!

Ab initio calculations: The nuclear structure hockey stick



Gaute Hagen, DNP 2016

- Why has the reach of precision structure calculations increased?
  - Application of effective field theory (EFT) and renormalization group (RG) methods $\Longrightarrow$ low-resolution ("softened") potentials
  - Explosion of many-body methods: GFMC/AFDMC, (IT-)NCSM, coupled cluster, lattice EFT, IM-SRG, SCGF, UMOA, MBPT, . . .

Uses of the renormalization group (RG) [cf. S. Weinberg (1981)]

- Improving perturbation theory; e.g., in QCD calculations
  - Mismatch of energy scales can generate large logarithms
  - Shift between couplings and loop integrals to reduce logs
- Identifying universality in critical phenomena
  - Filter out short-distance degrees of freedom
- Simplifying calculations of nuclear structure/reactions
  - Make nuclear physics look more like quantum chemistry!

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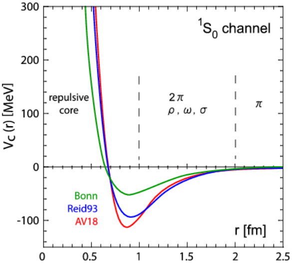
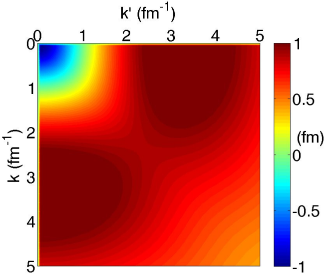


AV18, Bonn, Reid93



$\langle k | V_{\mathrm{AV18}} | k' \rangle$

Coupling of low-k/high-k modes: non-perturbative, strong correlations, . . .

Remedy: Use RG to decouple modes

$\Longrightarrow$ low resolution

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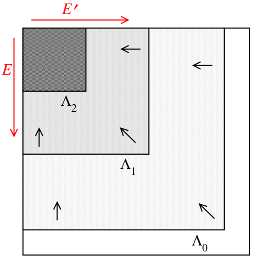
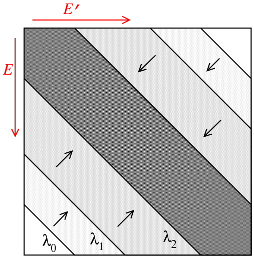


"$V_{\text{low } k}$"



Similarity RG

- $V_{\text{low } k}$: lower cutoff Λ_i in k, k' via $dT(k,k';k^2)/d\Lambda = 0$
- SRG: drive H toward diagonal with flow equation

$$dH_s/ds = [[G_s, H_s], H_s]$$

Continuous unitary transforms (cf. running couplings)

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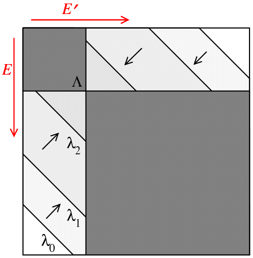
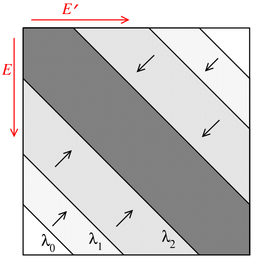


Block diagonal SRG



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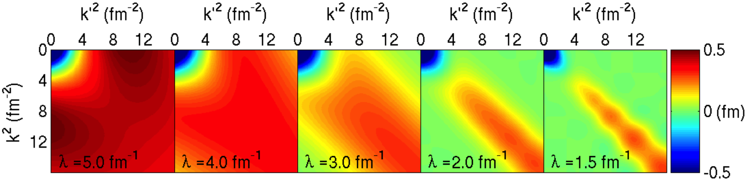
AV18:



- Decoupling naturally visualized in momentum space for $G_s = T$
  - Phase-shift equivalent! Width of diagonal given by $\lambda^2 = 1/\sqrt{s}$
  - What does this look like in coordinate space?

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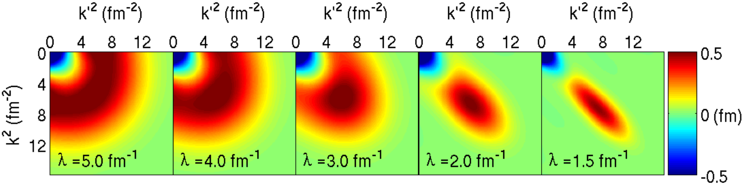
N^{3}LO:
(500 MeV)



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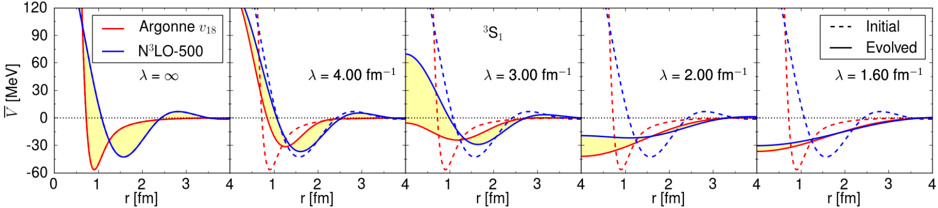
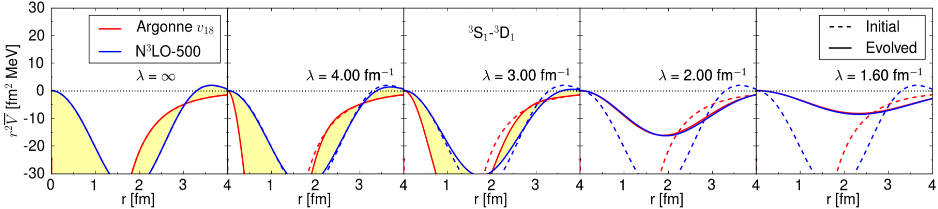
Visualizing the softening of NN interactions

- Project non-local NN potential: $\overline{V}_\lambda(r) = \int d^3r'\, V_\lambda(r, r')$
  - Roughly gives action of potential on long-wavelength nucleons
- Central part (S-wave) [Note: The V_λ's are all phase equivalent!]



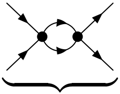
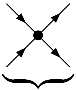
- Tensor part (S-D mixing) [graphs from K. Wendt et al., PRC (2012)]



$\Longrightarrow$ Flow to universal potentials!

Compare changing a cutoff in an EFT to RG decoupling

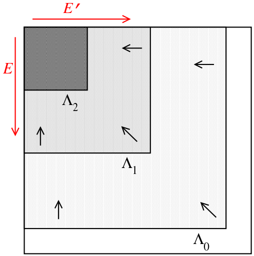
- (Local) field theory version in perturbation theory (diagrams)
  - Loops (sums over intermediate states) $\overset{\Delta\Lambda_c}{\Longleftrightarrow}$ LECs

$\dfrac{d}{d\Lambda_c}$ [ $+$ ] $= 0$

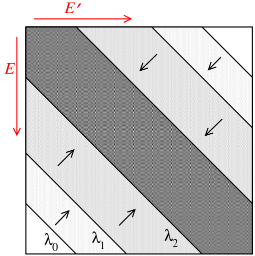
$$\int^{\Lambda_c} \frac{d^3q}{(2\pi)^3} \frac{C_0 M C_0}{k^2 - q^2 + i\epsilon} \qquad C_0(\Lambda_c) \propto \frac{\Lambda_c}{2\pi^2} + \cdots$$

  - Momentum-dependent vertices $\Longrightarrow$ Taylor expansion in k^2
  - This implements an operator product expansion!
- Claim: $V_{\text{low }k}$ RG and SRG decoupling work analogously

"$V_{\text{low }k}$"

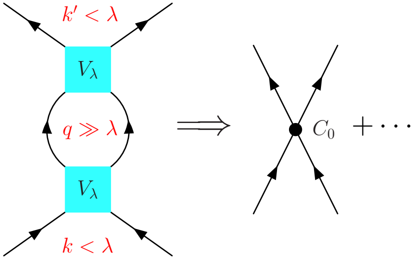


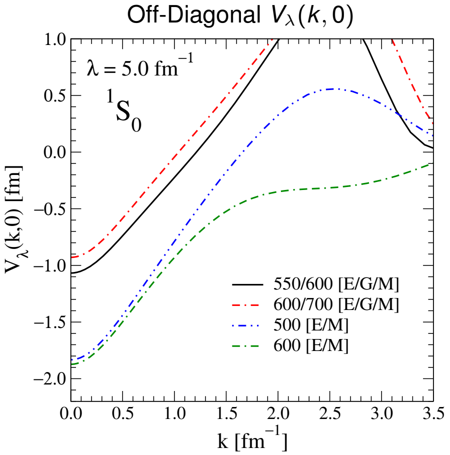
SRG ("T" generator)



Approach to universality (fate of high-q physics!)

