Implications Of EFT For Two-Body Coordinate-Space Nuclear Distributions

Quantitative Challenges in SRC and EMC Research

Joel E. Lynn

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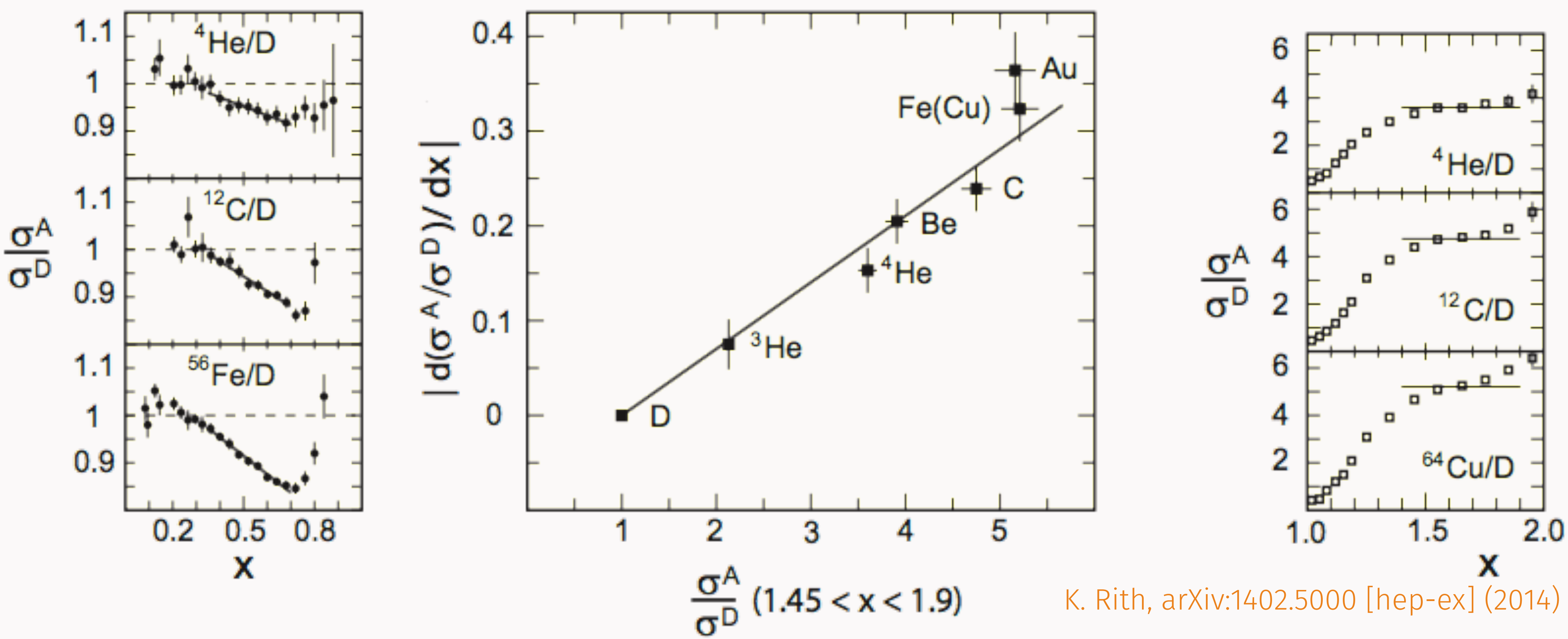
(I Know: Everyone Knows)

SRC scaling factor $a_2(A,x) \equiv \frac{2\sigma^A}{A\sigma^d}\Big|_{1.5<x<2}$.

