Factorization and Universality

Nir Barnea
The Racah institute for Physics
The Hebrew University, Jerusalem, Israel

December 2, 2016
Quantitative challenges in EMC and SRC Research and Data-Mining

האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

Short Range Correlations in a many-body system

Heavy Fermions

The Mara river, Kenya (2016).

Short Range Correlations in a many-body system

Heavy Fermions

The Mara river, Kenya (2016).

Short range interaction

We start with 2-body Schrodinger ...

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(\boldsymbol{r})\right]\psi = E\psi$$

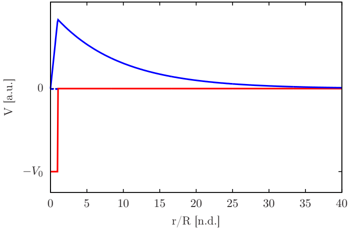
At vanishing distance, $r \longrightarrow 0$

- The energy becomes negligible $E \ll \hbar^2/mr^2$
- The w.f. ψ assumes an asymptotic **energy independent** form φ

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi(\boldsymbol{r}) = 0 \qquad r\varphi(r) = 0|_{r=0}$$

- Valid for any A-body system.

The Limit of vanishing energy



- Short range: only the s-wave survives
- Most of the wave function is outside the range of the potential.
- The wave funcion depends on a single length scale - the **scattering length a**
- The potential can be replaced by the boundary condition `[Bethe-Peierles]`

$$\boxed{\left.\frac{d\log(r\psi)}{dr}\right|_0 = \left.\frac{u'}{u}\right|_0 = -\frac{1}{a}}$$

- Consequently,

$$\boxed{\psi \longrightarrow \left(\frac{1}{r} - \frac{1}{a}\right)}$$

- Valid for **any** short range potential.

The Contact - Tan's Relations

A system of spin up - spin down fermions

Tan relations connects the contact C with:

1. **Tail of momentum distribution** $|a|^{-1}, d^{-1} \ll k \ll r_0^{-1}$

$$n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}$$

2. **The energy relation**

$$T + U = \sum_\sigma \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2 C}{4\pi m a}$$

3. **Adiabatic relation**

$$\frac{dE}{d1/a} = -\frac{\hbar^2 C}{4\pi m}$$

4. ...

The Contact - Tan's Relations

A system of spin up - spin down fermions

Tan relations connects the contact C with:

1. **Tail of momentum distribution** $|a|^{-1}, d^{-1} \ll k \ll r_0^{-1}$

$$n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}$$

2. **The energy relation**

$$T + U = \sum_\sigma \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2 C}{4\pi m a}$$

3. Adiabatic relation

$$\frac{dE}{d1/a} = -\frac{\hbar^2 C}{4\pi m}$$

4. ...

The Contact - Tan's Relations

A system of spin up - spin down fermions

Tan relations connects the contact C with:

1. **Tail of momentum distribution** $|a|^{-1}, d^{-1} \ll k \ll r_0^{-1}$

$$n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}$$

2. **The energy relation**

$$T + U = \sum_\sigma \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2 C}{4\pi m a}$$

3. **Adiabatic relation**

$$\frac{dE}{d1/a} = -\frac{\hbar^2 C}{4\pi m}$$

4. ...

The Contact - Tan's Relations

A system of spin up - spin down fermions

Tan relations connects the contact C with:

1. **Tail of momentum distribution** $|a|^{-1}, d^{-1} \ll k \ll r_0^{-1}$

$$n_\sigma(\boldsymbol{k}) \longrightarrow \frac{C}{k^4}$$

2. **The energy relation**

$$T + U = \sum_\sigma \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{C}{k^4} \right) + \frac{\hbar^2 C}{4\pi m a}$$

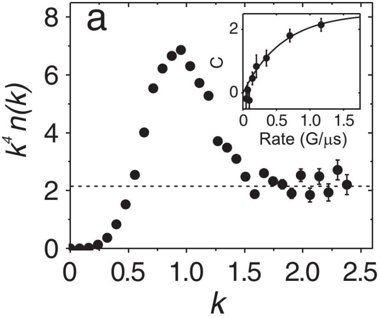
3. **Adiabatic relation**

$$\frac{dE}{d1/a} = -\frac{\hbar^2 C}{4\pi m}$$

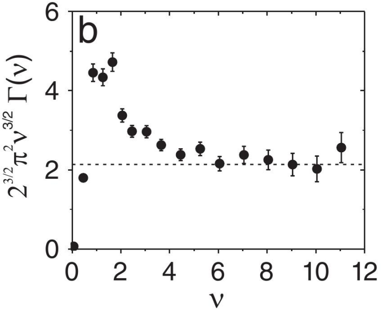
4. ...

