Factorization, universality and the nuclear contacts

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Collaborators: O. Hen, E. Piasetzki and R. Torres

The contact

- Zero-range condition: $r_0 \ll a, d$

- Many quantities are connected to the ***contact C***:

$$n(k) = \boldsymbol{C}/k\text{^}4 \text{ for } k \to \infty$$

$$T + U = \frac{\hbar^2}{4\pi m a}\boldsymbol{C} + \sum_{\sigma} \frac{d^3k}{(2\pi)^3}\frac{\hbar^2 k^2}{2m}\left(n_\sigma(\boldsymbol{k}) - \frac{\boldsymbol{C}}{k^4}\right)$$

And many more…

S. Tan, Ann. Phys. (N.Y.) 323, 2952 (2008); Ann. Phys. (N.Y.) 323, 2971 (2008); Ann. Phys. (N.Y.) 323, 2987 (2008)

The contact

- The basic factorization assumption:

$$\psi \xrightarrow{r_{ij}\to 0} \left(\frac{1}{r_{ij}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

$$n(k) = \frac{1}{k^4} \times \boldsymbol{C}$$

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NOT FOR NUCLEAR PHYSICS

$$r_0 \ll d, a$$

The Nuclear contacts

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$$\Downarrow$$

$$\psi \xrightarrow{r_{ij}\to 0} \varphi_{ij}(r_{ij}) \times A\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

The Nuclear contacts

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⇓

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$$\psi \xrightarrow{r_{ij}\to 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) \times A_{ij}^{\alpha}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

Channels $\alpha = (\ell_2 S_2) j_2 m_2$

“**universal**“ function

The pair kind $ij \in \{pp, nn, pn\}$

The Nuclear contact – Momentum

One-body momentum distribution – $\boldsymbol{n_N(k)}$ – The probability to find a proton/neutron with momentum k

Two-body momentum distribution – $\boldsymbol{F_{NN}(k)}$ – The probability to find an NN pairs with relative momentum k

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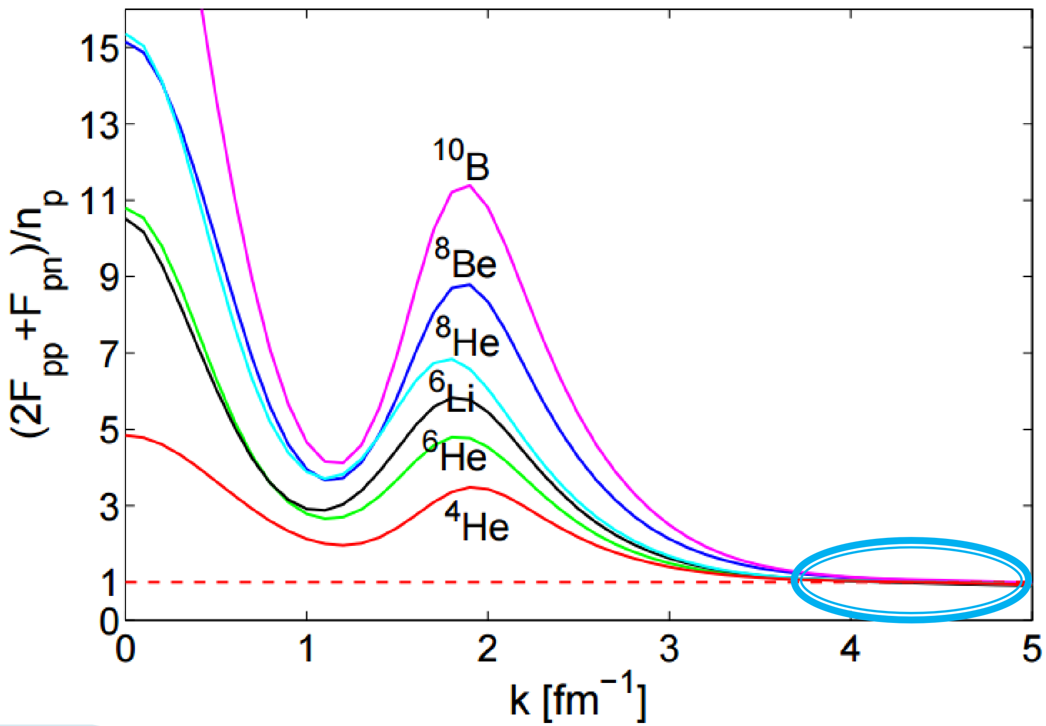

