Two-body Momentum Distributions

in $2 \leq A \leq 40$ nuclei

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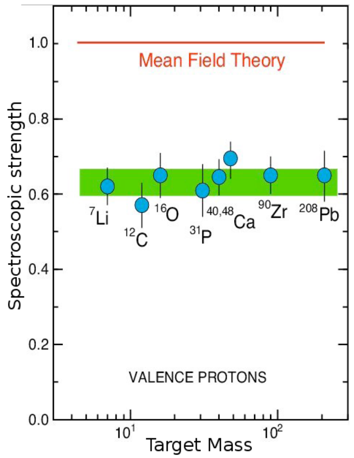
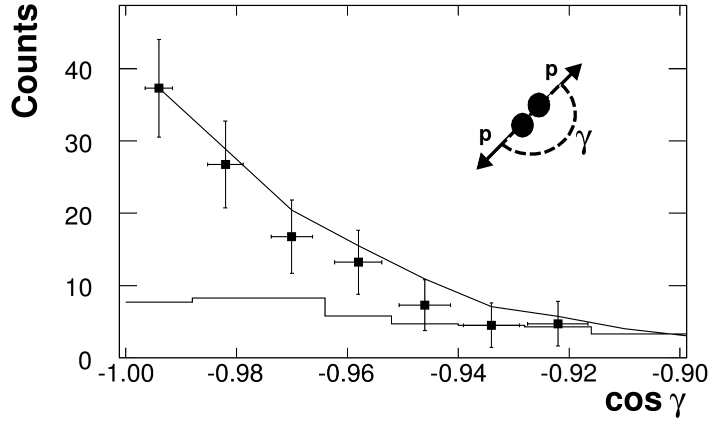
- **Relationship between $K_{c.m.}$-integrated and $K_{c.m.}$=0 2BMD**

2. Comparison with experimental data

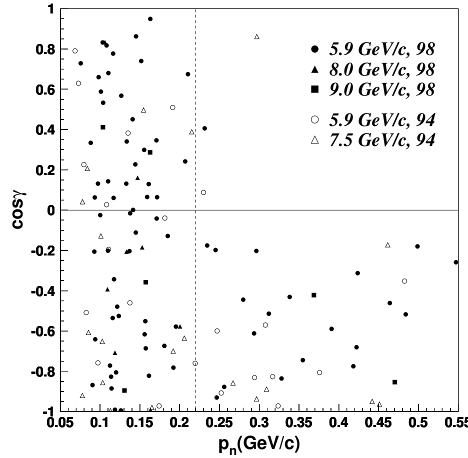
0. Comprehensive review of experimental results ☺



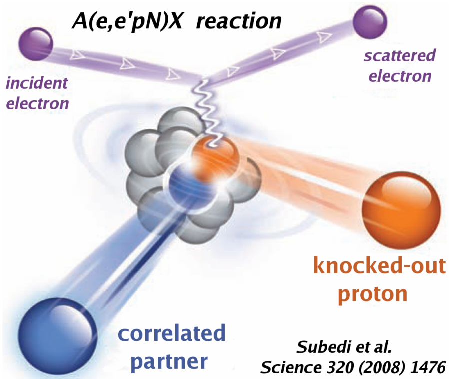
$A(e, e'p)X$

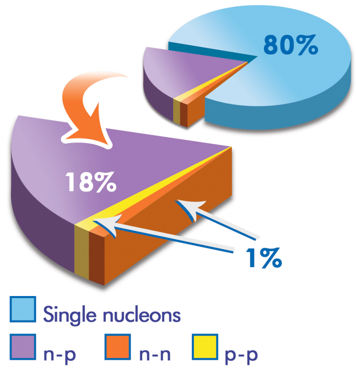


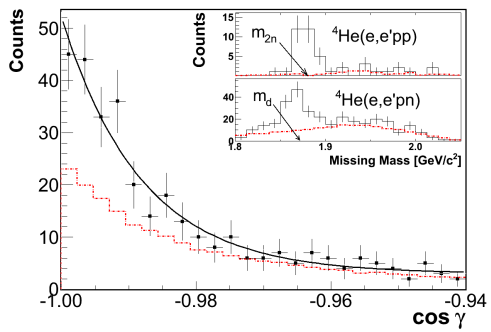
$A(p, 2p)$ Tang et al., PRL90 (2003)
theory: Ciofi et al., PRC53 (1996)

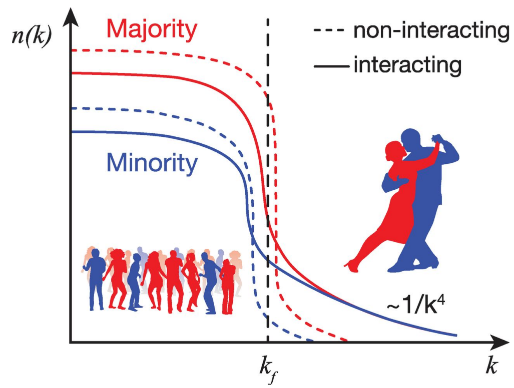


$A(p, ppn)$ Aclander et al., PLB (1999)
theory: Piasetzky et al., PLB (1999)









triple coincidence $A(e, e'pN)X$: **Carbon**
Subedi et al., **Science, 320** (2008)
Shneor et al., **PRL99** (2007)

Helium
Korover et al., **PRL 113** (2014)

heavier targets
Hen et al., **Science, 356** (2014)

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0. Nuclear Hamiltonian

- The non-relativistic nuclear many-body problem:

$$\hat{\mathbf{H}}\,\Psi_n = E_n\,\Psi_n\,, \qquad \hat{\mathbf{H}} = -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{ij}\hat{v}_{ij} + \ldots$$

- *Exact* ground-state wave functions obtained by various methods are available for ***light nuclei*** ($A \leq 12$);
  $\Longrightarrow$ calculations will be shown using ^2H, ^3He, ^4He WFs;
- Variational wave functions of nuclei can be obtained with approximated methods; usually difficult to use/generalize
  $\Longrightarrow$ we developed an easy-to-use *cluster expansion* technique for the calculation of basic quantities of ***medium-heavy nuclei***, ^{12}C, ^{16}O, ^{40}Ca;
- SRCs implemented MC generator for nuclear configurations for nuclei from ^{12}C to ^{238}U for the initialization of pA and AA collisions simulations

`http://sites.psu.edu/color`

0. Calculation of basic quantities

- one- and two-body densities:

$$\rho_N^{ST}(\boldsymbol{r}_1, \boldsymbol{r}'_1) = \int d\boldsymbol{r}_1 \sum\!\!\!\!\!\!\int \prod_{j=3}^{A} d\boldsymbol{r}_j \Psi_A^{o\dagger}(\boldsymbol{x}_1, ..., \boldsymbol{x}_A)\, \hat{P}_{pN}^{ST}\, \Psi_A^{o}(\boldsymbol{x}'_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_A)$$

$$\rho_{pN}^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1, \boldsymbol{r}'_2) = \sum\!\!\!\!\!\!\int \prod_{j=3}^{A} d\boldsymbol{r}_j \Psi_A^{o\dagger}(\boldsymbol{x}_1, ..., \boldsymbol{x}_A)\, \hat{P}_{pN}\, \Psi_A^{o}(\boldsymbol{x}'_1, \boldsymbol{x}'_2, \boldsymbol{x}_3, ..., \boldsymbol{x}_A)$$

- one- and two-body momentum distributions:

$$n_N^{ST}(k_1) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r}_1 d\boldsymbol{r}'_1\, e^{-\boldsymbol{k}_1 \cdot (\boldsymbol{r}_1 - \boldsymbol{r}'_1)} \rho_N^{ST}(\boldsymbol{r}_1, \boldsymbol{r}'_1)$$

$$n_{pN}^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) = \frac{1}{(2\pi)^6} \int d\boldsymbol{r}_1 d\boldsymbol{r}'_1 d\boldsymbol{r}_2 d\boldsymbol{r}'_2\, e^{-\boldsymbol{k}_1 \cdot (\boldsymbol{r}_1 - \boldsymbol{r}'_1)}\, e^{-\boldsymbol{k}_2 \cdot (\boldsymbol{r}_2 - \boldsymbol{r}'_2)} \;.$$

$$\cdot\, \rho_{pN}^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1, \boldsymbol{r}'_2) \quad \longleftrightarrow \quad n_{pN}^{(2)}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$$

0. Two-Body Momentum Distributions

$$\boldsymbol{k}_{rel} \equiv \boldsymbol{k} = \frac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2) \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 \qquad \boldsymbol{r}' = \boldsymbol{r}'_1 - \boldsymbol{r}'_2$$

$$\boldsymbol{K}_{CM} \equiv \boldsymbol{K} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \qquad \boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2) \qquad \boldsymbol{R}' = \frac{1}{2}(\boldsymbol{r}'_1 + \boldsymbol{r}'_2)$$

$$n^{(2)}(\boldsymbol{k}, \boldsymbol{K}) = n^{(2)}(k_{rel}, K_{CM}, \Theta) =$$
$$= \frac{1}{(2\pi)^6} \int d\boldsymbol{r}\, d\boldsymbol{r}'\, d\boldsymbol{R}\, d\boldsymbol{R}'\, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')}\, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R}')$$

$$n^{(2)}(\boldsymbol{k}) = \int d\boldsymbol{K}\, n(\boldsymbol{k}, \boldsymbol{K}) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r}\, d\boldsymbol{r}'\, d\boldsymbol{R}\, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R})$$

$$n^{(2)}(\boldsymbol{K}) = \int d\boldsymbol{k}\, n(\boldsymbol{k}, \boldsymbol{K}) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r}\, d\boldsymbol{R}\, d\boldsymbol{R}'\, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}; \boldsymbol{R}, \boldsymbol{R}')$$

$$n^{(2)}(\boldsymbol{k}, \boldsymbol{K} = 0) = \frac{1}{(2\pi)^6} \int d\boldsymbol{r}\, d\boldsymbol{r}'\, d\boldsymbol{R}\, d\boldsymbol{R}'\, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R}')$$

$\boldsymbol{K}_{CM} = 0$ corresponds to $\boldsymbol{k}_2 = -\boldsymbol{k}_1$, *i.e. back-to-back* nucleons

0. Using Realistic WFs of large nuclei: *Cluster Expansion*

- Cluster Expansion is a technique to reduce the computational effort in many many-body calculations; we use: $\Psi_o = \hat{F}\Phi_o = \prod_{ij}\sum_n \hat{f}_{ij}^{(n)}\Phi_o$
- Expectation value over Ψ_o of any one- or two-body operator $\hat{Q}$:

$$\frac{\langle\Psi_o|\hat{Q}|\Psi_o\rangle}{\langle\Psi_o|\Psi_o\rangle} = \frac{\langle\hat{F}^\dagger\hat{Q}\hat{F}\rangle}{\langle\hat{F}^2\rangle} = \frac{\langle\prod\hat{f}^\dagger\hat{Q}\hat{f}\rangle}{\langle\prod\hat{f}^2\rangle} = \frac{\langle\hat{Q}\prod(1+\hat{\eta})\rangle}{\langle\prod(1+\hat{\eta})\rangle} =$$