Run NN to lower λ via SRG $\Longrightarrow$ $\approx$Universal low-k V_{NN}





$q \gg \lambda$ (or Λ) intermediate states
$\Longrightarrow$ change is $\approx$ contact terms:
$C_0\delta^3(\mathbf{x}-\mathbf{x}') + \cdots$

[cf. $\mathcal{L}_{\rm eft} = \cdots + \frac{1}{2}C_0(\psi^\dagger\psi)^2 + \cdots$]

- Similar pattern with phenomenological potentials (e.g., AV18)

Factorization: $\Delta V_\lambda(k,k') = \int U_\lambda(k,q)V_\lambda(q,q')U^\dagger_\lambda(q',k')$ for $k,k' < \lambda,\ q,q' \gg \lambda$

$\overset{U_\lambda \to K\cdot Q}{\longrightarrow} K(k)[\int Q(q)V_\lambda(q,q')Q(q')]K(k')$ with $K(k) \approx 1$!

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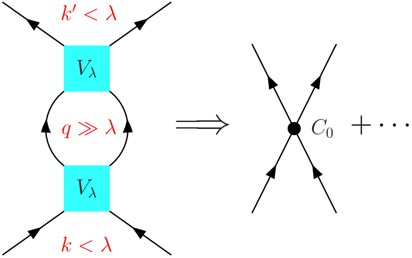

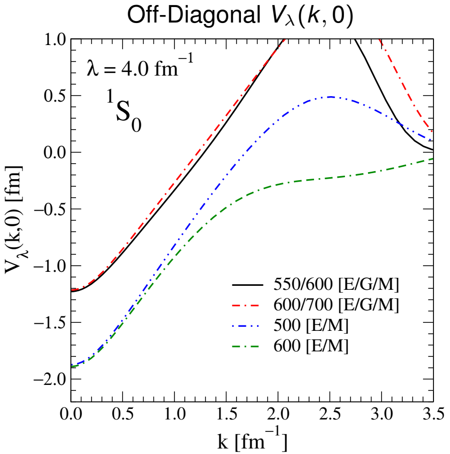

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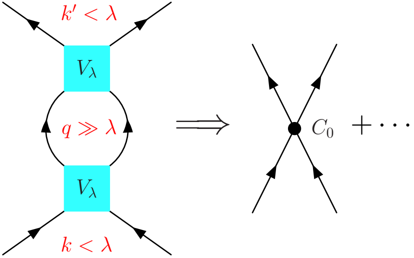
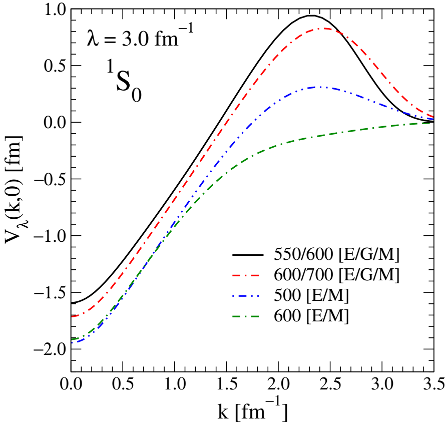


Off-Diagonal $V_\lambda(k,0)$



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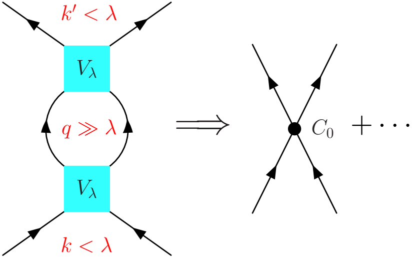

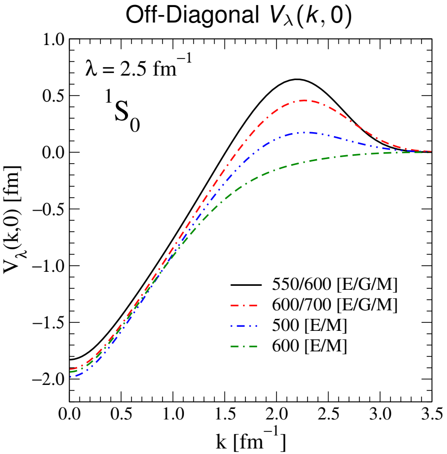

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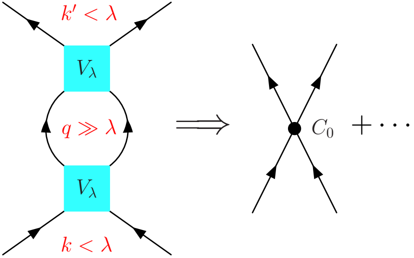

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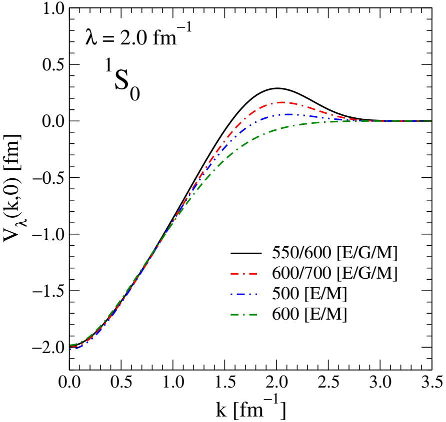

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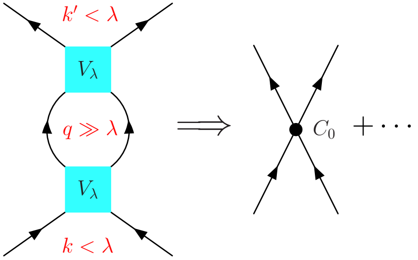

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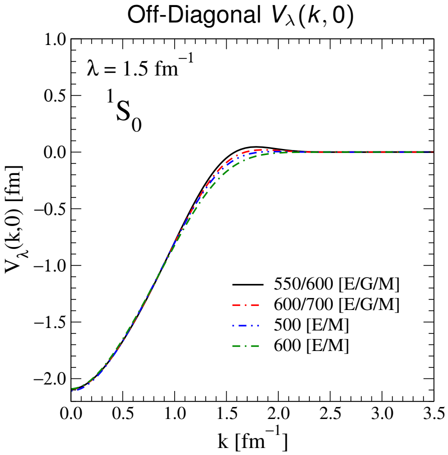

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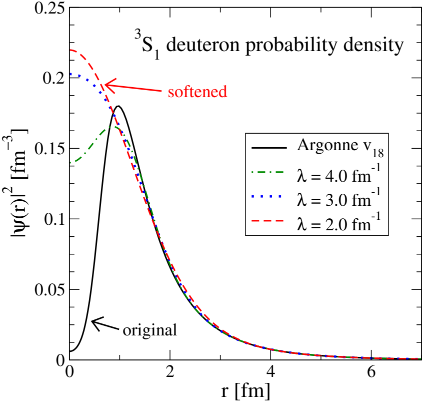
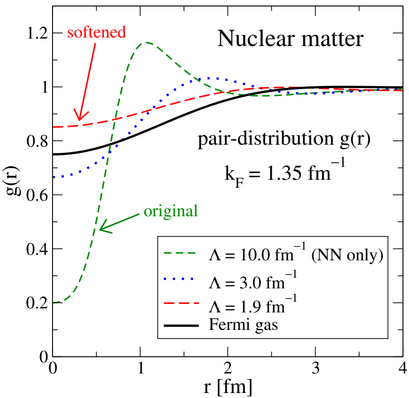
Nuclear structure natural with *low* momentum scale

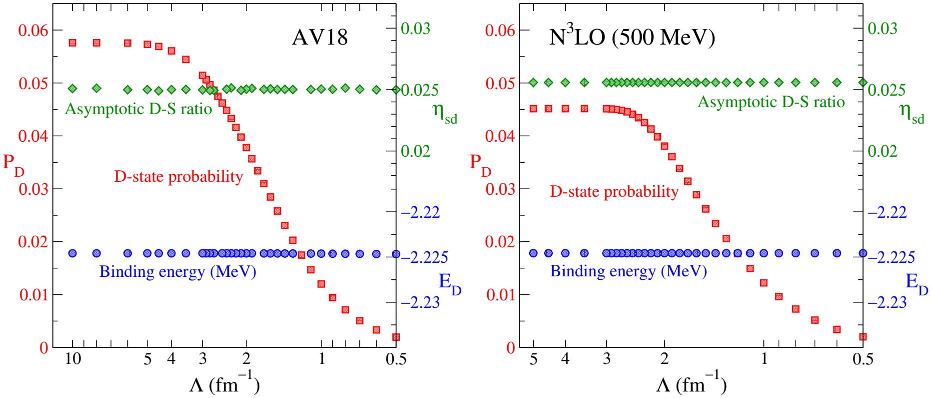
But lowering resolution reduces short-range correlations (SRCs)!