$dR_{EMC}/dx \propto a_2$



K. Rith, arXiv:1402.5000 [hep-ex] (2014)

Implications Of EFT

J.-W. Chen & W. Detmold, Phys. Lett. B **625**, 165 (2005):

Structure functions factorize: $F_2^A(x)/A = F_2^N(x) + g_2(A,\Lambda) f_2(x,\Lambda)$

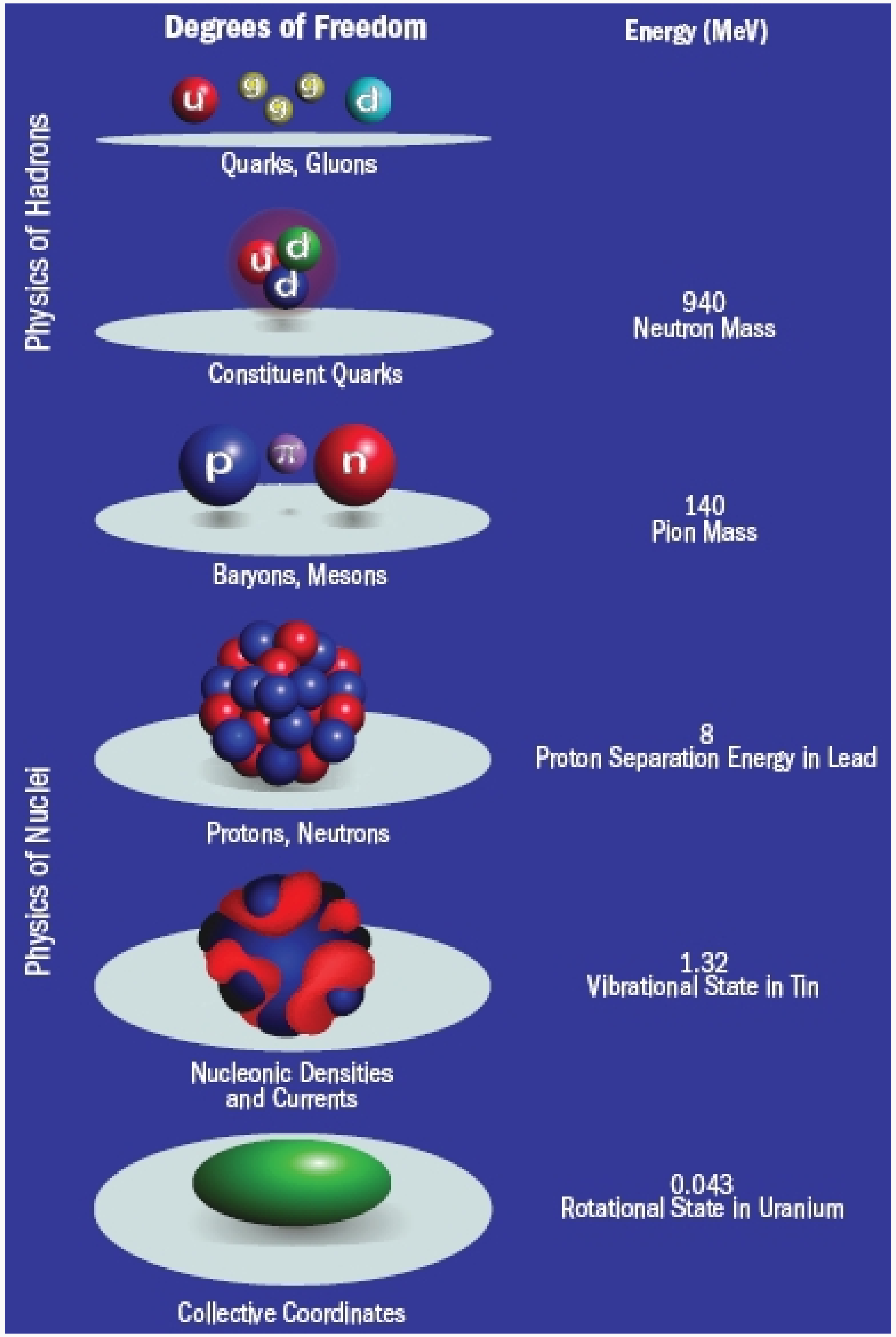
$$g_2(A,\Lambda) = \frac{1}{A} \langle A | (N^\dagger N)^2 | A \rangle_\Lambda$$

J.-W. Chen, W. Detmold, JEL, A. Schwenk,
arXiv:1607.03065 [hep-ph] (2016):

$$a_2(A, x > 1) = \frac{g_2(A,\Lambda)}{g_2(2,\Lambda)} \Rightarrow \frac{\mathrm{d}R_{\mathrm{EMC}}}{\mathrm{d}x} \propto a_2.$$

Motivation - Framework

Quantum chromodynamics (QCD) is ultimately responsible for strong interactions.