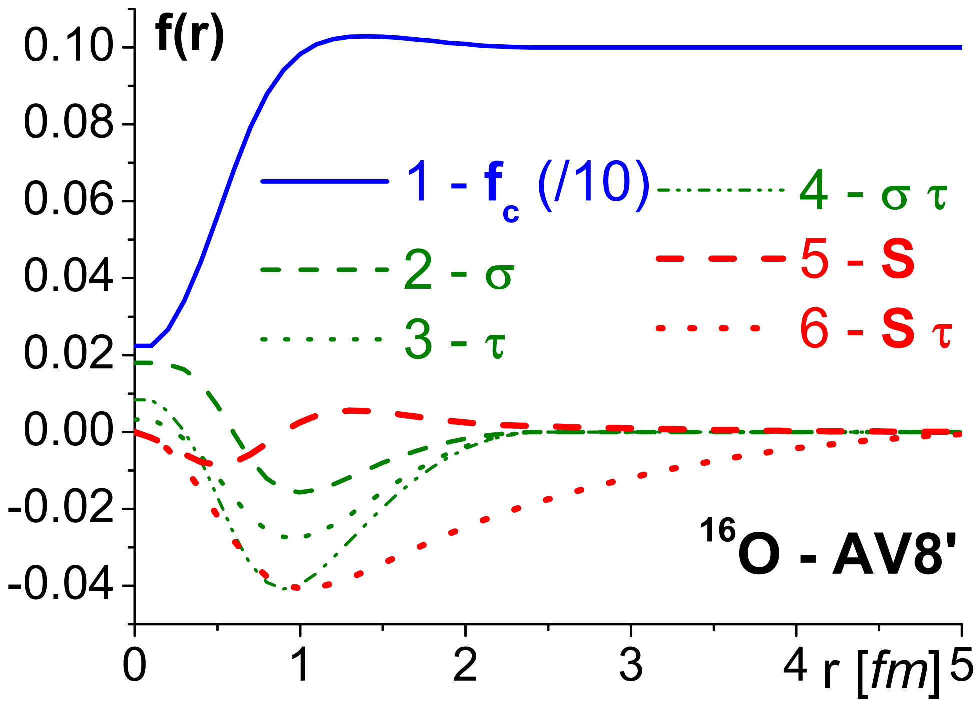
$$= \frac{\langle\hat{Q}\,(1+\sum\hat{\eta}+\sum\hat{\eta}\hat{\eta}+\ldots)\rangle}{\langle(1+\sum\hat{\eta}+\sum\hat{\eta}\hat{\eta}+\ldots)\rangle} \simeq \frac{\langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle}{1+\langle\sum\hat{\eta}\rangle} =$$

$$\simeq \left[\langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle\right]\left(1-\langle\sum\hat{\eta}\rangle+\ldots\right) \simeq \langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle_L$$

- $\langle\hat{\eta}\rangle = \langle\left[\hat{\mathbf{f}}^{\mathbf{2}}-\mathbf{1}\right]\rangle$ is the *small expansion parameter*; $\langle\hat{Q}\rangle \equiv \langle\Phi_o|\hat{Q}|\Phi_o\rangle$
- we end up with ***linked*** clusters; up to **4b diagrams needed for 2B density**, each involving the **square** of: $\hat{f} = \sum_n f_n(r_{ij})\hat{O}_n(ij)$

0. Using Realistic WFs of large nuclei: *Cluster Expansion*

correlation functions: *Central*, *Spin-Isospin*, *Tensor*



- $\langle \hat{\eta} \rangle = \langle \left[\hat{\mathbf{f}}^{\mathbf{2}} - \mathbf{1} \right] \rangle$ is the *small expansion parameter*
- we end up with ***linked*** clusters; up to **4b diagrams needed for 2B density**, each involving the **square** of: $\hat{f} = \sum_n f_n(r_{ij}) \hat{O}_n(ij)$

• at **first order** of the η−expansion, the **full correlated** `one-body mixed` density matrix expression is as follows:

$$\rho^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) + \rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) + \rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,,$$

with

$$\rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \int d\boldsymbol{r}_2\left[H_D(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2) - H_E(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\,\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\right]$$

$$\rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = -\int d\boldsymbol{r}_2 d\boldsymbol{r}_3\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\left[H_D(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\rho_o(\boldsymbol{r}_3) - H_E(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}_3)\rho_o^{(1)}(\boldsymbol{r}_3,\boldsymbol{r}'_1)\right]$$

and the functions H_D and H_E are defined as:

$$H_{D(E)}(r_{ij},r_{kl}) = \sum_{p,q=1}^{6} f^{(p)}(r_{ij})\,f^{(q)}(r_{kl})\,C_{D(E)}^{(p,q)}(r_{ij},r_{kl}) - C_{D(E)}^{(1,1)}(r_{ij},r_{kl})$$

with $C_{D(E)}^{(p,q)}(r_{ij},r_{kl})$ proper functions arising from spin-isospin traces;

(*Alvioli, Ciofi degli Atti, Morita*, ***PRC72*** (2005))

• at **first order** of the $\eta-$expansion, the **full correlated** `two-body mixed` density matrix expression is as follows:

$$\rho^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2)$$

with:

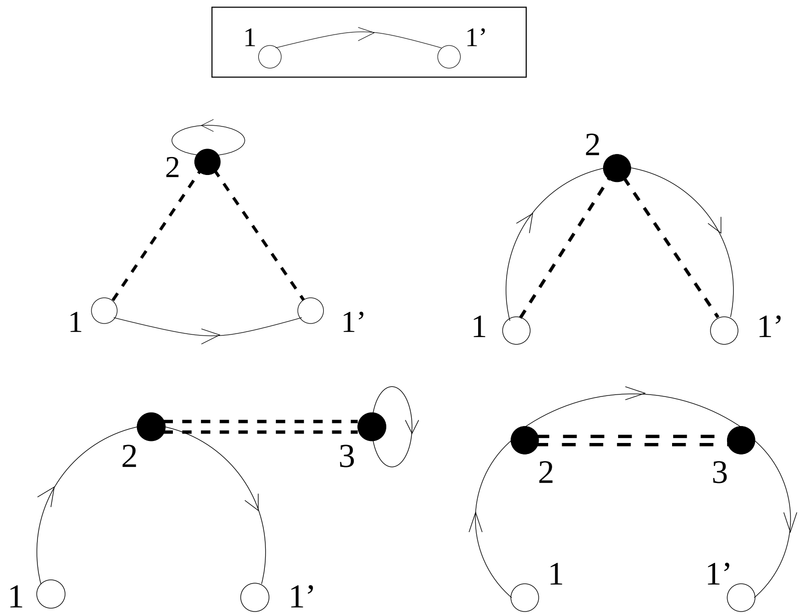
$$\rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = C_D\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - C_E\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)$$

$$\rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)$$

$$\begin{aligned}\rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \int d\boldsymbol{r_3}\,\hat{\eta}(r_{13},r_{1'3})\,[\,&\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3) + \\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2) + \\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2) + \\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1) + \\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1) + \\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3)\,]\end{aligned}$$

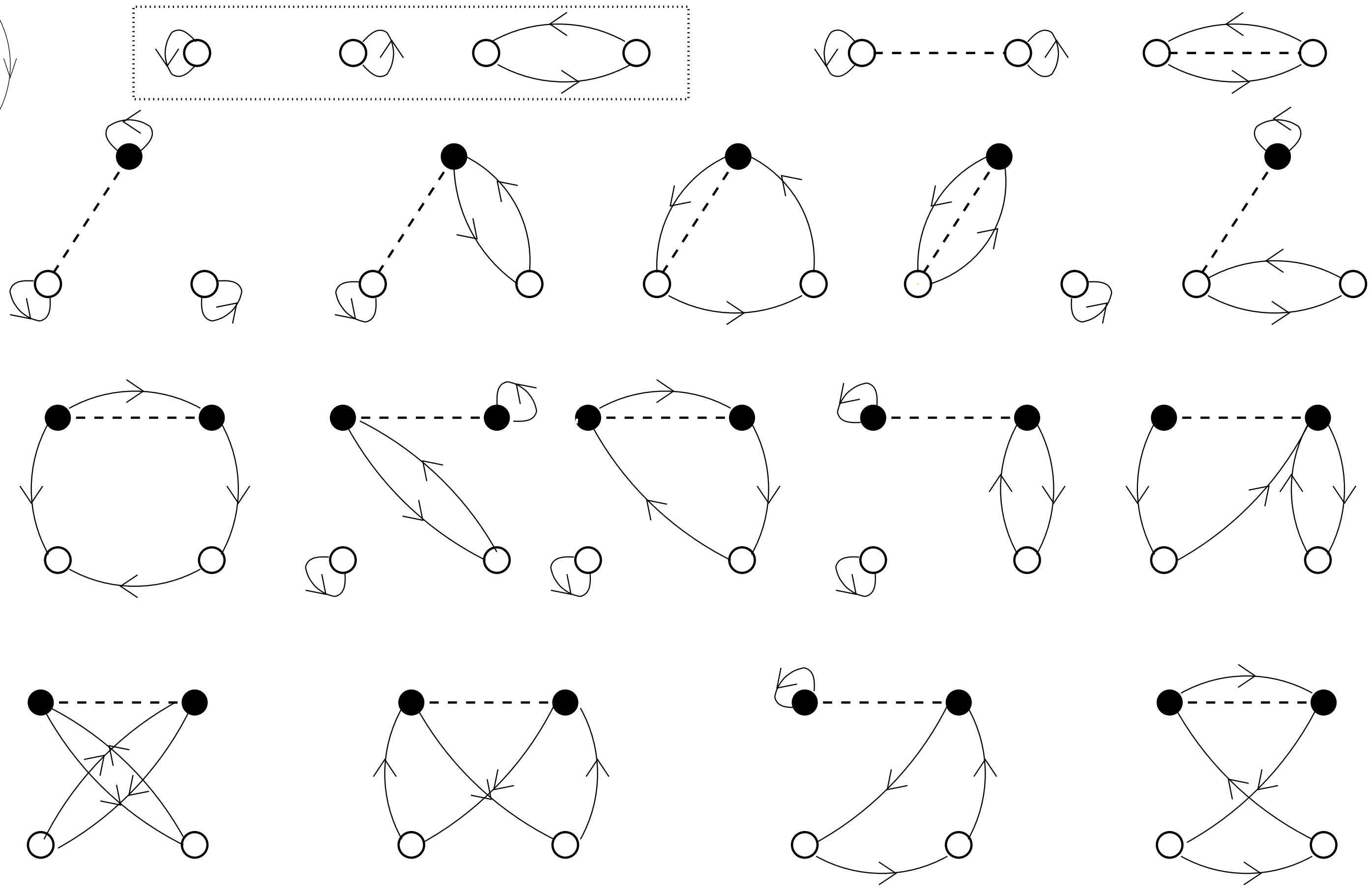
$$\begin{aligned}\rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= \frac{1}{4}\int d\boldsymbol{r_3}d\boldsymbol{r_4}\,\hat{\eta}(r_{34})\,\cdot \\ &\cdot\sum_{\mathcal{P}\in\mathcal{C}}(-1)^{\mathcal{P}}\,[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_{\mathcal{P}1'})\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_{\mathcal{P}2'})\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_{\mathcal{P}3})\,\rho_o(\boldsymbol{r}_4,\boldsymbol{r}_{\mathcal{P}4})\,]\end{aligned}$$

(*Alvioli, Ciofi degli Atti, Morita*, ***PRC72*** (2005))
(*Alvioli, Ciofi degli Atti, Morita*, ***PRL100*** (2008))