Using the variational Monte Carlo data (VMC)

Wiringa et al. Phys. Rev. C 89, 024305 (2014)

Extracting the contacts

- Assuming only **two significant channels**:

  The **deuteron** channel – L=0,2; S=1; J=1; T=0

  The **pure s-wave** channel – L=0; S=0; J=0; T=1

- We get:

$$F_{pn}(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^{2} + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^{2}$$

$$F_{nn}(k) \xrightarrow[k\to\infty]{} C_{nn}^{0}|\varphi_{nn}^{0}(k)|^{2}$$

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The VMC data

Zero-energy solution of the two-body system (AV18)

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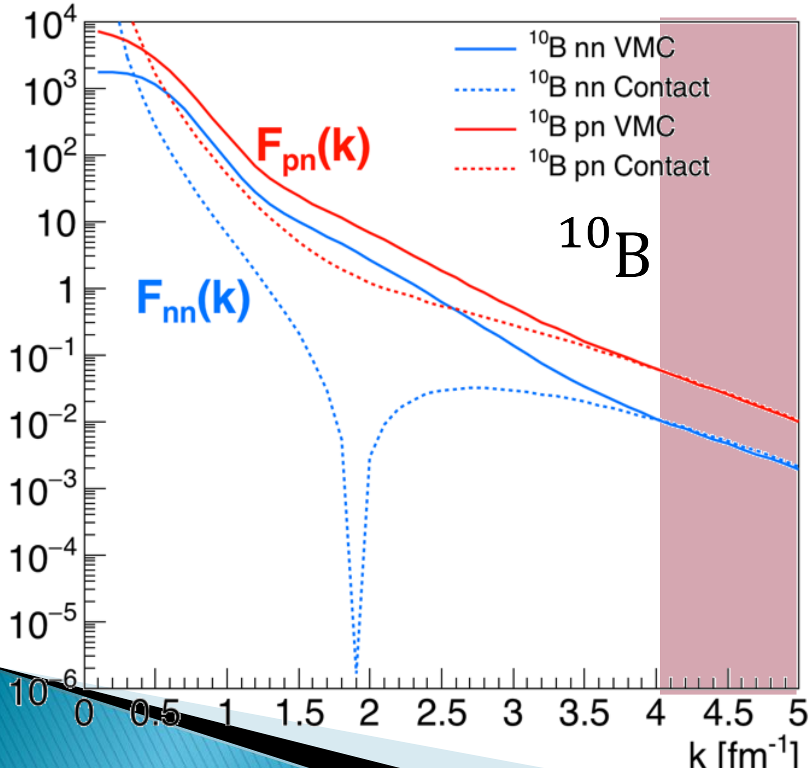
Momentum space