- Continuously transformed potential $\Longrightarrow$ variable SRCs in wfs!
- Therefore, it is would seem that SRCs are very resolution dependent
- But what does this mean for knock-out experiments that are said to measure (or be sensitive to) SRCs? Or momentum distributions?

Deuteron scale-(in)dependent observables

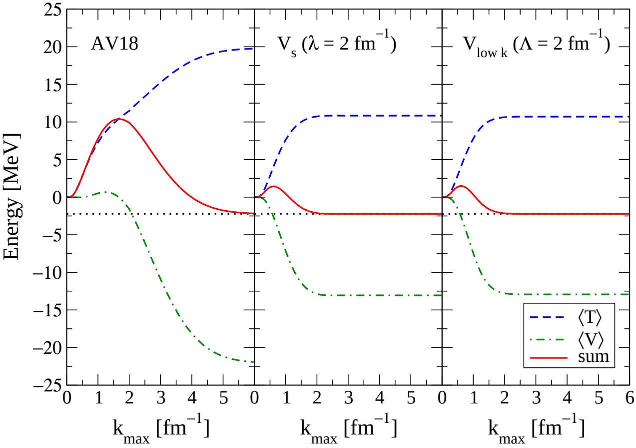


- $V_{\text{low }k}$ RG transformations labeled by Λ (different V_Λ's)
  $\Longrightarrow$ soften interactions by lowering resolution (scale)
  $\Longrightarrow$ reduced short-range and tensor correlations
- Energy and asymptotic D-S ratio are unchanged (cf. ANC's)
- But D-state probability changes (cf. spectroscopic factors)
- What about other quantities and other nuclei?

Distribution of kinetic and potential energy in the deuteron

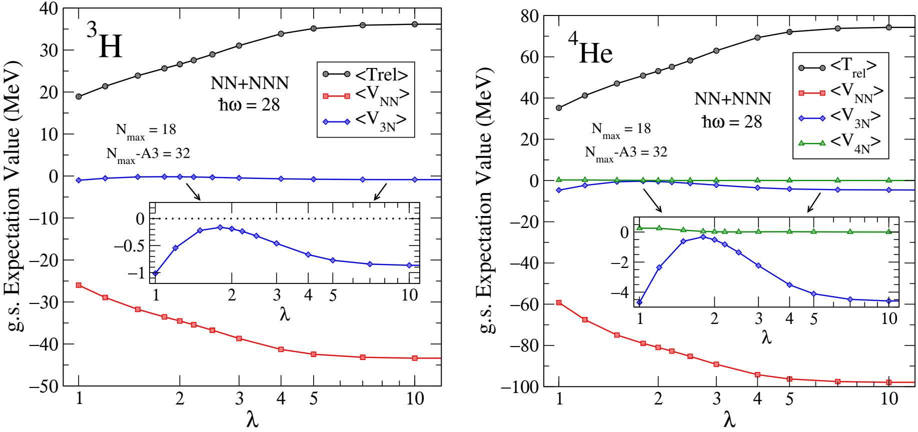
Look at expectation value of kinetic and potential energies cut off at $k_{\max}$

$$E_d(k < k_{\max}) = T_{\rm rel}(k < k_{\max}) + V_s(k < k_{\max})$$
$$= \int_0^{k_{\max}} d\mathbf{k} \int_0^{k_{\max}} d\mathbf{k}'\, \psi_d^\dagger(\mathbf{k};\lambda) \left(k^2 \delta^3(\mathbf{k}-\mathbf{k}') + V_s(\mathbf{k},\mathbf{k}')\right) \psi_d(\mathbf{k}';\lambda)$$



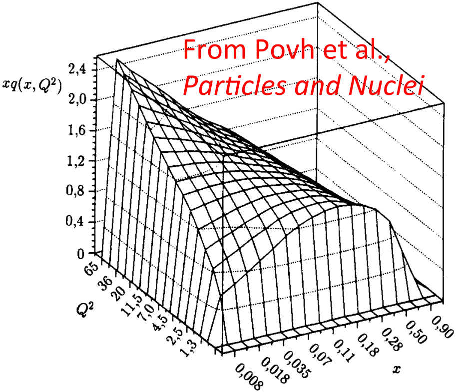
Contributions to the ground-state energy

- Look at ground-state matrix elements of KE, NN, 3N, 4N



- Clear hierarchy, but also strong cancellations at NN level
- What about the *A* dependence?
- Kinetic energy is resolution dependent!

Parton vs. nuclear momentum distributions



- The quark distribution $q(x, Q^2)$ is scale *and* scheme dependent
- $x\, q(x, Q^2)$ measures the share of momentum carried by the quarks in a particular x-interval
- $q(x, Q^2)$ and $q(x, Q_0^2)$ are related by RG evolution equations

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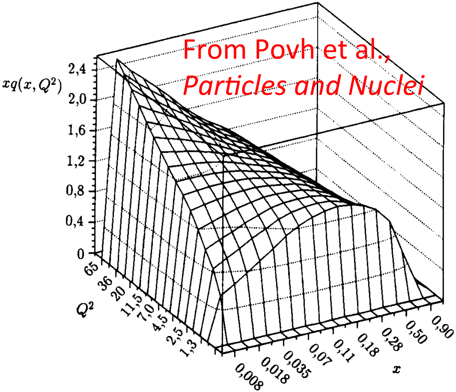

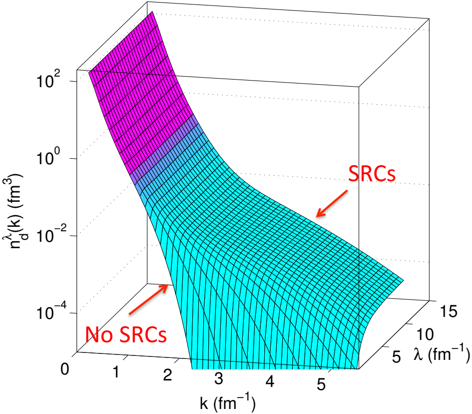

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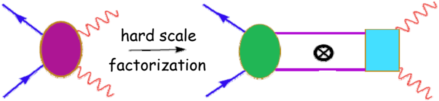
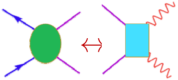
- Deuteron momentum distribution is scale *and* scheme dependent
- Initial AV18 potential evolved with SRG from $\lambda = \infty$ to $\lambda = 1.5\,\text{fm}^{-1}$
- High momentum tail shrinks as λ decreases (lower resolution)

Factorization: high-E QCD vs. low-E nuclear



$$F_2(x, Q^2) \sim \sum_a f_a(x, \mu_f) \otimes \widehat{F}_2^a(x, Q/\mu_f)$$



- Separation between long- and short-distance physics is not unique $\Longrightarrow$ introduce μ_f
- Choice of μ_f defines border between long/short distance
- Form factor F_2 is independent of μ_f, but pieces are not
- Q^2 running of $f_a(x, Q^2)$ comes from choosing μ_f to optimize extraction from experiment

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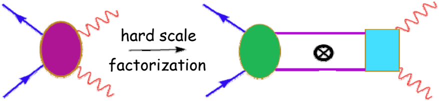

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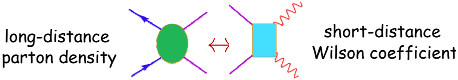

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- Also has factorization assumptions (e.g., from D. Bazin ECT* talk, 5/2011)

Observable: cross section — Structure model: spectroscopic factor — Reaction model: single-particle cross section

$$\sigma^{if} = \sum_{|J_f - J_i| \leq j \leq J_f + J_i} S_j^{if} \sigma_{sp}$$

- Is the factorization general/robust? (Process dependence?)
- What is the scale/scheme dependence of extracted properties?
- What are the trade-offs? (Does simpler structure always mean much more complicated reaction?)