one-body, *non-diagonal*

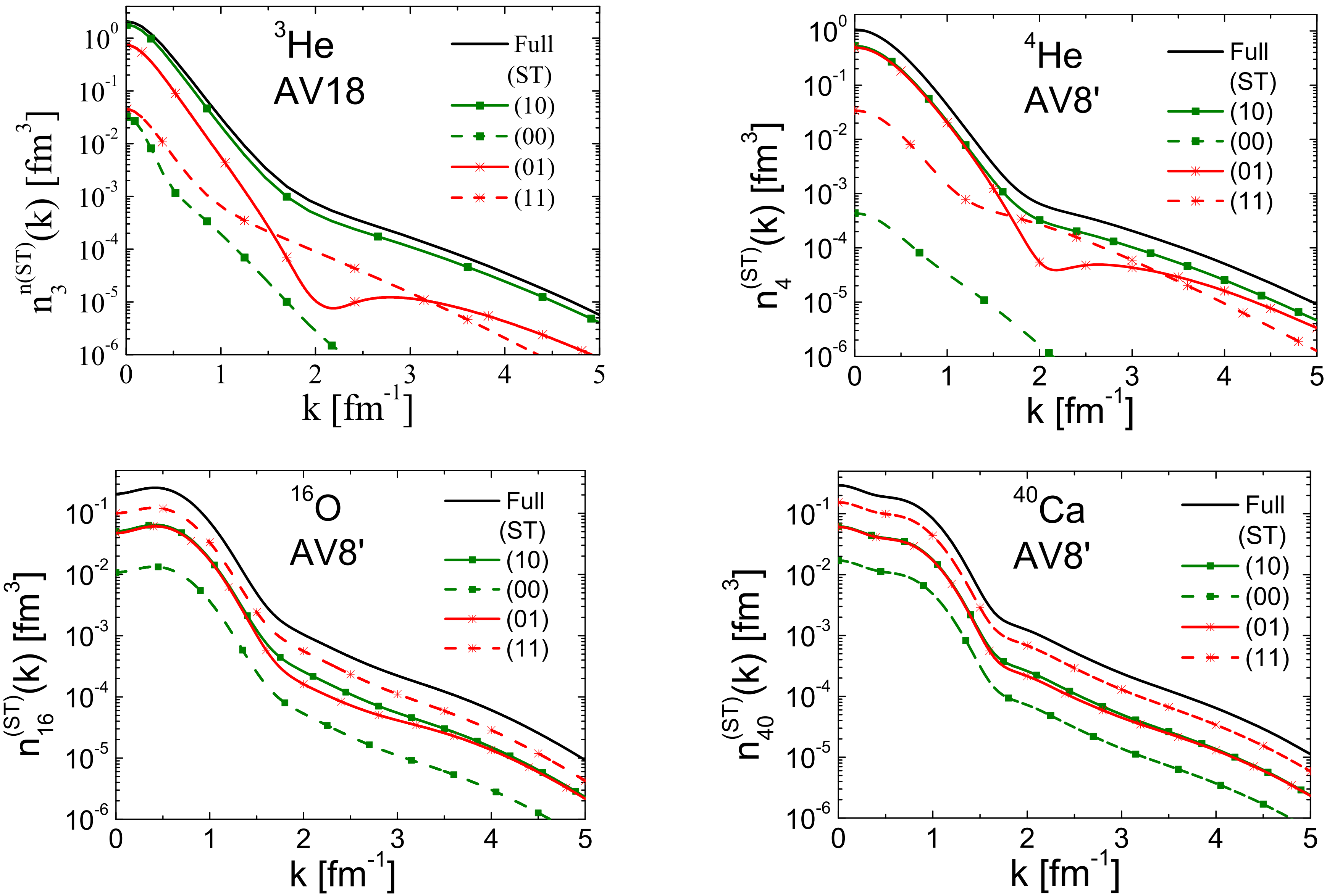
$\longleftarrow \rho(\boldsymbol{r}_1, \boldsymbol{r}'_1)$ *diagrams*



two-body, *diagonal*

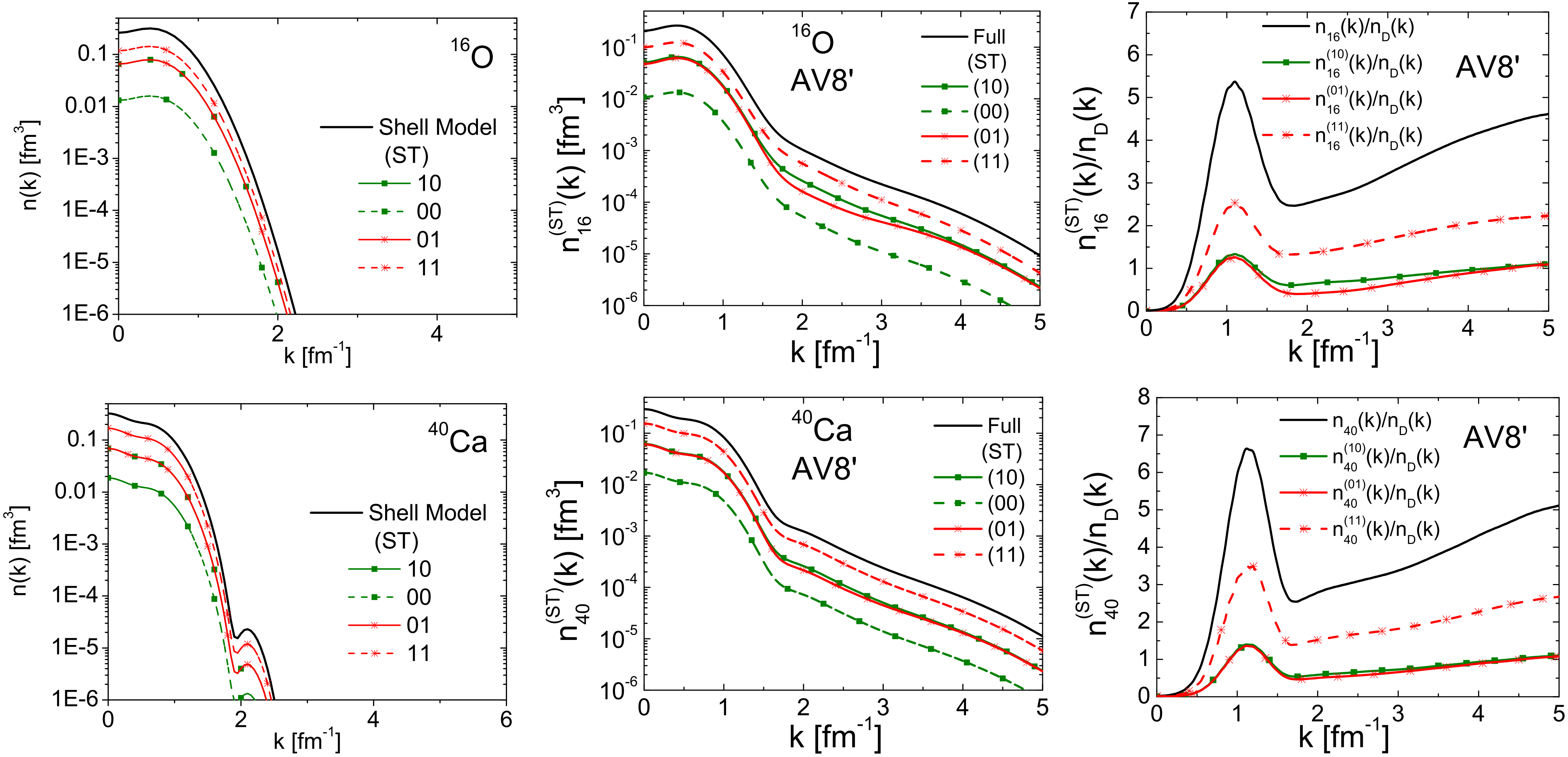
$\rho^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2)$ *diagrams* $\longrightarrow$

0. One-Body Mom distrs: Few- and Many-Body nuclei



M. Alvioli et al., ***PRC87*** (2013); ***IntJModPhys E22*** (2013)

0. One-Body Mom distrs: Many-Body nuclei



M. Alvioli et al., ***PRC87*** (2013); ***IntJModPhys E22*** (2013)

0. One-Body Mom distrs: ST pairs contribution

M. ALVIOLI *et al.*

TABLE III. The number of pairs $N^A_{(ST)}$, Eq. (9) in various spin-isospin states in the independent particle model (IPM) and taking into account SRCs within many-body theories with realistic interactions (in the approach of Ref. [46] pairs in relative $L = 0$ motion were identified as those prone to SRCs).

Nucleus		(ST)			
		(10)	(01)	(00)	(11)
^{2}H		1	—	—	—
^{3}He	IPM	1.50	1.50	—	—
	SRC (Present work)	1.488	1.360	0.013	0.139
	SRC [40]	1.50	1.350	0.01	0.14
	SRC [23]	1.489	1.361	0.011	0.139
^{4}He	IPM	3	3	—	—
	IPM (0*s* states) [46]	3	3	—	—
	SRC (Present work)	2.99	2.57	0.01	0.43
	SRC [40]	3.02	2.5	0.01	0.47
	SRC [23]	2.992	2.572	0.08	0.428
^{16}O	IPM	30	30	6	54
	IPM (0*s* states) [46]	20	18	—	—
	SRC (Present work)	29.8	27.5	6.075	56.7
	SRC [40]	30.05	28.4	6.05	55.5
^{40}Ca	IPM	165	165	45	405
	IPM (0*s* states) [46]	90	20	—	—
	SRC (Present work)	165.18	159.39	45.10	410.34

[23] Feldmeier, Horiuchi, Neff, Suzuki - PRC84 (2011) 054003

[40] Forest, Pandharipande, Pieper, Wiringa, Schiavilla, Arriaga PRC54 (1996) 646

[46] Vanhalst, Cosyn, Ryckebusch, PRC84 (2011) 031302

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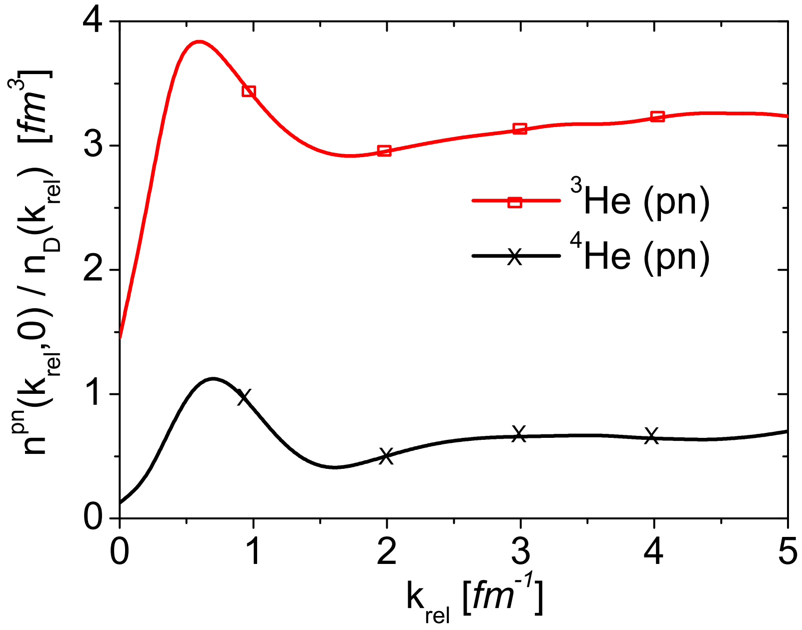
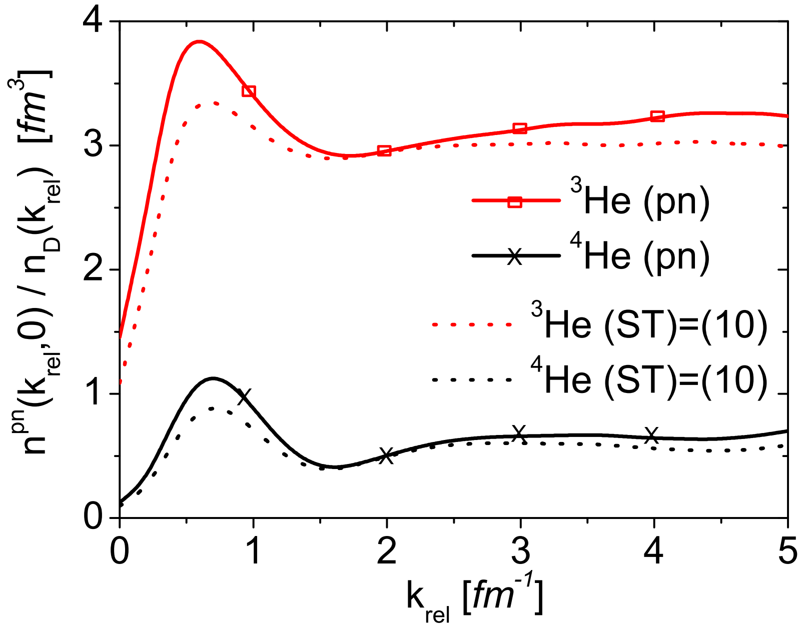
1. Two-Body Distributions: a closer look to deuteron scaling



M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev. C**85** (2012) 021001*

- Should a nucleus' $n^{pn}(k_{rel}, K_{CM} = 0)$ scale to ^2H's $n_D(k_{rel})$?

