White-light solitons

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Optical spatial solitons made from incoherent white light were experimentally observed in 1997 by Mitchell and Segev [Nature (London) **387,** 880 (1997)]. We present what is believed to be the first theory describing these solitons and find the characteristic features of their spatiotemporal coherence properties and their temporal power spectrum. © 2003 Optical Society of America

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The phenomenon of spatial solitons can be understood as a dynamic balance between two opposing tendencies, namely, the tendency for the beam to expand as a result of diffraction and the tendency for the beam to contract because of self-focusing.[1] This remarkable phenomenon has been studied in the context of nonlinear optics for a number of years. However, before the experiment of Mitchell *et al.*,[2] optical solitons were believed to be solely coherent entities. The experiment reported in Ref. 2 demonstrated solitons made from partially spatially incoherent yet quasi-monochromatic (temporally coherent) light; the light source was laser light passed through a rotating diffuser. One year later, Mitchell and Segev demonstrated solitons made from both spatially and temporally incoherent light; the light source was an incandescent light bulb.[3] These experimental results initiated a series of theoretical studies of incoherent solitons.[4–9] However, all these theories have considered only solitons made from spatially incoherent yet temporally coherent (quasi-monochromatic) light, like those observed in the study reported in Ref. 2. Hence, these theories cannot describe solitons made from incoherent white light (such as those reported in Ref. 3) and cannot describe their spatiotemporal coherence properties and features of their temporal spectral density.

Here we present what is believed to be the first theoretical (numerical) study of solitons made from temporally and spatially incoherent light, that is, white-light solitons. The evolution dynamics reveals that the spatiotemporal coherence properties of the input light beam change in a characteristic fashion and self-adjust to form a soliton. We identify the characteristic features of the temporal power spectrum and the spatiotemporal coherence properties of white-light solitons. Specifically, the spatial intensity profile of light within the bandwidth $[\omega, \omega + d\omega]$ is wider (less localized) at lower frequencies and narrower at higher frequencies. Furthermore, the spatial correlation distance (across the soliton) is always larger for lower frequencies and shorter for higher frequencies. We study white-light solitons in two generic types of nonlinearity, the saturable nonlinearity[3] and the Kerr nonlinearity,[10] when both have a noninstaneous response.

The light source used to construct white-light solitons generates spatially and temporally incoherent continuous-wave (CW) light.[3] Because of the noninstantaneous nature of the medium, the induced nonlinear index of refraction is unable to follow fast phase fluctuations of incoherent light but responds only to the time-averaged intensity, I, which is independent of time, $\partial I/\partial t = 0$; the time average is taken over the response time of material. The dynamic equation(s) describe the evolution of time-averaged intensity, and the coherence properties of light, along the propagation axes. One theoretical approach to this problem is through the evolution equation derived from the coherent density theory,[4] extended to include temporal incoherence[11]:

$$i\left(\frac{\partial f^\omega}{\partial z} + \theta\,\frac{\partial f^\omega}{\partial x}\right) + \frac{1}{2k_\omega}\,\frac{\partial^2 f^\omega}{\partial x^2} + \frac{k_\omega}{n_0}\,\delta n(I) f^\omega(x, z, \theta) = 0\,. \tag{1}$$

Here $f^\omega(x, z, \theta)$ represents the coherent density for each frequency constituent of a beam; $k_\omega = n_0\omega/c$, and θ represents an angle with respect to the z axes.[4] The spatiotemporal coherence properties of light can be described in terms of the mutual spectral density evaluated at a given transverse plane of the beam[11,12]: $B_\omega(x_1, x_2, z) = \int_{-\infty}^{\infty} d\theta \exp[ik_\omega(x_1 - x_2)]f^\omega(x_1, z, \theta)f^\omega(x_2, z, \theta)^*$. Equation (1) is derived to be equivalent[13] to the evolution of mutual spectral density formulated in Ref. 11. In deriving Eq. (1) we assume that the medium is dispersionless. Since the coupling term $\delta n(I)$ is independent of time, $\partial \delta n(I)/\partial t = 0$, dispersion can be included by substitution of n_0 with $n_0(\omega)$. In this Letter we focus on the collective self-focusing effect of the white-light beam and neglect the effect of dispersion. Since the light is CW ($\partial I/\partial t = 0$), dispersion is negligible if $n_0(\omega)$ does not vary significantly over the frequency span.

