The nonlinear effect from the interplay between the nonlinearity and the supercollimation of photonic crystal

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The authors theoretically and numerically investigate the beam propagation near the supercollimation frequency ω_s^0 in a photonic crystal made of nonlinear material. Since the value and sign of the equal-frequency-contour curvature which dominates the beam behaviors can be nonlinearly tuned near ω_s^0, a kind of nonlinear effect is generated. The envelope equation with unique form is also obtained. Beam-control mechanisms are theoretically predicted and observed in numerical experiments, such as tunable collimation, tunable beam-divergence angle, and self-lock of collimation. These mechanisms can be utilized to function as fiber, lens and coupler, or to design photonic devices. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2739413]

Controlling the propagation of beams or pulses in nonlinear materials is of central importance in nonlinear optics.[1] Generally it is described by a nonlinear Schrödinger equation (NSE) with the factors depending on the material dispersion, such as $\partial_{\mathbf{k}}\omega$ and $\partial_{\mathbf{k}}^2\omega$. On the other hand, photonic crystals (PhCs) (Refs. 2 and 3) have shown the strong *structural dispersion*, much different from natural materials. The combination of the nonlinearity and the PhC dispersion at special frequency ranges has drawn attentions, such as band-gap-edge modes or defect modes.[4–6] Due to its potential to steer light beam, the supercollimation (SC) (also called "self-collimation") phenomena have been intensively studied experimentally[7–9] and theoretically[10–16] in two and three dimensional (2D and 3D) PhCs. Some intrinsic advantages of SC in linear PhCs, i.e., the flexibility[7] and the zero-cross-talk at intersection,[10,13] have been discussed. Optical devices, such as the sharp bend and the splitter, can also be designed and made.[9,12,14] The SC is from the *zero-curvature* parts of the equal-frequency contour (EFC) in k space at certain frequency ω_s^0.[7] Since the group velocity $\mathbf{v}_g$ (or the Poynting vector) is always normal to the EFC, zero-curvature (flat) EFC means that the Bloch waves can have different $\mathbf{k}$ but same $\mathbf{v}_g$ direction. The SC beam composed of these Bloch waves can propagate without expanding (no diffraction).[7,8] Albeit, to the best of our knowledge, the nonlinear study around the SC frequency is still absent thus far. The interplay between nonlinearity and SC could be more interesting since the uniqueness that the EFC curvature and its sign, which is essential for beam evolution, is *very sensitive* to small frequency change around SC frequency (its sign changed from $\omega<\omega_s^0$ to $\omega>\omega_s^0$). Given the uniqueness, we anticipate the different nonlinear effect arising from the interplay, with important applications for beam control.

In this letter, 2D PhCs made of nonlinear Kerr material are studied around the SC frequency. A nonlinear effect, even much stronger than the traditional ones, merges as a result of nonlinear tune of the EFC curvature. The beam envelope equation with unique form is obtained with *a nonlinear factor* in front of the derivative operator on the beam transverse direction. Interesting beam-control mechanisms, such as the tunable SC frequency, the continuously tunable beam-divergent angle, and the self-locking of SC, are revealed.

To study the nonlinearity-SC interplay, a concrete 2D PhC is considered. Without losing generality, all concrete parameters in our model are just for numerical simulations. The PhC setup[12] is a square lattice of air holes in the nonlinear dielectric $n_1=n_1^0+\chi^{(3)}|E|^2/n_1^0$ (Kerr material), with lattice constant a and hole radius $r=0.35a$. Without nonlinearity $n_1=n_1^0=3.46$, the EFCs in k space for E_z-polarization mode are displayed in the inset (a), in which the SC EFC (with zero-curvature sections) is shown by the red curve. Along Γ-M direction (defined as k_x direction), the cross point of the SC EFC and k_x axis is the SC central point S. In the inset (b) we schematically show the divergent, collimated, and convergent beam behaviors for different frequencies with the negative ($\omega<\omega_s^0$), zero ($\omega=\omega_s^0$), and positive ($\omega>\omega_s^0$) EFC curvatures, respectively. From the inset (b), *the different* EFC-curvature sign means *qualitatively different* beam behavior with same initial condition. If the EFC-curvature sign can be *nonlinearly tuned* near SC, the nonlinear PhCs become the diffraction-quality-changeable material. The next study will show that this is true.