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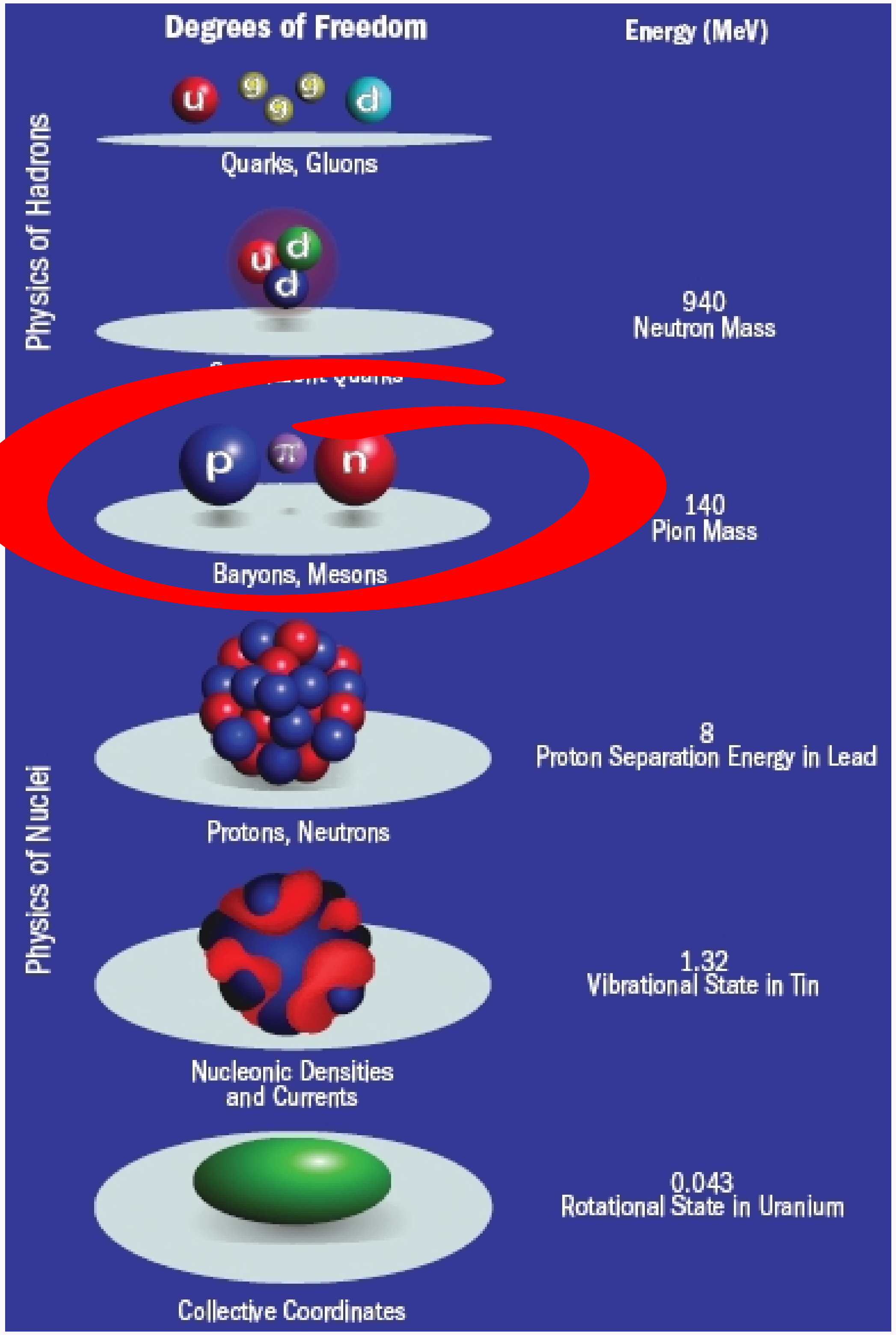
Nucleons are the relevant degrees of freedom for low-energy nuclear physics.



Motivation - A Hard Problem!