First, we seek white-light solitons in saturable self-focusing media, $\delta n(I) = -\alpha(1 + I)^{-1}$, where $I(x, z) = \int_0^\infty \mathrm{d}\omega \int_{-\infty}^\infty \mathrm{d}\theta |f^\omega(x, z, \theta)|^2$ denotes the time-averaged intensity expressed in units of the dark irradiance of the crystal.[4] For the photorefractive screening nonlinearity $\alpha = -0.5 n_0^3 r_{33} V/D$.[14] In our numerical experiments, we have taken realistic parameter values from Ref. 3; $n_0 = 2.3$ is the extraordinary refractive index, $r_{33} = 1022$ pm V^{-1} is the electro-optic coefficient, $V = 550$ V is the voltage applied on the crystal between electrodes separated by $D = 6$ mm. To find white-light solitons for the photorefractive screening nonlinearity we launch a beam with $f^\omega(x, z = 0, \theta) = \sqrt{\hat{r}\omega_0^{-1}\Delta^{-1}}\exp[-\theta^2/(2\theta_0^2)] \times \exp[-x^2/(2w^2)]$, where $\hat{r} = 3.32$ ($\hat{r}$ determines the ratio between the peak intensity of the soliton and the dark irradiance), $\theta_0 = 0.55°$, and $w = 7.74$ μm. The spectral density is rectangular within the interval $[\omega_{\min}, \omega_{\max}] = [\omega_0(1 - \Delta/2), \omega_0(1 + \Delta/2)]$, where the average frequency $\omega_0 = 3.86 \times 10^{15}$ Hz and the frequency bandwidth is $\Delta = 20\%$.

The propagation of the total-intensity profile of this beam is shown in Fig. 1(a) for $\simeq$38 diffraction lengths. (The diffraction length, $d \simeq 0.78$ mm, is calculated numerically; the nonlinearity is turned off, and d is the distance at which the width of the input Gaussian beam increases by a factor of $\sqrt{2}$.) During the formulation of the white-light soliton, the spatiotemporal coherence properties acquire characteristic properties. We observe the evolution of the spatial intensity profile, $I_\omega(x, z) = \int_{-\infty}^\infty \mathrm{d}\theta |f^\omega(x, z, \theta)|^2$, and the complex coherence factor $\mu_\omega(x_1, x_2, z) = B_\omega(x_1, x_2, z)/[B_\omega(x_1, x_1, z)B_\omega(x_2, x_2, z)]^{1/2}$ at a particular frequency ω. The quantity $I_\omega(x, z)$ provides information on the temporal spectral density, and $\mu_\omega(x_1, x_2, z)$ describes the spatial coherence properties of each frequency constituent of a beam.[12] By observing the quantities $I(x, z)$, $I_\omega(x, z)$, and $\mu_\omega(x_1, x_2, z)$ during propagation, we find that they self-adjust after a propagation distance of $\sim$10d and remain unchanged afterward. We conclude that the input has converged into a white-light soliton, a beam whose intensity profile and spatiotemporal properties exhibit stationary propagation in z. The quantities $I_\omega(x)$ and $\mu_\omega(x, 0)$ that are characteristic of a white-light soliton are shown in Figs. 2(a) and 2(b) at three representative frequencies, $\omega_{\min}$, ω_0, and $\omega_{\max}$; the functions are calculated at $z \simeq 38d$. We observe that the spatial correlation distance is larger for lower frequencies and smaller for higher frequencies, as can be seen from the width of $\mu_\omega(x, 0)$; the plot of $I_\omega(x)$ shows that the spatial intensity profile is slightly wider (less localized) at lower frequencies, and narrower, with a higher peak, at higher frequencies [see the insert in Fig. 2(b)]. The same feature is observed in the Kerr medium but is much more pronounced, as is discussed below.