The Contact - Experimental Results

Momentum Distribution



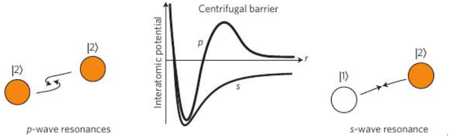
RF line shape



`Verification of Universal Relations in a Strongly Interacting Fermi Gas J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)`

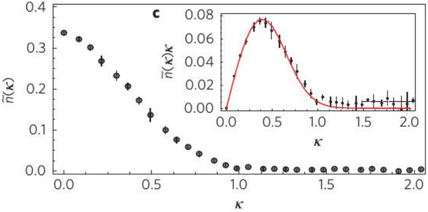
The contact formalism is not limited so s-waves

A system of one component fermions dominated by p-**wave interaction**



The asymptotic momentum distribution takes the form

$$n(\boldsymbol{k}) = \frac{16\pi^2}{V}\sum_m Y_{1m}^2(\hat{\boldsymbol{k}})\frac{C_p^m}{k^2}$$



C. Luciuk, *et al.*, Nature Phys. **12**, 599 (2016)

The short range factorization

[Tan, Braatan & Platter, Werner & Castin,...]

- The interaction is represented through the boundary condition

$$\left[\partial \log r_{ij}\Psi / \partial r_{ij}\right]_{r_{ij}=0} = -1/a$$

- Thus, when two particles approach each other

$$\Psi \xrightarrow[r_{ij}\to 0]{} \underbrace{\left(1/r_{ij} - 1/a\right)}_{\text{universal}} A_{ij}(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k\neq i,j})$$

- The contact C represents the probability of finding an interacting pair within the system

$$C \equiv 16\pi^2 \sum_{ij} \langle A_{ij} | A_{ij} \rangle$$

- Where

$$\langle A_{ij} | A_{ij} \rangle = \int \prod_{k\neq i,j} d\mathbf{r}_k \, d\mathbf{R}_{ij} \, A_{ij}^{\dagger}\left(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k\neq i,j}\right) \cdot A_{ij}\left(\mathbf{R}_{ij}, \{\mathbf{r}_k\}_{k\neq i,j}\right)$$

The short range factorization

[Tan, Braatan & Platter, Werner & Castin,...]

- The interaction is represented through the boundary condition

$$\left[\partial \log r_{ij}\Psi / \partial r_{ij}\right]_{r_{ij}=0} = -1/a$$

- Thus, when two particles approach each other

$$\Psi \xrightarrow[r_{ij}\to 0]{} \underbrace{(1/r_{ij} - 1/a)}_{\text{universal}} A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

- The contact C represents the probability of finding an interacting pair within the system

$$C \equiv 16\pi^2 \sum_{ij} \langle A_{ij} | A_{ij} \rangle$$

- Where

$$\langle A_{ij} | A_{ij} \rangle = \int \prod_{k\neq i,j} d\boldsymbol{r}_k \, d\boldsymbol{R}_{ij} \, A^{\dagger}_{ij}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right) \cdot A_{ij}\left(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}\right)$$

The 3-body contact

[Braatan & Platter, Werner & Castin,...]

The Efimov effect - 3 Bosons

- $a_s \longrightarrow \infty$, **NO** length scale
- The effective potential must scale as $1/r^2$
- The resulting spectrum

$$\frac{E_n}{E_0} = e^{-2\pi n/|\nu_0|} \approx 515^{-n}.$$

- At the origin $\rho \longrightarrow 0$

$$\Psi \longrightarrow c \sin\left[\nu_0 \log(\kappa\rho/2) - \phi\right]/\rho^2$$

- Consequntly $n(\boldsymbol{k})$ gets an oscillating $1/k^5$ tail

The Nuclear Contact(s)

Nuclear Scales

- The interaction range $r_0 = \hbar / m_\pi c \approx 1.4$ fm
- Scattering lengths $a_t \approx 5.4$ fm , $a_s \approx 20$ fm
- Interparticle distance $d \approx 2.4$ fm

Conclusions

- The Tan conditions are not strictly applicable in nuclear physics
- The interaction range is significant
- There could be different interaction channels - not only s-wave
- Therefore, we need replace the asymptotic form

$$\Psi \xrightarrow[r_{ij} \to 0]{} (1/r_{ij} - 1/a) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$

- Consequently we don't expect a $1/k^4$ tail

The Nuclear Contact(s)

Nuclear Scales

- The interaction range $r_0 = \hbar / m_\pi c \approx 1.4$ fm
- Scattering lengths $a_t \approx 5.4$ fm , $a_s \approx 20$ fm
- Interparticle distance $d \approx 2.4$ fm