First, we demonstrate that the SC frequency can be nonlinearly shifted. In Fig. 1, we draw the first-band diagram for the case without nonlinearity as the black curve, which is also the side view of inset (a) along k_x axis. The coordinates of the SC point S are $\{\omega_s^0=0.18(2\pi c/a),k_s^0=0.537(2\pi/a)\}$, where c is the light speed. Based on perturbation theory,[17,18] the main effect of a small index change (i.e., the nonlinearity) is just to vertically suppress or expand the photonic band. Numerical calculations also confirm this, shown by the dashed blue lines in Fig. 1, where the frequency ω_s of the new SC point S' (or S'') is tuned slightly lower $\omega_s<\omega_s^0$ (or higher $\omega_s>\omega_s^0$) with a larger(or smaller) $n_1(|E|^2)$. We emphasize that since the SC frequency always is the "*curvature*

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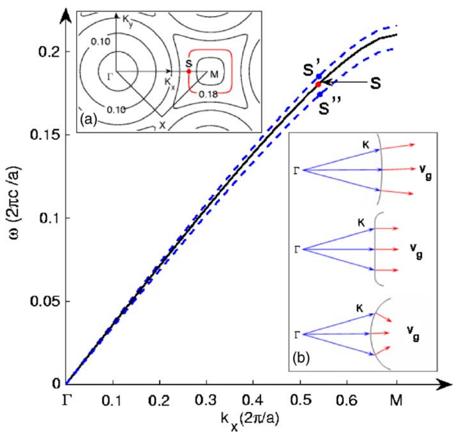

FIG. 1. (Color online) Frequency vs Bloch vector of the first band along Γ-*M* direction. The solid, upper dashed and lower dashed lines are the cases, without nonlinearity, with the n_1 tuned smaller and larger, respectively. The SC point for each case is marked by *S*, *S'*, and *S''*. Inset (a) shows the EFCs of the first band in *k* space without nonlinearity. The red line represents the EFC of SC. In the inset (b), the divergent, SC and convergent behaviors of beam propagation are schematically shown by the group velocity vectors $\mathbf{v}_g$ for EFCs with negative, zero, and positive curvatures, respectively.

zero point" and can be tuned $\omega_s=\omega_s(|E|^2)$, the EFC curvature of all near frequencies is also tuned simultaneously. If our working frequency ω is in the frequency range $[\omega_s^0,\omega_s]$ which is swept by the tuning SC point, then not only the value but also *the sign* of the curvature is changed, such as $\omega>\omega_s^0$ and positive EFC curvature originally, but now $\omega<\omega_s$ and the curvature becomes negative. Obviously, the curvature sign of EFC at ω is always same as $\omega-\omega_s$.

The envelope equation which can quantitatively describe the beam propagation is our next goal. An EFC around the SC point generally has the parabolic form $(k_x-k_x^0)=\kappa k_y^2/2$, where k_x^0 is the *k* vector of the cross point of the EFC and the k_x axis, and κ is the EFC curvature. From the general corresponding relations in the envelope-equation derivation, $(k_x-k_x^0)\rightarrow i(\partial/\partial x)$ and $k_y^2\rightarrow -\partial^2/\partial y^2$, we can phenomenologically introduce an envelope equation for a beam $E_z=U(x,y)e^{i(k_x^0x-\omega t)}$ around the SC frequency in nonlinear PhCs (Ref. 19),

$$i\frac{\partial U}{\partial x}+\frac{\kappa(\omega,|U|^2)}{2}\frac{\partial^2 U}{\partial y^2}+\gamma\chi^{(3)}|U|^2U=0, \quad (1)$$

where $\gamma=f\omega^2\mu_0/(c^2k_x^0)$ and $f=1-\pi r^2/a^2$ is the filling factor of PhC dielectric. Although Eq. (1) looks like the common NSC, actually its form and physical meaning are unique since besides the "traditional nonlinear term" $\gamma\chi^{(3)}|U|^2$ there is a new nonlinear factor $\kappa(\omega,|U|^2)$. Without nonlinearity, κ is the linear EFC curvature [Fig. 1 inset (a)] and can be expressed as $\kappa=\kappa_0(\omega)=(\omega-\omega_s^0)/\tilde{m}$, where $\tilde{m}$ is a positive value and depends only on ω, then Eq. (1) is a common NSE. The beam diffraction is dominated by κ_0. But with the nonlinearity turned on, κ becomes a nonlinear factor $\kappa=(\omega-\omega_s(|U|^2))/\tilde{m}$ in front of the transverse derivative operator $\partial^2/\partial y^2$, and it dominates the beam diffraction. The unique form of Eq. (1) demonstrates the new nonlinear optics that the *boundary* between the positive and negative diffraction regimes (defined by the κ sign) can be overcome by nonlinear tune. Nonlinear PhCs really are diffraction-quality-changeable material. The exact solutions of Eq. (1) can be quite complex. To simplify the problem, the last term $\gamma\chi^{(3)}|U|^2$ is neglected in next beam-control study due to its much weaker nonlinear effect, but will be discussed later.