1. Two-Body Distributions: a closer look to deuteron scaling



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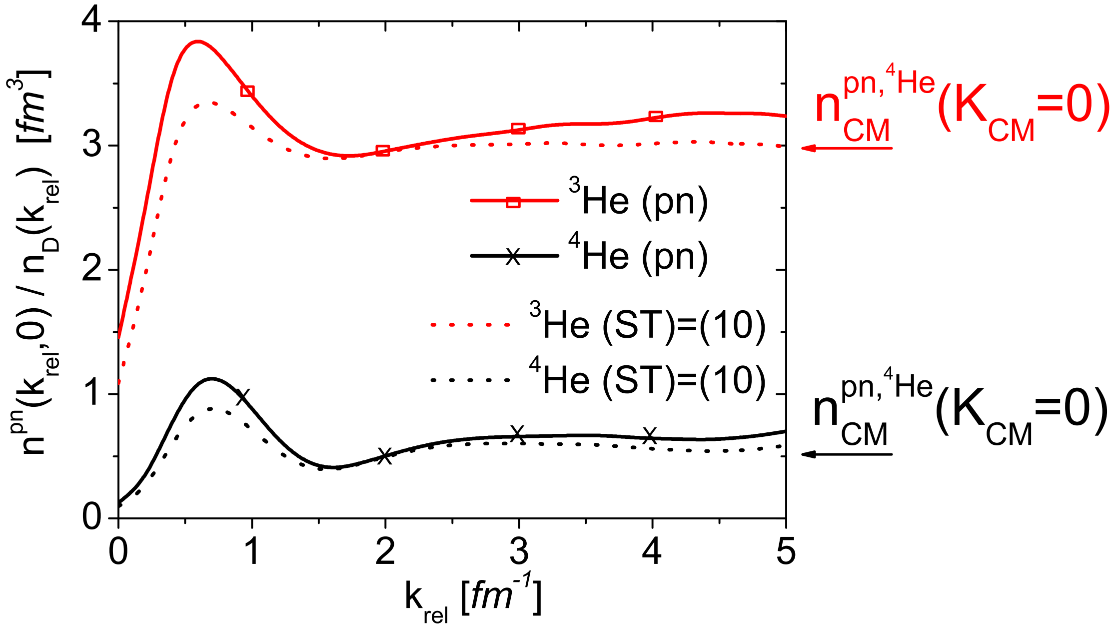
- Should a nucleus' $n^{pn}(k_{rel}, K_{CM} = 0)$ scale to ^2H's $n_D(k_{rel})$?
- Including only pairs with deuteron-like quantum numbers (ST)=(10) we find exact scaling!

1. Two-Body Distributions: a closer look to deuteron scaling



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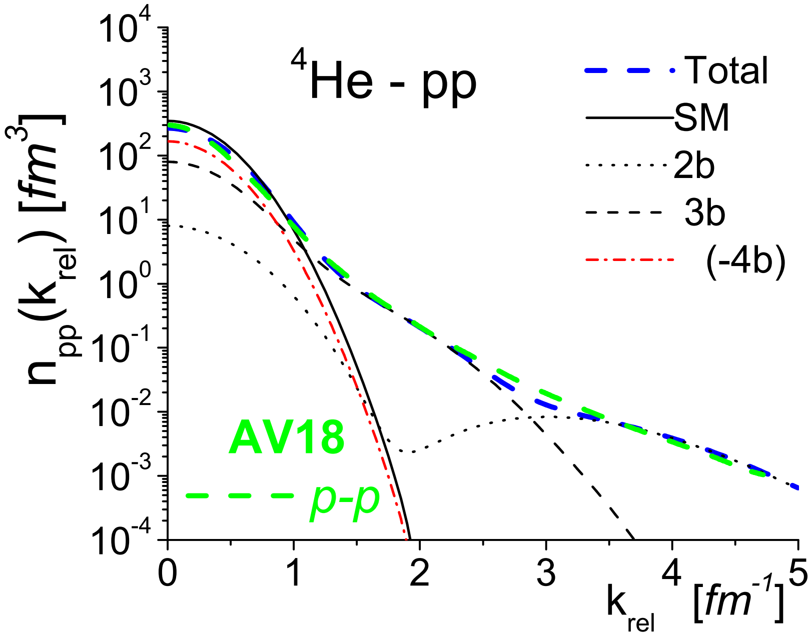
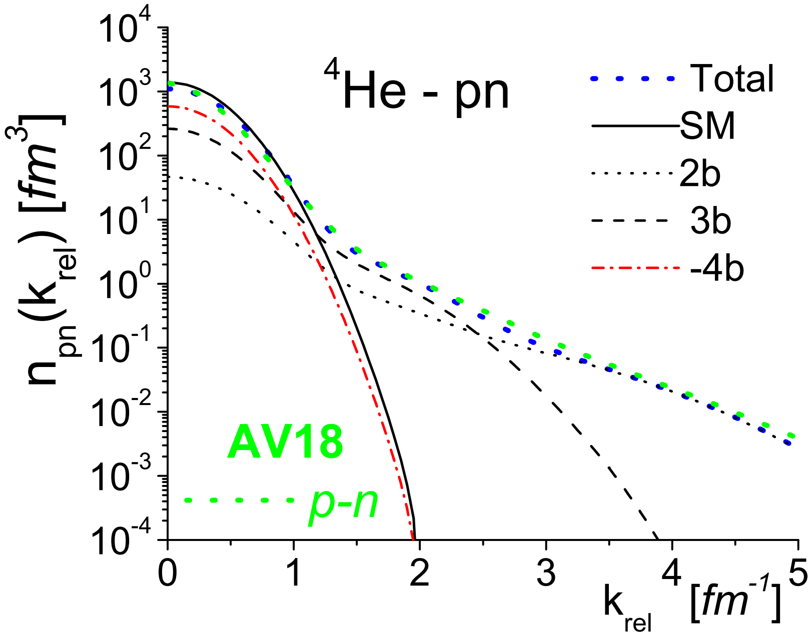
- Should a nucleus' $n^{pn}(k_{rel}, K_{CM}=0)$ scale to ^2H's $n_D(k_{rel})$?
- Including only pairs with deuteron-like quantum numbers (ST)=(10) we find exact scaling!
- $n(\boldsymbol{k}_{rel},0)/n^D(\boldsymbol{k}_{rel}) \simeq n^D(k_{rel})n_{CM}(0)/n^D(k_{rel}) = n_{CM}(K_{CM}=0)$!

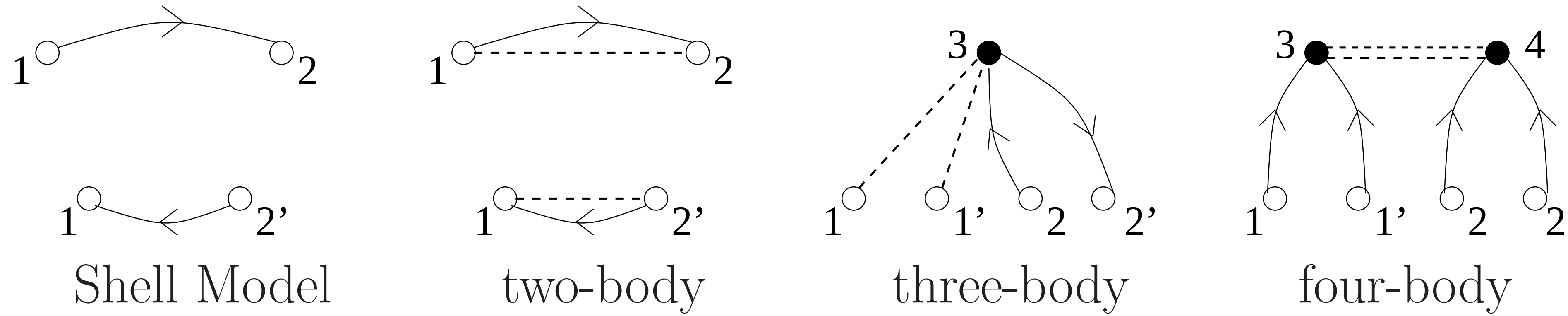
example: many-body contributions in ^{4}He 2BMD

$$n_{pN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\; n_{pN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$$





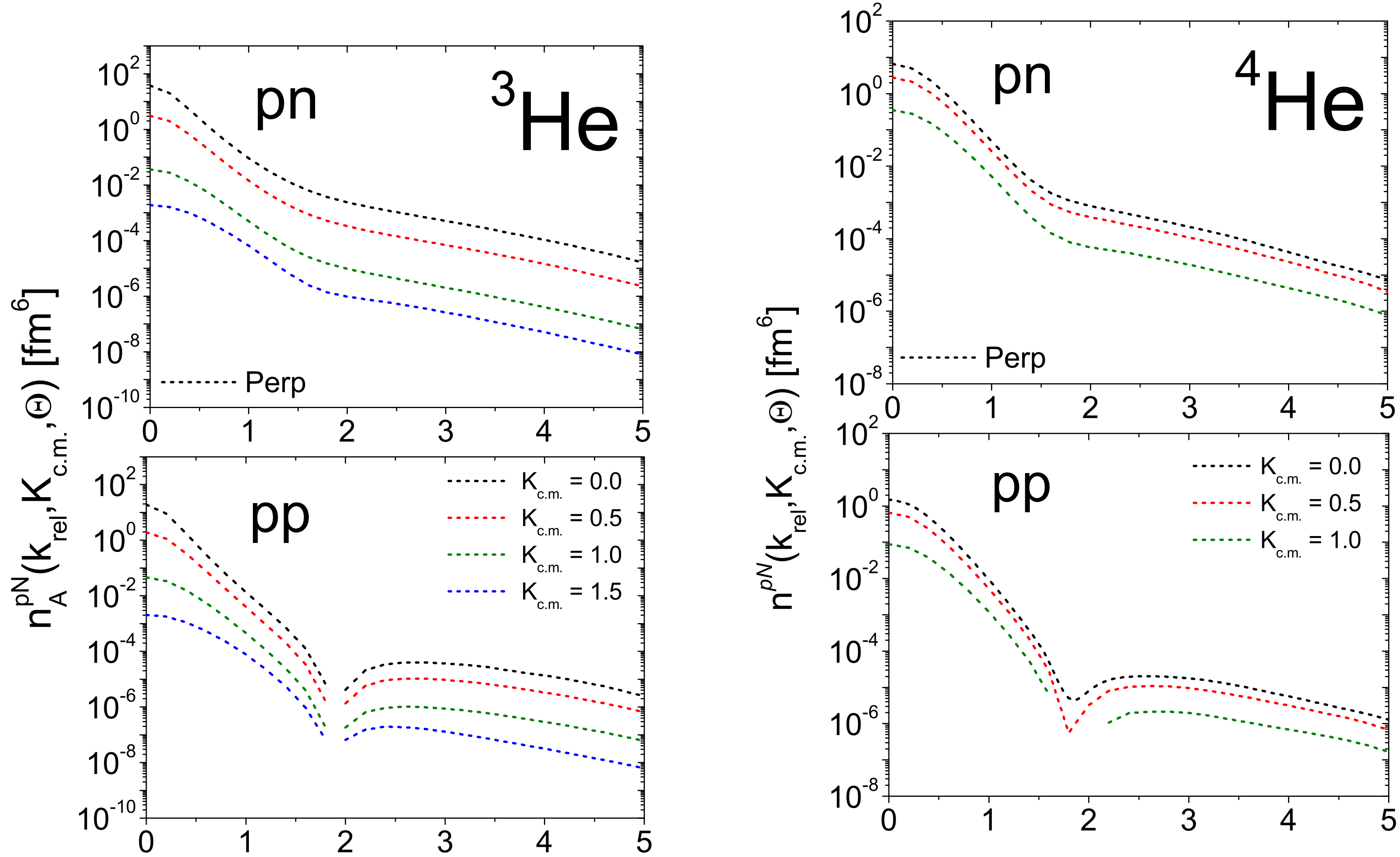
(**AV18**: *Schiavilla at al.* **PRL98** (2007))



