

## Color of Shock Waves in Photonic Crystals

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Unexpected and stunning new physical phenomena result when light interacts with a shock wave or shocklike dielectric modulation propagating through a photonic crystal. These new phenomena include the capture of light at the shock wave front and reemission at a tunable pulse rate and carrier frequency across the band gap, and bandwidth narrowing as opposed to the ubiquitous bandwidth broadening. To our knowledge, these effects do not occur in any other physical system and are all realizable under experimentally accessible conditions. Furthermore, their generality make them amenable to observation in a variety of time-dependent photonic crystal systems, which has significant technological implications.

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Photonic crystals [1–4], or periodic dielectric media, are a promising and versatile way to control the propagation of electromagnetic radiation. In this Letter, we consider the influence of a propagating shock wave, or shocklike modulation of the dielectric, in a photonic crystal on the electromagnetic radiation inside. We find that new physical effects arise when light reflects from the moving shocklike modulation of the dielectric. To our knowledge, these new effects are not possible to observe in any other system. Moreover, they preserve optical coherence and may even preserve quantum coherence given the nature of the shock-photon interaction. Through detailed numerical simulations and analytical theory, we have uncovered three new phenomena. The first is the transfer of light frequency from the bottom of a band gap to the top or vice versa. Significantly changing the frequency of light an arbitrary amount with high efficiency is a very challenging problem. Nonlinear materials are currently the only approach, but high intensities are required and the frequencies produced are limited by the range of input frequencies and phase-matching constraints. Furthermore, except for some special cases, more than one input frequency is required. In contrast, frequency conversion in the

shocked photonic crystal occurs at any intensity, the amount of frequency shift is tunable by adjusting the band gap size, and only one input frequency is required. The second new phenomenon is the capture of light of significant bandwidth at the shock front for a controlled period of time. Of interest in optical applications is the ability to trap and manipulate pulses of light, but existing approaches for stopping light for a period of time work only for bandwidths that are orders of magnitude too narrow for practical use and do not preserve quantum information [5–7]. The third effect is the increase or decrease of the bandwidth of light by orders of magnitude with 100% energy conservation. There is no existing non-quantum mechanical way to significantly narrow the bandwidth of an arbitrary optical signal.

*Computational experiments.*—To explore the phenomena associated with light scattering from a shocklike wave in a photonic crystal, we perform finite-difference time-domain [8] simulations of Maxwell's equations in one dimension, single polarization, and normal incidence for a system described by a time-dependent dielectric  $\epsilon(x, t)$ .

A typical model shocklike profile in a photonic crystal considered in this work is given by the dielectric function,

$$\epsilon\left(\hat{x} = \frac{x}{a}, \hat{t} = \frac{ct}{a}\right) = 7 + 6 \sin\left[\pi\left(3\hat{x} - \frac{v}{c}\hat{t}\right) - \frac{\pi}{\gamma} \log\left[2 \cosh\left(\gamma\left(\hat{x} - \frac{v}{c}\hat{t}\right)\right)\right]\right], \quad (1)$$

where  $v$  is the shock speed and  $a$  is the period of the preshocked crystal. This represents two photonic crystals of periods  $a$  and  $\frac{a}{2}$  which meet at an interface of width  $\gamma^{-1}$ , which we take to be 0.05. The photonic crystals on both sides of the shock front have periodic variations of  $\epsilon$  ranging from 1 to 13. As the interface moves, material in the crystal with lattice constant  $a$  is transferred to the crystal with lattice constant  $\frac{a}{2}$ . This choice of compression is for illustrative purposes only. The observed phenomena are reproducible using scenarios with consid-

erably less compression, to be discussed later. The shocked lattice is identical to the original but with a smaller lattice constant. The interface moves at the shock speed  $v$ .