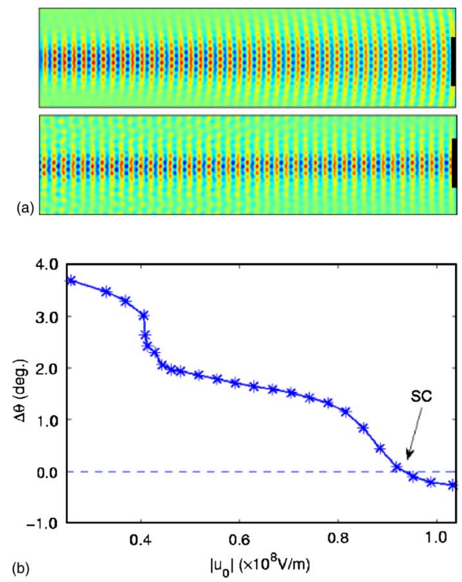

FIG. 2. (Color online) (a) Field vs *x* of stable beams in the nonlinear PhC of (upper panel) $|U_0|=2.6\times10^7$ V/m (weak nonlinearity) and (lower panel) $|U_0|=9.2\times10^7$ V/m (the tunable SC is achieved), where $|U_0|$ is the peak amplitude of the Gaussian beam. The thick black lines represent the detection area. (b) The divergence angle $\Delta\theta$ vs the incident field $|U_0|$, where the divergence angle is defined as $\Delta\theta=\tan^{-1}(\Delta W/L)$. $L=36.75\sqrt{2}a$ is the distance from the source to the detection plane; ΔW is the change of the beam half-width on the detecting plane. $\chi^{(3)}=1\times10^{-17}$ m^2/V^2.

The tunable SC frequency is confirmed by our finite difference time domain numerical experiments. In the simulation, a Gaussian beam is excited using an adjacent single-mode slab waveguide at the left end,[12] then the incident beam propagates into the nonlinear PhC. We observe the beam-propagating behavior (divergent, convergent, or collimated) after the incidence. The beam frequency is chosen as $\omega=0.17(2\pi c/a)<\omega_s^0$, so the beam should be divergent without nonlinearity or with a weak nonlinearity $(\omega-\omega_s<0)$, which is demonstrated in Fig. 2(a) (upper panel). But if the field intensity is strong enough to tune the SC frequency as to satisfy $\omega-\omega_s=0$, the beam becomes collimated. Such tunable SC over a long distance is shown in Fig. 2(a) (lower panel). The tunable SC means that the SC can be realized in a much wider frequency range now. With certain beam central amplitude U_0, the new SC frequency ω_s can be approximately obtained from the perturbation theory $\omega_s\approx\omega_s^0-\alpha\chi^{(3)}|U_0|^2k_s^0$, where α is a constant which depends on the linear PhC structure ($\alpha=0.24$ in our model). From the condition $\omega_s=\omega$, the required beam central intensity $I_s=|U_0|^2$ to reach the tunable SC is approximately $I_s\simeq(n_1)\varepsilon_0|U_s|^2c\simeq(\omega_s^0-\omega)(n_1)^3\varepsilon_0c/\alpha\chi^{(3)}k_s^0$.

The EFC curvature, tuned by nonlinearity, also provides a mechanism to *continuously* modulate the beam-divergence angle. Such a *divergence-angle continuous control* (DACC) is also confirmed by our numerical results, which are shown in Fig. 2(b). DACC may function as a focal-length-tunable lens or a width-tunable coupler between different devices in a photonic microcircuit.