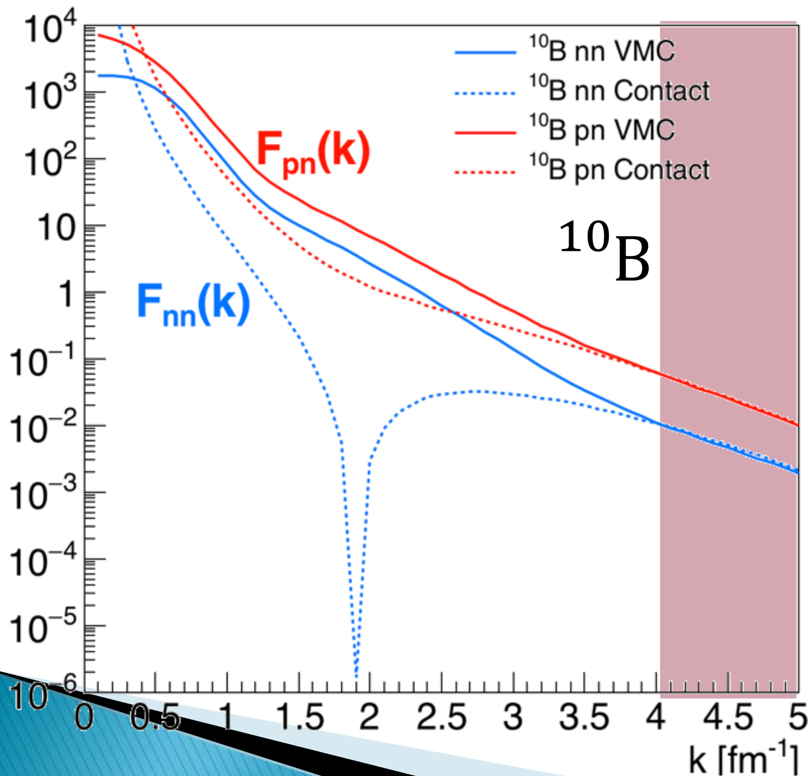


Extracting the contacts

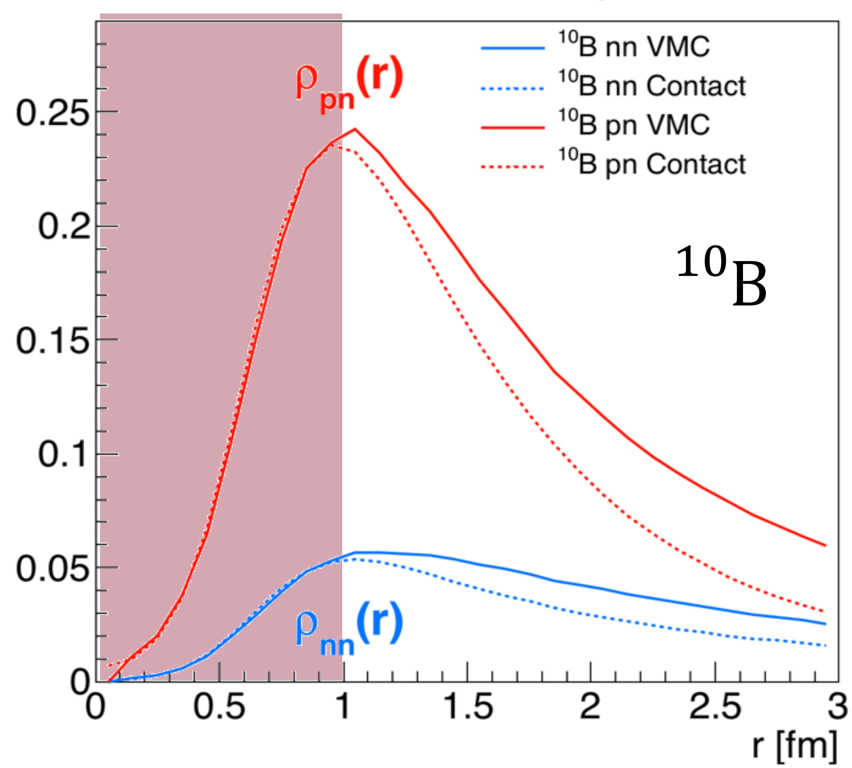
$$F_{pn}(k) \xrightarrow[k\to\infty]{} C^{d}_{pn}\left|\varphi^{d}_{pn}(k)\right|^2 + C^{0}_{pn}\left|\varphi^{0}_{pn}(k)\right|^2$$

$$F_{nn}(k) \xrightarrow[k\to\infty]{} C^{0}_{nn}|\varphi^{0}_{nn}(k)|^2$$

Momentum space



Coordinate space



Extracting the contacts

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

Universal functions – Calculated for the two-body system

Extracting the contacts

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Fitted to $F_{ij}\,(k)$ for $k > 4\ \mathrm{fm}^{-1}$