The band gaps for the crystals which exist in front of and behind the shock front are depicted in Fig. 1. Consider now electromagnetic radiation incident from the right (opposite to the direction of shock propagation) with frequency just below the second band gap, as shown

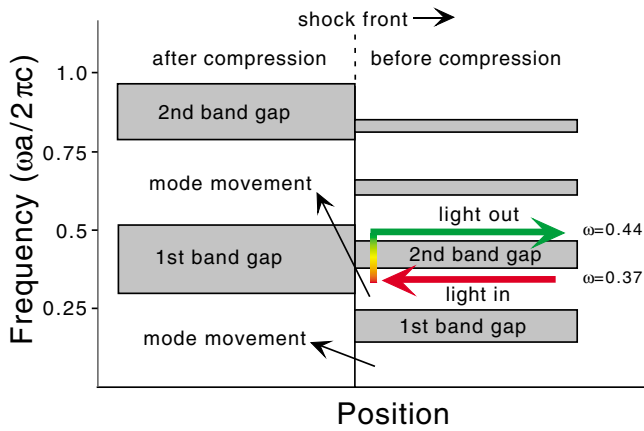


FIG. 1 (color). Schematic of a shock wave moving to the right which compresses the lattice by a factor of 2. Light incident from the right (red arrow) will be converted up in frequency at the shock front and escape to the right. The black arrows indicate the adiabatic evolution of the modes for the lowest two bands.

in Fig. 1. We find that radiation is converted up in frequency to the top of the band gap where it propagates away from the shock. For example, Fig. 2 shows the magnetic field for a simulation where the shock front (dashed line) propagates to the right with  $v = 3.4 \times 10^{-4}c$ . The panels in Fig. 2 are obtained by Fourier transforming the magnetic field over windows of time ( $\Delta t = 200a/c$ ) centered at the times shown in the upper right-hand corners. Light is initially spatially contained in a Gaussian distribution centered around  $x/a = 40$  at the band edge frequency. As the Gaussian distribution moves and broadens to the left, Fig. 2 indicates that most of the incident light is trapped in a localized state at the shock front in the overlapping band gap frequency region of the two photonic crystals. While it is trapped, it evolves up in frequency and is released to the right of the shock with about 20% change in frequency in this case. The amount of frequency shift in this example can be tuned by adjusting the size of the band gap of the preshocked crystal. The shock wave propagates about  $0.5a$  in Fig. 2, and this frequency conversion process is observed once for every lattice unit the shock wave traverses, resulting in a pulsed output for a continuous wave input. The pulse rate, typically around  $10^{10}$  Hz, can be controlled with the shock speed, and the width of each pulse can be controlled by altering the crystal structure. Qualitatively similar effects were observed for a slower shock velocity of  $v = 3.4 \times 10^{-5}c$ .

An additional consequence of this scenario is the localization of light for a controlled period of time. Light is confined in the gap region for a time that is determined by the shock speed and details of the crystal structure and shock front. The propagation speed of light is near zero while trapped at the shock front, which may have useful applications in telecommunications or quantum optics.