Nuclei are strongly interacting many-body systems.

1. **How do we solve the many-body Schrödinger equation?**

$$H|\Psi\rangle = E|\Psi\rangle$$

$2^A\binom{A}{Z}$ coupled differential equations in $3A-3$ variables.

^{4}He $\rightarrow$ 96 equations in 9 variables
^{12}C $\rightarrow$ 3 784 704 equations in 33 variables

2. **What is the Hamiltonian?**

Motivation - A Hard Problem!

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2. **What is the Hamiltonian?** $\rightarrow$ **Chiral EFT!**

QMC Methods

QMC methods in two lines:

$$H|\Psi\rangle = E|\Psi\rangle$$

$$\lim_{\tau\to\infty} e^{-H\tau}|\Psi_T\rangle \to |\Psi_0\rangle$$

QMC methods in more than two lines:

J. Carlson et al, RMP **87**, 1067 (2015).

QMC Methods - Variational Monte Carlo (VMC) Method

1. Guess a trial wave function Ψ_T and generate a random position: $\mathrm{R} = \mathrm{r}_1, \mathrm{r}_2, \ldots, \mathrm{r}_A$.

2. Use the Metropolis algorithm to generate new positions R' based on the probability $P = \frac{|\Psi_T(\mathrm{R}')|^2}{|\Psi_T(\mathrm{R})|^2}$. (Yields a set of "walkers" distributed according to $|\Psi_T|^2$).

3. Invoke the variational principle: $E_T = \frac{\langle\Psi_T|H|\Psi_T\rangle}{\langle\Psi_T|\Psi_T\rangle} > E_0$.

QMC Methods - Diffusion Monte Carlo Method

- The wave function is imperfect: $|\Psi_T\rangle = \sum_{i=0}^{\infty} \alpha_i |\Psi_i\rangle$.
- Propagate in imaginary time to project out the ground state $|\Psi_0\rangle$.

$$
\begin{aligned}
|\Psi(\tau)\rangle &= e^{-(H-E_T)\tau} |\Psi_T\rangle \\
&= e^{-(E_0-E_T)\tau}[\alpha_0 |\Psi_0\rangle + \sum_{i\neq 0} \alpha_i e^{-(E_i-E_0)\tau} |\Psi_i\rangle].
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\end{aligned}
$$

$$
|\Psi(\tau)\rangle \xrightarrow{\tau\to\infty} |\Psi_0\rangle .
$$

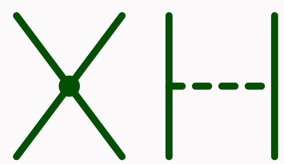
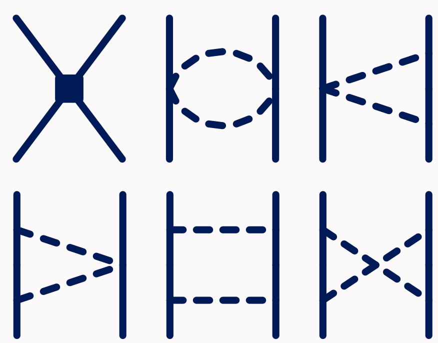
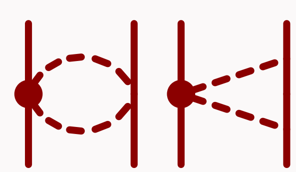
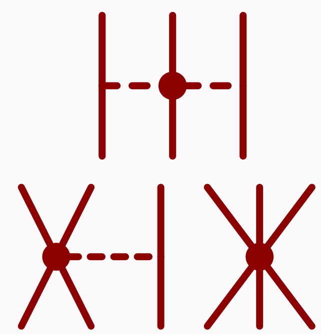
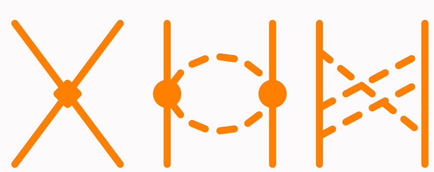
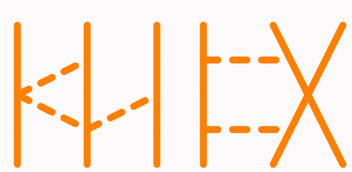
The Hamiltonian

Of course, the nuclear Hamiltonian is complicated.

$$H = \sum_{i=1}^{A} \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} V_{ijk} + \cdots$$

Where should it come from?