1. Two-Body momentum Distributions of Few-Body Nuclei

Pisa
AV18
+UIX

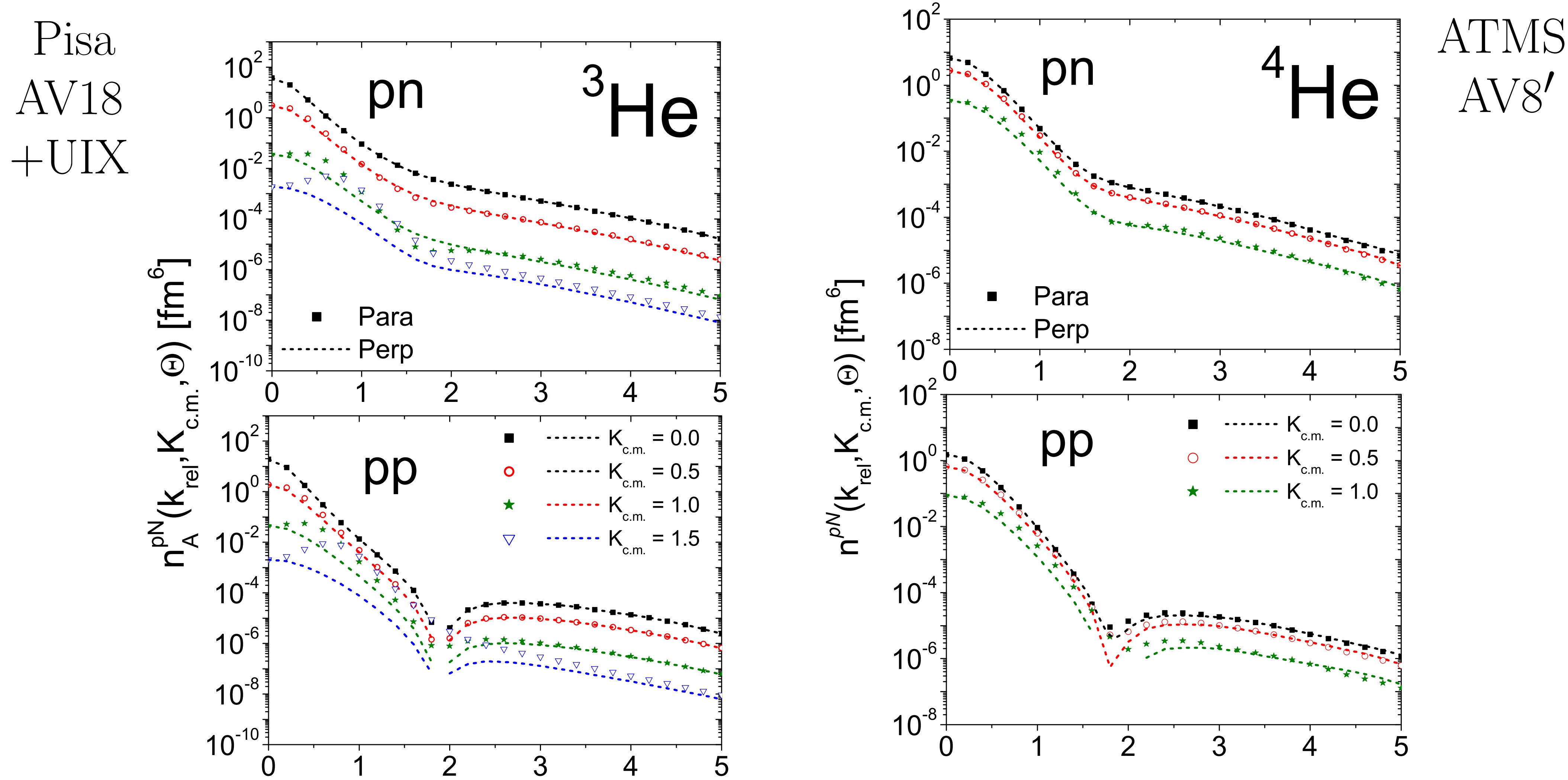
ATMS
AV8′



here $\boldsymbol{k}_{rel}$ is perpendicular to $\boldsymbol{K}_{CM}$

Alvioli, Ciofi, Kaptari, Mezzetti, Morita, Scopetta; *PRC**85** (2012)*
Alvioli, Ciofi, Morita; *Phys. Rev. C**94** (2016) 044309*

1. Two-Body momentum Distributions of Few-Body Nuclei

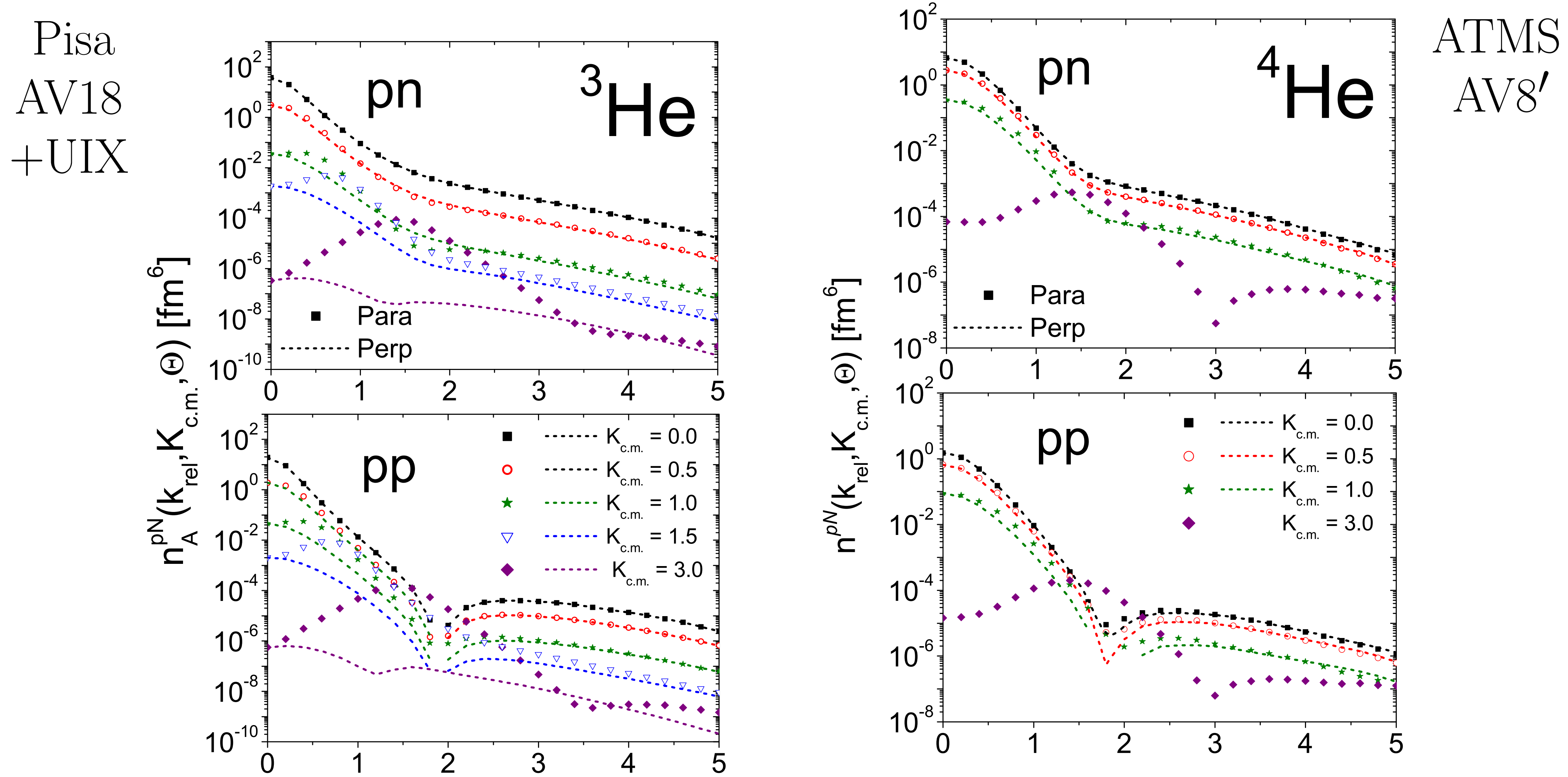


$n^{pN}(k_{rel}, K_{CM}, \Theta)$ is *angle independent* for **large** k_{rel} and **small** K_{CM}

Alvioli, Ciofi, Kaptari, Mezzetti, Morita, Scopetta; *PRC**85** (2012)*

Alvioli, Ciofi, Morita; *Phys. Rev. C**94** (2016) 044309*

1. Two-Body momentum Distributions of Few-Body Nuclei



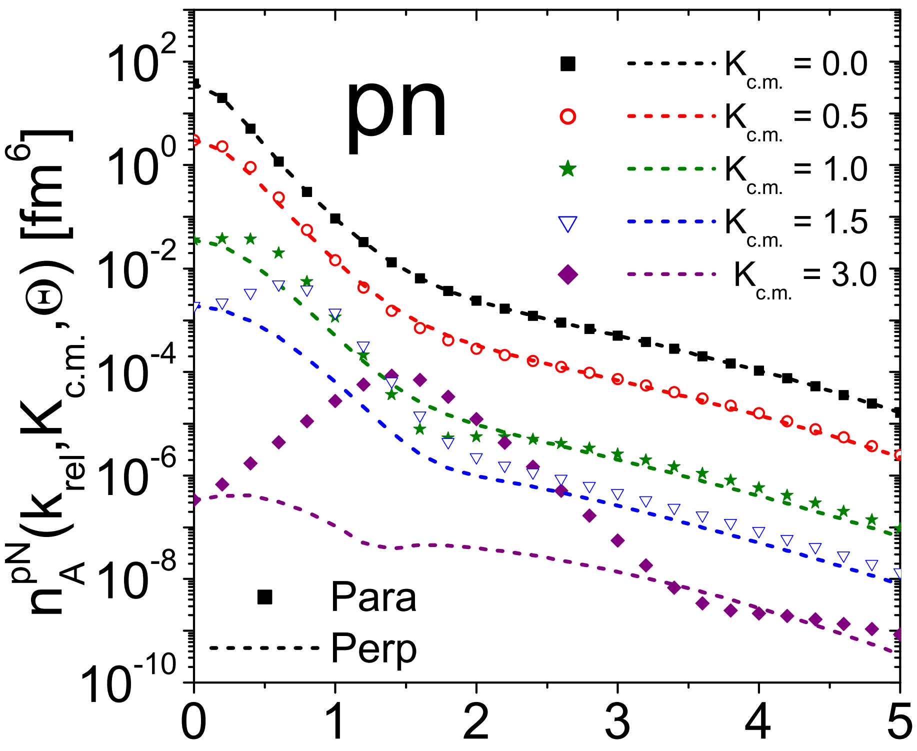
three-body correlations must be in the ***large*** K_{CM} *region*

Alvioli, Ciofi, Kaptari, Mezzetti, Morita, Scopetta; *PRC**85** (2012)*

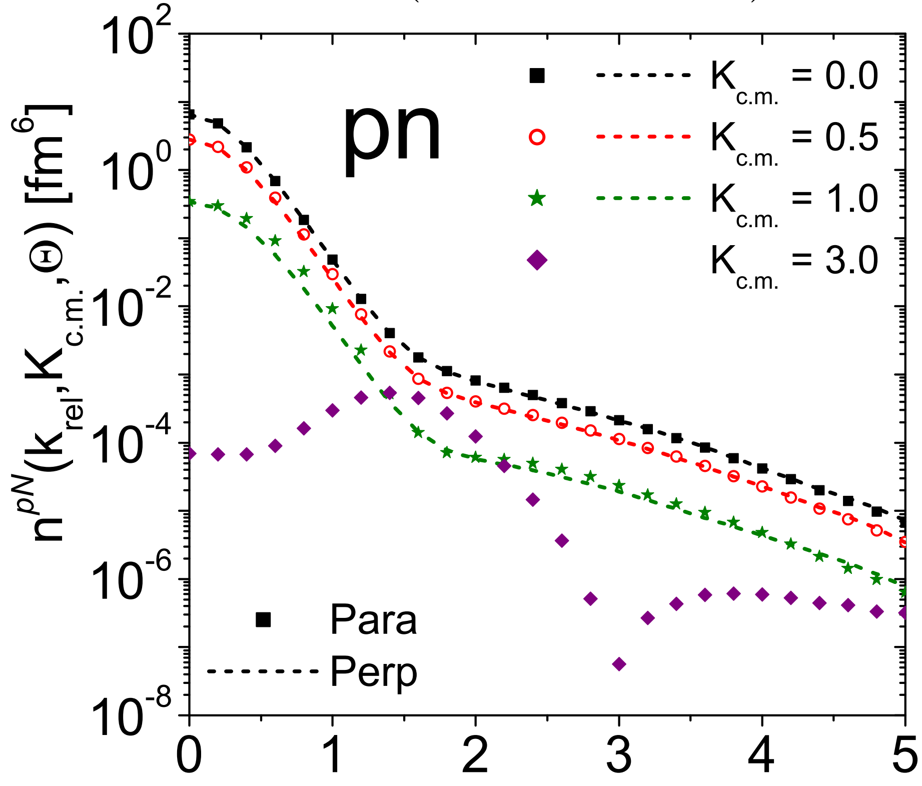
Alvioli, Ciofi, Morita; *Phys. Rev. C**94** (2016) 044309*

1. Two-Body momentum Distributions of Few-Body Nuclei

^{3}He (Pisa AV18+UIX)