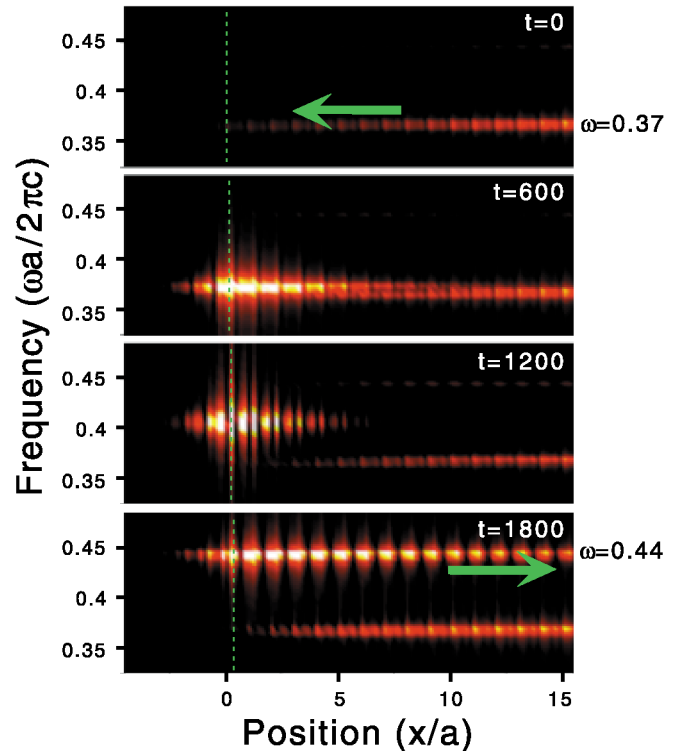


FIG. 2 (color). Large frequency shift. Depicted are four moments in time during a computer simulation of the shock in Eq. (1) moving to the right with  $v = 3.4 \times 10^{-4}c$ . Time is given in units of  $a/c$ . The shock front location is indicated by the dotted green line. The light begins the simulation below the gap in the unshocked material at  $\omega = 0.37$ , as in Fig. 1. As the light propagates to the left, most of it is trapped at the shock front until it escapes to the right at  $\omega = 0.44$ .

The shocklike dielectric modulation of Eq. (1) can also be used to *narrow* the bandwidth of a pulse of light. Bandwidth broadening can be obtained in nonlinear and other systems, but narrowing of an arbitrary signal cannot. Here, bandwidth narrowing is accomplished with 100% efficiency by confining the light between the moving shock front on the left and a reflecting surface of the right. Figure 3 contains results of such a simulation where the bandwidth of input light is reduced by a factor of 4 with no losses. The shock is moving with  $v = 10^{-4}c$  and  $\gamma^{-1} = 2$ . The bandwidth can be narrowed more for slower shock velocities, narrower initial bandwidths, and larger separations between the shock front and reflecting surface.

*Analysis.*—The phenomenon observed in Fig. 2 can be largely understood by considering the time evolution of the various allowed modes as the shock propagates. Each time the shock wave propagates through one lattice unit, the crystal on the right-hand side in Fig. 1 is reduced in length by one lattice unit and the crystal on the left-hand side is increased by one lattice unit. Since for each crystal the number of allowed values of  $k$  in a band is equal to the number of lattice periods in that crystal (ignoring polarization degeneracy), each time the shock wave passes over

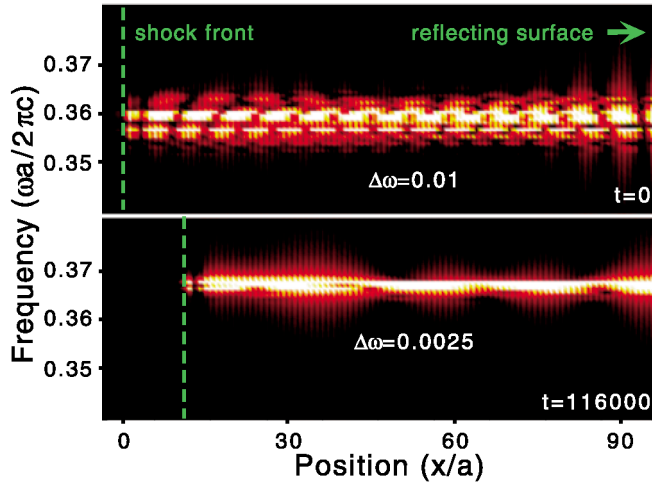


FIG. 3 (color). Bandwidth narrowing. Depicted are two moments in time during computer simulation of the shock in Eq. (1). The shock front is indicated by the dotted green line, and time is given in units of  $a/c$ . Light is confined between the reflecting shock front on the left and a fixed reflecting surface on the right. As the shock moves to the right with  $v = 10^{-4}c$ , the bandwidth of the confined light is decreased by a factor of 4.

one lattice unit the number of states in each band must decrease by one in the preshocked crystal and increase by one in the postshocked crystal. This transfer process is indicated by the mode movement arrows in Fig. 1 for the lowest two bands. Note that, to accomplish this in the case of the second band, it is necessary for a mode to move up through the overlapping gap formed by the second band gap in the preshocked region and the first band gap in the postshocked region.

Indeed, the large frequency change in Fig. 2 is a direct result of the adiabatic evolution of the light through this overlapping band gap. The light is essentially trapped in a cavity which is “squeezed” as the shock compresses the lattice, thereby increasing the frequency. This occurs once each time the shock propagates through a lattice unit.