Scheming for parton distributions

Need schemes for both renormalization and factorization

From the "Handbook of perturbative QCD" by G. Sterman et al.

"Short-distance finite parts at higher orders may be apportioned arbitrarily between the C's and ϕ's. A prescription that eliminates this ambiguity is what we mean by a factorization scheme. . . . The two most commonly used schemes, called DIS and $\overline{MS}$, reflect two different uses to which the freedom in factorization may be put."

"The choice of scheme is a matter of taste and convenience, but it is absolutely crucial to use schemes consistently, and to know in which scheme any given calculation, or comparison to data, is carried out."

Specifying a scheme in low-energy nuclear physics includes specifying a potential and *consistent* currents, including regulators, and how a reaction is analyzed.

Source of scale-dependence for low-E structure

- Measured cross section as convolution: reaction $\otimes$ structure
  - but separate parts are not unique, *only* the combination
- Short-range unitary transformation U leaves m.e.'s invariant:

$$O_{mn} \equiv \langle \Psi_m | \widehat{O} | \Psi_n \rangle = \left(\langle \Psi_m | U^\dagger \right) U \widehat{O} U^\dagger \left(U | \Psi_n \rangle \right) = \langle \widetilde{\Psi}_m | \widetilde{O} | \widetilde{\Psi}_n \rangle \equiv \widetilde{O}_{\widetilde{m}\widetilde{n}}$$

Note: matrix elements of operator $\widehat{O}$ itself between the transformed states are in general modified:

$$O_{\widetilde{m}\widetilde{n}} \equiv \langle \widetilde{\Psi}_m | O | \widetilde{\Psi}_n \rangle \neq O_{mn} \quad \Longrightarrow \quad \text{e.g., } \langle \Psi_n^{A-1} | a_\alpha | \Psi_0^A \rangle \text{ changes}$$

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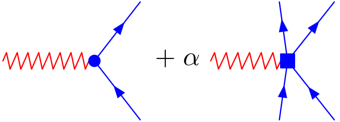
- In a low-energy effective theory, transformations that modify *short-range* unresolved physics $\Longrightarrow$ equally valid states. So $\widetilde{O}_{mn} \neq O_{mn} \Longrightarrow$ scale/scheme dependent observables.
- RG unitary transformations change the decoupling scale $\Longrightarrow$ change the factorization scale. Use to characterize and explore scale and scheme and process dependence!

All pieces mix with unitary transformation

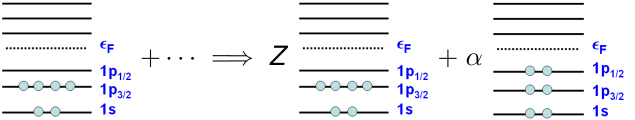
- A one-body current becomes many-body (cf. EFT current):

$$\widehat{U}\widehat{\rho}(\mathbf{q})\widehat{U}^\dagger = \quad [\text{diagram}] \; + \alpha \; [\text{diagram}] \; + \cdots$$



- New wf correlations have appeared (or disappeared):

$$\widehat{U}|\Psi_0^A\rangle = \widehat{U} \; [\text{diagram}] \; + \cdots \Longrightarrow Z \; [\text{diagram}] \; + \alpha \; [\text{diagram}] \; + \cdots$$



  - Similarly with $|\Psi_f\rangle = a_{\mathbf{p}}^\dagger|\Psi_n^{A-1}\rangle$
  - Thus *spectroscopic factors* are scale dependent

- Final state interactions (FSI) are also modified by $\widehat{U}$

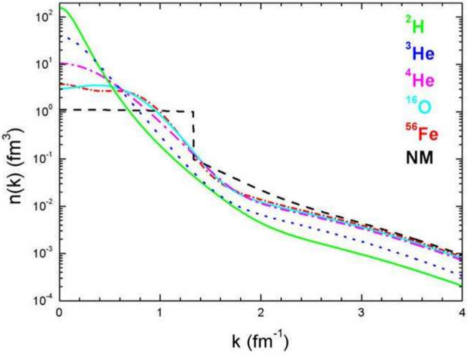
- Bottom line: the cross section is unchanged *only* if all pieces are included, *with the same U*: $H(\lambda)$, current operator, FSI, …

Nuclear scaling from RG factorization (schematic!)

- RG unitary transformation with scale separation: $\widehat{U} \to U_\lambda(k, q)$
- Factorization: when $k < \lambda$ and $q \gg \lambda$, $U_\lambda(k, q) \to K_\lambda(k) Q_\lambda(q)$

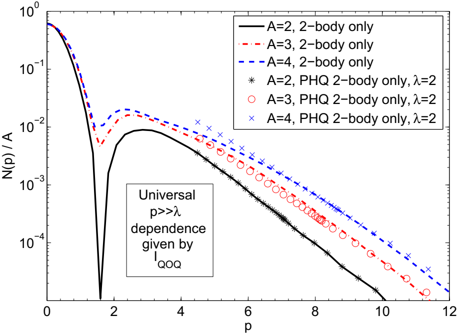
$$\frac{n_A(q)}{n_d(q)} = \frac{\langle A | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | A \rangle}{\langle d | a^\dagger_{\mathbf{q}} a_{\mathbf{q}} | d \rangle} \xrightarrow[\widehat{U}^\dagger \widehat{U}=1]{\text{RG}} \quad \widehat{U}|d\rangle \to |\widetilde{d}\rangle\ ,\ \widehat{U}|A\rangle \to |\widetilde{A}\rangle\ ,\ \widehat{U} a^\dagger_{\mathbf{q}} a_{\mathbf{q}} \widehat{U}^\dagger$$

$\Longrightarrow n_A(q) \approx C_A n_D(q)$ at large q



[From C. Ciofi degli Atti and S. Simula]

Test case: A bosons in toy 1D model



[Anderson et al., arXiv:1008.1569]
[also Bogner, Roscher, arXiv:1208.1734]

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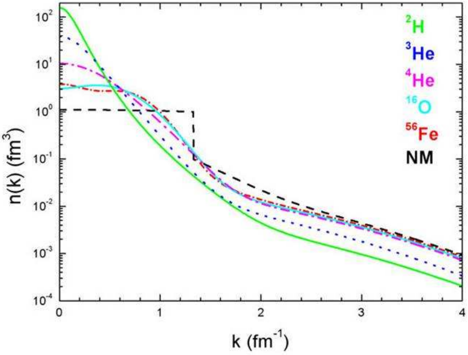

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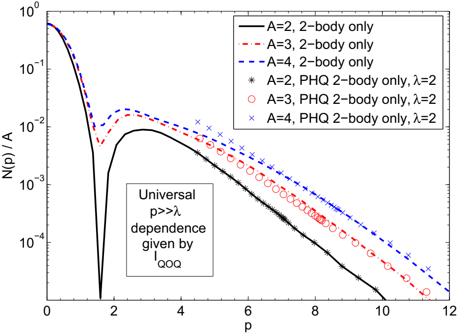

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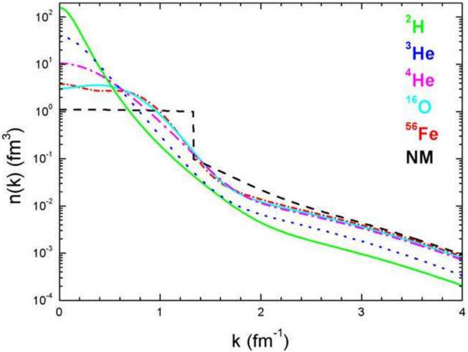

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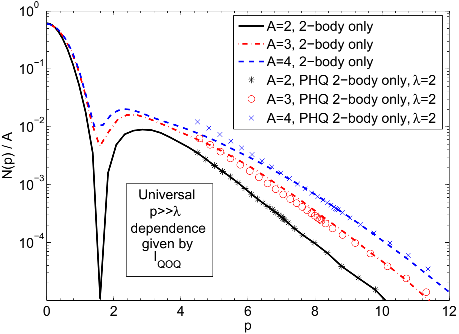