The simulations of the evolution of white-light solitons in a Kerr medium are based on the modal theory,[5] extended to include broad spectral density:

$$\frac{\partial^2 u_m{}^\omega}{\partial x^2} + 2ik_\omega \frac{\partial u_m{}^\omega}{\partial z} + \frac{2\delta n(I) k_\omega{}^2}{n_0} u_m{}^\omega(x, z) = 0\,. \quad (2)$$

Here, $u_m{}^\omega(x, z)$ is the profile of the mth mode at frequency ω ($k_\omega = n_0\omega/c$). The mutual spectral density is $B_\omega(x_1, x_2, z) = \sum_m \lambda_m{}^\omega u_m{}^\omega(x_1, z) u_m{}^\omega(x_2, z)^*$, where $\lambda_m{}^\omega$ denotes the time-averaged modal weights[5]; Eq. (2) is equivalent to the evolution of mutual spectral density.[11,13] The Kerr medium responds as $2n_0\delta_n(I) = n_2 I$, where $I(x, z) = \int_0^\infty \mathrm{d}\omega \sum_m \lambda_m{}^\omega |u_m{}^\omega(x, z)|^2$ is the time-averaged intensity.

We seek a white-light soliton by launching an optical field with the following parameters: The total-intensity profile of the input beam is $I_0\,\mathrm{sech}^2(x/w)$, with a characteristic width $w = 9.51$ μm and height defined by $n_2 I_0 = 4 \times 10^{-4}$. The spectral density is rectangular within the same interval $[\omega_{\min}, \omega_{\max}]$ as for the saturable nonlinearity. At $z = 0$, we set $u_1{}^\omega = \sqrt{I_0\omega_0{}^{-1}\Delta^{-1}}\,\mathrm{sech}^2(x/w)$, $u_2{}^\omega = \sqrt{I_0\omega_0{}^{-1}\Delta^{-1}}\,\mathrm{sech}(x/w)\tanh(x/w)$, $\lambda_1{}^\omega = \lambda_2{}^\omega = 1$. These parameters are chosen such that in the limit $\Delta \to 0$ this input converges to the two-mode spatially incoherent, quasi-monochromatic soliton.[10] However, since the power spectrum is broad, this input evolves, and we observe whether it self-traps. Figure 1(b)

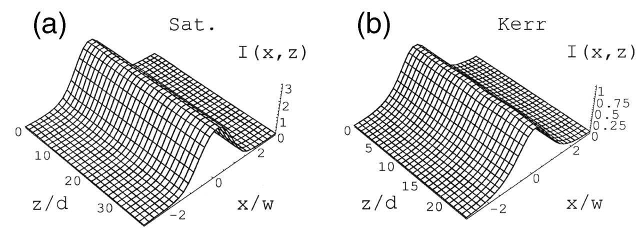

Fig. 1. Evolution of the intensity structure of white-light solitons in (a) saturable and (b) Kerr nonlinearity. The propagation distance, z, is expressed in diffraction length units, and spatial coordinate x is in characteristic width units. In the saturable case, the parameters are taken from Ref. 3.

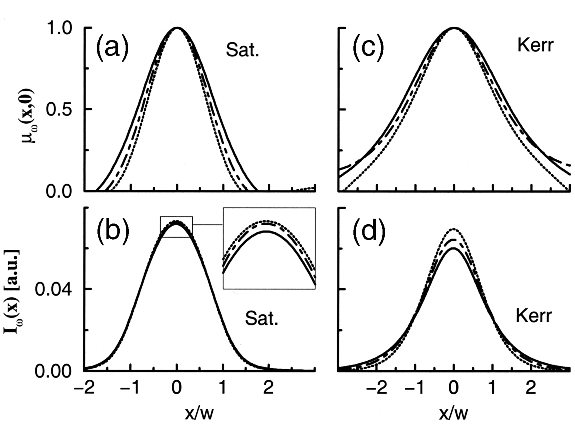

Fig. 2. Complex coherence factor $\mu_\omega(x, 0)$ and the intensity profile $I_\omega(x)$ of a white-light soliton at three representative frequencies: $\omega_{\min}$ (solid curve), ω_0 (dotted–dashed curve), and $\omega_{\max}$ (dotted curve).