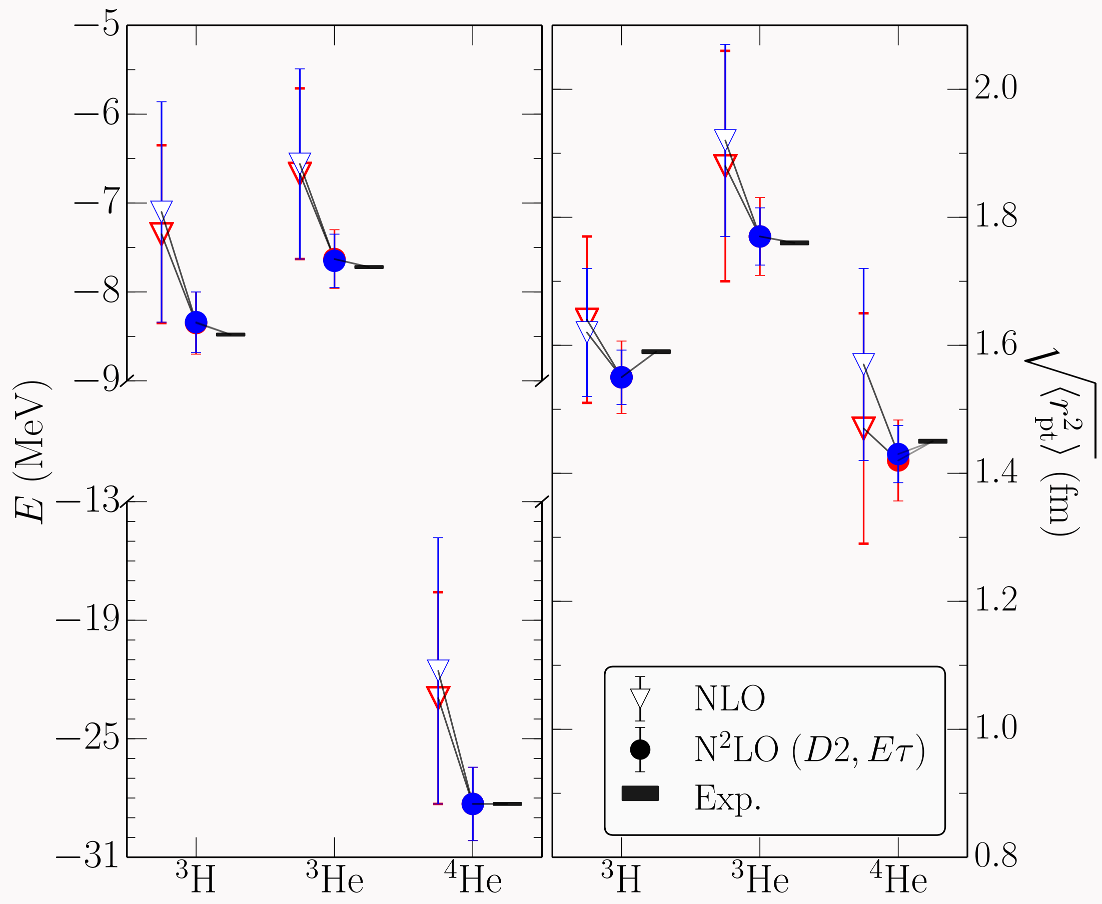
Chiral EFT

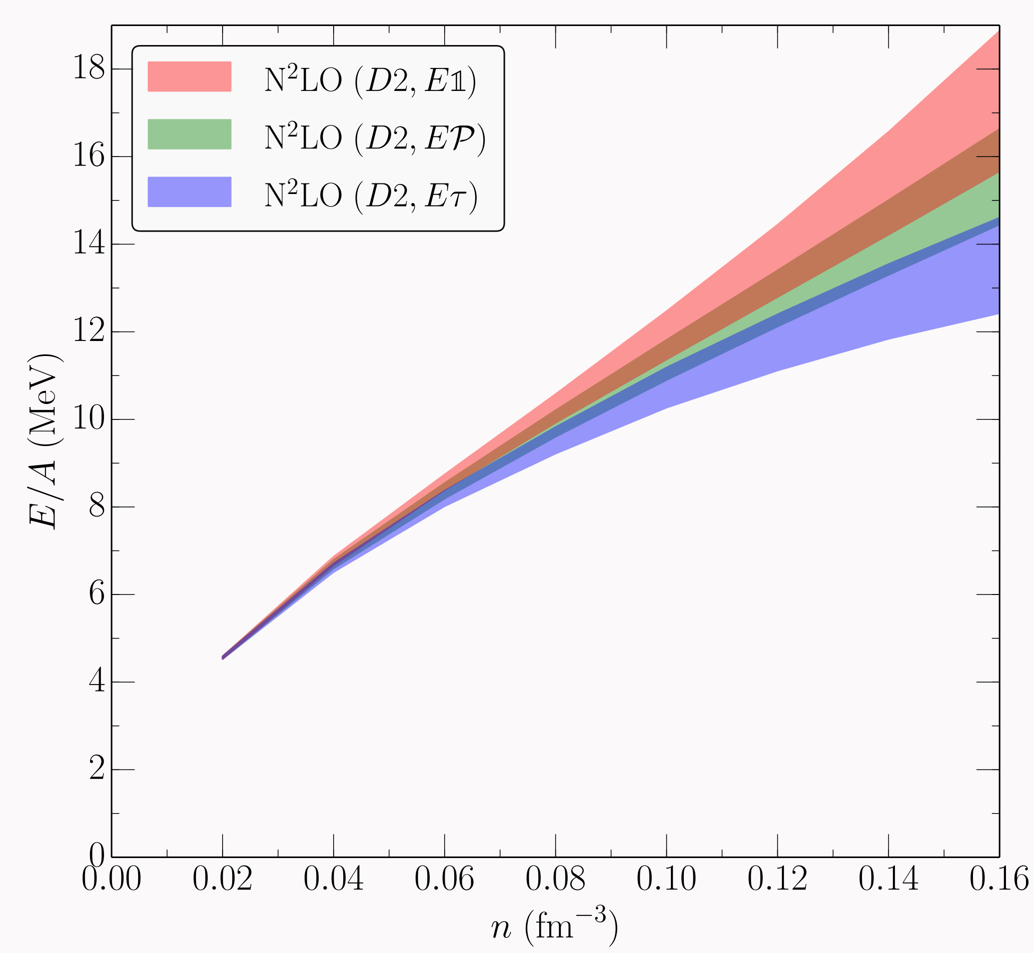
		NN	NNN
LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$		–
NLO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$		–
N^2LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$	 $+\cdots$	
N^3LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^4$	 $+\cdots$	 $+\cdots$

- Chiral EFT: Expand in powers of Q/Λ_b.
  $Q \sim m_\pi \sim 100$ MeV
  $\Lambda_b \sim 500$ MeV
- Long-range physics: π exchanges.
- Short-range physics: Contacts × LECs.
- Many-body forces & currents enter systematically.

Results

A simultaneous description of properties of light nuclei, n-α scattering and neutron matter is possible. Uncertainty analysis as in E. Epelbaum et al, EPJ **A51**, 53 (2015).





JEL et al, PRL **116**, 062501 (2016)

Uncertainty Estimates

Define

$$X^{(i)} = X^{(0)} + \Delta X^{(2)} + \cdots + \Delta X^{(i)},$$
$$\Delta X^{(2)} = X^{(2)} - X^{(0)}, \ \Delta X^{(i)} = X^{(i)} - X^{(i-1)}, \ i > 2.$$