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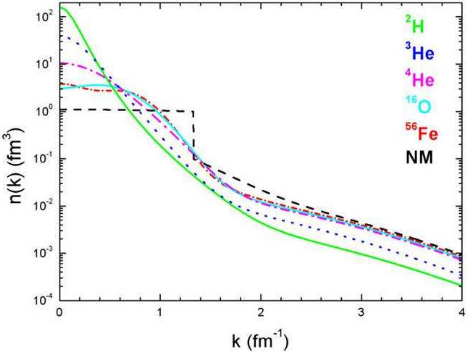

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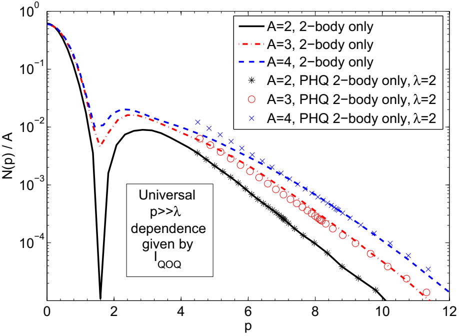

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U-factorization with SRG [Anderson et al., arXiv:1008.1569]

- Factorization: $U_\lambda(k,q) \to K_\lambda(k)Q_\lambda(q)$ when $k < \lambda$ and $q \gg \lambda$
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q)\int_0^\lambda p^2dp\, Z(\lambda)\Psi_\alpha^\lambda(p) + \eta^\lambda(q)\int_0^\lambda p^2dp\, p^2\, Z(\lambda)\,\Psi_\alpha^\lambda(p) + \cdots$$

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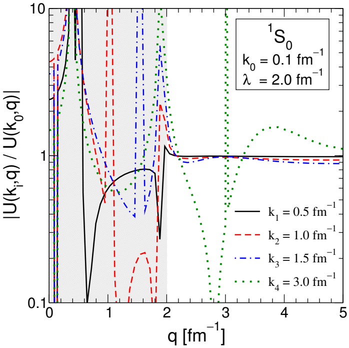
- Test of factorization of U:

$$\frac{U_\lambda(k_i,q)}{U_\lambda(k_0,q)} \to \frac{K_\lambda(k_i)Q_\lambda(q)}{K_\lambda(k_0)Q_\lambda(q)},$$

so for $q \gg \lambda \Rightarrow \dfrac{K_\lambda(k_i)}{K_\lambda(k_0)} \xrightarrow{\text{LO}} 1$

- Look for plateaus: $k_i \lesssim 2\,\text{fm}^{-1} \lesssim q$ $\Longrightarrow$ it works!
- Leading order $\Longrightarrow$ contact term!



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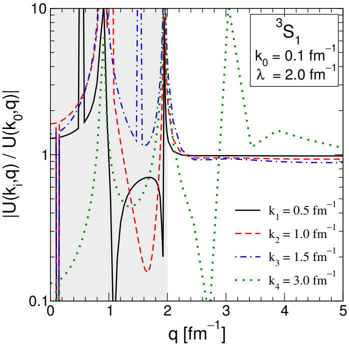

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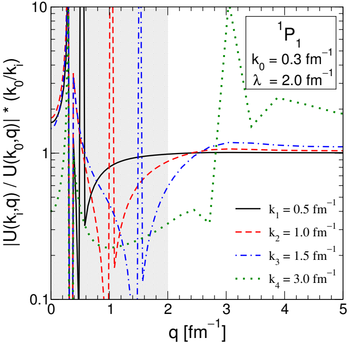

How should one choose a scale and/or scheme?

- To make calculations easier or more convergent
  - QCD running coupling and scale: improved perturbation theory; choosing a gauge: e.g., Coulomb or Lorentz
  - Low-*k* potential: improve many-body convergence, or to make microscopic connection to shell model or . . .
  - (Near-) local potential: quantum Monte Carlo methods work
- Better interpretation or intuition $\Longrightarrow$ predictability
  - SRC phenomenology?
- Cleanest extraction from experiment
  - Can one "optimize" validity of impulse approximation?
  - Ideally extract at one scale, evolve to others using RG
- Plan: use range of scales to test calculations and physics
  - Find (match) Hamiltonians and operators with EFT
  - Use renormalization group to consistently relate scales and quantitatively probe ambiguities (e.g., in spectroscopic factors)

Summary: Precision nuclear structure and reactions

- We're in a golden age for low-energy nuclear physics
  - Many complementary methods able to incorporate 3NFs
  - Synergies of theory and experiment
  - Large-scale collaborations facilitate progress
  - Many opportunities and challenges for precision physics

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  - Robust uncertainty quantification is a frontier
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- EFT and RG have become important tools for precision
  - Robust uncertainty quantification is a frontier
  - Scale and scheme dependence is inevitable $\Longrightarrow$ deal with it!
- Challenges for which EFT/RG perspective + tools can help
  - Can we have controlled factorization at low energies?
  - How should one choose a scale/scheme in particular cases?
  - What *is* the scheme-dependence of SF's and other quantities?
  - What are the roles of short-range/long-range correlations?
  - How do we consistently match Hamiltonians and operators?
  - . . . and many more. Calculations are in progress!

Backups

EMC effect from the EFT perspective

- Exploit scale separation between short- and long-distance physics
  - *Match* complete set of operator matrix elements (power count!)
  - Cf. needing a *model* of short-distance nucleon dynamics
  - Distinguish long-distance nuclear from nucleon physics
- EMC and effective field theory (examples)
  - "DVCS-dissociation of the deuteron and the EMC effect"
    [S.R. Beane and M.J. Savage, Nucl. Phys. A 761, 259 (2005)]

    "By constructing all the operators required to reproduce the matrix elements of the twist-2 operators in multi-nucleon systems, one sees that operators involving more than one nucleon are not forbidden by the symmetries of the strong interaction, and therefore must be present. While observation of the EMC effect twenty years ago may have been surprising to some, in fact, its absence would have been far more surprising."
  - "Universality of the EMC Effect"
    [J.-W. Chen and W. Detmold, Phys. Lett. B 625, 165 (2005)]
  - "SRCs and the EMC Effect in EFT" [Chen et al., arXiv:1607.03065]

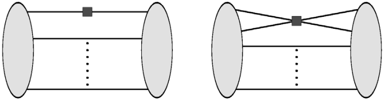
A dependence of the EMC effect is long-distance physics!

- EFT treatment by Chen and Detmold [Phys. Lett. B 625, 165 (2005)]

$$F_2^A(x) = \sum_i Q_i^2 \, x q_i^A(x) \qquad \Longrightarrow \qquad R_A(x) = F_2^A(x)/AF_2^N(x)$$

"The x dependence of $R_A(x)$ is governed by short-distance physics, while the overall magnitude (the A dependence) of the EMC effect is governed by long distance matrix elements calculable using traditional nuclear physics."

- Match matrix elements: leading-order nucleon operators to isoscalar twist-two quark operators



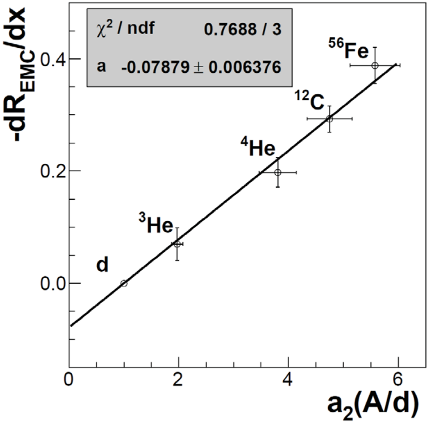
$$\Longrightarrow \langle x^2 \rangle_q v^{\mu_0} \cdots v^{\mu_n} N^\dagger N [1 + \alpha_n N^\dagger N] + \cdots$$

$$R_A(x) = \frac{F_2^A(x)}{AF_2^N(x)} = 1 + g_{F_2}(x)\mathcal{G}(A) \quad \text{where} \quad \mathcal{G}(A) = \langle A | (N^\dagger N)^2 | A \rangle / A\Lambda_0$$

$\Longrightarrow$ the slope $\frac{dR_A}{dx}$ scales with $\mathcal{G}(A)$ [Why is this not cited more?]