^{4}He (ATMS AV8′)



for the pn case we see clear scaling to the deuteron n_D

Alvioli, Ciofi, Kaptari, Mezzetti, Morita, Scopetta; *PRC**85** (2012)*
Alvioli, Ciofi, Morita; *Phys. Rev. C**94** (2016) 044309*

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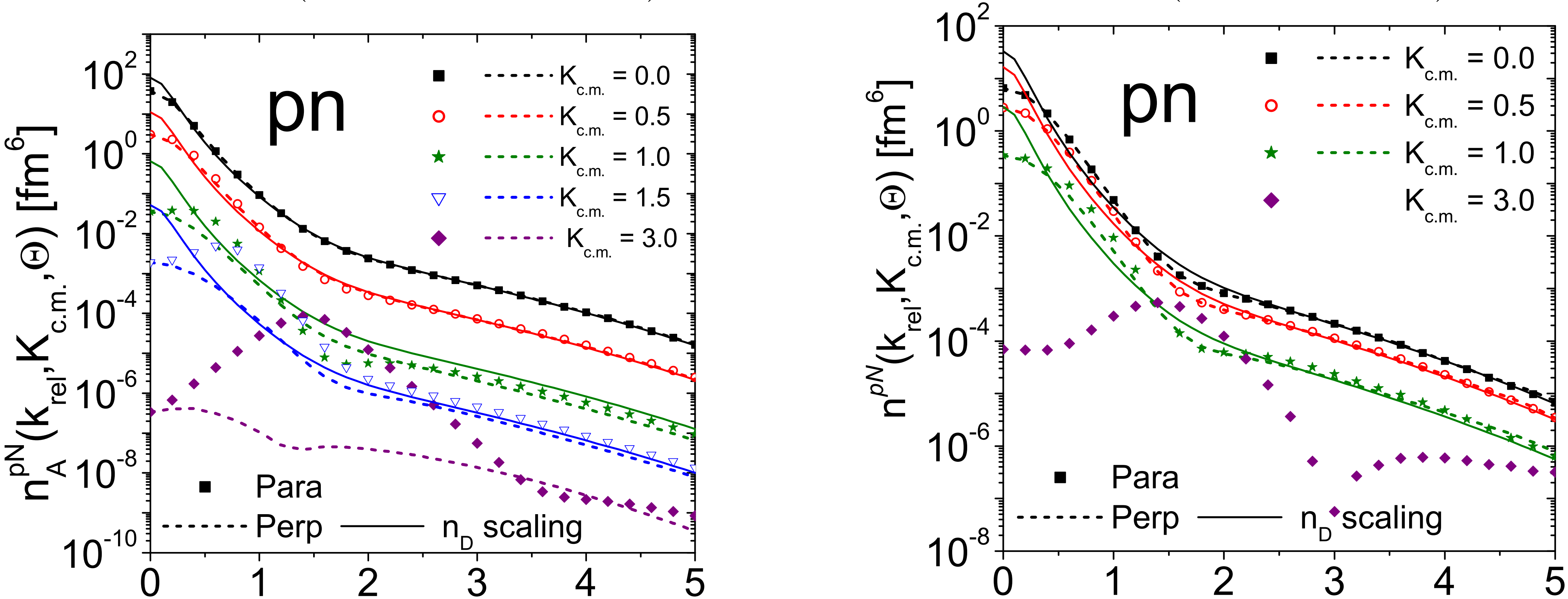

solid curves, the TNC model: rescaling of the deuteron by $n^A_{CM}(K_{CM})$!

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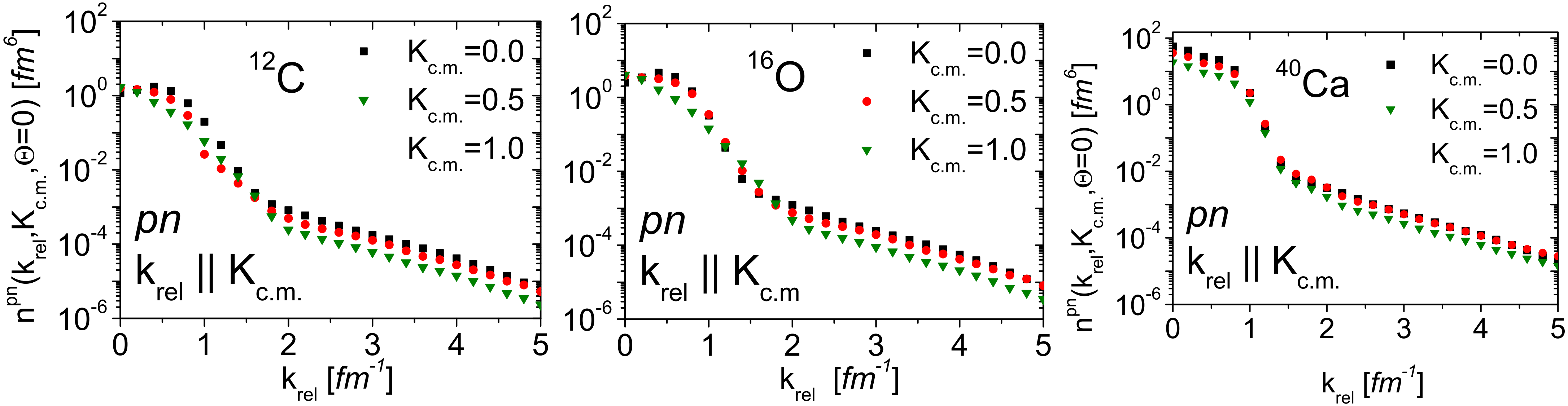
1. Two-Body momentum Distributions of Many-Body Nuclei

cluster expansion $\rho(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1, \boldsymbol{r}'_2), \Longrightarrow n^{pn}(k_{rel}, K_{CM}, \Theta)$



M. Alvioli, C.Ciofi degli Atti, H. Morita, *PRL* ***100*** *(2008) 162503*
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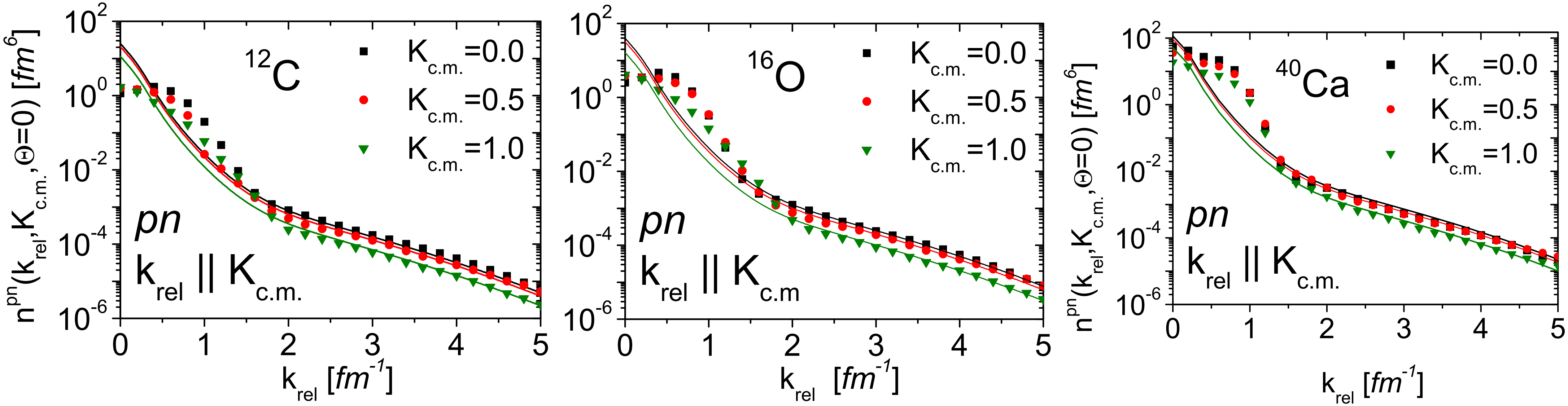

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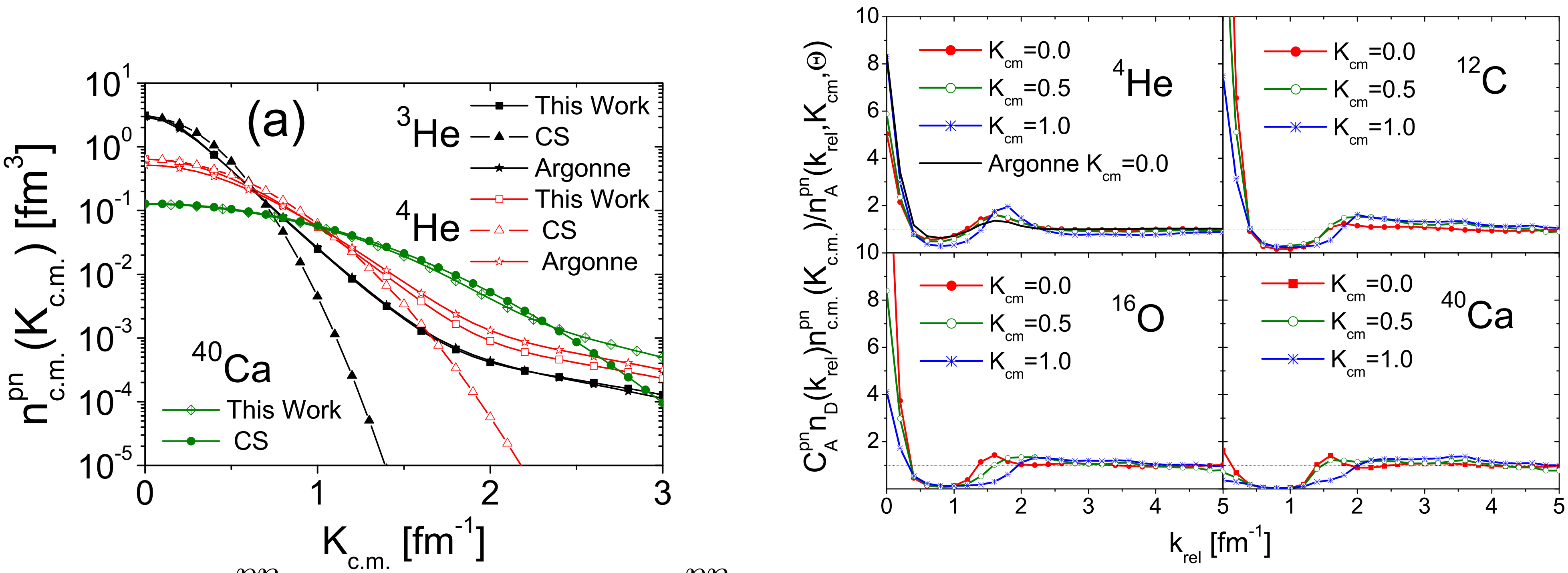
- solid curves are the deuteron scaled with the corresponding $n^{pn}_{c.m.}(K_{c.m.})$
- same behaviour & conclusions as in few-body ($K_{CM} > 1$ not shown)
- ***universality of NN correlations***
- we can *update* the TNC model with many-body quantities

1. Factorization in momemtum space for 3<A<40



$$
\begin{aligned}
n^{pn}_A(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) &= n^{pn}_A(k_{rel}, K_{c.m.}, \Theta) \\
&\simeq \frac{n^{pn}_A(k_{rel}, K_{c.m.}=0)}{n^{pn}_{c.m.}(K_{c.m.}=0)}\, n^{pn}_{c.m.}(K_{c.m.}) \\
&\simeq C^{pn}_A\, n_D(k_{rel})\, n^{pn}_{c.m.}(K_{c.m.}),
\end{aligned}
$$

$k_{rel} \gtrsim k^{-}_{rel} = 1.5\ fm^{-1} + K_{c.m.}$ and $K_{c.m.} \lesssim 1.0 - 1.5\, fm^{-1}$