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The VMC data

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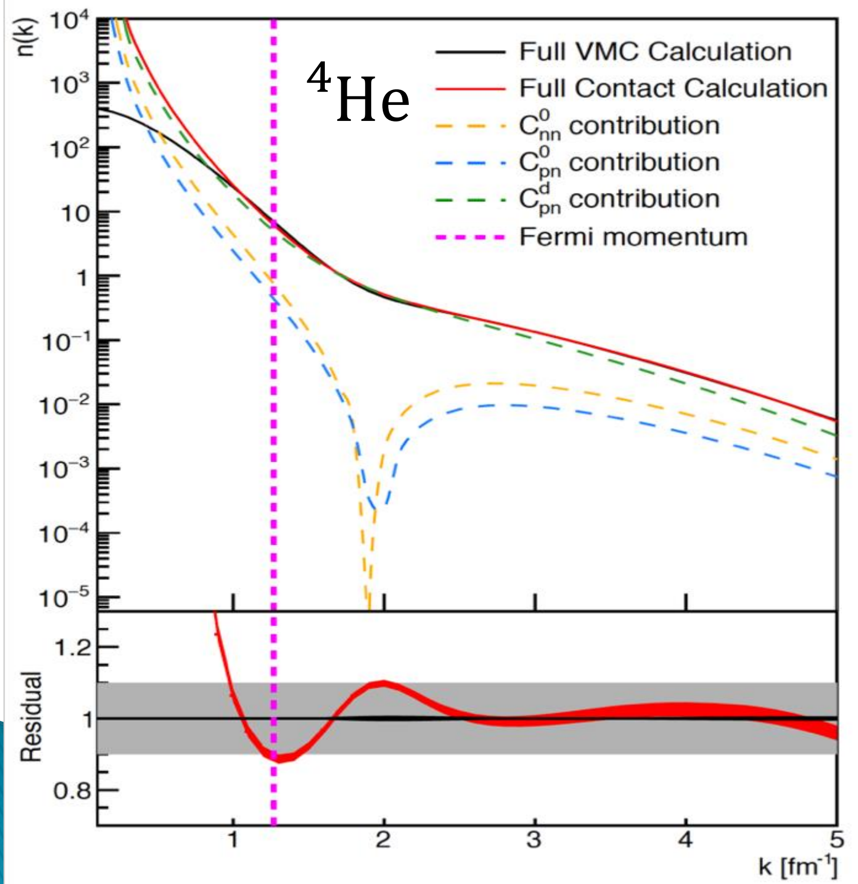
$n_p(k)$



Extracting the contacts

$$n_p(k) \xrightarrow[k\to\infty]{} C^d_{pn}\left|\varphi^d_{pn}(k)\right|^2 + C^0_{pn}\left|\varphi^0_{pn}(k)\right|^2 + 2C^0_{pp}\left|\varphi^0_{pp}(k)\right|^2$$

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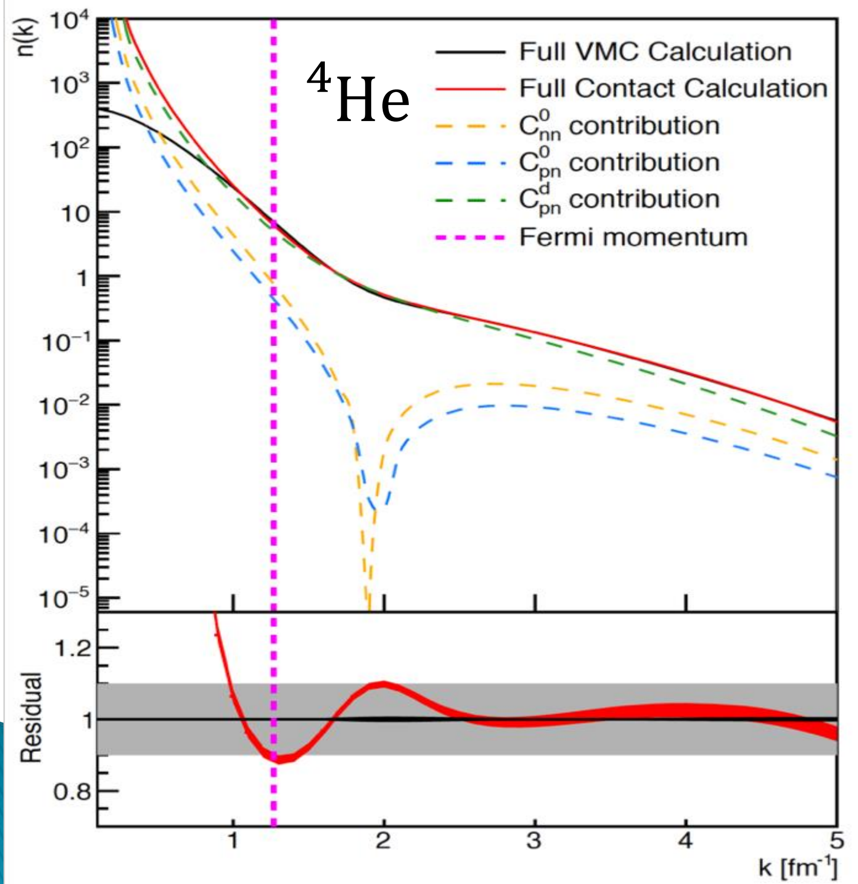
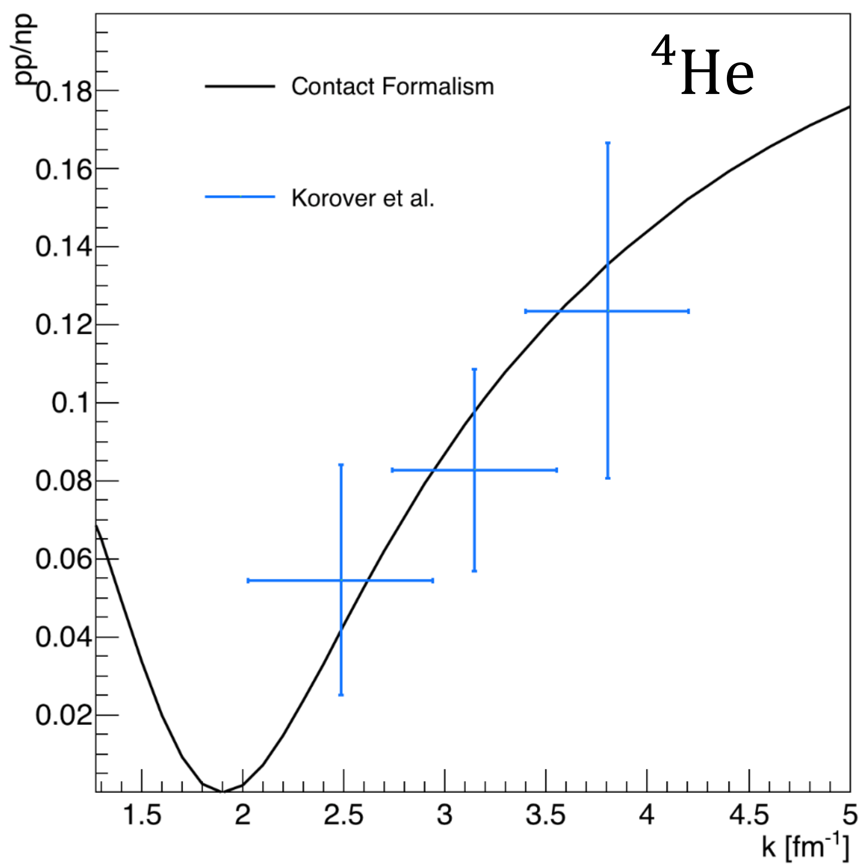


