Superprism effect based on phase velocities

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The superprism effect has been studied in the past by use of the anomalous group velocities of optical waves in photonic crystals. We suggest the possibility of realizing agile beam steering based on purely phase-velocity effects. We present designs of photonic crystal prisms that might make experimental observation of this effect possible.

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The unusual dispersion properties of photonic crystals outside their forbidden bandgaps have attracted much recent attention.[1–8] One particular example is the superprism effect.[2] This is most commonly understood as an effect due to group-velocity dispersion: a large change in the propagation direction of the refracted ray within the photonic crystal is achieved with respect to a small variation in incident parameters. As confirmed by several recent authors,[9–13] this superprism effect occurs mostly within sharp-corner regions of the dispersion surfaces of photonic crystals. It is, however, important to note that in this group-velocity-based effect agile beam steering occurs for light waves traveling inside the crystal. As a result, a large crystal whose size is of the order of centimeters[11] is necessary when this effect is used to spatially separate the slightly different incident light waves, e.g., in semicircular-shaped designs.[10] A natural question is whether the superprism effect can arise in prism-shaped photonic crystals to manipulate the directions of light beams in free space. Here Bloch wave vector $\mathbf{k}$, which measures the phase velocity of light waves in the crystal, is the key quantity connecting the incident and the outgoing directions: the component of $\mathbf{k}$ that is parallel to the incident and the exit facets should be conserved as a rule of thumb. When the superprism effect occurs in a high-curvature region of the dispersion surface, the change in $\mathbf{k}$ inside the crystal is actually small. As a result, for beams entering a prism-shaped photonic crystal on one facet and exiting on another facet, the large angular difference created by the group-velocity dispersion disappears once the light is outside the photonic crystal.

In this Letter we discuss the possibility of a superprism effect based on phase-velocity dispersion, i.e., an effect that will induce large changes in Bloch wave vector $\mathbf{k}$ with respect to small changes in the incident parameters. The pioneering work on exploring the phase-velocity dispersion is due to Lin *et al.*,[1] who experimentally measured the dispersion properties of a photonic crystal prism. The magnitude of dispersion achieved in that experiment is comparable to that in a classical grating, and approximately 2 orders of magnitude smaller than the largest dispersion reported using group-velocity dispersion effects.[3] However, if one is willing to accept large insertion losses, a classical grating can also have increased dispersion in its grazing-angle limit, perhaps by an order of magnitude. Here we show that photonic crystals can be used to realize a magnitude of phase-velocity dispersion much larger than that of classical gratings in their grazing-angle limit and thus comparable to that achieved with the group-velocity dispersion effects. We present designs of photonic crystal prisms that might make experimental observation of this effect possible.

For definitiveness we now focus on the propagation direction as the parameter for continuous-wave (cw) radiation incident from a uniform medium onto a photonic crystal. The incident radiation is specified by a plane wave with wave vector $\mathbf{k}_{\text{inc}}$, which couples with one or more Bloch waves with Bloch wave vector $\mathbf{k}$ inside the crystal. Since the component of $\mathbf{k}$ parallel to the interface, $\mathbf{k}_{\parallel}$, will be Bloch equivalent to that of $\mathbf{k}_{\text{inc}}$ after refraction, a small difference in the incident directions will produce a small change in this component. Thus, a large change in $\mathbf{k}$ is possible only if the $\mathbf{k}$ component perpendicular to the interface, $\mathbf{k}_{\perp}$, undergoes a large variation. For this amplification to happen for a continuous range of $\mathbf{k}_{\parallel}$, the corresponding region on the dispersion surface should be almost perpendicular to the crystal interface and flat. This criterion points to a regime of dispersion surfaces with a very small curvature, in contrast to the group-velocity-based effect that relies on the sharp-corner regions. In Figs. 1(a) and 1(b) we compare the refraction analysis for both a sharp-corner region and a flat region of the dispersion surface. It is clear that, for the same change in the incident directions, the induced change in the Bloch wave vector $\mathbf{k}$ is much larger in the latter case. On the dispersion surface of a photonic crystal, generally there can be many regions where the curvature is small. For the simplest example, consider a crystal with a one-dimensional (1D) periodicity with a weak dielectric contrast. The dispersion surfaces will approximately be multiple intersecting spheres centered on the reciprocal lattice sites, which are distorted and connected by the bandgap effect [Fig. 1(c)]. As a result, near the intersections, regions of both large and small curvatures appear, indicating that both the group-velocity-based and phase-velocity-based superprism effect may be possible near a bandgap. However, in this example, an interface that is perpendicular to the flat region of the dispersion surface [near B in Fig. 1(c)] does

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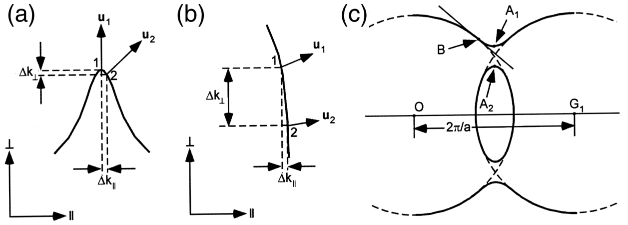