Conclusions

- The Tan conditions are not strictly applicable in nuclear physics
- The interaction range is significant
- There could be different interaction channels - not only s-wave
- Therefore, we need replace the asymptotic form

$$\Psi \xrightarrow[r_{ij}\to 0]{} (1/r_{ij} - 1/a) A_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

- Consequently we don't expect a $1/k^4$ tail

The Nuclear Contact(s)

Nuclear Scales

- The interaction range $r_0 = \hbar/m_\pi c \approx 1.4$ fm
- Scattering lengths $a_t \approx 5.4$ fm , $a_s \approx 20$ fm
- Interparticle distance $d \approx 2.4$ fm

Conclusions

- The Tan conditions are not strictly applicable in nuclear physics
- The interaction range is significant
- There could be different interaction channels - not only s-wave
- Therefore, we need replace the asymptotic form

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi_\alpha(\boldsymbol{r}_{ij}) A^\alpha_{ij}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

- Consequently we don't expect a $1/k^4$ tail

The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels

$$\alpha = (s, \ell)jm$$

- For each pair we define the contact matrix

$$C_{ij}^{\alpha\beta} \equiv 16\pi^2 N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

- For $\ell = 0$ we need consider only **4** contacts

$$P = \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}, (np)_{S=1}\}$$

- Adding isospin symmetry the number of contacts is reduced to **2**,

$$C_s \longleftrightarrow \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}\}$$
$$C_t \longleftrightarrow \{(np)_{S=1}\}$$

The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels

$$\alpha = (s, \ell)jm$$

- For each pair we define the contact matrix

$$\boxed{C_{ij}^{\alpha\beta} \equiv 16\pi^2 N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle}$$

- For $\ell = 0$ we need consider only **4** contacts

$$P = \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}, (np)_{S=1}\}$$

- Adding isospin symmetry the number of contacts is reduced to **2**,

$$C_s \longleftrightarrow \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}\}$$
$$C_t \longleftrightarrow \{(np)_{S=1}\}$$

The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels

$$\alpha = (s, \ell)jm$$

- For each pair we define the contact matrix

$$\boxed{C_{ij}^{\alpha\beta} \equiv 16\pi^2 N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle}$$

- For $\ell = 0$ we need consider only **4** contacts

$$P = \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}, (np)_{S=1}\}$$

- Adding isospin symmetry the number of contacts is reduced to **2**,

$$C_s \longleftrightarrow \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}\}$$
$$C_t \longleftrightarrow \{(np)_{S=1}\}$$

The Nuclear Contact(s)

- In nuclear physics we have **3** possible particle pairs

$$ij = \{pp,\ nn,\ pn\}$$

- For each pair there are different channels

$$\alpha = (s, \ell)jm$$

- For each pair we define the contact matrix

$$\boxed{C_{ij}^{\alpha\beta} \equiv 16\pi^2 N_{ij} \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle}$$

- For $\ell = 0$ we need consider only **4** contacts

$$P = \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}, (np)_{S=1}\}$$

- Adding isospin symmetry the number of contacts is reduced to **2**,

$$C_s \longleftrightarrow \{(pp)_{S=0}, (nn)_{S=0}, (np)_{S=0}\}$$
$$C_t \longleftrightarrow \{(np)_{S=1}\}$$

The Nuclear Contact

The nuclear contact relations/applications

1. **The nuclear photoabsorption cross-section - The quasi-deutron model**

   `R. Weiss, B. Bazak, N. Barnea, PRL` **114**`, 012501 (2015)`

2. **The 1-body and 2-body momentum distributions**

   `R. Weiss, B. Bazak, N. Barnea, PRC` **92**`, 054311 (2015)`

   `M. Alvioli et al., arXiv:1607.04103 [nucl-th] (2016)`

   `R. Weiss, E. Pazy, N. Barnea, Few-Body syst. (2016)`

3. **Generalized treatment of the photoabsorption cross-section**

   `R. Weiss, B. Bazak, N. Barnea, EPJA (2016)`

4. **Electron scattering**

   `O. Hen et al., PRC` **92**`, 045205 (2015)`

5. **Symmetry energy**

   `B.J. Cai, B.A. Li, PRC` **93**`, 014619 (2016)`

6. ...

Open questions and points for discussion

Factorization and Universality in nucler physics

- What is the meaning of **universality** in nuclear physics?
- Can we derive model **independent** results?
- To what extent the **contact** formalism applicable to nuclei?
- How do we fix the **contact** experimentally?
- How do we fix the **contact** using *ab-initio* calculations?
- What is the role of 3-body correlations?
- Formal derivation of the W.F. factorization.
- Validity range, small parameter?
- Can we develop a simple theory to account for **all** SRC dominated reactions and observables? (probably yes)

Thank you !