pp/np



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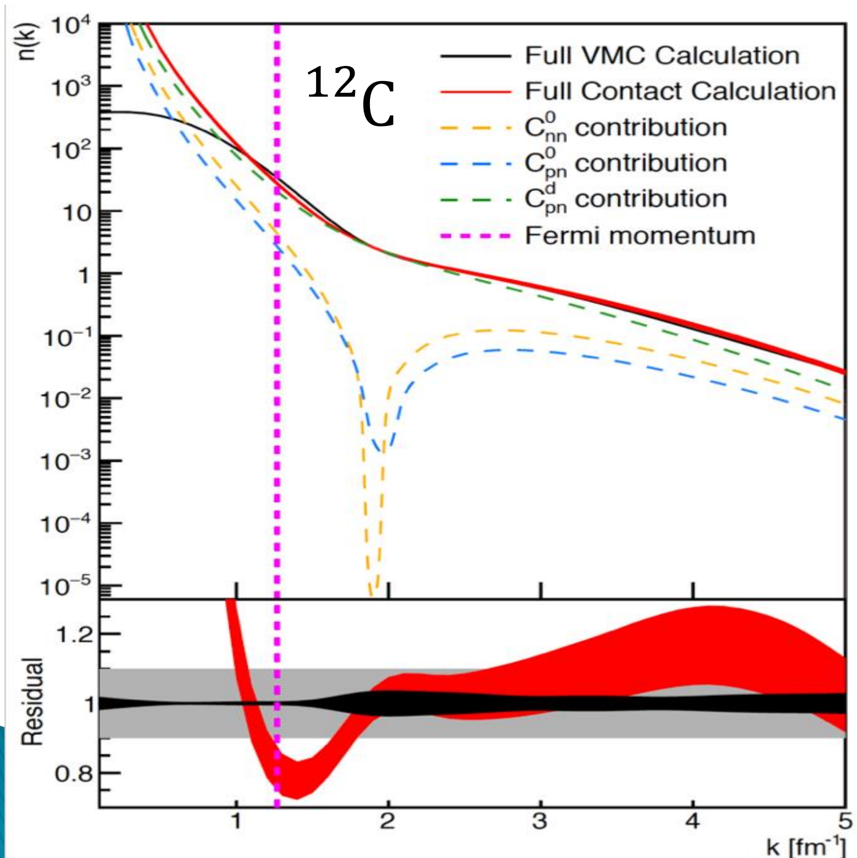