Fig. 1. (a) A sharp-corner region of the dispersion surface makes possible large changes in the direction of $\mathbf{u} = \partial\omega/\partial\mathbf{k}$ for small changes in $\mathbf{k}_\parallel$. (b) A flat region can produce a large change in $\mathbf{k}_\perp$ by a small change in $\mathbf{k}_\parallel$. The $\parallel$ direction is parallel to the crystal interface, and the $\perp$ direction points toward the inside of the crystal. (c) Example dispersion surfaces of a 1D photonic crystal with period a. The background dashed lines represent the periodically shifted dispersion surfaces of a uniform medium. Near their intersections, regions of both large curvature (near A_1 and A_2) and small curvature (near B) appear on the photonic-crystal dispersion surface. O and G_1 are reciprocal-lattice sites.

not correspond to a major symmetry direction of the crystal. This may complicate the application of this analysis because of possible undesired diffractions.

A better approach is to use a two-dimensional (2D) photonic crystal with a strong dielectric contrast. Such crystals are known to exhibit very flat dispersion surfaces along major symmetry directions, and this has been shown to produce beam collimation effects.[4] The present effect can be regarded as essentially a reverse of the collimation process through the use of a prism-shaped crystal. A specific example is shown in Fig. 2 for the TE modes (electric field in the plane) traveling inside a 2D square lattice (period a) of air holes of radius $r = 0.35a$ in dielectric $\epsilon = 12$. The dispersion surfaces of this system were calculated in Ref. 14, and Fig. 2(b) shows a surface of frequency $0.17(2\pi c/a)$ with extremely flat regions along the ΓM direction. A possible way to demonstrate the angular-amplification effect is then to use a device as shown in Fig. 2(a) that couples light to the flat dispersion surface. A tiny change in the incident angle can thus give rise to a huge change in the refracted wave vector, steering the beam exiting the device within a large range of directions.

How different is our 2D example from elementary processes of beams traveling at the grazing angles, i.e., in refraction from a high-index medium to a low-index medium or in one of the sidelobes of diffraction gratings? In these latter possibilities, a significant change in wave vector $\mathbf{k}$ of the outgoing beam can also occur for a small change in the incident directions. In fact, it is straightforward to derive that in the outgoing beam a given small variation of interface-parallel wave vector $\Delta k_\parallel$ from the exact grazing-angle point will induce a change in interface-perpendicular wave vector $\Delta k_\perp$ by the relation $\Delta k_\perp/\Delta k_\parallel \approx [2/(\sigma\Delta k_\parallel)]^{1/2}$, where σ is the curvature of the dispersion surface at the exact grazing-angle point of the outgoing medium. A large angular amplification can thus also be expected in these elementary processes in the limit of $\Delta k_\parallel \to 0$. The key difference lies in the shape of the dispersion surface. In a uniform material the surfaces are always spherical, having a constant curvature everywhere, but in a photonic crystal they become anisotropic and very different shapes can emerge.[5] In the 2D example above we can have $\sigma \to 0$ at a $\mathbf{k}$ point along the ΓM direction, producing a dispersion surface flatter than that of any uniform medium. In this way, for a given $\Delta k_\parallel$ a much higher sensitivity of beam steering can result than for the elementary processes in their grazing-angle limit. Therefore, the magnitude of dispersion here should in principle be able to reach a level comparable to the largest reported value obtained by use of group-velocity effects. Moreover, since light of slightly different incident angles does not need to be spatially separated within the crystal, the present effect permits the use of crystals of a smaller size. Another interesting point is that, while the group-velocity dispersion effect seems to be favored in triangular-lattice photonic crystals with strong directional anisotropy, our effect demonstrates better performance in square-lattice crystals, in which the flat dispersion surfaces occupy a large phase-space region.

To explore the effects of phase velocities in more detail, we numerically simulated light transmission at the two interfaces of the 2D photonic crystal, using the finite-difference time-domain method. The incident wave coming from a high-index medium is modeled as a finite-sized Gaussian beam. Because light is expected to travel at grazing angles in the crystal, as with all the elementary processes in this limit the direct coupling efficiency is usually quite small. Some particular interface designs that were recently proposed[15] may provide some improvement to this problem. Here, we adopted a method in the waveguide field that can systematically improve the coupling efficiency through adiabatic tapering.[16] To create a smooth transition between the medium and the crystal, we added between the crystal and the high-index medium several intermediate hole layers