While a significant change in the frequency of electromagnetic radiation through mechanical means usually requires the interaction with objects that are moving at a significant fraction of the speed of light, the adiabatic approach we describe does not have this requirement. The adiabatic nature of the evolution of the radiation up in frequency through the total system band gap has the property that it can be done arbitrarily slowly with the same large shifts in frequency, resulting in a velocity-independent Doppler-like shift. We also note that a time reversed, frequency lowering effect also occurs in this adiabatic picture.

The argument presented here can be put on a firm analytical foundation. Since the frequency of the incident light lies within the band gap of the compressed photonic crystal, an effective model of the shock front is a mirror with a space-dependent  $E$  field reflection coefficient,  $R(x)$ , where  $x$  is the mirror position. Since the incident light

reflects completely from the band gap of the postshocked crystal, we let  $R(x) \equiv e^{i\theta(x)}$ . If the shock front is stationary, the  $E$  field in the preshocked crystal has the Bloch form,

$$E_1 e^{ik_1 x} w_{k_1, n}(x) e^{-i\omega_1 t} = E_0 R(x) e^{ik_0 x} w_{k_0, n}(x) e^{-i\omega_0 t}, \quad (2)$$

where  $k_0$  and  $k_1$  correspond to the incident and reflected states, respectively,  $E_0$  and  $E_1$  are constants,  $n$  is the band index, and  $w_{k, n}(x+a) = w_{k, n}(x)$ .

For light near a band edge ( $k=0$  for simplicity), the frequency has the form  $\omega = \omega_0 + \alpha k^2$ . If we assume that the shock is moving sufficiently slowly that the reflected light has the form of a single Bloch state, we can make the substitution  $x \rightarrow x_0 - vt$  to obtain a relation for the boundary condition at the shock front. We assume that the light bounces repeatedly between the slowly moving shock and a fixed reflector positioned a distance  $L \gg a$  away in the crystal, and take the limit  $v/v_g \rightarrow 0$  to compute the evolution of  $k$ . Now let  $e^{iP(x_0 - vt)} \equiv [w_{k_0, n}(x_0 - vt)/w_{k_1, n}(x_0 - vt)]$ . Assigning unit magnitude for the latter term is acceptable for all time in the limit of small  $v$  where  $k_1 \rightarrow -k_0$ . Then the time dependence of Eq. (3) requires

$$\alpha k_1^2 + k_1 v = \alpha k_0^2 + k_0 v + \theta'(x_0 - vt_0) v + P'(x_0 - vt_0) v, \quad (3)$$

where  $\theta$  and  $P$  have been linearized about  $t_0$ , which is valid in the limit  $v/v_g \rightarrow 0$ . Primes denote derivatives. The number of bounces of the light  $p$  that occur during a time  $a/v$  when the reflector moves through one lattice constant is  $p = av_g/2Lv$  with each bounce described by Eq. (3). The variation of  $v_g$  over this time can be neglected in the limit  $L \gg a$ .

The periodicity of the crystal gives the property that  $\theta(x+a) - \theta(x) = 2\pi\ell$ , and the periodicity of  $w_{k, n}$  gives the property that  $P(x+a) - P(x) = 2\pi m$ , where  $\ell$  and  $m$  are integers. In the limits  $v/v_g \rightarrow 0$  ( $p \rightarrow \infty$ ) and  $L \ll a$ , these substitutions and some simplification give the final result for the net change in  $k$  during the propagation of the shock over one lattice constant,

$$\Delta k = \frac{1}{2L} \int_{x_0}^{x_0+a} [\theta'(x) + P'(x)] dx = \frac{\pi(\ell + m)}{L}. \quad (4)$$

For any crystal structure, it can be shown that  $m = -1$  above and below the first band gap,  $m = -2$  above and below the second band gap, and so on. It can also be shown that, for any crystal structure,  $\ell = 1$  for the first band gap,  $\ell = 2$  for the second band gap, and so on. While quantum numbers are preserved in an adiabatic evolution, the  $k$  values we refer to here change during an adiabatic evolution because they are convenient labels, not quantum numbers. When Eq. (4) is applied to the scenario in Fig. 1,  $\ell + m = 1 - 2 = -1$  which indicates that the  $k$  crystal momentum of light will adiabatically decrease one  $k$  quantum ( $\frac{\pi}{L}$ ) for each lattice constant the shock wave moves. When combined with the fact that modes

cannot collect or be annihilated at the band edge, this is consistent with the picture presented above which shows that one mode of the system must move up through the total system gap per lattice unit the shock traverses in Fig. 1.