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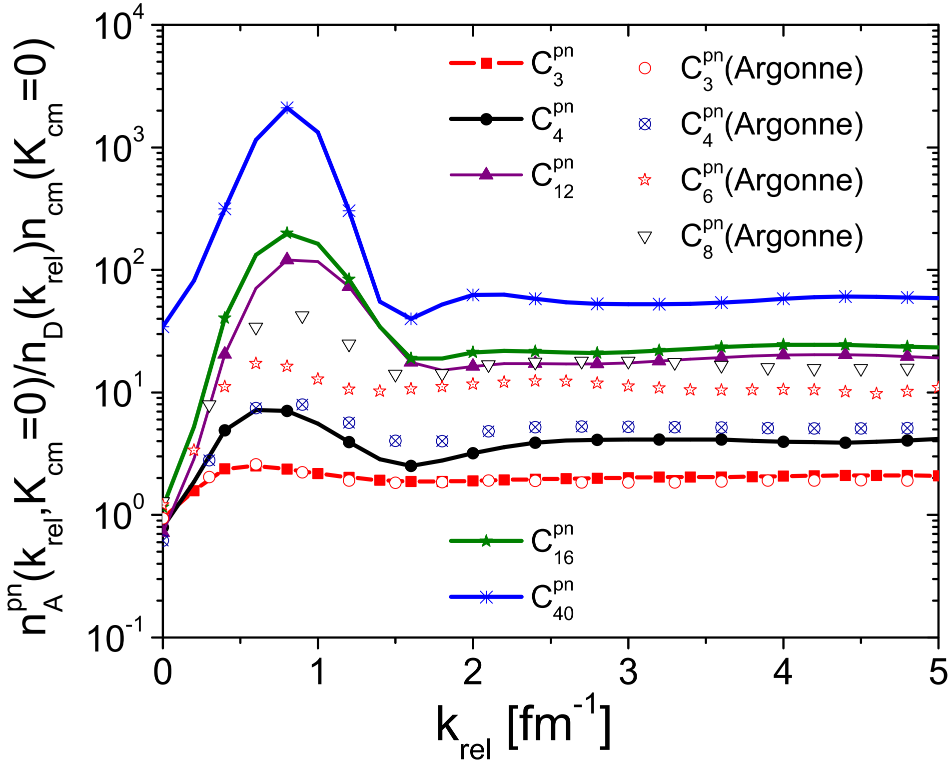
1. Scaling coefficients to the deuteron distribution n_D



$$\lim_{k_{rel}\to k_{rel}^-} \frac{n_A^{pn}(k_{rel}, K_{c.m.}=0)}{n_D(k_{rel})\, n_{c.m.}^{pn}(K_{c.m.}=0)} = C_A^{pn}$$

C_A^{pn} are completely specified by many-body calculation, only depend on NN potential, approximations, ..

	^{2}H	^{3}He	^{4}He	^{6}Li	^{8}Be	^{12}C	^{16}O	^{40}Ca
VMC	1.0	2.0 ± 0.1	4.0 ± 0.1	–	–	20 ± 1.6	24 ± 1.8	60 ± 4.0
PRC94	1.0	(2.0 ± 0.1)	(5.0 ± 0.1)	(11.1 ± 1.3)	(16.5 ± 1.5)	(–)	(–)	(–)

C_A^{pn} is a measure of the *number of BB deuteron-like pairs*

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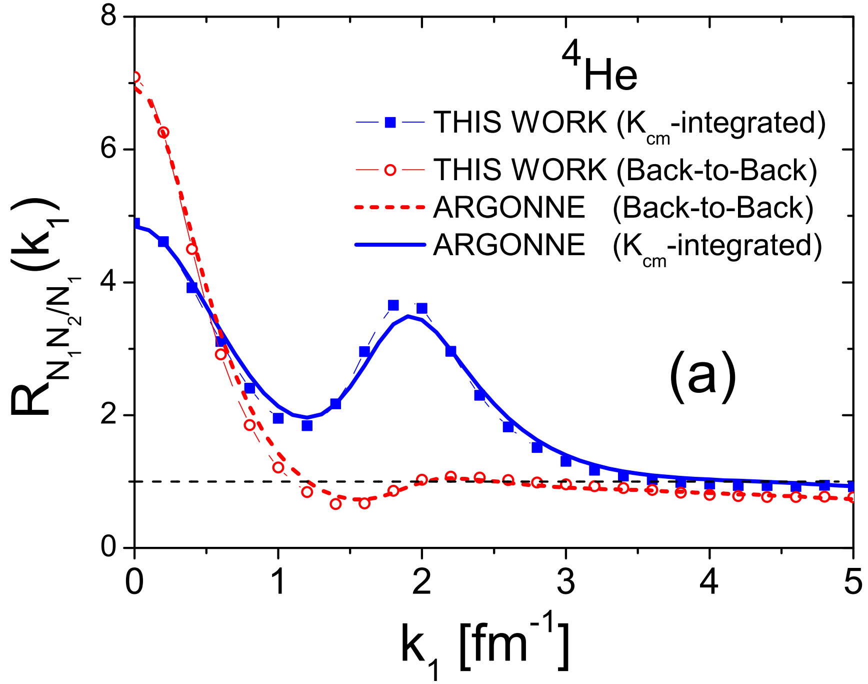
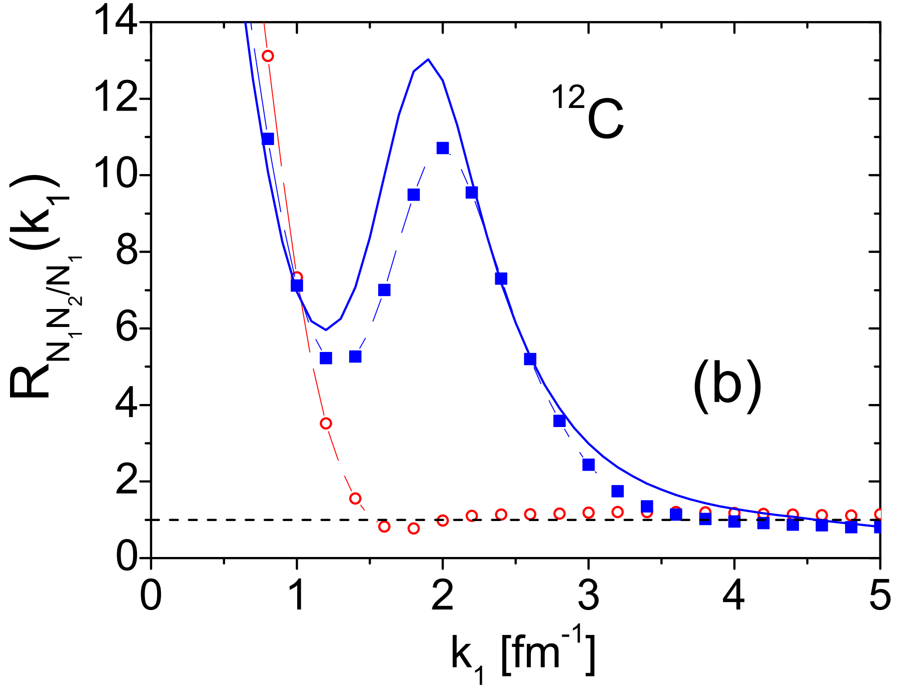
1. One- and two-body momentum distributions relationship





$$R^{BB}_{N_1N_2/N_1}(k_1) = \frac{1}{n^p_A(k_1)}\left[\frac{n^{pn}_A(k_{rel}=k_1, K_{\rm c.m.}=0)}{n^{pn}_{\rm c.m.}(K_{\rm c.m.=0})} + 2\frac{n^{pp}_A(k_{rel}=k_1, K_{\rm c.m.}=0)}{n^{pp}_{\rm c.m.}(K_{\rm c.m.=0})}\right]$$

$$R^{int}_{N_1N_2/N_1}(k_1) = \frac{n^{pn}_A(k_{rel}=k_1) + 2n^{pp}_A(k_{rel}=k_1)}{n^p_A(k_1)}$$

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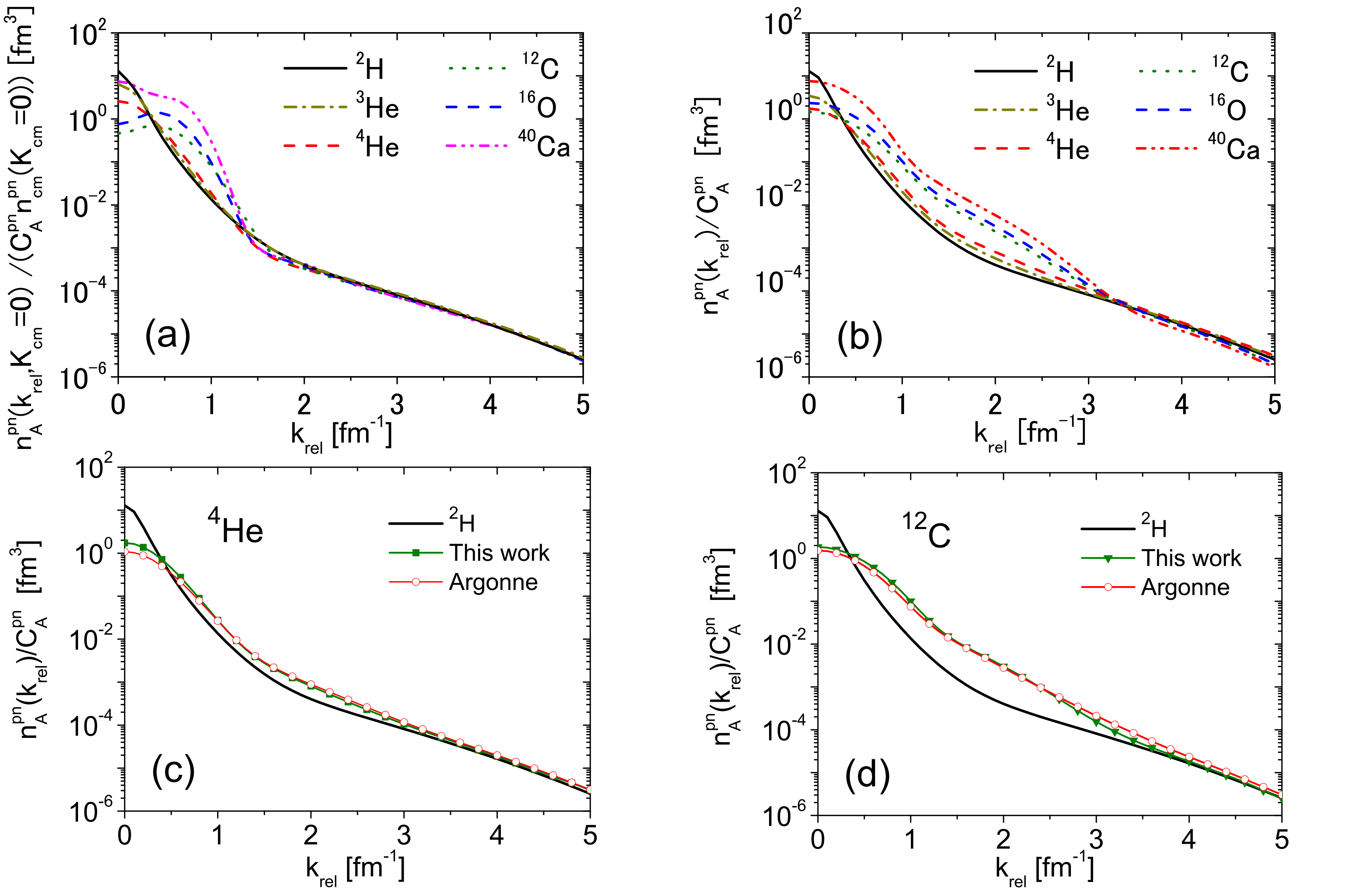
1. Relationship between $K_{c.m.}$-integrated and $K_{c.m.} = 0$