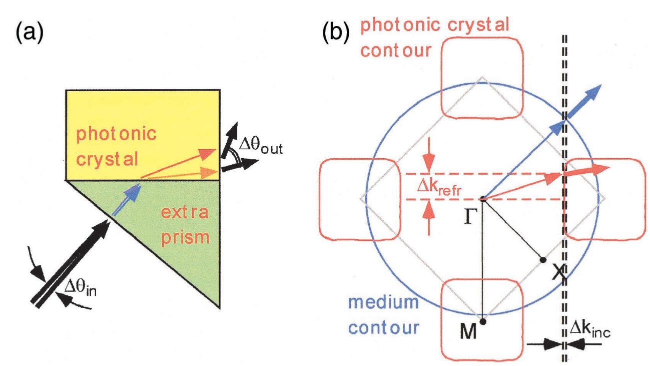

Fig. 2. (a) Schematic illustration of a prism setup in which light enters an extra prism of index $\epsilon = 12$ and goes through the photonic crystal prism. (b) Analysis of the refraction process at the incident (ΓM) crystal interface. The red contours are the dispersion surfaces of the photonic crystal, and the blue contour is that of the incident medium. The gray lines indicate the first Brillouin zone with high-symmetry points Γ, X, and M. The thin arrows represent the Bloch wave vectors of the modes, and the thick arrows are their group velocities.

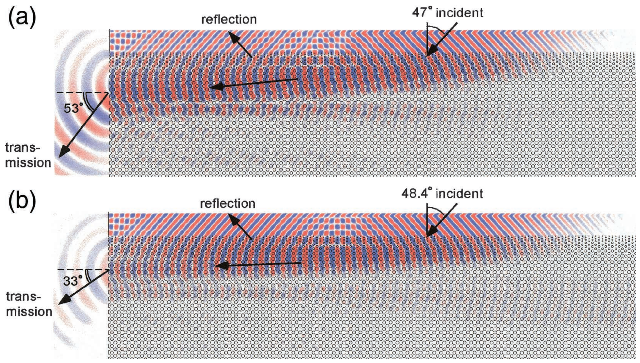

Fig. 3. Finite-difference time-domain simulation snapshots of the magnetic field perpendicular to the plane for cw waves [$\omega = 0.17(2\pi c/a)$] with incident angles shown. The arrows in the outgoing waves indicate the directions of the peak Poynting vectors. Red, white, and blue correspond to positive, zero, and negative field values.

whose transverse periodicity is the same as that of the crystal interface and whose longitudinal periodicity is gradually tapered. These additional structures seem to increase the coupling efficiency into the bulk crystal to an observable level (with actual numbers shown below), at least for the modes of our interest in Fig. 2, which have $k_{\perp} > 0$ and can be tapered to incidence wave vector $\mathbf{k}_{\rm inc}$ without passing through $k_{\perp} = 0$.[17] An example simulation result of beam steering is shown in Fig. 3. Here, a Gaussian beam of half-waist $w = 15.8a$ is used as the incidence from the high-index medium, and beams of this size are approximately at the upper limit of simulations that we can perform on an ordinary computer. The beam enters the photonic crystal on the tapered interface, travels inside the crystal, and exits the device into the air from a perpendicular direct interface as in Fig. 2(a). As the incident angle from the high-index medium changes slightly from 47° to 48.4°, a barely noticeable variation in the incident side, the field pattern of the outgoing beam does experience a significant change: the intensity becomes weaker, and the overall direction swings toward the normal of the output facet. The angular change in the output beam is ~20°, and the energy carried by the output beam with respect to the incident beam is estimated to be 5% in Fig. 3(a) and 0.4% in Fig. 3(b). We note that in these simulations there are significant cylindrical-wave patterns accompanying the outgoing beam, suggesting that diffraction is serious, and this is due to the finite-size effect, i.e., the reduced waist of the beam traveling at grazing angles in the crystal. Nevertheless, the general trend in the simulated transmitted beam qualitatively follows our simple analysis in Fig. 2. The issues associated with beam-width reduction are a necessary consequence of the large dispersion and the linearity of the present effect, and their complete solution, e.g., based on nonlinear effects, is beyond the scope of this Letter. We anticipate that the drawbacks may, however, be partly overcome in realistic experiments simply by employing a sufficiently wide incident beam. Using a beam-waist reduction ratio of the order of 1:20 as in Fig. 3(b), we estimate that a width of 800 wavelengths in the incident beam should be sufficient to reduce the diffraction in an outgoing distance of the order of 1 mm. The size of such a device operating in the infrared and optical regimes can thus be a few millimeters. Given the large dispersion of such devices, this size compares favorably with those of gratings and conventional prisms.

Although we focused on beam steering with respect to incident directions, very similar conclusions can also be drawn for beam steering with respect to changes in the incident frequency, if the dispersion relations for a fixed incident angle are analyzed. This effect can thus be used to separate beams of slightly different frequencies as well. The 2D example can be reproduced in experiments using guided modes in 2D photonic-crystal slabs. Similar effects can also be expected in three-dimensional photonic crystals, from which free-space beam steering in full space may become possible.

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17. We omitted from Fig. 2 another mode that has $k_{\perp} < 0$. The coupling of this mode can be reduced by the taper structure.