Scaling and EMC correlation via low resolution

- SRG factorization, e.g.,
  $U_\lambda(k,q) \rightarrow K_\lambda(k) Q_\lambda(q)$
  when $k < \lambda$ and $q \gg \lambda$
  - Dependence on high-q independent of A
    $\Longrightarrow$ universal [cf. Neff et al.]
  - A dependence from low-momentum matrix elements $\Longrightarrow$ calculate!
- EMC from EFT using OPE:
  - Isolate A dependence, which factorizes from x
  - EMC A dependence from long-distance matrix elements



L.B. Weinstein, et al., Phys. Rev. Lett. 106, 052301 (2011)

If the same leading operators dominate, then does linear A dependence of ratios follow immediately?

Need to do quantitative calculations to explore!

What about long-range correlations?

- SF calculations with FRPA
- Chiral N^3LO Hamiltonian
  - Soft $\Longrightarrow$ small SRC
  - SRC contribution to SF changes dramatically with lower resolution
- Compare short-range correlations (SRC) to long-range correlations from particle-vibration coupling
- LRC $\gg$ SRC!!

- How scale/scheme dependent are long-range correlations?
- Additional microscopic calculations are needed!

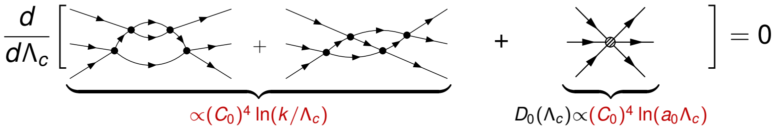
C. Barbieri, PRL 103 (2009)

TABLE I. Spectroscopic factors (given as a fraction of the IPM) for valence orbits around ^{56}Ni. For the SC FRPA calculation in the large harmonic oscillator space, the values shown are obtained by including only SRC, SRC and LRC from particle-vibration couplings (full FRPA), and by SRC, particle-vibration couplings and extra correlations due to configuration mixing (FRPA + ΔZ_α). The last three columns give the results of SC FRPA and SM in the restricted $1p0f$ model space. The ΔZ_αs are the differences between the last two results and are taken as corrections for the SM correlations that are not already included in the FRPA formalism.

	10 osc. shells			Exp. [29]	$1p0f$ space		
	FRPA (SRC)	Full FRPA	FRPA $+\Delta Z_\alpha$		FRPA	SM	ΔZ_α
^{57}Ni:							
$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	−0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	−0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	−0.03
^{55}Ni:							
$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	−0.03
^{57}Cu:							
$\pi 1p_{1/2}$	0.96	0.66	0.62		0.80	0.76	−0.04
$\pi 0f_{5/2}$	0.96	0.60	0.58		0.80	0.78	−0.02
$\pi 1p_{3/2}$	0.96	0.67	0.65		0.81	0.79	−0.02
^{55}Co:							
$\pi 0f_{7/2}$	0.95	0.73	0.71		0.89	0.87	−0.02

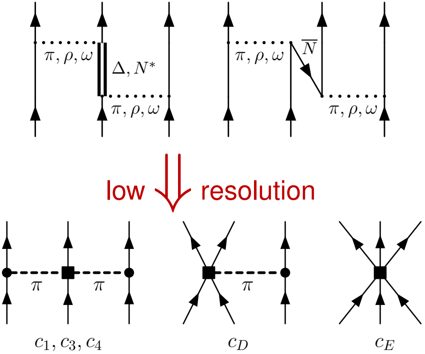
What can we say about the flow of NN$\cdots$N potentials?

- Can arise from counterterm for new UV cutoff dependence, e.g., changes in Λ_c must be absorbed by 3-body coupling $D_0(\Lambda_c)$



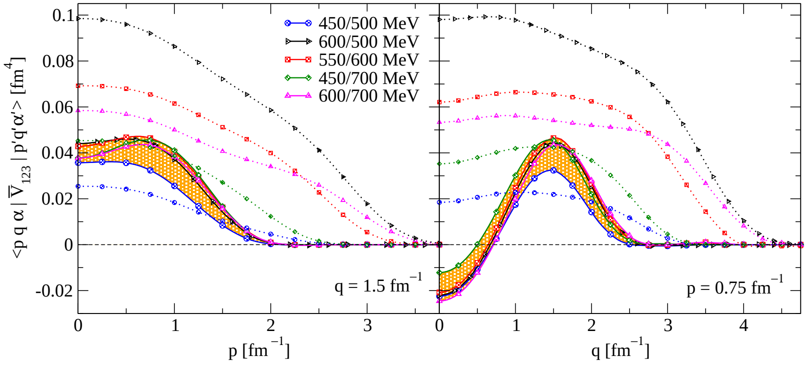
RG invariance dictates 3-body coupling flow [Braaten & Nieto]

- General RG: 3NF from integrating out *or* decoupling high-k states



Is there 3NF universality?

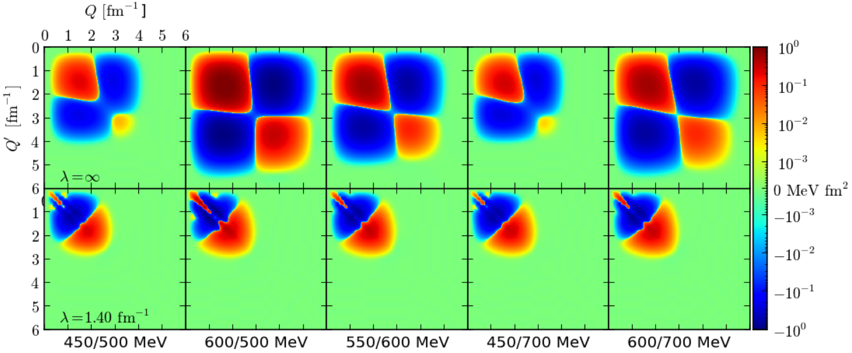
- Evolve chiral NNLO EFT potentials in momentum plane wave basis to $\lambda = 1.5\,\text{fm}^{-1}$ [K. Hebeler, Phys. Rev. C85 (2012) 021002]
- In one 3-body partial wave, fix one Jacobi momentum (p, q) and plot vs. the other one:



- Collapse of curves includes non-trivial structure

Is there 3NF universality?

- Evolve in discretized momentum-space hyperspherical harmonics basis to $\lambda = 1.4\,\mathrm{fm}^{-1}$ [K. Wendt, Phys. Rev. C87 (2013) 061001]
- Contour plot of *integrand* for 3NF expectation value in triton

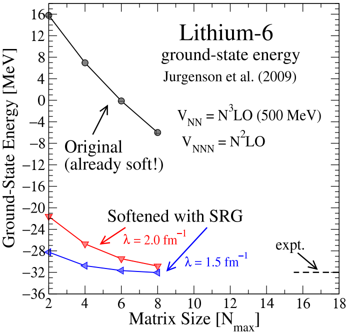


- Local projections of 3NF also show flow toward universal form
- Can we exploit universality à la Wilson? Stay tuned!

Nuclear structure natural with *low* momentum scale

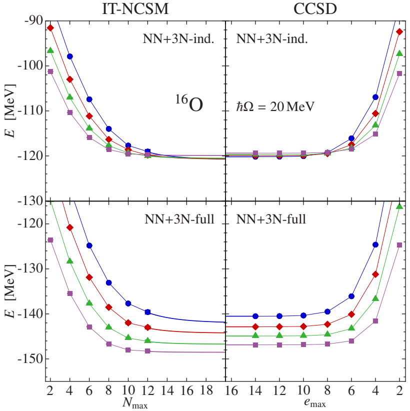
Softened potentials (SRG, $V_{\mathrm{low}\,k}$, UCOM, ...) enhance convergence

- Convergence for no-core shell model (NCSM):



- (Already) soft chiral EFT potential and evolved (softened) SRG potentials, including NNN

- Softening allows importance truncation (IT) and converged coupled cluster (CCSD)



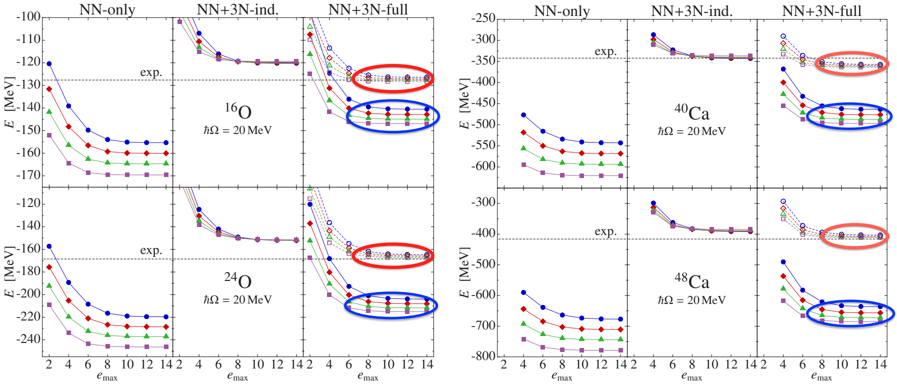
[Roth et al., PRL 109, 052501 (2012)]

- Also enables ab initio nuclear reactions with NCSM/RGM [Navratil et al.]