M. Alvioli, C.Ciofi degli Atti, H. Morita; *Phys. Rev. C**94** (2016) 044309*

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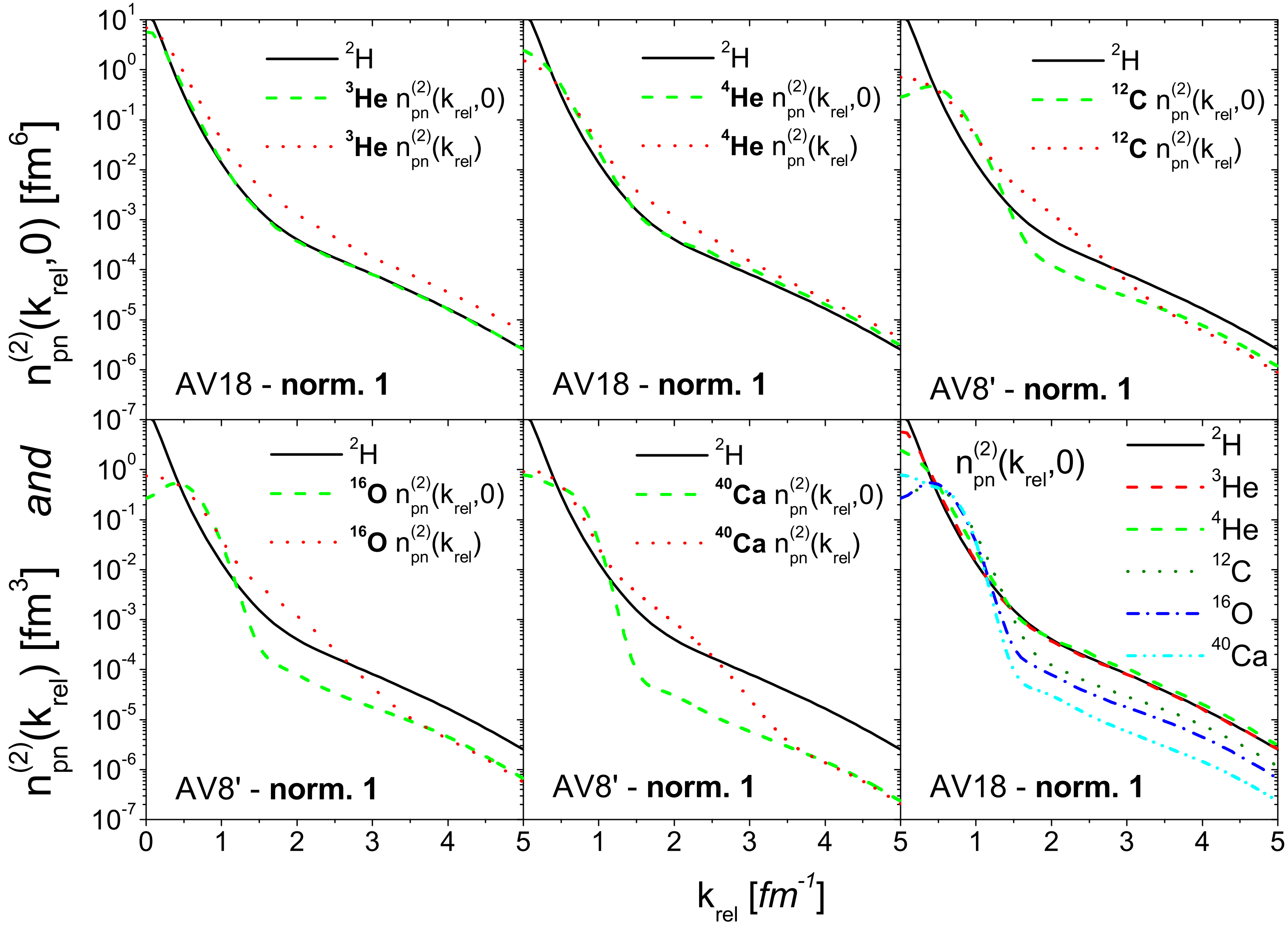
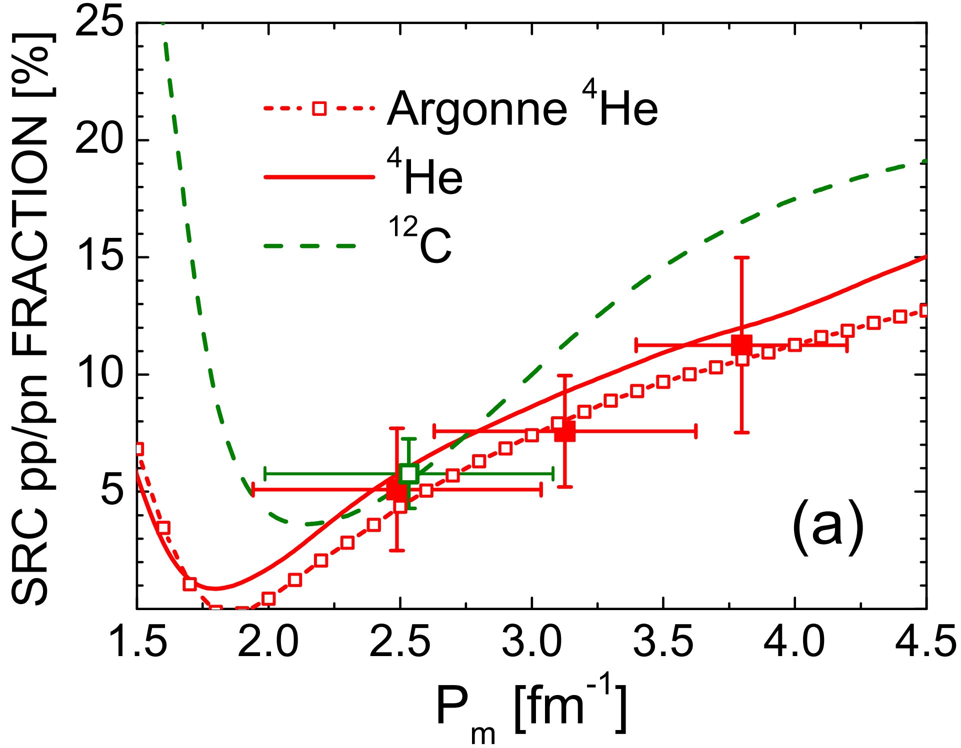


2. Two-body mom distrs results - comparison with data



$$n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0)\,/\,n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0)$$

Alvioli, Ciofi, Morita; *Phys. Rev. C**94** (2016) 044309*

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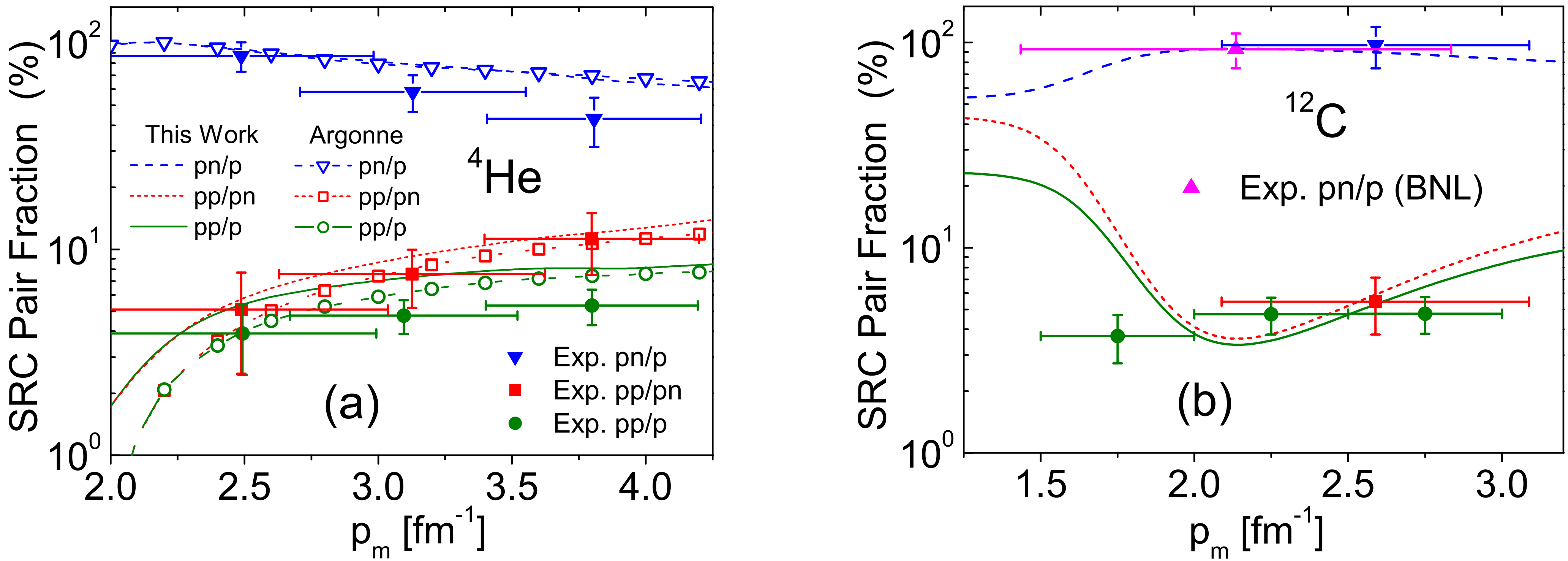

$$n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0) \,/\, n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0)$$

$$n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0) \,/\, \left(n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0) + 2\, n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0)\right)$$

$$n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0) \,/\, \left(n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0) + 2\, n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0)\right)$$

Alvioli, Ciofi, Morita; *Phys. Rev. C* ***94*** *(2016) 044309*

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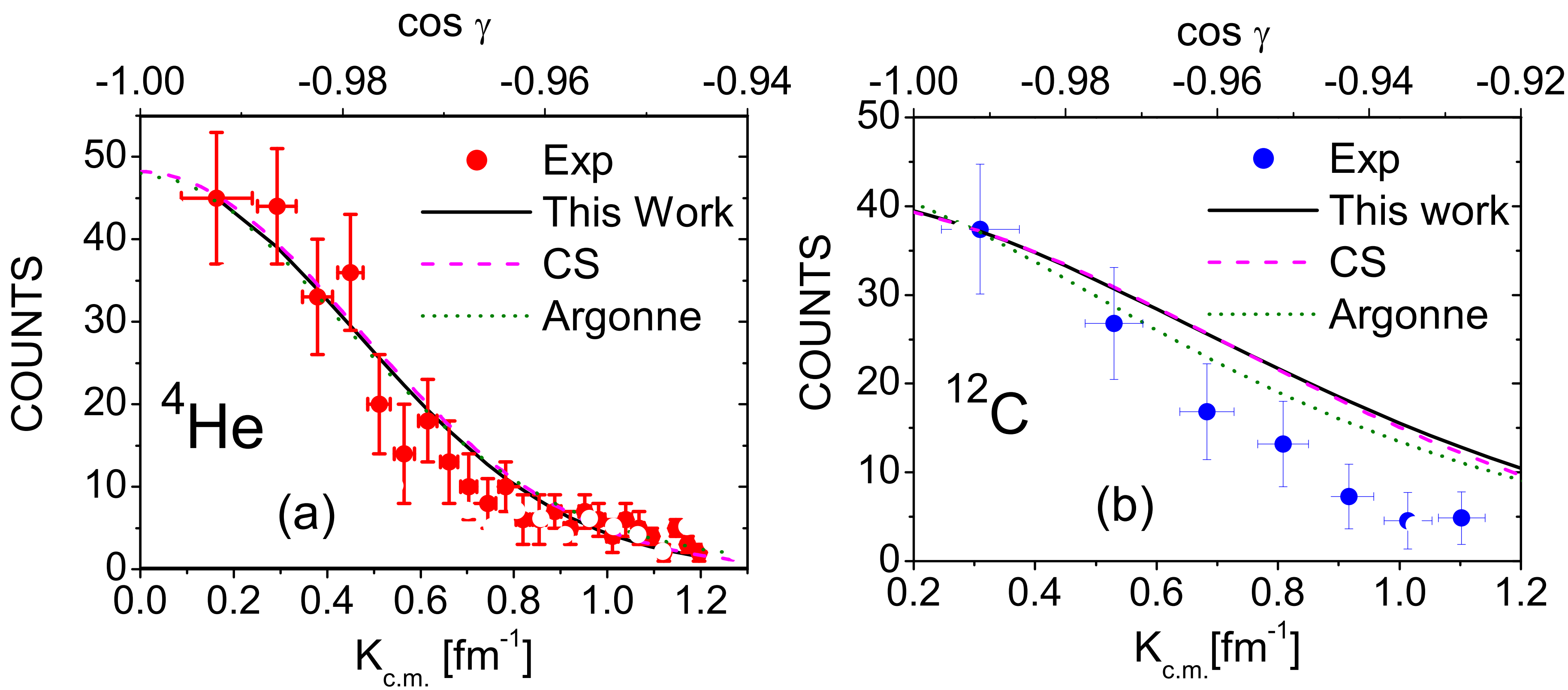

$n^{pN}_{c.m.}(K_{CM})$ (k_{rel}-integrated)
and normalized to the first data point

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Thank you