Expected size

$$\Delta X^{(i)} = \mathcal{O}(Q^i X^{(0)}).$$

Then,

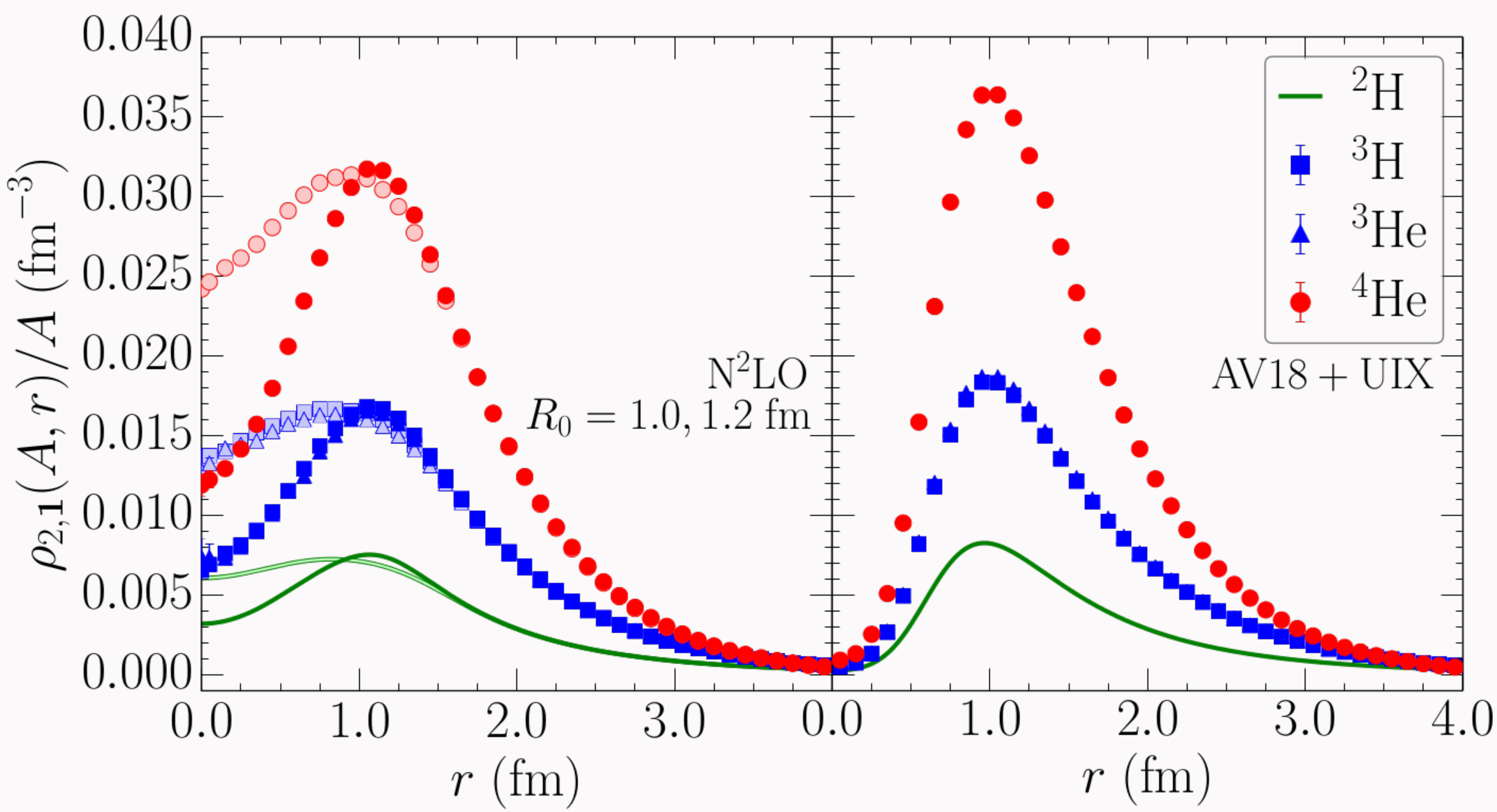
$$\delta X^{(0)} = Q^2 |X^{(0)}|, \ \delta X^{(i)} = \max(Q^{i+1}|X^{(0)}|, Q^{i+1-j}|\Delta X^{(j)}|), \ 2 \leq j \leq i$$

$$Q = \max(p/\Lambda_b, m_\pi/\Lambda_b).$$

Two-Body Distribution Functions (g_2)

$$g_2(A,\Lambda) = \rho_{2,1}(A, r=0)/A, \;\; \rho_{2,1}(A,r) \equiv \frac{1}{4\pi r^2}\left\langle \Psi_0 \middle| \sum_{i<j} \delta(r - r_{ij}) \middle| \Psi_0 \right\rangle$$