Nuclear structure natural with *low* momentum scale

Team Roth: SRG-evolved N^3LO with NNN [PRL 109, 052501 (2012)]

- Coupled cluster with interactions $H(\lambda)$: λ is a decoupling scale
  - *Only* when NNN-induced added to NN-only $\Longrightarrow$ λ independent
  - With initial NNN: predictions from fit only to $A = 3$ properties
- Open questions: red (400 MeV) works, blue (500 MeV) doesn't!



- Same predictions for λ's! (issues about NNN resolved by 4N?)

Every operator flows

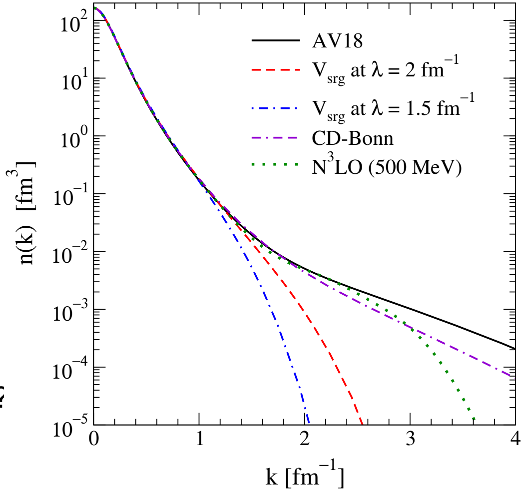
- Evolution with s of any operator O is given by:

$$O_s = U_s O U_s^\dagger$$

so O_s evolves via

$$\frac{dO_s}{ds} = [[G_s, H_s], O_s]$$

- $U_s = \sum_i |\psi_i(s)\rangle\langle\psi_i(0)|$
- Matrix elements of evolved operators are unchanged $\Longrightarrow$ How does this play out?
- Example: momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ (in deuteron)



Flow equations lead to many-body operators

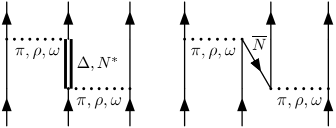
- Consider a's and $a^\dagger$'s wrt s.p. basis and reference state:

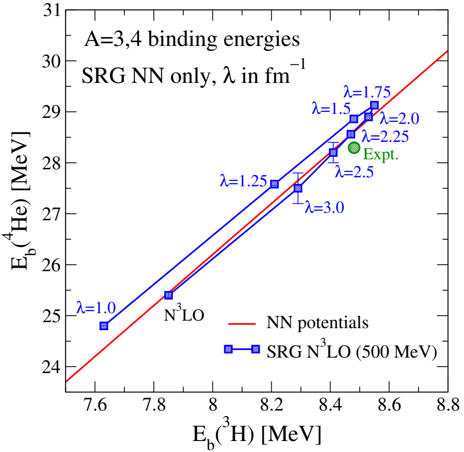
$$\frac{dV_s}{ds} = \Big[\Big[\sum \underbrace{a^\dagger a}_{G_s}, \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}\Big], \sum \underbrace{a^\dagger a^\dagger aa}_{\text{2-body}}\Big] = \cdots + \sum \underbrace{a^\dagger a^\dagger a^\dagger aaa}_{\text{3-body!}} + \cdots$$

  so there will be A-body forces (and operators) generated
- Is this a problem?
  - Ok if "induced" many-body forces are same size as natural ones
  - Alternative: choose a non-vacuum reference state [Scott]
- Nuclear 3-body forces already needed in unevolved potential
  - In fact, there are A-body forces (operators) initially
  - Natural hierarchy from chiral EFT
    $\Longrightarrow$ stop flow equations before unnatural 3-body size
  - Many-body methods must deal with them!
- SRG is a tractable method to evolve many-body operators

Observations on three-body forces

- Three-body forces arise from eliminating/decoupling dof's
  - excited states of nucleon
  - relativistic effects
  - high-momentum intermediate states
- Omitting 3-body forces leads to model dependence
  - observables depend on Λ/λ
  - cutoff dependence as tool
- NNN at different Λ/λ can be evolved or fit to χEFT
  - how large is 4-body?





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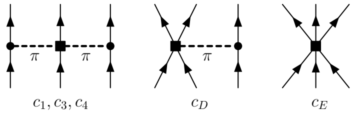
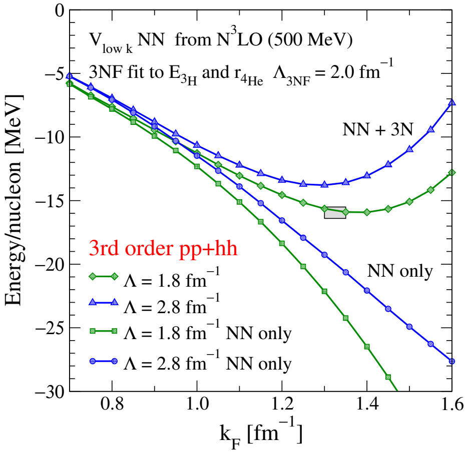
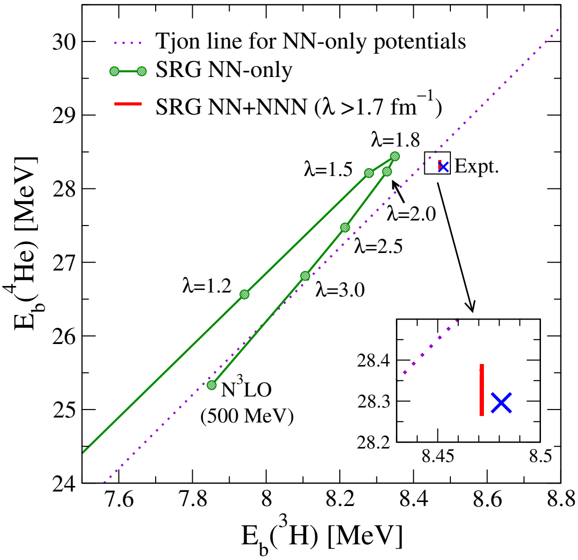
- NNN at different Λ/λ can be evolved or fit to χEFT
  - how large is 4-body?
  - saturation of nuclear matter (K. Hebeler — corrected + improved 3NF treatment)





Tjon line revisited



Every operator flows [see Anderson et al., arXiv:1008.1569]

- Evolution with s of any operator O is given by:

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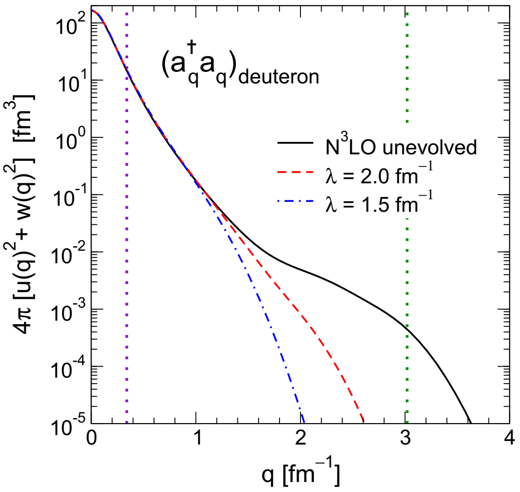
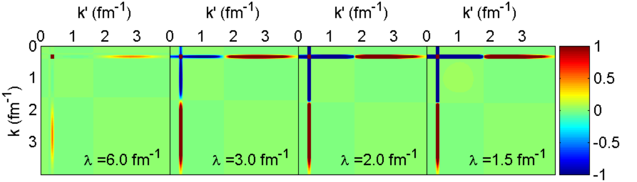
- $U_s = \sum_i |\psi_i(s)\rangle\langle\psi_i(0)|$
- Matrix elements of evolved operators are unchanged
- Consider momentum distribution $< \psi_d | a_q^\dagger a_q | \psi_d >$ at $q = 0.34$ and $3.0\,\mathrm{fm}^{-1}$