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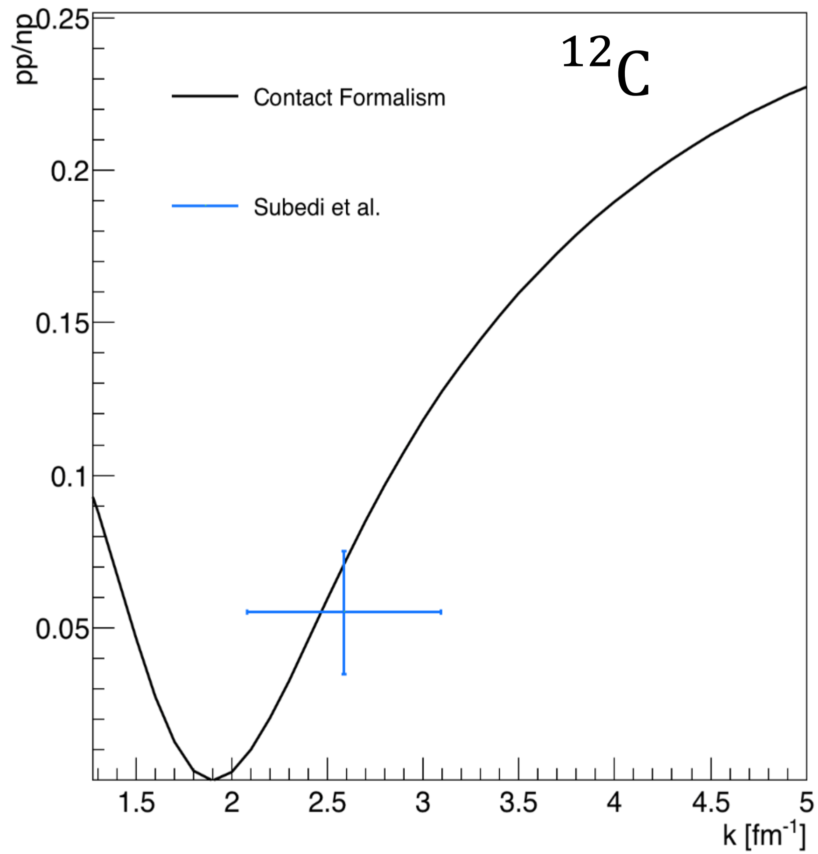

Counting the SRCs (symmetric nuclei)

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{d}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

Normalization: $\int_{k_F}^{\infty}\left|\varphi_{ij}^{\alpha}\right|^2 d^3k = 1$

$$\%SRC \equiv \frac{1}{Z}\int_{K_F}^{\infty} n_p(\boldsymbol{k})d^3k = \frac{1}{Z}\left[C_{pn}^{d} + C_{pn}^{0} + 2C_{nn}^{0}\right]$$

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^{4}He → Total number of pairs:

pp – 1   np–4

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	C^{0}_{pp}/Z (%)	C^{0}_{pn}/Z (%)	C^{d}_{pn}/Z (%)
k-space	0.65 ± 0.03	0.69 ± 0.03	12.3 ± 0.1

Non-combinatorial isospin symmetry (T=1)

Neutron-proton dominance

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	C^0_{pp}/Z (%)	C^0_{pn}/Z (%)	C^d_{pn}/Z (%)	%SRCs
k–space	0.65 ± 0.03	0.69 ± 0.03	12.3 ± 0.1	14.3%
r–space	0.567 ± 0.004		11.61 ± 0.03	13.3%

Similar results are obtained for all the available nuclei in the VMC data

The nuclear contact relations

- **Momentum distributions**

  *R. Weiss, B. Bazak, N. Barnea, PRC **92**, 054311 (2015)*
  *M. Alvioli, CC. Degli Atti, H. Morita, PRC **94**, 044309 (2016)*

- **The Levinger constant**

  *R. Weiss, B. Bazak, N. Barnea, PRL **114**, 012501 (2015)*
  *R. Weiss, B. Bazak, N. Barnea, EPJA **52**, 92 (2016)*