An important implication of Eq. (4) is that the bandwidth of a pulse of light can be modified in a controlled fashion while bouncing between the moving shock wave and a fixed reflecting surface as in Fig. 3. Equation (4) indicates that the width of a wave packet in crystal momentum space  $\delta k$  should be preserved after many bounces because every  $k$  state moves by the same amount. The dispersion relation near a band edge,  $\omega = \omega_0 + \alpha(k - k_{\text{edge}})^2$ , indicates that the bandwidth  $\delta\omega$  for a wave packet of width  $\delta k$  is a function of  $k$  near the band edge,  $\delta\omega = 2\alpha(k - k_{\text{edge}})\delta k$ . Therefore, the bandwidth of a pulse will be altered as it evolves through  $k$  space during the bouncing as shown in Fig. 3. This fact enables the compression of the bandwidth of a pulse of light to a vanishing amount in the limits of a large separation between the reflecting surface and the slowly moving shock front. Note the bandwidth is modified without a large change in average frequency. However, if the wave packet reaches the band edge, it will be trapped at the shock front and converted up in frequency until it is expelled at the frequency of the top of the band gap. In this case, there is a large average frequency shift in addition to a narrowing in bandwidth and the output will be pulsed.

Finally, we speculate that quantum coherence may be conserved to a large extent in all the processes described above. This is because the photon-shock interaction is essentially a Doppler shift (albeit from a time-dependent interface boundary condition) where the photon is never absorbed and reemitted because we operate at a frequency far from any material resonances. Moreover, since the system in no way performs a “measurement” on the photons involved, the state of the interacting photon may be changed without significant destruction of a quantum entanglement with another photon.

*Practical considerations.*—The new physical phenomena presented in this work are quite general and also observable in 2D and 3D systems. The generalized shock-like profiles of the dielectric of the type discussed in the previous sections may be generated by a variety of means. It is possible to launch a physical shock wave into the photonic crystal. Materials are routinely shocked to GPa and higher pressures using lasers and gun facilities. Optical techniques involving the reflection of light from a moving shock front are used as diagnostics in shock experiments [9,10]. A photonic crystal with any type of structure may be used to observe the predicted effects, including a multilayer film. While we have considered large compressions for illustrative purposes, the same phenomena are readily observable with compressions of a few percent or less through the use of deliberately designed defect bands or overlapping band gaps formed

by higher frequency band gaps in the pre- and post-shocked crystals. Typical shock speeds of  $10^4$  m/sec and a readily achievable  $Q$  for the trapped mode of  $10^3$  are sufficient to observe all the effects described in this work. For example, a photonic crystal consisting of a silicon/silicon-dioxide multilayer film can be fabricated and shocked to a strain of a few percent without significantly increasing the material absorption or changing the dielectric response. If light is shined into this shocked crystal with frequency components within roughly  $10^{-3}\omega_{\text{edge}}$  of the lower edge of a higher band gap  $\omega_{\text{edge}}$  (e.g., the 30th band gap, depending on the amount of strain), reflected light with a frequency shift on the order of 1% will be observed. The pulse rate will be around 10 GHz for light of wavelength  $1\ \mu\text{m}$ . The bandwidth narrowing scheme as presented in Fig. 3 can also be realized with 1000 bounces in a photonic crystal 100 lattice units in length. For this scenario, a bandwidth narrowing on the order of a factor of 10 is expected for input light, of frequency  $\omega$ , farther from the band gap edge and bandwidth  $10^{-3}\omega$ .

Alternatively, nonlinear materials which undergo changes in the dielectric constant under an applied electric field or applied change in temperature can be modulated in a time-dependent shocklike fashion. Also, light in a defect state can be adiabatically transferred between the top and bottom of a band gap in time-dependent microelectrical-mechanical system devices.

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