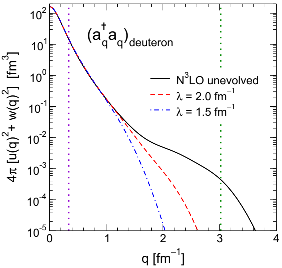


High and low momentum operators in deuteron

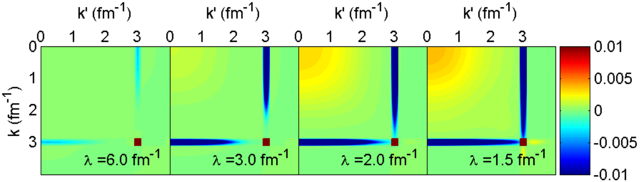
- **Integrand** of $(U a_q^\dagger a_q U^\dagger)$ for $q = 0.34\,\mathrm{fm}^{-1}$



- Momentum distribution



- **Integrand** for $q = 3.02\,\mathrm{fm}^{-1}$



- **Decoupling** $\Longrightarrow$ High momentum components suppressed
- Integrated value does not change, but nature of operator does
- Similar for other operators: $\langle r^2 \rangle$, $\langle Q_d \rangle$, $\langle 1/r \rangle$ $\langle \frac{1}{r} \rangle$, $\langle G_C \rangle$, $\langle G_Q \rangle$, $\langle G_M \rangle$

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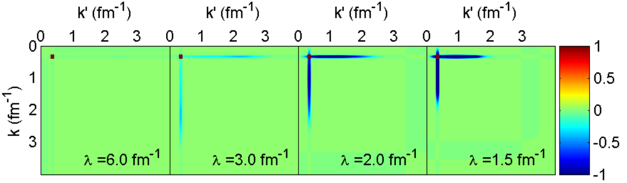

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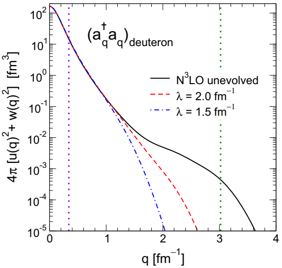

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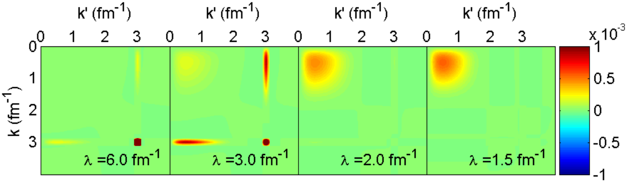

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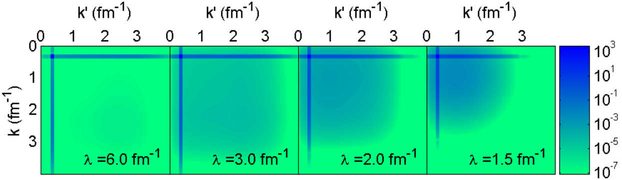
High and low momentum operators in deuteron

- **Integrand** of $\langle\psi_d|\,(Ua_q^\dagger a_q U^\dagger)\,|\psi_d\rangle$ for $q = 0.34\,\mathrm{fm}^{-1}$



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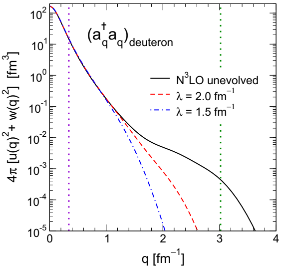

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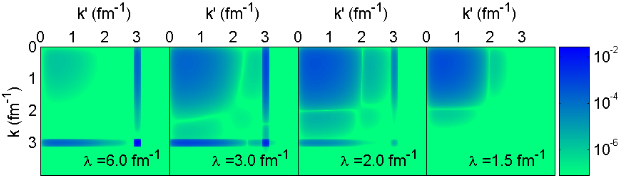

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Factorization [Anderson et al., arXiv:1008.1569]

- If $k < \lambda$ and $q \gg \lambda \Longrightarrow$ factorization: $U_\lambda(k,q) \to K_\lambda(k) Q_\lambda(q)$?
- Operator product expansion for nonrelativistic wf's (see Lepage)

$$\Psi_{true}(r) = \overline{\gamma}(r) \int d\mathbf{r}' \, \Psi_{eff} \delta_a(\mathbf{r}') + \overline{n}(r) a^2 \int d\mathbf{r}' \, \Psi_{eff} \nabla^2 \delta_a(\mathbf{r}') + \mathcal{O}(a^4)$$

- Similarly, in momentum space

$$\Psi_\alpha^\infty(q) \approx \gamma^\lambda(q) \int_0^\lambda p^2 dp \, Z(\lambda) \Psi_\alpha^\lambda(p) + \eta^\lambda(q) \int_0^\lambda p^2 dp \, p^2 \, Z(\lambda) \, \Psi_\alpha^\lambda(p) + \cdots$$

- By projecting potential in momentum subspace, recover OPE via:

$$\gamma^\lambda(q) \equiv - \int_\lambda^\infty q'^2 dq' \, \langle q | \frac{1}{\widehat{Q}_\lambda H^\infty \widehat{Q}_\lambda} | q' \rangle V^\infty(q', 0)$$

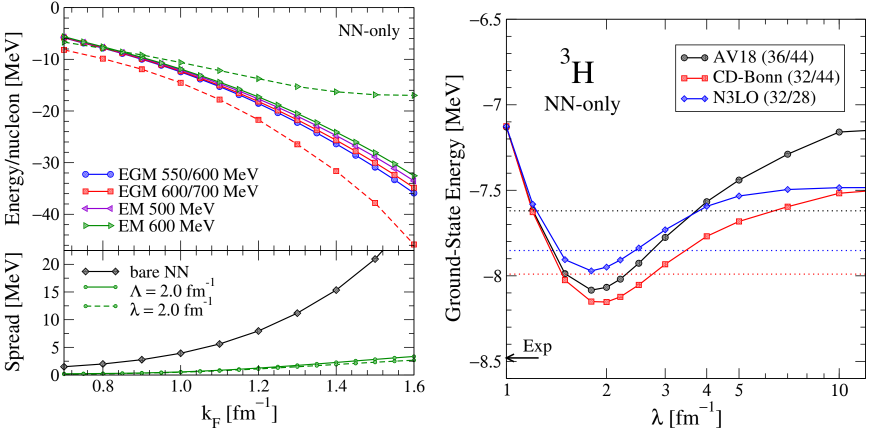
$$\eta^\lambda(q) \equiv - \int_\lambda^\infty q'^2 dq' \, \langle q | \frac{1}{\widehat{Q}_\lambda H^\infty \widehat{Q}_\lambda} | q' \rangle \frac{\partial^2}{\partial p^2} V^\infty(q', p)|_{p^2=0}$$

- Construct unitary transformation to get $U_\lambda(k, q) \approx K_\lambda(k) Q_\lambda(q)$

$$U_\lambda(k,q) = \sum_\alpha \langle k | \psi_\alpha^\lambda \rangle \langle \psi_\alpha^\infty | q \rangle \to \Big[\sum_\alpha^{\alpha_{low}} \langle k | \psi_\alpha^\lambda \rangle \int_0^\lambda p^2 dp \, Z(\lambda) \Psi_\alpha^\lambda(p) \Big] \gamma^\lambda(q) + \cdots$$

Impact of V_{NN} "collapse" on $A \geq 3$ observables

- Limited cases so far and NN-only: [K. Hebeler, E. Jurgenson]



- Nuclear matter spread ($V_{\mathrm{low}\,k}$ shown) sizable at $\lambda \approx 2\ \mathrm{fm}^{-1}$
- Binding energy collapse in light nuclei only for $\lambda \leq 1.5\ \mathrm{fm}^{-1}$