- **Electron scattering**

  *O. Hen et al., PRC **92**, 045205 (2015)*

- **Symmetry energy**

  *BJ. Cai, BA. Li, PRC **93**, 014619 (2016)*

- **The Coulomb sum rule (and a review)**

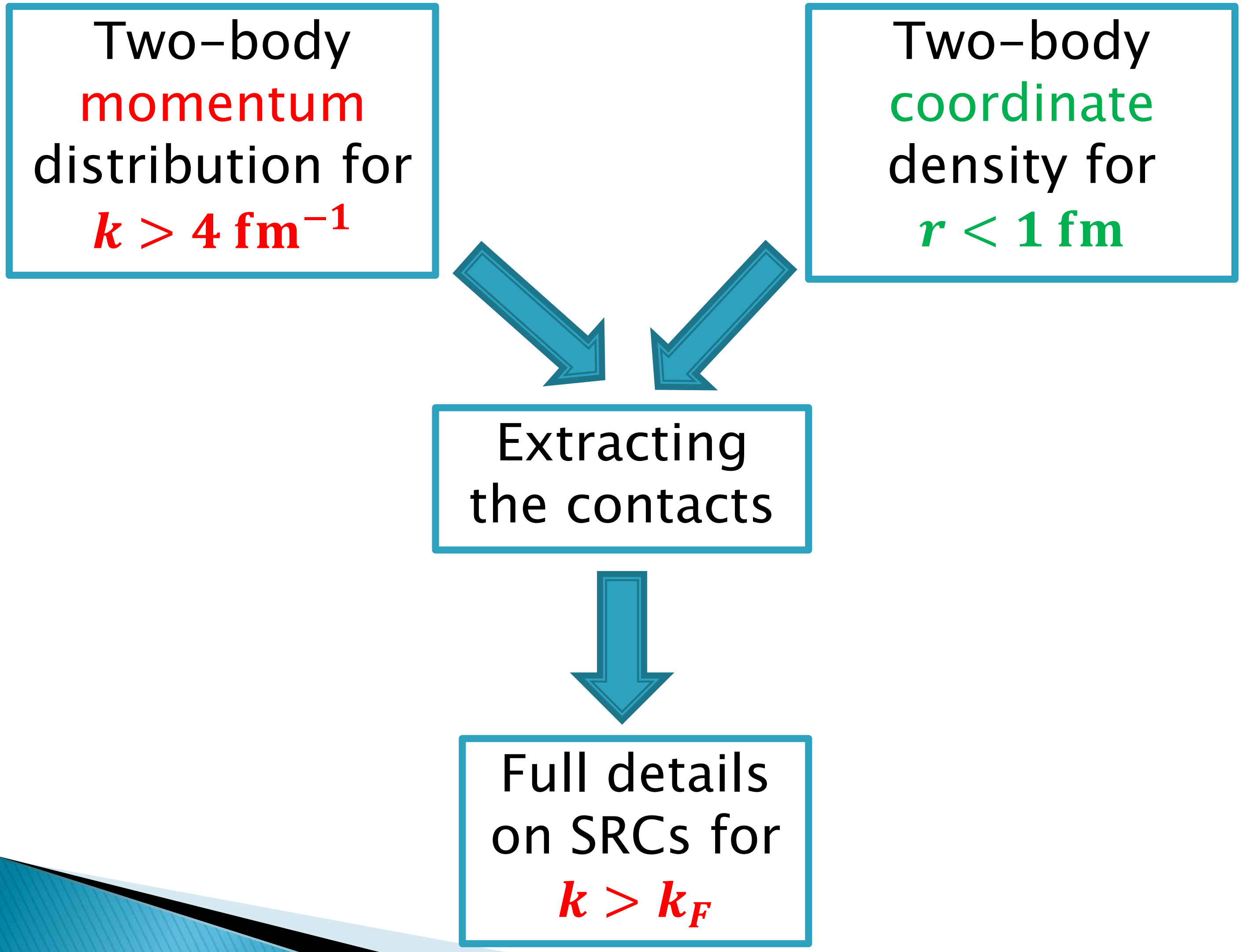
  R. Weiss, E. Pazy, N. Barnea, Few-Body Systems (2016)