displays the evolution of the total-intensity profile of the beam from $z = 0$ to $z \simeq 24\ d$ of propagation; in this case, the diffraction length is $d \simeq 1.1$ mm. The total-intensity profile changes only slightly throughout propagation. Because of the small oscillation of the peak of the beam [see Fig. 1(b)], we refer to this self-trapped and stable beam as a white-light quasi-soliton. The power is not radiated from this beam during propagation.

Although the total intensity profile is practically unchanged during evolution, the spatiotemporal statistical properties change more significantly again in a characteristic fashion. Our choice for the input beam corresponds to having identical $I_\omega(x, z = 0)$ and $\mu_\omega(x_1, x_2, z = 0)$ for all frequencies. However, these quantities evolve differently for different frequencies, and we observe a shift of higher (lower) frequency constituents toward the peak (tails) of the self-trapped beam. Similarly to the saturable case, after a few diffraction lengths the spatiotemporal coherence properties have self-adjusted and do not change significantly afterward. Figures 2(c) and 2(d) show the complex coherence factor $\mu_\omega(x, 0)$ and the intensity profile $I_\omega(x)$ at the frequencies $\omega_{\min}$, ω_0, and $\omega_{\max}$; the functions are calculated at $z \simeq 24d$. We observe that the spatial intensity profile is narrower at higher frequencies. The spatial correlation distance is shorter for higher frequencies, as can be seen from the width of $\mu_\omega(x, 0)$.

For numerical simulations, the coordinates x, θ, and ω are represented on uniform grids consisting of 2048, 256, and 41 points, respectively; the points are uniformly spaced in the intervals $\sim[-30w, 30w]$, $[-2.5\theta_0, 2.5\theta_0]$, and $[\omega_{\min}, \omega_{\max}]$, respectively. For the integration, we use the standard split-step Fourier technique. To check the consistency of numerical simulation, we check the conservation of total power, the conservation of power within each frequency constituent of the beam, and the conservation of transverse momentum.[4,15]

The results of our simulations for both the Kerr and the saturable cases can be interpreted as follows: The white-light incoherent soliton is fundamentally a collective phenomenon, as all frequencies contribute to its formation. That is, the total intensity of the beam induces (via the nonlinearity) a multimode waveguide (induced potential) that, at the same time, guides the light at the various frequencies by populating the modes (bound states) of the waveguide. The characteristic width of the soliton is, say, w. However, every frequency constituent sees this width in terms of its own characteristic length scale–the wavelength. Hence, the induced waveguide is effectively broader (has a larger numerical aperture) at the higher frequencies (shorter wavelengths). Consequently, a larger amount of intensity at higher frequencies is guided inside the waveguide, i.e., closer to the soliton peak, whereas the intensity at smaller frequencies has a smaller confinement factor, i.e., spreads more away from the center of the waveguide. These results explain the behavior of $I_\omega(x)$. Let us now explain the behavior of the complex coherence factor. A spatially incoherent soliton occurs when incoherent diffraction (diffusion) is balanced by nonlinearity.[9] White-light solitons are made up of many wavelengths; however, they are all trapped within the same waveguide and have approximately the same incoherent diffraction angle, $\theta_0 \propto \lambda / l_s(\lambda)$, where $l_s(\lambda)$ is the spatial correlation distance at wavelength λ. From this argument, we immediately obtain $l_s(\lambda) \propto \lambda$; i.e., the spatial correlation distance in a white-light soliton is generally shorter for shorter wavelengths (higher frequencies).

In conclusion, we have theoretically identified incoherent white-light solitons in a noninstantaneous nonlinear medium and characterized their main features. We find that the spatial intensity profiles at different (temporal) frequencies of the soliton are wider (less localized) at lower frequencies and narrower at higher frequencies. At the same time, the spatial correlation distance is larger at lower frequencies and shorter at higher frequencies.

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