So far the nonlinearity in our system is just engaged to tune the index of PhC material and then the EFC curvature. This goal can also be realized by other tunable materials,[20] i.e., liquid crystals. Nevertheless, the nonlinearity around the

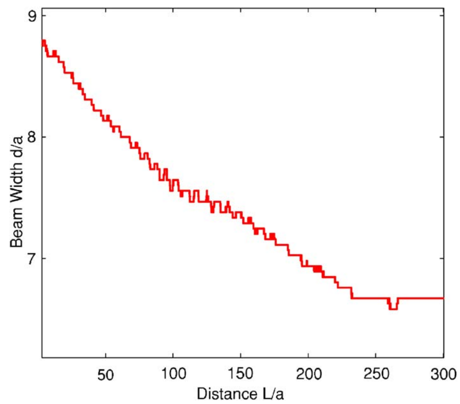

FIG. 3. (Color online) Beamwidth vs the beam propagating distance. The wide incident beam gradually becomes a SLOC beam (with constant width). The parameters are $\omega=0.1835(2\pi c/a)>\omega_s^0$, $|U_0|=3.2\times10^7$ V/m for the incident beam and $\chi^{(3)}=-2.28\times10^{-17}$ m^2/V^2.

SC frequency can cause further novel properties since there is a *feedback* from the EFC curvature to the nonlinearity. The EFC curvature *dominates* the beam convergent (or divergent) behavior, which results in a stronger (or weaker) beam intensity, implying a stronger (or weaker) nonlinearity. Such feedback means the nontrivial interplay between the nonlinearity and the SC, which is also implied in Eq. (1). The nontrivial interplay can cause a novel phenomena, the self-lock of collimation (SLOC), if two conditions (i) $\chi^{(3)}<0$ (the ω_s shifts up with stronger field) and (ii) $\omega>\omega_s^0$ (without nonlinearity, the curvature is positive and the beam is convergent) are satisfied. The SLOC works in the following way. If the beam intensity $I<I_s$ (or $I>I_s$), ω_s is not tuned large enough (or overtuned), thus $\omega>\omega_s$ (or $\omega<\omega_s$), and the beam should be convergent (or divergent). Then, owing to the convergence (or divergence), the beam intensity I becomes larger (or smaller) and the evolution goes on *until* $I=I_s$ and $\omega_s=\omega$, so that a stable collimated mode is locked. Obviously, the physical mechanism of SLOC is attributed to *the negative feedback* when the beam frequency ω deviates from ω_s. The SLOC can make the SC beam significantly robust, which is essential for the SC use in real systems. In our numerical experiment, the SLOC process is shown in Fig. 3 and one can indeed see the effect that is theoretically predicted. Furthermore, we will demonstrate that the SLOC is also a method to control the collimated beamwidth d. Suppose the total energy flux of the beam is W, the SLOC beamwidth can be estimated by

$$d_{\mathrm{SLOC}} \simeq W/I_s \propto W\chi^{(3)}/|\omega-\omega_s^0|. \quad (2)$$

Obviously, the SLOC beamwidth can be tuned by the beam total flux W. The SLOC can function as the *width-tunable fiber*, which does not exist in macrooptic systems. If the gain media is introduced in, W can be controlled by the outside pumping power, therefore the SLOC beamwidth can be dynamically modulated. The SLOC can also be realized by the positive $\chi^{(3)}$ which appears in most nonlinear materials, but then one needs to use the SC point in a high-order band, such as the second band. SLOC can be used to design devices too. For example, SLOC can be exploited as a width-tunable coupler between PhC chips and waveguides.

Since all the related beam-control properties are from the nonlinear shift of the *linear* PhC dispersion around SC, a relatively weak nonlinearity, whose magnitude can be approached by realistic materials, is needed to obtain strong effects. But in our simulation, since the limited computer resources, our numerical system is smaller than real ones[7–9] and a quite strong nonlinearity ($\sim10^{-2}$ order index change) has been used. After optimizing the structure and parameters to make the EFC curvature more sensitive to frequency shift, we expect that the needed nonlinearity is one order weaker. The usage of the SC on high-order bands can also reduce the needed nonlinearity. Furthermore, our study reveals the additional advantages of nonlinear PhC around SC. First, the SC is valid in a wider frequency range or even self-locked, which makes the phenomena very robust. Second, both DACC and SLOC can be dynamically modulated by external variables, i.e., the input beam power or the outside pumping power. Third, multifunctions (lenses, cables, couplers) can be realized on a *single* PhC chip depending on the usage. Such versatility is essential for the integration in a photonic circuit. In the previous study, we have neglected the last term in Eq. (1). If we include the effect of the term, then we can have other new solutions, such as solitons. Since the curvature around SC can be much smaller than the EFC of vacuum, the solitons should be quite different from common ones. Other novel results, i.e., new solitons and new dynamical processes, can be expected in further studies. Such studies can be broaden to other fields of nonlinear periodic systems, such as electronic systems. So our study does open a window for new nonlinear studies.

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