- **The EMC effect**

  JW. Chen, W. Detmold, J. E. Lynn, A. Schwenk, arxiv 1607.03065 [hep-ph] (2016)

and more…

Summary


Two-body momentum distribution for $k > 4\ \mathrm{fm}^{-1}$
Two-body coordinate density for $r < 1\ \mathrm{fm}$
Extracting the contacts
Full details on SRCs for $k > k_F$

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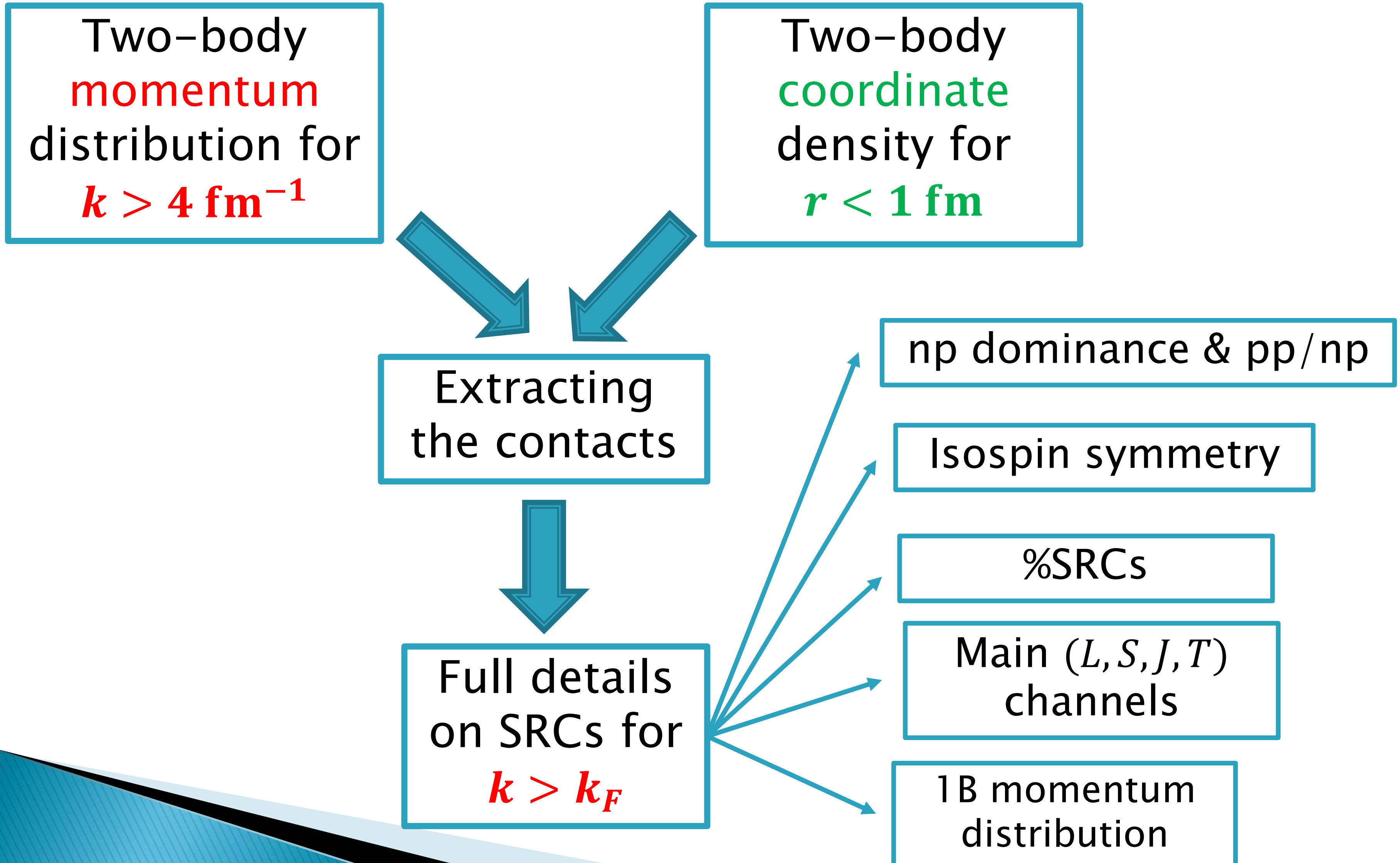
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np dominance & pp/np
Isospin symmetry
%SRCs
Main (L,S,J,T) channels
1B momentum distribution

Questions?