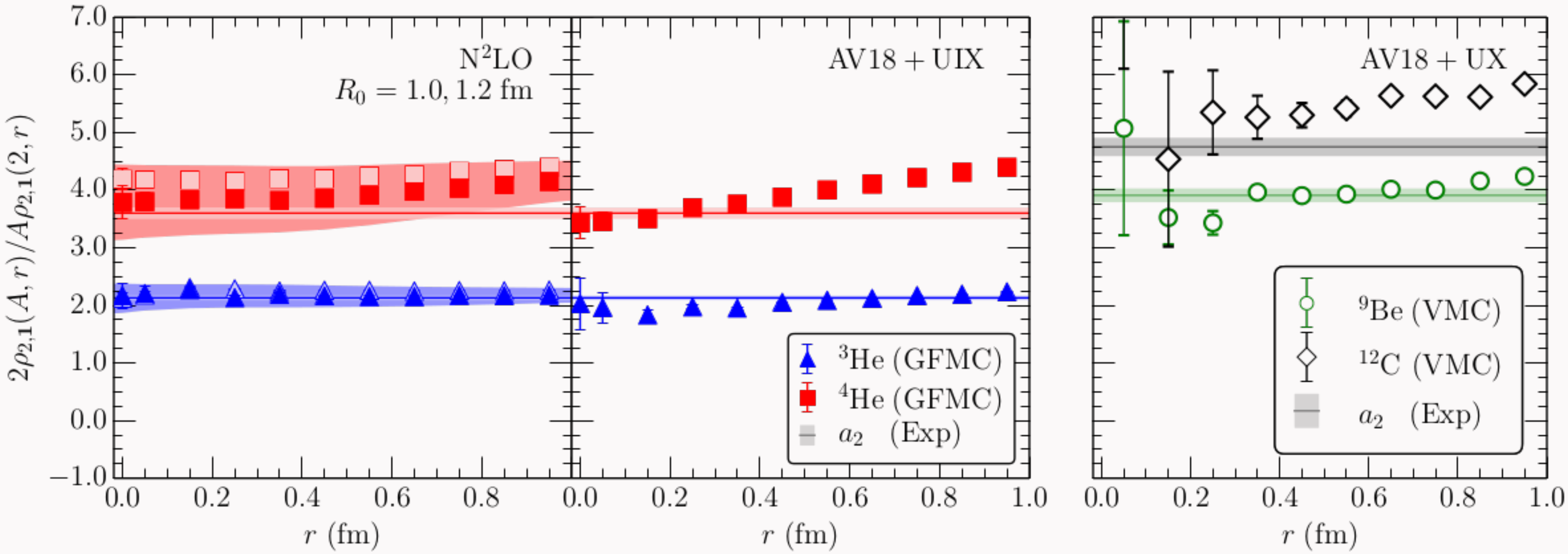
Scale and scheme dependent



SRC Correlation Factors

$$a_2 \equiv \lim_{r\to 0} \frac{2\rho_{2,1}(A,r)}{A\rho_{2,1}(2,r)}$$

Scale and scheme *independent*!



SRC Correlation Factors

Detailed comparison of experiment and theory for a_2.

	N^2LO ($R_0 = 1.0 - 1.2$ fm)	AV18+UIX	Exp
^3H	2.1(2) – 2.3(3)	2.0(4)	
^3He	2.1(2) – 2.1(3)	2.0(4)	2.13(4)
^4He	3.8(7) – 4.2(8)	3.4(3)	3.60(10)

SRCs & EMC Effect Summary

- EFT explains the linear relationship between $\mathrm{d}R_{\mathrm{EMC}}/\mathrm{d}x$ and a_2, and further implies that a_2 is calculable in “traditional nuclear physics”.
- Two-body distributions are scheme and scale dependent, but ratios are observable.
- Our results suggest that EFT can shed light on the existence of a_3.

Two Purposely Provocative Questions

What does “short-range” in SRC mean?

Should a_2 be called the SRC factor?